Appendix A.

Noether's Theorem in Both Linearly Accelerated and Inertial Frames

Aa. Noether's Theorem

In physics, Noether's theorem is of fundamental importance because it reveals an intimate relation between conservation laws or conserved quantities and symmetries of a physical system. It was first proved by Emmy Noether (1882–1935) in 1918.[1] It is usually employed in inertial frames. There was little discussion of its implications in non-inertial frames because the symmetry properties of a physical system viewed from non-inertial frames are not obvious. Furthermore, the conventional Noether's theorem based on symmetry in spacetime is generalized to the case with symmetry in the *phase space.*

In classical mechanics, Noether's Theorem can be stated as follows: Suppose a physical system is described by the Lagrangian equation with the Lagrangian $L = L(q_i,\dot{q}_i,w)$, where $\dot{q}_i = dq_i/dw$. Let

$$\begin{aligned} w' &= w + \varepsilon T(q_i,\dot{q}_i,w) , \\ q'_i &= q_i + \varepsilon Q(q_i,\dot{q}_i,w) , \end{aligned} \tag{A.1}$$

be a set of infinitesimal transformations with a continuous parameter ε. If the Lagrangian $L(q'_i,\dot{q}'_i,w')$, where $\dot{q}'_i = dq'_i/dw'$, satisfies

$$\left\{\frac{\partial}{\partial\varepsilon}\left[\frac{dw'}{dw}L(q'_i,\dot{q}'_i,w')\right]\right\}_{\varepsilon=0} = \frac{df}{dw} , \tag{A.2}$$

where f is some function, $f = f(q_i,\dot{q}_i,w)$, then

$$\sum_i p_i Q_i - HT - f \tag{A.3}$$

is a constant of motion, where

$$p_i = \partial L/\partial \dot{q}_i \quad \text{and} \quad H = \sum_i p_i \dot{q}_i - L. \tag{A.4}$$

To see the result (A.2) it suffices to calculate $(dw'/dw)L(q'_i, \dot{q}'_i, w')$ in (A.2) to the first order in ε. Since

$$\dot{q}'_i = \frac{dq_i'}{dw'} = \dot{q}_i + \varepsilon \frac{dQ_i}{dw} - \varepsilon \dot{q}_i \frac{dT}{dw}, \tag{A.5}$$

we have

$$\left\{\frac{\partial}{\partial\varepsilon}\left[\frac{dw'}{dw} L(q'_i, \dot{q}'_i, w')\right]\right\}_{\varepsilon=0}$$

$$= \left\{\frac{\partial}{\partial\varepsilon}\left[\left(1 + \varepsilon\frac{dT}{dw}\right)L\left(q_i + \varepsilon Q_i,\ \dot{q}_i + \varepsilon\frac{dQ_i}{dw} - \varepsilon\dot{q}_i\frac{dT}{dw},\ w + \varepsilon T\right)\right]\right\}_{\varepsilon=0}$$

$$= \left\{\frac{\partial}{\partial\varepsilon}\left[\left(1 + \varepsilon\frac{dT}{dw}\right)L(q_i, \dot{q}_i, w) + \frac{\partial L}{\partial q_i}\varepsilon Q_i + \varepsilon(\partial L/\partial\dot{q}_i)\left(\frac{dQ_i}{dw} - \dot{q}_i\frac{dT}{dw}\right) + \frac{\partial L}{\partial w}\varepsilon T\right]\right\}_{\varepsilon=0}$$

$$= L\frac{dT}{dw} + \frac{\partial L}{\partial q_i}Q_i + (\partial L/\partial\dot{q}_i)\left(\frac{dQ_i}{dw} - \dot{q}_i\frac{dT}{dw}\right) + \frac{\partial L}{\partial w}T = \frac{df}{dw}. \tag{A.6}$$

Using (A.4), one has

$$T\frac{dH}{dw} = T\frac{d}{dw}\left([\partial L/\partial\dot{q}_i]\,\dot{q}_i\right) - T\frac{\partial L}{\partial q_i}\dot{q}_i - T\,[\partial L/\partial\dot{q}_i]\frac{d\dot{q}_i}{dw} - T\frac{\partial L}{\partial w}. \tag{A.7}$$

Substituting $(\partial L/\partial w)T$ in (A.6) into (A.7) and using (A.4), one obtains

$$\frac{d}{dw}\left(\sum_i p_i Q_i - HT - f\right) = (Q_i - T\dot{q}_i)\left(\frac{d}{dw}[\partial L/\partial\dot{q}_i] - \frac{\partial L}{\partial q_i}\right) = 0, \tag{A.8}$$

which vanishes because of the Lagrangian equation

$$\frac{d}{dw}[\partial L/\partial\dot{q}_i] - \frac{\partial L}{\partial q_i} = 0. \tag{A.9}$$

The result (A.8) implies that the quantity in (A.3) is a constant of motion.

To see the intimate relation among Noether's theorem and symmetry of a physical system, we observe that, if df/dw = 0, the condition (A.2) is equivalent to the property that the action of a physical system is invariant under the transformation (A.1) because

$$\left\{\frac{\partial}{\partial\varepsilon}\left[\frac{dw'}{dw}L(q'_i,\dot{q}'_i,w')\right]\right\}_{\varepsilon=0}$$

$$=\left\{\frac{\partial}{\partial\varepsilon}\Big[L(q'_i,\dot{q}'_i,w')dw' - L(q_i,\dot{q}_i,w)dw\Big]\frac{1}{dw}\right\}_{\varepsilon=0}=0. \qquad \text{(A.10)}$$

In general, even if the difference, $[L(q'_i,\dot{q}'_i,w')dw' - L(q_i,\dot{q}_i,w)dw]$, is a total differential of some function $f(q_i,\dot{q}_i,w)$ rather than zero, we still have a conservation law, according to Noether's theorem. This is related to the fact that a total differential in the action of a physical system does not affect the equation of motion.

Now, let us consider a few specific cases to illustrate Noether's theorem.

Ab. Symmetry of Time and Space Translations in Inertial Frames

Let us consider a simple action,

$$S=\int_a^b -mds=\int_{w_{Ia}}^{w_{Ib}} L\ dw_I, \qquad \text{(A.11)}$$

$$L = L(\mathbf{r}_I,\dot{\mathbf{r}}_I,w_I) = -m\frac{ds}{dw_I} = -m\sqrt{1-\dot{x}_I^2-\dot{y}_I^2-\dot{z}_I^2}\ ,$$

where $\mathbf{r}_I = (x_I, y_I, z_I)$ is the Cartesian coordinate and the velocity is $\dot{\mathbf{r}}_I = d\mathbf{r}_I/dw_I$. This Lagrangian L depends only on the velocities and does not depend on space and time. Suppose we consider infinitesimal translations in time and space

$$w_I \to w'_I = w_I + \varepsilon T_O,$$
$$x_I \to x'_I = x_I + \varepsilon Q_O, \qquad y_I \to y'_I = y_I, \qquad z_I \to z'_I = z_I, \qquad \text{(A.12)}$$

where T_O and Q_O are constant. One can verify that df/dw in (A.2) vanishes,

$$\left\{\frac{\partial}{\partial \varepsilon}\left[\frac{dw'_I}{dw_I} L(\mathbf{r}'_I, \dot{\mathbf{r}}'_I, w'_I))\right]\right\}_{\varepsilon=0}$$

$$= \left\{\frac{\partial}{\partial \varepsilon}\left[-m\sqrt{1 - \dot{x}_I^2 - \dot{y}_I^2 - \dot{z}_I^2}\right]\right\}_{\varepsilon=0} = 0, \qquad (A.13)$$

so that f is simply a constant. Therefore, the result (A.3) of Noether's theorem implies

$$p_x Q_o - H T_o = \text{constant}. \qquad (A.14)$$

If one considers only the time translation, i.e., $T_o = 1$ and $Q_o = 0$ in (A.12), the result (A.14) leads to the conservation of energy,

$$H = \text{constant}, \qquad (A.15)$$

where H = E. Similarly, the symmetry of the Lagrangian (A.11) under space translations along the x-axis, i.e., $T_o = 0$ and $Q_o = 1$ in (A.12), implies by Noether's theorem the conservation of momentum, p_x = constant.

Ac. Symmetry of the Lorentz Group for Inertial Frames

We know that the action (A.11) is invariant under the infinitesimal 4-dimensional transformation

$$w'_I = w_I - \beta x_I, \qquad x'_I = x_I - \beta w_I, \qquad y'_I = y_I, \qquad z'_I = z_I. \qquad (A.16)$$

Indeed, it follows from (A.11) and (A.16) that

$$\left\{\frac{\partial}{\partial \beta}\left[\frac{dw'_I}{dw_I} L(\mathbf{r}'_I, \dot{\mathbf{r}}'_I, w'_I))\right]\right\}_{\beta=0}$$

$$= \left\{\frac{\partial}{\partial \beta}\left(-m(1 - \beta\dot{x}_I)\left[(1 - \dot{r}_I^2)(1 - 2\beta\dot{x}_I)\right]^{1/2}\right)\right\}_{\beta=0} = 0. \qquad (A.17)$$

Thus, f in (A.2) is a constant. Noether's theorem (A.3) implies

$$p_{Ix}w_I - Hx_I = \text{constant}\,, \tag{A.18}$$

where

$$p_{Ix} = \partial L/\partial \dot{x}_I = m(d\dot{x}_I/dw_I)/[1-\dot{r}_I^2]^{1/2}\,, \qquad H = \mathbf{p}_I \cdot \dot{\mathbf{r}}_I - L = m/[1-\dot{r}_I^2]^{1/2}\,.$$

The invariance of the quantity (A.18) can also be directly verified by using the 4-dimensional transformations of coordinates and momenta, (7.4) and (10.5). The form (A.18) resembles that of angular momentum because, mathematically, it is related to the property that the Lorentz transformation (A.16) can be viewed as a rotation of w_I and x_I axes in a 4-dimensional space with y_I and z_I fixed.

