Einstein’s Tests of General Relativity through the Eyes of Newton

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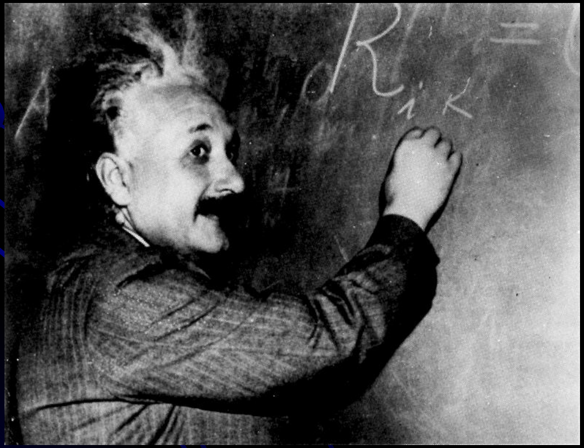
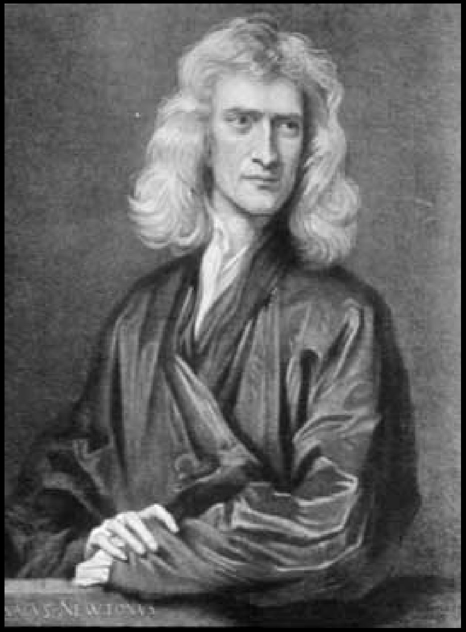


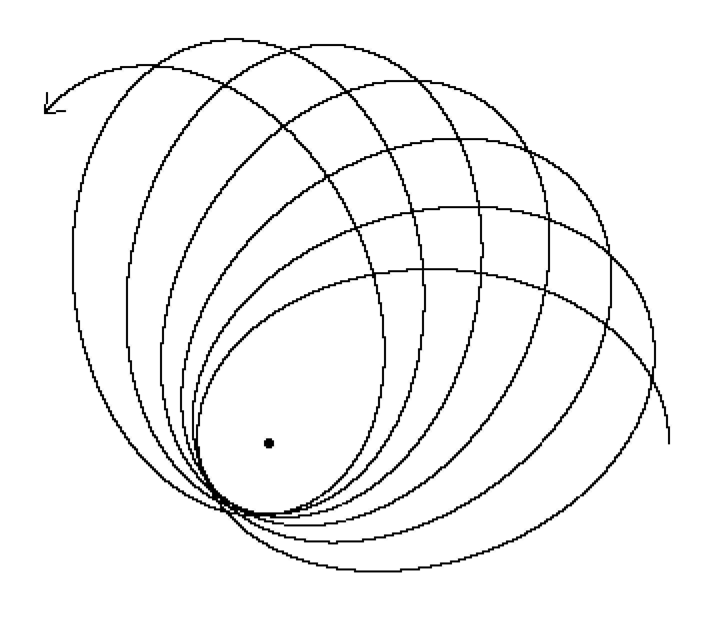


Outline

I. Experimental Overview
II. Newtonian Philosophy
III. Gravitational Redshift
IV. Ritzian Emission Theory
V. Precession of Mercury
VI. Bending of Starlight
VII. Conclusions
VII. Future Work

Experimental Overview

1845 – Urbain LeVerrier observes a 35" per century excess precession of Mercury's orbit



Modern accepted value is 43.11" ± .45" per century

Would take over 3,000,000 years to complete a circle

1915 - Einstein's General Relativity predicts 42.9" precession

1919 – Eddington measures deflection of starlight during total solar eclipse – 1.75”

1960 – Pound and Rebka measure a redshift of 2.5×10^{-15}

Newtonian Philosophy

- Absolute space and time
- Atomic Hypothesis
- Determinism
- Three laws of motion
- Action-at-a-Distance
- “The whole burden of philosophy seems to consist in this- from the phenomena to investigate the forces of Nature and then from these forces to demonstrate the other phenomena”- Newton
- Unification of Force is implicit

Historical Interlude

- Gravitational Redshift

  1784 - Rev. John Michell proposes using gravitational redshift to measure the mass of stars
- Bending of Starlight

  1785 – Henry Cavendish calculates the deflection of light due to Sun’s gravity, getting half the value of the modern value.
- Precession of Mercury

  1875 – Tisserand uses gravitational version of Weber’s law to arrive at correct value.