If one pauses and reflects for a moment concerning the physical implications of (A.18), one cannot refrain from wondering: Why a splendid and useful Lorentz invariance turns out to be associated with such a dull and useless conservation law (A.18)?!

Ad. Generalized Translational Symmetry of the Wu Group in Accelerated Frames

In a non-inertial frame such as F(w,x,y,z) with a constant-linear-acceleration along the x axis, the energy p_0 and momentum p_x of a "free" (or non-interacting) particle are clearly not constant, as one can see from (22.6) and (22.9). Evidently, this is intimately related to the fact that the action (22.1) in F for a "free" particle, i.e., $a_\mu=0$, does not have the usual symmetries of translations in time w and space in the x direction, in contrast to the action in (A.11) in F_I.

However, there are more sophisticated symmetries associated with the action (22.1) with $a_\mu=0$ in non-inertial frames. They are the generalizations of translations (A.12) from an inertial frame F_I to an non-inertial frame F. Let us substitute the infinitesimal translations in (A.12) into the inverse Wu transformation (21.47)

$$w' = \frac{(w'_I + \beta_o/\alpha\gamma_o)}{(\alpha x'_I + 1/\gamma_o)} - \frac{\beta_o}{\alpha}\,,$$

$$x' = \sqrt{(x'_I + 1/\gamma_o\alpha)^2 - (w'_I + \beta_o/\alpha\gamma_o)^2} - \frac{1}{\alpha\gamma_o^2}\,, \quad y = y_I\,, \quad z = z_I\,. \tag{A.19}$$

Using (A.19) and (A.12), we have the generalized translations in a CLA frame F(w,x,y,z),

$$w' = w + \varepsilon T,$$
$$x' = x + \varepsilon Q, \qquad y' = y, \qquad z' = z, \tag{A.20}$$

where ε is an infinitesimal parameter and the functions T and Q are given by

$$T = \frac{T_0}{\alpha(x_I + 1/\alpha\gamma_0)} - \frac{Q_0(w_I + \beta_0/\alpha\gamma_0)}{\alpha(x_I + 1/\alpha\gamma_0)^2} = T_0\left(\frac{\gamma}{W}\right) - Q_0\left(\frac{\beta\gamma}{W}\right), \tag{A.21}$$

$$Q = \frac{Q_0(x_I + 1/\alpha\gamma_0) - T_0(w_I + \beta_0/\alpha\gamma_0)}{\sqrt{(x_I + 1/\gamma_0\alpha)^2 - (w_I + \beta_0/\alpha\gamma_0)^2}} = \gamma Q_0 - \gamma\beta T_0. \tag{A.22}$$

Let us evaluate (A.2) with the Lagrangian (22.4) with $\bar{e}=0$ and primed variables. We find that df/dw = 0,

$$\left\{\frac{\partial}{\partial\varepsilon}\left[\frac{dw'}{dw} L(\mathbf{r}',\dot{\mathbf{r}}',w')\right]\right\}_{\varepsilon=0}$$

$$= \left\{\frac{1}{dw}\frac{\partial}{\partial\varepsilon}\left(\sqrt{W'^2dw'^2 - dr'^2}\right)\right\}_{\varepsilon=0}$$

$$= \frac{Wdw^2(2\alpha\beta\gamma^2WT + \alpha\gamma^2Q) + W^2dwdT - dxdQ}{dw\sqrt{W^2dw^2 - dr^2}} = 0, \tag{A.23}$$

where we have used

$$\left\{\frac{\partial}{\partial\varepsilon}W\right\}_{\varepsilon=0} = 2W\alpha\beta\gamma^2T + \alpha\gamma^2Q, \tag{A.24}$$

$$\left\{\frac{\partial}{\partial\varepsilon}dw'^2\right\}_{\varepsilon=0} = 2dwdT, \qquad \left\{\frac{\partial}{\partial\varepsilon}dr'^2\right\}_{\varepsilon=0} = 2dxdQ, \tag{A.25}$$

$$dT = -\frac{\alpha\beta\gamma^3T_0\,dw}{W} - \frac{\alpha\gamma^3T_0\,dx}{W^2} - \frac{Q_0}{W}[\alpha\gamma - \alpha\beta^2\gamma^3]dw + \frac{Q_0}{W^2}\alpha\beta\gamma^3dx, \tag{A.26}$$

$$dQ = \alpha\beta\gamma^3Q_0\,dw - \alpha\gamma^3T_0\,dw. \tag{A.27}$$

Thus, Noether's theorem implies

$$p_x(\gamma Q_o - \gamma\beta T_o) - H\left(\frac{T_o\gamma}{W} - \frac{Q_o\beta\gamma}{W}\right) = \text{constant}, \tag{A.28}$$

in CLA frames. This is the generalization of the conservation (A.14) of inertial frames. For the generalized time translational symmetry, i.e., $T_o=1$, $Q_o=0$, the result (A.28) leads to

$$p_x(-\gamma\beta) - H\left(\frac{\gamma}{W}\right) = -\gamma\left(\frac{p_o}{W} + \beta p_x\right) = \text{constant}, \tag{A.29}$$

where p_o, p_x and W are functions of space **r** and time w. Similarly, the generalized space translational symmetry (i.e., $T_o=0$, $Q_o=1$) implies

$$p_x(\gamma) - H\left(-\frac{\beta\gamma}{W}\right) = \gamma\left(p_x + \beta\frac{p_o}{W}\right) = \text{constant}. \tag{A.30}$$

Equations (A.29) and (A.30) are consistent with the constant energy p_{Io} and momentum $p_{I1}=-p_I{}^1=-p_{Ix}$ in an inertial frame, as shown in the 4-momentum transformation (22.10).

Ae. Classical and Quantum Rings (or Closed Strings) in a Central Force Field

Some conservation laws for the motion of physical objects in a potential field are not related to Noether's theorem. For example, a string-like object, which has been extensively discussed in recent years, may have a different type of symmetry from that of ordinary particles. Let us consider a simple string which is closed and moves in a potential field with a constant radius. Based on a formal analogy between the equation of the Nambu string[2] and the massless Klein-Gordon equation with cyclic radial momentum p_r and cyclic angular momentum p_θ, it has been suggested that a closed quantum string with a constant radius could be described by a Hamiltonian with cyclic radial momenta p_r and p_θ.[3] The Lagrangian L_r for such a "ring" moving in a Coulomb-like potential V(r) was assumed to have the form

$$L_r(r,\theta,\phi,\dot{\phi}) = -m(1-r^2\dot{\phi}^2\sin^2\theta)^{1/2} - V(r)\,, \tag{A.31}$$

which has cyclic velocities $\dot{r}=dr/dw$ and $\dot{\theta}=d\theta/dw$. Since the generalized coordinates are $q_i=(q_1,q_2,q_3)=(r,\theta,\phi)$, their conjugate momenta are given by

$$p_r = \frac{\partial L_r}{\partial \dot{r}} = 0\,, \quad p_\theta = \frac{\partial L}{\partial \dot{\theta}} = 0\,, \quad p_\phi = \frac{\partial L_r}{\partial \dot{\phi}} = \frac{mr^2\dot{\phi}\sin^2\theta}{(1-r^2\dot{\phi}^2\sin^2\theta)^{1/2}}\,. \tag{A.32}$$

Thus, one cannot obtain a relation for the velocity $\dot{r}$ (or $\dot{\theta}$) in terms of p_r (or p_θ) and coordinates. In some cases, the lack of these relations will render the Legendre transformation and, hence, the Hamiltonian undefined.[4] However, in this case one can follow Routh's procedure for treating cyclic variables[5] and define a new Hamiltonian H_r for the ring

$$H_r(r,\theta,\phi,p_\phi) = p_\phi\,\dot{\phi} - L_r = \sqrt{m^2+p_\phi^2(r^2\sin^2\theta)} + V(r)\,, \tag{A.33}$$

where we have used the equations in (A.32). The usual Hamiltonian equations for ϕ and p_ϕ can be obtained. The momenta p_r and p_θ are cyclic in H_r. We also have the following equations (in which H_r plays the role of the Lagrangian) for the ring's motion:

$$\frac{\partial H_r}{\partial r} = \frac{d}{dw}(\partial H_r/\partial \dot{r}) = 0\,, \qquad \frac{\partial H_r}{\partial \theta} = \frac{d}{dw}(\partial H_r/\partial \dot{\theta}) = 0\,. \tag{A.34}$$

These equations determine the constant values of r and θ,

$$r = \alpha_r\,, \qquad \theta = \alpha_\theta\,. \tag{A.35}$$

For an arbitrary central potential V(r), the second equation in (A.34) leads to $\alpha_\theta=\pi/2$, while the numerical value of r depends on the specific form of the potential V(r) in the model.

Classically, the Hamiltonian (A.33) describes a particle with a mass m moving in a circular orbit. Since a rotating ring with a constant radius can be pictured as a collection of N particles moving in the same orbit, we can also interpret (A.33) as the Hamiltonian for a rotating ring with a total rest mass m.

The Hamiltonian (A.33) has the cyclic momenta p_r and p_θ which can be used to construct a "ring model of quarks" with permanent confinement of the quantum ring.[3]

Af. A Generalized Noether's Theorem for Symmetry in Phase Space

The constants of motion in (A.35) for inertial frames are not covered by the conventional Noether's theorem based on symmetry in spacetime. However, one can generalize Noether's theorem to imply (A.35) as a special case by considering symmetry in the *phase space.*

Let us introduce a new function called "Jingsian" J_s which symmetrizes the p and q variables explicitly and plays a double role of the Lagrangian and the Hamiltonian:

$$J_s(q_1,..q_n,p_1,..p_s,\dot{q}_{s+1},..\dot{q}_n,t) \equiv L(q_i,\dot{q}_i,t) - \sum_{i=1}^{s}\frac{1}{2}(p_i\,\dot{q}_i - q_i\,\dot{p}_i)$$

$$-\frac{d}{dt}\sum_{i=1}^{s}\frac{1}{2}(p_i\,q_i), \qquad 0\le s\le n. \tag{A.36}$$

This form enables us to treat p_i and q_i on a more equal footing in performing variational calculations than the usual Hamiltonian form. That is, the Jingsian J_s makes the p-q symmetry more explicit and does not depend on $\dot{p}_i$, so that one can deal with symmetry related to the momenta p_1, p_2,....,p_s and the coordinates q_1, q_2,....,q_s.

When s=0, the Jingsian J_0 is just the Lagrangian L, which is defined in a configuration space formed from the n generalized coordinates. And when s=n, J_n reduces to the negative Hamiltonian -H(q,p,t) defined in the phase space. The general Jingsian J_s, $1 \le s \le n-1$, is defined in a combined phase space (p_1,..,p_s, q_1,..,q_s.) and configuration space (q_{s+1}, q_{s+2},....,q_n), which may be called "Jingsian space" for short. The Hamilton's principle $\delta S \equiv \delta\int_1^2 L dt = 0$ indicates that one can have the modified Hamilton's principle in the Jingsian space by expressing L in terms of the Jingsian J_s in (A.36). We have

$$\delta S = \delta \int_1^2 \left\{ \sum_{i=1}^{s} \left(\frac{1}{2} p_i \frac{dq_i}{dt} - \frac{1}{2} q_i \frac{dp_i}{dt} \right) + \frac{d}{dt} \sum_{i=1}^{s} \frac{1}{2} (p_i q_i) \right.$$

$$\left. + J_s(q_1,...q_n,p_1,...,p_s,\dot{q}_{s+1},...\dot{q}_n,t) \right\} dt ,$$

$$= \int_1^2 \left\{ \sum_{i=1}^{s} (\dot{q}_i \delta p_i - \dot{p}_i \delta q_i) + \sum_{i=1}^{s} \left[\frac{\partial J_s}{\partial q_i} \delta q_i + \frac{\partial J_s}{\partial p_i} \delta p_i \right] \right.$$

$$\left. + \sum_{i=s+1}^{n} \left[\frac{\partial J_s}{\partial q_i} - \frac{d}{dt} \frac{\partial J_s}{\partial q_i^*} \right] \delta q_i \right\} dt = 0 , \qquad q_i^* \equiv \dot{q}_i , \qquad (A.37)$$

where we have used the following conditions in the Jingsian space,

$$\delta q_i(t_1) = \delta q_i(t_2) = 0, \quad i = 1,2,...n ;$$

$$\delta p_i(t_1) = \delta p_i(t_2) = 0, \quad i = 1,2,...s . \qquad (A.38)$$

We obtain the desired equations of motion

$$\frac{dp_k}{dt} = \frac{\partial J_s}{\partial q_k}, \qquad \frac{dq_k}{dt} = -\frac{\partial J_s}{\partial p_k}, \qquad k=1,2,...,s . \qquad (A.39)$$

Equations (A.39) involving the coordinates $q_1,...,q_s$ and the momenta $p_1,...,p_s$ are in the form of Hamilton's equations with the negative Jingsian, $-J_s$, as the Hamiltonian. However, the (n-s) coordinates and velocities obey the Lagrange equations[2]

$$\frac{d}{dt} (\partial J_s/\partial \dot{q}_i) - (\partial J_s/\partial q_i) = 0 , \qquad i = s+1,....,n . \qquad (A.40)$$

Suppose a physical system is described by the equations of motion (A.39) and (A.40). The last term involving a total differential in (A.37) does not contribute to the equations of motion and it can be ignored in the action functional without affecting physics. Thus, the action functional S of a physical system can be written in the following symmetrized form:

$$S = \int_1^2 \{\sum_{i=1}^{s} (\frac{1}{2}p_i\dot{q}_i - \frac{1}{2}q_i\dot{p}_i) + J_s(q_1,...q_n,p_1,...,p_s,\dot{q}_{s+1},.. \dot{q}_n,t)\}dt. \quad (A.41)$$

Suppose this action S is invariant under the following infinitesimal transformations in the Jingsian space,

$$p'_k = p_k + \varepsilon P_k^*, \qquad k=1,...s;$$

$$q'_i = q_i + \varepsilon Q_i^*, \qquad i=1,...,n; \quad (A.42)$$

$$t' = t + \varepsilon T.$$

That is,

$$\{\frac{\partial}{\partial\varepsilon}[(\sum_{i=1}^{s} (\frac{1}{2}p'_i\,\dot{q}'_i - \frac{1}{2}q'_i\,\dot{p}'_i) + J_s(q'_1,...q'_n;p'_1,...,p'_s,\dot{q}'_{s+1},....\dot{q}'_n,t'))\frac{dt'}{dt}]\}_{\varepsilon=0}$$

$$= \frac{df}{dt}, \qquad \dot{p}'_k \equiv \frac{dp'_k}{dt'}, \quad (A.43)$$

where P_k^*, Q_i^*, T and f are functions of $(q_1,..q_n,p_1,.,p_s,\dot{q}_{s+1},..,\dot{q}_n,t)$. Following similar steps from (A.5) to (A.9), we have the generalized Noether's theorem for the case with symmetry in the phase space:

$$\frac{d}{dt}\left[HT + f - \sum_{i=1}^{n} p_i Q_i + \sum_{k=1}^{s} q_k P_k\right] = 0, \quad (A.44)$$

where $P_i = (\frac{1}{2}P_1^*,..,\frac{1}{2}P_s^*)$ and $Q_i = (\frac{1}{2}Q_1^*,..,\frac{1}{2}Q_s^*,Q_{s+1}^*,...,Q_n^*)$.

References

1. She was born in Erlangen, Germany. Due to the strong prejudice existing in the early 20th century against women being professors, even the great Mathematician Hilbert could not gain her a permanent position at Göttingen University. His repeated attempts were frustrated for years until 1922 when she was named an "unofficial associate professor."
2. Y. Nambu, Lecture at the Copenhagen Summer Symposium, 1970 and *Proceedings of International Conference on Symmetries and Quark Model*, Wayne State University, 1969 (Gordon and Breach, New York, 1970), p. 269.
3. J. P. Hsu, J. Math. Phys. 30, 2682 (1989). Within the 4-dimensional symmetry framework, the time variables related to conservation laws are flexible and not unique, as discussed in chapters 7 and 12 and 17.
4. When there is a cyclic velocity in a Lagrangian, the Legendre transformation has a problem because it cannot be defined in the usual sense. See J. Kevorkian, *Partial Difference Equations: Analytic Solution Techniques* (Wadsworth & Brooks/Cole 1990; Pacific Grove, CA), pp. 332–333. But in certain cases, one still can define a Hamiltonian for a quantum ring model in the absence of two velocities $\dot{r}$ and $\dot{\theta}$. See, for example, J. P. Hsu, J. Math. Phys. in ref. 3.
5. H. Goldstein, *Classical Mechanics* (2nd. Ed. Addison-Wesley, Reading MA., 1981), pp. 351–356.

Appendix B.

Quantum Electrodynamics in Both Linearly Accelerated and Inertial Frames

Ba. Quantum Electrodynamics of Bosons in CLA and Inertial Frames

Quantum scalar field operators obey equation (24.35) in CLA frames. This suggests that we use the taiji-time w in a general frame as the evolution variable for a state $\Phi^{(S)}(w)$ in the Schrödinger representation:

$$i\frac{\partial\Phi^{(S)}(w)}{\partial w} = H^{(S)}(w)\Phi^{(S)}(w)\,, \qquad H^{(S)} = H_o^{(S)} + H_I^{(S)}, \qquad J = 1\,. \tag{B.1}$$

The reason is that the evolution of a physical system is assumed to be described by a Hamiltonian operator which has the same transformation property as $\partial/\partial w$. A covariant partial derivative is the same as an ordinary partial derivative, $D_\mu = \partial_\mu$, when they operate on scalar functions. We may remark that the form (B.1) is no longer true if the Hamiltonian involves spinor fields; in this case, the time derivative ∂_0 has to be replaced by the gauge covariant derivative ∇_0, according to equation (24.56).