Gravitational Redshift

$$E_{bottom} = E_{top}\left(1 + \frac{gh}{c^2}\right)$$

$$1 + z = \frac{\lambda_{top}}{\lambda_{bottom}} = \frac{h\nu_{bottom}}{h\nu_{top}} = \frac{E_{bottom}}{E_{top}} = 1 + \frac{gh}{c^2}$$

From Pound & Rebka (1960)…

$$h = 22.5m \Rightarrow z = \frac{\Delta\lambda}{\lambda} = 2.5x10^{-15}$$

Unification Scheme

- *Gravity*
- *Thermodynamics*
- *Atomic Physics*
- *Electromagnetism*

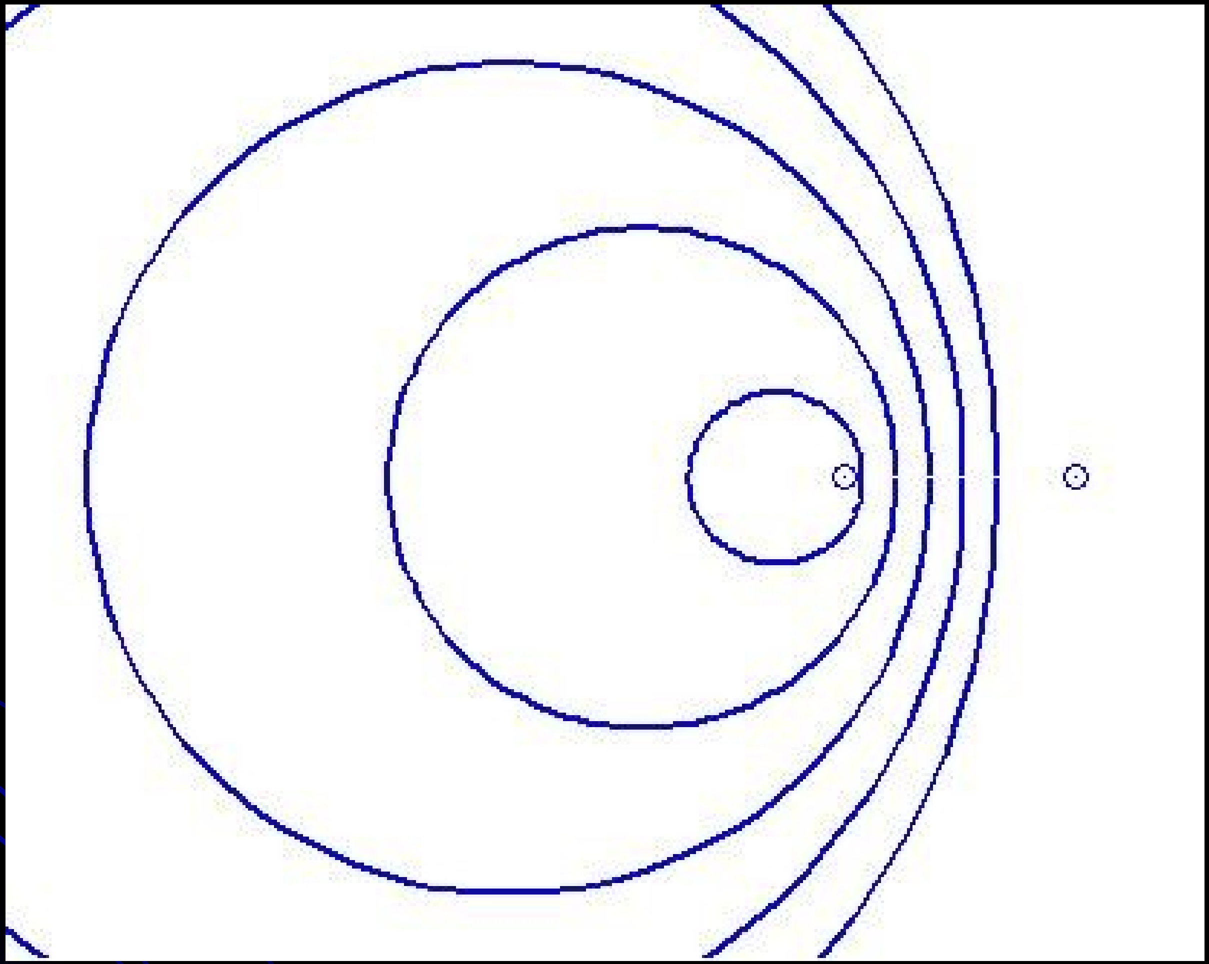
Force Transmission

- Ballistic method

  Action-at-a-distance
- Medium method

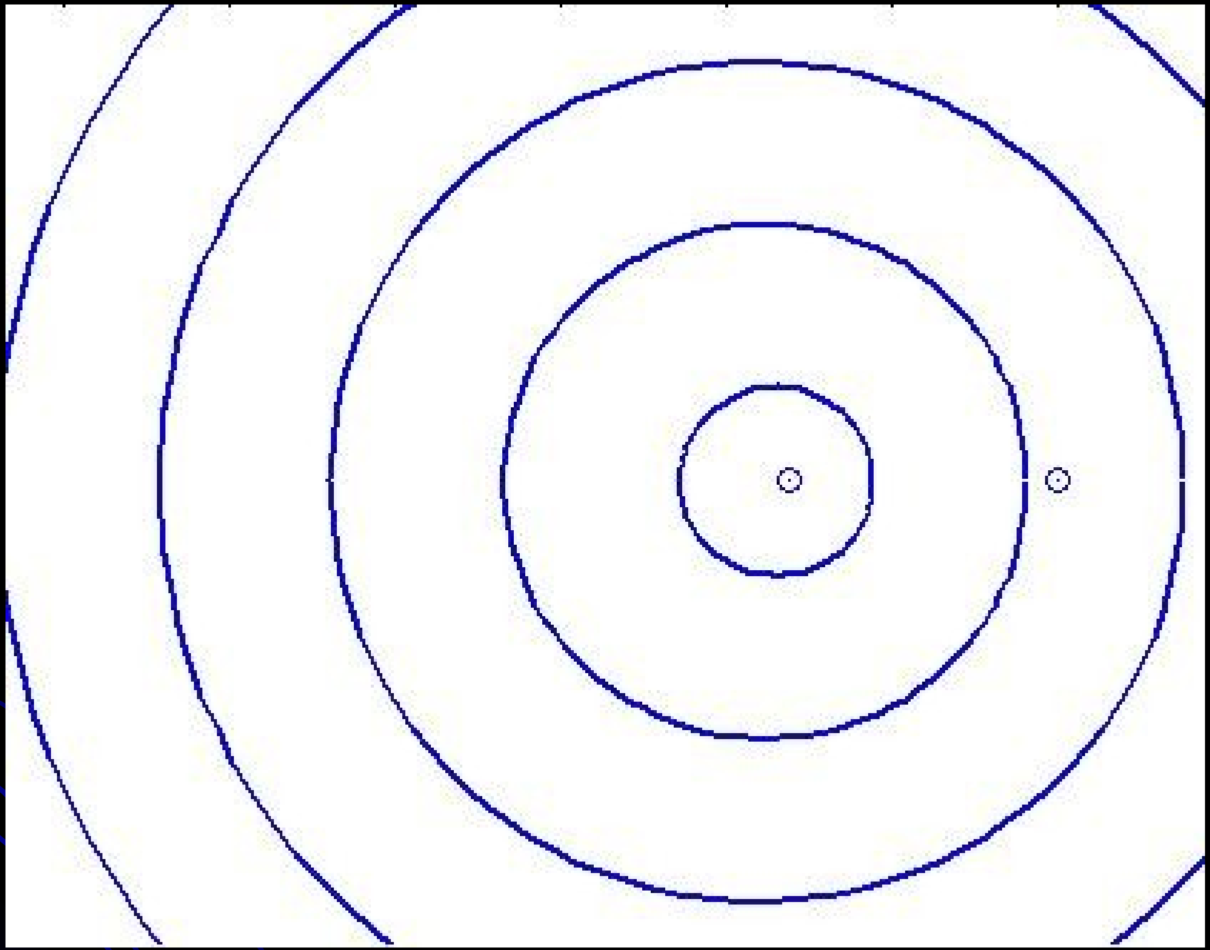
  Ether

Medium Picture



Waves move outward at velocity c, independent of source velocity

Ballistic Picture



Particles move outward at velocity c+v'