It is natural to assume that the usual covariant formalism of perturbation theory[1] can also be applied to QED of scalar bosons in CLA frames, which are smoothly connected to inertial frames in the limit of zero acceleration. Let us briefly consider the interaction representation and the S-matrix in CLA frames. The transformations of the state vector $\Phi(w)$ and operator $O(w)$ from the Schrödinger representation to the interaction representation are defined as

$$\Phi(w) \equiv \Phi^{(I)}(w) = \exp[iH_o^{(S)}w]\Phi^{(S)}(w)\,, \tag{B.2}$$

$$O(w) \equiv O^{(I)}(w) = \exp[iH_o^{(S)}w]O^{(S)}\exp[-iH_o^{(S)}w]\,. \tag{B.3}$$

Because $O^{(S)}$ and $O(w)$ are the same for $w = 0$, we have

$$i\frac{\partial\Phi(w)}{\partial w} = H_I(w)\Phi(w), \quad H_I = \exp[iH_o^{(S)}w]H_I^{(S)}\exp[-iH_o^{(S)}w]\,, \tag{B.4}$$

$$O(w) = \exp[iH_o^{(S)}w]O(0)\exp[-iH_o^{(S)}w] \,. \tag{B.5}$$

The U-matrix can be defined in terms of the time w: $\Phi(w) = U(w,w_o)\Phi(w_o)$, $U(w_o,w_o) = 1$. It follows from (B.4) and (B.5) that

$$i\frac{\partial U(w,w_o)}{\partial w} = H_I(w)U(w,w_o) \,. \tag{B.6}$$

If a physical system is in the initial state Φ_i at time w_o, the probability of finding it in the final state Φ_f at a later time w is

$$|\langle\Phi_f|U(w,w_o)\,\Phi_i\rangle|^2 = |U_{fi}(w,w_o)|^2 . \tag{B.7}$$

Evidently, the average transition probability per unit time for $\Phi_f \to \Phi_i$ is

$$|U_{fi}(w,w_o) - \delta_{fi}|^2/(w - w_o) \,. \tag{B.8}$$

As usual, we can express the S-matrix in terms of the U-matrix, i.e. $S = U(\infty, -\infty)$ and obtain the following form

$$S = 1 - i\int_{-\infty}^{\infty} H_I(w)dw \;+\; (-i)^2\int_{-\infty}^{\infty} H_I(w)dw \int_{-\infty}^{w} H_I(w')dw' \;+ \ldots\ldots \,. \tag{B.9}$$

For w-dependent operators, one can introduce a w-product W^* (corresponding to the usual chronological product), so that (B.9) can be written in an exponential form:

$$S = W^*\{\exp[-i\int_{-\infty}^{\infty} H_I(x^\mu)dwd^3r]\} \,, \qquad \int_{-\infty}^{\infty} H_I(x^\mu)d^3r = H_I(w) \,. \tag{B.10}$$

Since J is a truly universal constant, we can have the "natural units" J = 1 in both CLA and inertial frames. Thus, we have the relation of dimensions

$$[a_\mu] = [\psi^{2/3}] = [\text{mass}] = [1/\text{length}] \,, \tag{B.11}$$

in CLA and inertial frames.

To obtain the rules for Feynman diagrams for scalar QED in CLA frames, we follow the usual procedure[1] and assume L_{SQED} to be

$$L_{SQED} = L_{sp} - \sqrt{-g}\,(\partial^\mu a_\mu)^2/(2\rho)\,, \tag{B.12}$$

$$L_{sp} = \sqrt{-g}\,\{g^{\mu\nu}[(i\partial_\mu - \bar{e}a_\mu)\Phi^*][(i\partial_\nu - \bar{e}a_\nu)\Phi] - m^2\Phi^*\Phi\} - \frac{1}{4}\sqrt{-g}\,f_{\mu\nu}f^{\mu\nu}\,,$$

$$f_{\mu\nu} = D_\mu a_\nu - D_\nu a_\mu = \partial_\mu a_\nu - \partial_\nu a_\mu\,, \quad \bar{e} = -1.6021891\times 10^{-20}\sqrt{4\pi}\,(g\cdot cm)^{1/2}\,,$$

where ρ is a gauge parameter. The Lagrangian L_{sp} is gauge invariant and observable results are independent of the gauge parameter ρ.

To see that there is a "conservation" of 4–momentum at each vertex of the Feynman diagram in CLA frames, let us consider the wave function $\Phi(w,\mathbf{x}) = \Phi(x)$ for a "free particle" given by (24.23) with the phase P given by (24.19) and the condition (24.21) for a plane wave. In CLA frames, one can verify that

$$\frac{\partial}{\partial w} P = k_{I0}(x^1 + 1/\alpha\gamma_o^2)\gamma^3\alpha + k_{I1}(x^1 + 1/\alpha\gamma_o^2)\gamma^3\alpha\beta$$

$$= (\gamma k_{I0} + \gamma\beta k_{I1})W = k_0\,, \tag{B.13}$$

$$\frac{\partial}{\partial x^1} P = k_{I0}\gamma\beta + k_{I1}\gamma = k_1\,, \quad \frac{\partial}{\partial x^2} P = k_2\,, \quad \frac{\partial}{\partial x^3} P = k_3\,,$$

where we have used $(\partial/\partial w)\gamma\beta = \gamma^3\alpha$ and $(\partial/\partial w)\gamma = \gamma^3\alpha\beta$. Thus, the relation

$$i\frac{\partial}{\partial x^\mu}\, e^{-iP} = k_\mu\, e^{-iP} \tag{B.14}$$

holds for a "free wave" in CLA frames, just as in inertial frames.

However, the zeroth component p_0 (or k_0) of the covariant 4–momentum depends on the Wu alteration function W(w,x), as shown in (22.9), and is not conserved in a particle collision process. Fortuately, the conservation of "momentum" in a collision process, e.g., a+b→c+d, as observed in CLA frames can be expressed in terms of

$$(p_0/W, p_1, p_2, p_3) = \bar{p}_\mu = (\bar{p}_0, \bar{p}_1, \bar{p}_2, \bar{p}_3), \tag{B.15}$$

$$g^{\mu\nu}p_\mu p_\nu = (p_0/W)^2 - (p_1)^2 - (p_2)^2 - (p_3)^2 = \eta^{\mu\nu}\bar{p}_\mu\bar{p}_\nu, \quad \eta^{\mu\nu} = (1,-1,-1,-1).$$

This $\bar{p}_\mu$ may be termed "alteration momentum" which is not exactly a 4-vector under the Wu or the MWL transformation. But the momentum space of $\bar{p}_\mu$ is formally closer to the space of the 4-momentum $p_{I\mu}$ in inertial frames than that of the true 4-momentum p_μ as far as the S-matrix and Feynman rules are concerned. For the scattering process a+b→c+d, we have the following relations for momentum:

$$\bar{p}_{0a} + \bar{p}_{0b} = \gamma_B(p_{I0a} + p_{I0b} - \beta_B[p_{I1a} + p_{I1b}]),$$

$$\bar{p}_{1a} + \bar{p}_{1b} = \gamma_B(p_{I1a} + p_{I1b} - \beta_B[p_{I0a} + p_{I0b}]),$$

$$p_{2a} + p_{2b} = p_{I2a} + p_{I2b}, \qquad p_{3a} + p_{3b} = p_{I3a} + p_{I3b}; \tag{B.16}$$

$$\bar{p}_{0c} + \bar{p}_{0d} = \gamma_A(p_{I0c} + p_{I0d} - \beta_A[p_{I1c} + p_{I1d}]),$$

$$\bar{p}_{1c} + \bar{p}_{1d} = \gamma_A(p_{I1c} + p_{I1d} - \beta_A[p_{I0c} + p_{I0d}]),$$

$$p_{2c} + p_{2d} = p_{I2c} + p_{I2d}, \qquad p_{3c} + p_{3d} = p_{I3c} + p_{I3d}; \qquad \bar{\mathbf{p}} = \mathbf{p},$$

which can be derived from the inverse transformation of (22.10) and (B.15). Since the 4-monentum is conserved in the inertial frame F_I, i.e., $p_{I0c} + p_{I0d} = p_{I0a} + p_{I0b}$ = constant and $\mathbf{p}_{Ic} + \mathbf{p}_{Id} = \mathbf{p}_{Ia} + \mathbf{p}_{Ib}$ = constant, we have the conservation of the alteration momentum in CLA frames:

$$\bar{p}_{0c} + \bar{p}_{0d} = \bar{p}_{0a} + \bar{p}_{0b} \qquad \text{and} \qquad \bar{\mathbf{p}}_c + \bar{\mathbf{p}}_d = \bar{\mathbf{p}}_a + \bar{\mathbf{p}}_b, \tag{B.17}$$

at the "instant of collision," so to speak. The reason for their conservation is that, although both sides are not constant as shown in (B.16), they must be the same when $\beta_B = \beta_A$ which is realized at the instant of collision.

Based on the Wu transformation for coordinates and 4-momenta, we have

$$\int d^4x_I \; \exp(-ip_{I\mu}x_I^{\mu}) = (2\pi)^4\delta^4(p_I) = \int\sqrt{-g}d^4x \; e^{-iP(x)}$$

$$= (2\pi)^4\delta(\gamma\bar{p}_0 - \gamma\beta\bar{p}_1)\delta(\gamma\bar{p}_1 - \gamma\beta\bar{p}_0)\delta(\bar{p}_2)\delta(\bar{p}_3) \tag{B.18}$$

$$= (2\pi)^4\delta(\bar{p}_0)\delta(\bar{p}_1)\delta(\bar{p}_2)\delta(\bar{p}_3) = (2\pi)^4\delta^4(\bar{p}) = (2\pi)^4 W\delta^4(p) \, ,$$

$$\delta(p_{I0})\delta(p_{I1}) = \delta(\gamma\bar{p}_0 - \gamma\beta\bar{p}_1)\delta(\gamma\bar{p}_1 - \gamma\beta\bar{p}_0) = \frac{\delta(\bar{p}_0)\delta(\bar{p}_1)}{J(p_{I\lambda}/\bar{p}_\lambda)} = \delta(\bar{p}_0)\delta(\bar{p}_1) \, ,$$

where we have used the 'free-wave' (24.9), the momentum transformation (22.10) and the Wu transformation (21.46). In the last equation, $J(p_{I\lambda}/\bar{p}_\lambda)$ is the Jacobian of the $p_{I\lambda}$ with respect to the $\bar{p}_\lambda$ which can be calculated by using (22.10). This result is the 2-dimensional generalization of the 1-dimensional case given by (24.20) with $\kappa = 0$.[2] In a CLA frame, the integral of a "plane wave" over the "whole spacetime" is limited and complicated by the presence of a "black wall" (i.e., a wall singularity) at $x = -1/(\alpha\gamma_o^2)$. The integration can be carried out by a change of variables and this amounts to using variables in an inertial frame as a crutch to obtain the result. The relation (B.17) or (B.18) implies that we have a conservation of momentum at a vertex in the generalized Feynman rules in CLA frames. Those properties in (B.13)–(B.18) are convenient for writing down the generalized Feynman rules for quantum electrodynamics in CLA frames.

As usual, if there are no identical particles in the final state, we define the relationship between the *M*- and S-matrices for initial (i) and final (f) states as follows:

$$S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(\bar{p}_f^{(tot)} - \bar{p}_i^{(tot)})[\Pi_{ext\ par}(n_j/V)]^{1/2}M_{fi} \, , \tag{B.19}$$

where "ext par" denotes external particles, $n_j = m_j/\omega_{kj}$ for spin 1/2 fermions and $n_j = 1/(2\omega_{kj})$ for bosons. Note that the S-matrix elements for physical processes which are observed and measured in CLA frames are defined only for those cases in which the momenta of the initial and final states are constant in an inertial frame.

Because of the 4-dimensional symmetry in (B.12) and (B.19), the generalized Feynman rules for writing the amplitude M_{fi} are formally the same as those in the usual QED, except that certain quantities (e.g., w, J and $\tilde{e}$) have different dimensions from the corresponding quantities in conventional QED.

One can use a more intuitive method of Feynman to obtain the generalized rules for Feynman diagrams in CLA frames. For example, the scalar boson propagator can be obtained from the scalar wave equations (24.1) and the relation (B.14) for a "free boson" with the momentum p_μ. The vertex of interaction can be obtained from the interaction Lagrangian density L_{SQED} in (B.12).[3]

The generalized Feynman rules for the amplitude M_{fi} in both constant-linear-acceleration frames and inertial frames are as follows:

(a) The covariant photon propagator is given by

$$\frac{-i\left\{g_{\mu\nu} - (1-\rho)\dfrac{k_\mu k_\nu}{(k^\lambda k_\lambda + i\varepsilon)}\right\}}{(k^\sigma k_\sigma + i\varepsilon)}, \tag{B.20}$$

where $\rho = 1$ is the Feynman gauge, and $\rho = 0$ the Landau gauge.

(b) The scalar boson propagator is

$$\frac{i}{(p^\mu p_\mu - m^2 + i\varepsilon)}. \tag{B.21}$$

(c) The vertex $\Phi(p)+\gamma(k,\mu) \to \Phi(p')$ is

$$-i\,\tilde{e}(p_\mu + p'_\mu), \tag{B.22}$$

where $\gamma(k,\mu)$ is an incoming photon line toward the vertex with the momentum k_λ and a polarization index μ.

(d) The vertex $\Phi+\gamma(\mu) \to \Phi+\gamma(\nu)$ has the factor

$$2i\,\tilde{e}^2 g_{\mu\nu}. \tag{B.23}$$

(e) Each external photon line with an index μ has a polarization vector ε_μ.

(f) A factor 1/2 for each closed loop containing only two photon lines, e.g., $\Phi+\Phi \to \gamma(\mu)+\gamma(\nu) \to \Phi+\Phi$.

Other rules such as a integration with $W^{-1}d^4k/(2\pi)^4 = d^4\bar{k}/(2\pi)^4$ over a momentum k_μ not fixed by the "conservation" of momentum at each vertex are the same as the usual rules in inertial frames.

Bb. Feynman Rules for QED in both CLA and Inertial Frames

To obtain the rules for Feynman diagrams of spinor QED in CLA frames, we have to replace the time derivative $\partial_0 = \partial/\partial w$ by the gauge covariant time derivative ∇_0, according to equation (24.56). We follow the usual quantization procedure and define L_{TQED} by adding a gauge fixing term in the Lagrangian density,

$$L_{TQED} = L - \sqrt{-g}\,(D^\mu a_\mu)^2/2\rho\,, \tag{B.24}$$

$$L = \sqrt{-g}\,\bar{\psi}\Gamma^\mu(i\nabla_\mu - \bar{e}a_\mu)\psi - \sqrt{-g}\,m\bar{\psi}\psi\,, \tag{B.25}$$

$$\nabla_\mu = (\partial_0 + \frac{1}{2}(\partial_k W)\gamma^0\gamma^k, \partial_1, \partial_2, \partial_3)\ ,$$

where ρ is a gauge parameter. As usual, the M-matrix is defined in (B.19). One can verify that the plane-wave solution (24.57) of a free fermion satisfies

$$\nabla_\mu e^{(-iP(x) - G(w))} = k_\mu e^{(-iP(x) - G(w))} \tag{B.26}$$

in CLA frames.

The generalized Feynman rules for the amplitude M_{fi} of QED in both CLA frames and inertial frames are as follows:

(a) The covariant photon propagator is given by (B.20).

(b) The electron propagator is

$$\frac{-i}{(\Gamma^\mu p_\mu - m + i\varepsilon)}\ . \tag{B.27}$$

(c) The electron–photon vertex is

$$-i\bar{e}\Gamma^\mu\,. \tag{B.28}$$

(d) Each external photon line has an additional factor ε_μ. Each external electron line has u(s,p) for the annihilation of an electron and $\overline{u}$(s,p) for the creation of an electron. Each external positron line has v(s,p) for the annihilation of a positron and $\overline{v}$(s,p) for the creation of a positron.

Other rules such as taking the trace with a factor (-1) for each closed electron loop, integration with $d^4k/[W(2\pi)^4]$ over a momentum k_μ not fixed by the conservation of alteration momentum at each vertex are the same as the usual.

Thus, if one calculates scattering cross sections and decay rates (with respect to the taiji-time w) of a physical process, one will get formally a similar result as that in conventional QED. For example, let us consider the decay rate $\Gamma(1\rightarrow2+3+...+N)$ for a physical process $1\rightarrow2+3+...+N$. It is given by the expression

$$\Gamma(1\rightarrow2+3+...+N) = \lim_{w\rightarrow\infty}\int \frac{|\langle f|S|i\rangle|^2}{w}\frac{d^3x_2d^3p_2}{(2\pi)^3}\frac{d^3x_3d^3p_3}{(2\pi)^3}\cdots\frac{d^3x_Nd^3p_N}{(2\pi)^3},$$

$$= \int\frac{1}{2\omega_{p_1}}|M_{fi}|^2\Big[\prod_{\text{ext fer}}(2m_{fer})\Big]\frac{d^3p_2}{(2\pi)^3 2\omega_{p_2}}\cdots\cdots\cdots$$

$$\times\frac{d^3p_N}{(2\pi)^3 2\omega_{p_N}}(2\pi)^4\delta^4(\bar{p}_1+\bar{p}_2-\bar{p}_3-\bar{p}_4\cdots-\bar{p}_N)S_O, \qquad \text{(B.29)}$$

where $\langle f|S|i\rangle = S_{fi}$ is the S-matrix element between the initial state i and the final state f given by (B.19). The decay rate $\Gamma(1\rightarrow2+3+...+N)$ has the dimension of inverse length and S_O denotes a factor 1/n! for each kind of (n) identical particles in the final state. When there is no external fermion in a process, then $[\Pi_{\text{ext fer}}(2m_{fer})]$ in (B.29) is replaced by 1. The decay length D is given by $D = 1/\Gamma(1\rightarrow2+3+...+N)$.

For a scattering process $1+2\rightarrow3+4+....+N$, the differential cross section $d\sigma$, which has the dimension of area, is given by

$$d\sigma = \frac{1}{4[(p_1\cdot p_2)^2-(m_1m_2)^2]^{1/2}}|M_{fi}|^2\Big[\prod_{\text{ext fer}}(2m_{fer})\Big]\frac{d^3p_3}{(2\pi)^3 2\omega_{p_3}}\cdots\cdots$$

$$\cdots\cdots\frac{d^3p_N}{(2\pi)^3 2\omega_{p_N}}(2\pi)^4\delta^4(\bar{p}_1+\bar{p}_2-\bar{p}_3-\bar{p}_4\cdots-\bar{p}_N)S_O, \qquad \text{(B.30)}$$

where $p_i = \bar{p}_i$, i = 1, 2, 3 and $\bar{p}_0 = \omega_p = (\mathbf{p}^2+m^2)^{1/2}$. If the initial particles are unpolarized, one takes the average over initial spin states. When there is no external fermion in a process, then $[\Pi_{ext\ fer}(2m_{fer})]$ in (B.30) is replaced by 1.