Ritzian Emission Theory

$$\vec{F}_{2\rightarrow 1} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\left\{\left[1+\frac{3-k}{4}\left(\frac{v}{c}\right)^2 - \frac{3(1-k)}{4c^2}\left(\vec{v}\cdot\hat{r}\right)^2 - \frac{r}{2c^2}\left(\vec{a}_2\cdot\hat{r}\right)\right]\hat{r} - \frac{k+1}{2c^2}\left(\vec{v}\cdot\hat{r}\right)\vec{v} - \frac{r}{c^2}\vec{a}_2\right\}$$

Explains:

- *Electrostatics*
- *Magnetostatics*
- *Electrodynamics*
- *Radiation/Optics*
- *Radiation Reaction*

Gravitational Version

$$Q \to M$$

$$\frac{1}{4\pi\varepsilon_0} \to G$$

$$\vec{F}_{2\to1} = G\frac{M_1 M_2}{r^2}\left\{\left[1+\frac{3-k}{4}\left(\frac{v}{c}\right)^2\ldots\right]\right\}$$

Insert into Newton's *2nd Law*

$$M_1 \vec{a}_1 = G\frac{M_1 M_2}{r^2}\left\{\left[1+\frac{3-k}{4}\left(\frac{v}{c}\right)^2\ldots\right]\right\}$$

Precession of Mercury

$$\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3}\left\{1+\frac{3-k}{4c^2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]-\frac{3(1-k)}{4c^2}\left(\frac{dr}{dt}\right)^2\right\}+\frac{\mu(k+1)}{2c^2r^2}\frac{dx}{dt}\frac{dr}{dt}$$

$$\frac{d^2y}{dt^2} = -\frac{\mu y}{r^3}\left\{1+\frac{3-k}{4c^2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]-\frac{3(1-k)}{4c^2}\left(\frac{dr}{dt}\right)^2\right\}+\frac{\mu(k+1)}{2c^2r^2}\frac{dy}{dt}\frac{dr}{dt}$$

Ugly… But not a tensor!

$$\frac{d}{dt}\left(y\frac{dx}{dt}-x\frac{dy}{dt}\right)=\frac{\mu(k+1)}{2c^2r^2}\frac{dr}{dt}\left(y\frac{dx}{dt}-x\frac{dy}{dt}\right)$$

$$\left(y\frac{dx}{dt}-x\frac{dy}{dt}\right)=\alpha e^{\frac{-(k+1)\mu}{2c^2r}}$$

Angular Momentum

$$r^2\frac{d\phi}{dt}=\alpha\left[1-\frac{(k+1)\mu}{2c^2r}\right]$$

Mercury Cont’d

Add higher order terms to make equation of motion integrable

$$\mu \frac{d^2x}{dt^2}\left(\frac{k+1}{2c^2r}\right)+\mu x\frac{1-k}{2c^2r^2}\left(\frac{x}{r}\frac{d^2x}{dt^2}+\frac{y}{r}\frac{d^2y}{dt^2}\right)$$

$$\mu \frac{d^2y}{dt^2}\left(\frac{k+1}{2c^2r}\right)+\mu y\frac{1-k}{2c^2r^2}\left(\frac{x}{r}\frac{d^2x}{dt^2}+\frac{y}{r}\frac{d^2y}{dt^2}\right)$$

Multiply each by $\frac{dx}{dt},\frac{dy}{dt}$*, add, integrate to find first integral of motion*

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2+\frac{1}{2}\left(\frac{dy}{dt}\right)^2-\frac{\mu(k+1)}{2r}\left[1-\frac{1}{2c^2}\left(\frac{dx}{dt}\right)^2-\frac{1}{2c^2}\left(\frac{dy}{dt}\right)^2\right]$$
$$-\frac{\mu(1-k)}{2r}\left[1-\frac{1}{2c^2}\left(\frac{dr}{dt}\right)^2\right]-\frac{\mu^2}{2c^2r^2}=\beta=\text{constant}$$