Bc. Some QED Results in Both CLA and Inertial Frames

Let us consider some well-known physical processes in QED[4] to illustrate the generalized Feynman rules in both CLA frames and inertial frames, and see how the conventional results in inertial frames are modified if they are measured in a laboratory with constant-linear-acceleration.

A. Electron Scattering from a Point-Like Proton

According to the generalized Feynman rules, the amplitude M_{fi} for such an electron scattering from a point-like proton, $e(p_i + p(P_i) \to e(p_f) + p(P_f)$ with the exchange of a photon $\gamma(q)$, where $q_\mu = p_{f\mu} - p_{i\mu}$, is given by

$$M_{fi} = \bar{u}(s_f,p_f)[-\,i\,\bar{e}\Gamma^\mu]u(s_i,p_i)\left[\frac{-ig_{\mu\nu}}{(q_\sigma q^\sigma+i\varepsilon)}\right]\bar{u}(S_f,P_f)[-\,i\,\bar{e}\Gamma^\mu]u(S_i,P_i)\ , \qquad (B.31)$$

where we have used the Feynman gauge for the photon propagator ($\rho = 1$). The S-matrix element S_{fi} in (B.19) takes the form

$$S_{fi} = -\,i(2\pi)^4\delta^4(\bar{P}_f + \bar{p}_f - \bar{P}_i - \bar{p}_i)\left[\frac{m}{\omega_{pf}}\frac{m}{\omega_{pi}}\frac{M}{\omega_{Pf}}\frac{M}{\omega_{Pi}}\frac{1}{V^4}\right]^{1/2} M_{fi}\ , \qquad (B.32)$$

where m and M are, respectively, masses for the electron and the proton. The differential cross section is given by (B.30),

$$d\sigma = \frac{mM}{[(\bar{p}_i\cdot\bar{P}_i)^2-(mM)^2]^{1/2}}|M_{fi}|^2\frac{md^3\bar{p}_f}{(2\pi)^3\omega_{pf}}\frac{Md^3\bar{P}_f}{(2\pi)^3\omega_{Pf}}$$

$$\times(2\pi)^4\delta^4(\bar{p}_f + \bar{P}_f - \bar{p}_i - \bar{P}_i)\ , \qquad (B.33)$$

where $\bar{p}_0 = \omega_{pf} = (\mathbf{p}_f^2+m^2)^{1/2}$, $\omega_{Pf} = (\mathbf{P}_f^2+M^2)^{1/2}$ and $|M_{fi}|^2$ is given by

$$|M_{fi}|^2 = \frac{\bar{e}^4}{2m^2M^2\bar{q}^4}\ [\bar{P}_f\cdot\bar{p}_f\bar{P}_i\cdot\bar{p}_i + \bar{P}_f\cdot\bar{p}_i\bar{P}_i\cdot\bar{p}_f - m^2\bar{P}_f\cdot\bar{P}_i$$

$$- M^2\bar{p}_f\cdot\bar{p}_i + 2M^2m^2]\ , \qquad (B.34)$$

Since $\bar{P}_f\cdot\bar{p}_f = \bar{P}_{f\mu}\bar{p}_f^{\mu}$, etc. we see that the differential cross section is formally the same as the usual one in an inertial frame, except that each individual momentum is not constant in CLA frames,

$$d\sigma(\text{CLA frame}) = d\sigma(\text{inertial frame})\ . \qquad (B.35)$$

The origin of this identical result is the limiting 4–dimensional symmetry which dictates the invariance of the action or the S–matrix.

After integration, (B.35) gives the total cross section which can be pictured as the effective size of the target particle, i.e., proton. This effective size of the proton depends of the strength of the interaction. For the electromagnetic interaction, the coupling strength is $\alpha_e \sim 1/137 \sim 10^{-2}$, the weak interaction coupling strength is about 10^{-12}. The size (or cross section) of a proton is about 10^{-24} cm^2 from the 'viewpoint' of the electron. But from the 'viewpoint' of a neutrino, which has only weak interactions with the proton, the size of a proton is extremely small, about 10^{-44} cm^2.

B. Compton Scattering

The S–matrix element for the Compton scattering process, $\gamma(k) + e(p_i) \rightarrow \gamma(k') + e(p_f)$, is given by

$$S_{fi} = -\,i(2\pi)^4\delta^4(\bar{k}' + \bar{p}_f - \bar{k} - \bar{p}_i)\left[\frac{m}{\omega_{pf}}\frac{m}{\omega_{pi}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k'}}\frac{1}{V^4}\right]^{1/2} M_{fi}\ . \qquad (B.36)$$

where $\omega_k = |\bar{\mathbf{k}}| = |\mathbf{k}|$, $\omega_{pf} = \sqrt{\ \bar{p}_f^2 + m^2}$ and the M–matrix element is given by

$$M_{fi} = \bar{u}(s_f, p_f)\Big\{[-i\bar{e}\Gamma^{\alpha}\varepsilon'_{\alpha}]\left[\frac{-i}{\gamma^{\mu}(\bar{p}_{i\mu} + \bar{k}_{\mu}) - m + i\varepsilon}\right][-i\bar{e}\Gamma^{\nu}\varepsilon_{\nu}]$$

$$+ [-i\bar{e}\Gamma^{\alpha}\varepsilon_{\alpha}]\left[\frac{-i}{\gamma^{\mu}(\bar{p}_{i\mu} - \bar{k}'_{\mu}) - m + i\varepsilon}\right][-i\bar{e}\Gamma^{\nu}\varepsilon'_{\nu}]\ \Big\}u(s_i, p_i)\ , \qquad (B.37)$$

according to the generalized Feynman rules.

We have seen that the differential cross section for the Compton scattering is also the same as that in an inertial frame,

$$d\sigma_{Compton}(\text{CLA frame}) = d\sigma_{Compton}(\text{inertial frame}) \ . \tag{B.38}$$

C. Self-Mass of the Electron

The self-mass of the electron is given by the expression

$$\delta m = \int \frac{d^4k}{W(2\pi)^4} \frac{-ig_{\mu\nu}}{[k_\sigma k^\sigma + i\varepsilon]} [-i\bar{e}\Gamma^\mu] \frac{-i}{[\Gamma^\rho(p_\rho - k_\rho) - m + i\varepsilon]} [-i\,\bar{e}\Gamma^\nu]$$

$$= \delta m(\text{inertial frame}) \ , \tag{B.39}$$

where we have set $\rho=1$ for simplicity and used (24.43) and (B.15). This is consistent with the fact that the (rest) mass of the electron in an inertial frame is the same as that of the electron in CLA frames, as shown in (23.8).

In all these calculations, the particles must move with constant velocities as measured in an inertial frame. This is the condition imposed in defining the S-matrix and for obtaining the generalized Feynman rules in both linearly accelerated and inertial frames. These discussons for QED can also be applied to non-inertial frames with a constant rotational motion, since we have the taiji rotational transformations (25.11) with limiting 4-dimensional symmetry.

References

1. See, for example, J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, MA, 1967), pp. 171–172 and pp. 181–188; S. Weinberg, *The Quantum Theory of Fields*, vol. I, Foundations (Cambridge University Press, New York, N.Y., 1995), pp. 134–147.
2. I. M. Gel'fand and G.E. Shilov, *Generalized Functions* Vol. 1 (transl. by E. Saletan, Academic Press, New York, 1964), p. 185.
3. As usual, one can use the interaction terms in iL_{SQED} and replace the field operators by appropriate 'free particle' wave functions. Omit $\exp(\pm iP(x))$ and the factors for external lines. The remainder is the vertex factor. The S-matrix expansion can be formulated by using the Lagrangian. See N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (transl. by G. M. Volkoff, Interscience Publishers, New York, 1959), pp. 206-226.
4. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), chapter 7.

Appendix C.

De Sitter and Poincaré Gauge-Invariant Fermion Lagrangians and Gravity*

We present a new fermion lagrangian which possesses exact symmetry under the local de Sitter group. The lagrangian involves new "scale gauge fields" related to the newtonian force and the usual Yang-Mills "phase gauge fields" related to a new "gravitational spin force" between two fermions. Generalization of the usual gauge theory for *external* symmetry groups is also discussed.

It has been suggested that gravity is related to gauge fields of four-dimensional symmetry such as the de Sitter group [1,2]. The idea is quite interesting because the de Sitter group possesses the maximum four-dimensional symmetry [3] and is the unique generalization of the Poincaré group. It also suggests the existence of a new "gravitational spin force" between objects with nonzero net spin densities. The de Sitter group is a rotational group in de Sitter space, which is the hypersurface of a four-dimensional sphere of a hyperbolic character in one direction, embedded in a five-dimensional space. The radius of the sphere is denoted by L. The de Sitter group reduces to the Poincaré group in the flat space limit $L \to \infty$.

One important ingredient in a realistic gauge theory of gravity is the fermion field — a source of the gravitational field. But in previous discussions [1,2] one either ignored the fermion field or discussed a fermion lagrangian which has only approximate symmetry under local de Sitter gauge transformations. It appears that one cannot get a fermion lagrangian with exact external gauge symmetry if one just employs the usual Yang-Mills fields, i.e. "phase gauge fields" [4].

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In this paper, we present a new fermion lagrangian, which has exact symmetry under the local de Sitter group. It is necessary that the lagrangian involves new "scale gauge fields" in addition to the usual Yang–Mills "phase gauge fields". They have different transformation property and, therefore, must be treated as different and independent fields.

Let us consider the generalization of $\gamma^\mu \partial_\mu \psi$ in the form for a non-abelian external symmetry group:

$$\Gamma^\mu D_\mu \psi \tag{C.1}$$

where Γ^μ involves both the Dirac matrices and scale gauge fields e^μ_A and the gauge–covariant derivative D_μ contains phase gauge fields $b^A_\mu = (b^i_\mu, b^a_\mu)$:

$$\Gamma^\mu \equiv e^\mu_A \gamma^A = e^\mu_k \gamma^k + e^\mu_{jk}\, i(\gamma^j\gamma^k - \gamma^k\gamma^j)/4L \equiv E^\mu_A Z^A, \tag{C.2}$$

$$D_\mu \equiv \partial_\mu - igb_\mu, \qquad b_\mu \equiv b^A_\mu Z_A, \tag{C.3}$$

$$Z_A = (Z_i, Z_a) = (\gamma_i/2L,\ i(\gamma_j\gamma_k - \gamma_k\gamma_j)/4), \qquad a \equiv jk,$$

$$\{\gamma_j, \gamma_k\} = 2\eta_{jk}, \qquad \eta_{jk} = (1, -1, -1, -1),$$

$$E^\mu_A = (2Le^\mu_i,\ e^\mu_{jk}/L).$$

Tile quantity Z_A is the matrix representation of the SO(3,2) de Sitter group generators:

$$[Z_B, Z_C] = if^A_{BC} Z_A, \qquad A = i, jk; \quad \text{etc.} \tag{C.4}$$

The local de Sitter gauge transformations are given by

$$\psi \to \psi' = \xi_d \psi,$$

$$b_\mu \to b'_\mu \equiv b_\mu + (\partial_\mu \xi_d)\, \xi_d^{-1}/(ig), \tag{C.5}$$

$$\Gamma^\mu \to \Gamma'^\mu = \xi_d \Gamma^\mu \xi_d^{-1},$$

where

$$\xi_d = \exp[i\omega^A(x)Z_A] .$$

The gauge functions $\omega^A(x)=(\omega^i(x), \omega^a(x))$ are real and arbitrary.

It can be shown that $\bar{\psi}(\Gamma^\mu D\mu+m)\psi$ is invariant under the local de Sitter gauge transformations (C.5):

$$\bar{\psi}'(\Gamma'^\mu D'_\mu+m)\psi' = \bar{\psi}(\Gamma^\mu D_\mu+m)\psi . \tag{C.6}$$

We stress that this symmetry property holds if and only if both e_i^μ and e_{jk}^μ are introduced.

Note that the field e_A^μ is dimensionless and is related to a change in the scale rather than a change in the phase [4], so that e_A^μ may be termed a "scale gauge field". In view of the presence of this new scale gauge field, the present gauge theory is a generalization of the Yang-Mills theory. Such a generalization appears to be necessary because the de Sitter group is an external symmetry group, in which the generator Z_A does not commute with γ_k, in contrast to the case of an internal symmetry group.

The phase field strength $F_{\mu\nu}^A$ is given by

$$(D_\mu D_\nu - D_\nu D_\mu)\psi = igF_{\mu\nu}^A Z_A \psi , \tag{C.7}$$

i.e.,

$$F_{\mu\nu}^A = \partial_\mu b_\nu^A - \partial_\nu b_\mu^A + gif^A_{BC} b_\mu^B b_\nu^C . \tag{C.8}$$

One can verify that $F_{\mu\nu} \equiv F_{\mu\nu}^A Z_A$ is gauge covariant and transforms as follows:

$$F_{\mu\nu} \to F'_{\mu\nu} = \xi_d F_{\mu\nu} \xi_d^{-1} . \tag{C.9}$$

Thus, $Tr(F_{\mu\nu}F_{\alpha\beta})$ is a gauge-invariant quantity:

$$Tr(F'_{\mu\nu}F'_{\alpha\beta}) = Tr(F_{\mu\nu}F_{\alpha\beta}) , \tag{C.10}$$

which is usually used as the lagrangian for the phase field b_ν^A.

We observe that $Tr(\Gamma^\mu \Gamma^\nu)$ is gauge invariant:

$$Tr(\Gamma'^\mu \Gamma'^\nu) = Tr(\Gamma^\mu \Gamma^\nu) . \qquad (C.11)$$

Thus, we expect that e^μ_A enters the lagrangian for the scale field through the combination

$$\bar{g}^{\mu\nu} \equiv Tr(\Gamma^\mu \Gamma^\nu /4)$$

$$= \eta^{ik} e^\mu_i e^\nu_k + e^\mu_{ij} e^\nu_{kn} \eta^{ik} \eta^{jn}/(2L)$$

$$\rightarrow g^{\mu\nu} \quad \text{as} \quad L \rightarrow \infty. \qquad (C.12)$$

$$g^{\mu\nu} \equiv \eta^{ik} e^\mu_i e^\nu_k .$$

For large L, $\bar{g}^{\mu\nu}$ is approximately the same as $g^{\mu\nu}$. In the limit $L \rightarrow \infty$, it is natural to interpret e^μ_i as the vierbein component and $g^{\mu\nu}$ as the spacetime metric. Thus we can interpret $\bar{g}^{\mu\nu}$ as the spacetime metric in the present theory with the de Sitter gauge group. We are able to define the affine connection $\bar{\Gamma}^\alpha_{\mu\nu}$ and the Riemann curvature tensor $\bar{R}^\alpha{}_{\lambda\mu\nu}$ in terms of the new gauge-invariant metric $\bar{g}^{\mu\nu}$ by the usual relations:

$$\bar{\Gamma}^\alpha_{\mu\nu} = \frac{1}{2} \bar{g}^{\lambda\alpha}(\partial_\nu \bar{g}_{\mu\lambda} + \partial_\mu \bar{g}_{\lambda\nu} - \partial_\lambda \bar{g}_{\nu\mu}) , \qquad \bar{g}^{\mu\lambda} \bar{g}_{\lambda\nu} = \delta^\mu{}_\nu ,$$

$$\bar{R}^\alpha{}_{\lambda\mu\nu} = \partial_\nu \bar{\Gamma}^\alpha_{\lambda\mu} - \partial_\mu \bar{\Gamma}^\alpha_{\lambda\nu} + \bar{\Gamma}^\beta_{\lambda\mu} \bar{\Gamma}^\alpha_{\beta\nu} - \bar{\Gamma}^\beta_{\lambda\nu} \bar{\Gamma}^\alpha_{\beta\mu} . \qquad (C.13)$$

In this way, $\bar{\Gamma}^\alpha_{\mu\nu}$, $\bar{R}^\alpha{}_{\lambda\mu\nu}$ and $\bar{g}^{\mu\lambda}$ are all invariant under the local de Sitter gauge transformations (C.5). The invariant lagrangian for these fields is uniquely determined by the principle of gauge invariance and the principle of general covariance:

$$\int d^4x (\det \bar{g}_{\mu\nu})^{1/2} \left[\frac{1}{2}(i\bar{\psi}\Gamma^\mu D_\mu \psi + \text{h.c.}) - \bar{\psi} m \psi - \frac{1}{8} Tr(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{8\pi G}\bar{R}\right] , \qquad (C.14)$$

where G is a constant and $\bar{R} = \bar{R}^{\alpha}{}_{\mu\nu\alpha}\bar{g}^{\mu\nu}$. Field equations for e^{μ}_{i}, b^{A}_{μ} and ψ can be determined from the lagrangian (C.14).