Mercury Cont'd

$$\frac{\alpha^2}{r^4}\left(\frac{dr}{d\phi}\right)^2 = -\frac{\alpha^2}{r^2}\left[1+\frac{\mu(k-1)}{2c^2r}\right]+\frac{2\mu}{r}\left[1+\frac{\mu(2k+1)}{2c^2r}\right]+2\beta\left(1+\frac{\mu k}{c^2r}\right)$$

$$=$$

$$\left[1+\frac{\mu(k-1)}{2c^2r}\right]\times\left[-\frac{\alpha^2}{r}+\frac{\mu^2(k+2)}{c^2r^2}+\frac{2\mu}{r}+\frac{\beta\mu(k+1)}{c^2r}+2\beta\right]$$

let $\frac{1}{r}=p$

$$A=\frac{2\beta}{\alpha^2} \quad B=\frac{2\mu}{\alpha^2}+\frac{\beta\mu(k+1)}{\alpha^2c^2} \quad C^2=1-\frac{\mu^2(k+2)}{\alpha^2c^2}$$

$$\phi-\phi_0=\int\frac{dp\left[1-\frac{\mu(k-1)p}{4c^2}\right]}{\sqrt{-C^2p^2+Bp+A}}$$

$$\phi-\phi_0=\left[1+\frac{\mu^2(k+5)}{4\alpha^2c^2}\right]\arcsin\left[\frac{2c^2p-B}{\sqrt{B^2+4AC^2}}\right]+\frac{\mu(k-1)}{4c^2}\sqrt{-C^2p^2+Bp+A}$$

ϕ advances not by π after half a revolution, but by $\pi\left[1+\frac{\mu^2(k+5)}{4\alpha^2c^2}\right]$

Mercury Cont'd

If N is the number of revolutions per century, the angle of precession is

$$\delta = \frac{\mu^2 \pi (k+5) N}{2\alpha^2 c^2}$$

$$\frac{\mu}{\alpha^2} = \frac{1}{a(1-e^2)}$$

$a_0 \equiv$ average distance between Earth & Sun $= 1$ Au

$v_0 \equiv$ orbital speed $= 29.77 \frac{\text{km}}{\text{s}}$

$a \equiv$ average distance between Planet & Sun $= .38703$ Au

$e \equiv$ eccentricity of Planet $= .206$

$N = 414.9378 \frac{\text{revs.}}{\text{century}}$

$\mu = a_0 {v_0}^2$

$$\delta = \frac{\pi(k+5)}{2(1-e^2)} \left(\frac{v_0}{c}\right)^2 \frac{a_0}{a} N$$

$$\frac{2\delta(1-e^2)}{N\pi} \left(\frac{c}{v_0}\right)^2 \frac{a}{a_0} - 5 = k$$

k = 7 results in a precession of 43.1” per century

…On to the Bending of Starlight…

Bending of Starlight

$$\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3}\left\{1+\frac{3-k}{4c^2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]-\frac{3(1-k)}{4c^2}\left(\frac{dr}{dt}\right)^2\right\}+\frac{\mu(k+1)}{2c^2r^2}\frac{dx}{dt}\frac{dr}{dt}$$

$$\frac{d^2y}{dt^2} = -\frac{\mu y}{r^3}\left\{1+\frac{3-k}{4c^2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]-\frac{3(1-k)}{4c^2}\left(\frac{dr}{dt}\right)^2\right\}+\frac{\mu(k+1)}{2c^2r^2}\frac{dy}{dt}\frac{dr}{dt}$$

Add higher order terms to x-component to make equation integrable

$$\frac{\mu}{c^2r}\left(\ddot{x}-\frac{\mu x}{r^3}\right)+\frac{\mu x}{c^2r^3}\left(x\ddot{x}+y\ddot{y}-\frac{\mu}{r}\right)$$

Similar term for y-component

Energy Conservation Equation

$$\frac{1}{2}v^2+\frac{(1+k)\mu}{2r}\left(1-\frac{v^2}{2c^2}\right)+\frac{(1-k)\mu}{2r}\left(1-\frac{\dot{r}^2}{2c^2}\right)-\mu^2/2c^2r^2=\text{constant}=\frac{1}{2}\omega^2$$

Bending of Starlight

Angular Momentum Conservation Equation

$$J = h\left(1 + \frac{1+k}{2}\frac{\mu\rho}{c^2}\right)$$

$$\text{where } h = \omega b\,,\ \ \rho = \frac{1}{r}\,,\ \ b \equiv \text{impact parameter}$$

Using b = 1 solar radius (6.96×10^8 m) and k=7

Angular Deflection Equation

$$\Delta\theta = \frac{4GM_S}{bc^2}$$ *= 1.75”*

Conclusions

- *Explanations of the precession of Mercury,the bending of starlight and gravitational redshift do <u>not</u> require the General Theory of Relativity.*

- *Ritz's Theory explains both observations if the constant k has the value 7.*

Future Work

- *Gravitational Lensing*
- *Gravitational Waves*
- *Unify Gravity and Electromagnetism*

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G.R. Parameters vs. Our Parameters

Param	k=7