When we take the limit $L \to \infty$, some components of gauge fields, i.e. b^{i}_{μ} and e^{μ}_{ij} disappear from the theory and we obtain the following lagrangian L:

$$L = (\det \mathrm{g}_{\mu\nu})^{1/2}\left[L_{\psi} - \frac{1}{8}\mathrm{Tr}(F^{a}_{\mu\nu}Z_a F^{b}_{\alpha\beta}Z_b)g^{\mu\alpha}g^{\nu\beta} + \frac{1}{8\pi G}R\right], \qquad \text{(C.15)}$$

$$R = R^{\alpha}{}_{\mu\nu\alpha}g^{\mu\nu} = (\partial_{\alpha}\Gamma^{\alpha}_{\mu\nu} - \partial_{\nu}\Gamma^{\alpha}_{\mu\alpha} + \Gamma^{\lambda}_{\mu\nu}\Gamma^{\alpha}_{\lambda\alpha} - \Gamma^{\lambda}_{\mu\alpha}\Gamma^{\alpha}_{\lambda\nu})g^{\mu\nu},$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\lambda\alpha}(\partial_{\nu}g_{\mu\lambda} + \partial_{\mu}g_{\lambda\nu} - \partial_{\lambda}g_{\nu\mu}),$$

$$g^{\mu\nu} = e^{\mu}_{i}e^{\nu}_{k}\eta^{ik}, \quad g_{\mu\nu} = e^{i}_{\mu}e^{k}_{\nu}\eta_{ik},$$

$$e^{i}_{\mu}e^{\nu}_{i} = \delta^{\nu}{}_{\mu}, \qquad e^{i}_{\mu}e^{\mu}_{k} = \delta^{i}{}_{k},$$

$$\mathrm{Tr}(F^{a}_{\mu\nu}Z_a F^{b}_{\alpha\beta}Z_b) = 2F^{ik}_{\mu\nu}F^{mn}_{\alpha\beta}\eta_{im}\eta_{kn}, \qquad a=ik,\ b=nm,$$

$$L_{\psi} = \frac{1}{2}i\bar{\psi}\Gamma^{\mu}(\partial_{\mu} - igb_{\mu})\psi + \frac{1}{2}i\bar{\psi}(\overleftarrow{\partial}_{\mu} - igb_{\mu})\Gamma^{\mu}\psi - \bar{\psi}m\psi,$$

where the last term in (C.15) is identical with Einstein's lagrangian. The lagrangian (C.15) is invariant under local Poincaré gauge transformations (i.e. the transformations (C.5) with $L \to \infty$) and general coordinate transformations (with the spacetime metric tensor $g^{\mu\alpha}$). The Poincaré gauge-invariant lagrangian (C.15) differs from that discussed by Kibble [5]. From the viewpoint of symmetry, the lagrangian (C.15) is more satisfactory than Kibble's lagrangian involving fermions.

Physically, the radius L of the de Sitter space is probably very large. In this case, physical effects of b^{i}_{μ} and e^{μ}_{ij} are negligible. Experimentally, the difference between the de Sitter gauge-invariant lagrangian (C.14) and the Poincaré gauge-invariant lagrangian (C.15) cannot be distinguished in the near future. So the *important fields* are b^{a}_{μ} and e^{μ}_{i}, which are generated by the spin density and mass density, respectively. This can be seen from the field equations derived from (C.15). For example, the source of b^{a}_{μ} in the approximation of static and weak fields is

$$b_\mu^{jk} = g\varepsilon^{ikj}\bar{U}(r)\sigma_j U(r)\,, \qquad \mu = 0\,,$$

$$= 0\,, \qquad \mu \neq 0\,, \qquad \text{(C.16)}$$

where U(r) and σ_j are respectively the positive-energy Pauli spinor and the Pauli matrices. Of course, the result (C.16) can also be derived from the de Sitter gauge–invariant lagrangian (C.14) with the approximation of very large L.

These gauge fields are interpreted as follows: *Gravity is related to scale gauge fields rather than the usual Yang-Mills gauge fields* because the scale gauge field is generated by the mass density, according to gauge–invariant lagrangians. The massless Yang-Mills field b_μ is generated by the spin density of fermions and corresponds to a new long-range force between two bodies with nonzero spin densities. The strength of this new force is determined by a new dimensionless coupling constant g^2, which is independent of the newtonian gravitational constant G. These hold for both finite G. These hold for both finite L (i.e. the de Sitter group) and infinite L (i.e. the Poincaré group).

Our interpretation of gauge fields for external symmetry groups differs from previous interpretations by Kibble and others [5-7] (see also refs. [2,8]). We may remark that a "contact spin force" between fermions has been discussed by Kibble based on a non-gauge–invariant fermion lagrangian. Since the external four–dimensional symmetry group is a fundamental symmetry of nature, the prediction of the new long-range gravitational spin force should be taken seriously.

We conclude that gauge field theory based on external four-dimensional symmetry groups dictates the presence of a new "scale gauge field", which differs from the Yang-Mills "phase gauge field" [9]. Furthermore, the theory predicts a new long-range gravitational spin force between fermions. It appears that the quantization of these fields cannot be accomplished by a straightforward application of the usual quantization procedure for Yang-Mills fields. This needs further investigation.

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Appendix D.

The Relativity of Lifetime Dilatation and an Experimental Test of "Twin Particles" Involving Linear Accelerations

Da. Three Relations of t' and t ($\Delta t' = \gamma \Delta t$, $\Delta t' = \Delta t/\gamma$, $\Delta t'_{20} = \Delta t_{10}$) for "Twin Particles" Under Different Conditions of Measurements in Special Relativity

Let us consider the *relativity* of the lifetime dilatation in special relativity (or, equivalently, that of decay-length dilatation in taiji relativity) and experimental tests of "twin particles" involving linear accelerations. The discussion can also illustrate some interesting and puzzling aspects of problems related to the so-called "clock paradox" or "twin paradox."[1,2] Some physicists[3] insist that the effect of acceleration on the twin is very important and must be taken into account, in sharp contrast to the conventional interpretation, which will be discussed below. In view of different and incompatible views in the literature, it is highly desirable that the matter can be resolved by a direct and unambiguous experimental test with linear accelerators. This can be done in the near future.

When one talks about the lifetime of unstable particles such as pions, it is understood that one is talking about the *mean lifetime* which is measured by observing the decays of many pions. The basic reason for this is that the decay of a single unstable pion is dictated by quantum-mechanical laws of probability and does not have a single fixed value of lifetime for all pions. Nevertheless, *the physical time should have the same property as the mean lifetime of unstable particles.*

In order to observe relativistic motions and effects, clocks and twins must be accelerated to speeds comparable to that of light, but since they are macroscopic objects, the task is difficult. However, it is useful to note that as far as "twins" are concerned, no pair of human identical twins are more twin-like than two identical unstable particles. At the present time, the decay lifetime dilatation of unstable particles in flight has been experimentally established beyond a reasonable doubt. Furthermore, *if the acceleration is neglected in numerical calculations*, then the "twin-particle paradox" can be treated and calculated completely within special relativity.

Within the conceptual framework of special relativity, some people have used the experimental results of the lifetime dilatation of unstable particles to support the conventional interpretation of the twin paradox.[4] This goes as follows: The traveler twin's rocket lifts off and reaches a constant velocity V in a negligibly short time. After traveling for a very long distance L_o as measured by the stay-at-home twin on the earth, the rocket reverses its velocity, comes back to the earth and stops. Reversal and stopping again occur in a negligibly short time. The stay-at-home twin records an elapsed time $T_o=2L_o/V$. However, the traveler twin will have recorded an elapsed time of $2T_o\sqrt{1-V^2/c^2}$, and will be younger than his stay-at-home twin.[5] This result agrees with the experimental evidence for the lifetime dilatation of particles decaying in flight.[4]

However, this argument is not completely satisfactory because the relation for the lifetime dilatation involving *constant linear velocity* is completely relative and is symmetric (or reciprocal) between the twins or any two inertial frames. Thus, it cannot be used to conclude that the stay-at-home twin is *absolutely* younger than his traveler twin within the framework of special relativity.

To see the flaw in the preceding line of reasoning, let us consider the "twin-particle paradox" in detail within the framework of special relativity. Suppose an unstable pion π_1 is at rest in an inertial frame F and another pion π_2 is at rest in a second inertial frame F' which moves relative to F with a constant velocity V along the +x direction. Let us consider the pion π_1. Its mean lifetime is Δt_{10} as measured by observers in F and $\Delta t'_1$ as measured by observers in F'. These two time intervals are related by

$$\Delta t'_1 = \frac{\Delta t_{10}}{\sqrt{1-V^2/c^2}}, \qquad \Delta x=0, \tag{D.1}$$

because π_1 is at rest in F (i.e. $\Delta x=0$ in the Lorentz transformations (5.7)) and Δt_{10} is the "proper lifetime." Similarly, for the second pion π_2, its lifetimes as measured by observers in F and F' (i.e. Δt_2 and $\Delta t'_{20}$) satisfy the relation

$$\Delta t_2 = \frac{\Delta t'_{20}}{\sqrt{1-V^2/c^2}}, \qquad \Delta x'=0, \tag{D.2}$$

because π_2 is at rest in F' (i.e. $\Delta x'=0$). It should be stressed that the result (D.1) [or (D.2)] is a relationship between the lifetime of a single pion as measured by two different observers and thus is not yet related to the experimental result, in which a single observer compares the lifetimes of two different pions π_1 and π_2, one at rest and one in motion. For example, if F is the laboratory frame, experiments show that

$$\Delta t_2 = \frac{\Delta t_{10}}{\sqrt{1-V^2/c^2}}. \tag{D.3}$$

i.e., the lifetime of π_2 decaying in flight is longer than that of π_1, which is at rest in the laboratory frame F. It follows from relations (D.1)–(D.3) that

$$\Delta t_{10} = \Delta t'_{20}, \tag{D.4}$$

and

$$\Delta t'_1 = \Delta t_2. \tag{D.5}$$

Result (D.4) implies that the lifetime Δt_{10} of π_1 at rest in F as measured by observers at rest in F is the same as the lifetime $\Delta t'_{20}$ of π_2 at rest in F' as measured by observers at rest in F'. The physical reason for the equality in (D.4) is exactly the same as that for the equality of meter sticks in equation (5.12). This is completely in harmony with the equivalence of the two inertial frames F and F' in special relativity. *The two time intervals,* Δt_{10} *and* $\Delta t'_{20}$, *in (D.4) are not related by the Lorentz transformations.*

The results in (D.1), (D.2), (D.4) and (D.5) bring out the most puzzling aspect of relativity theory.

Logically, (D.4) is *directly* implied by the first principle of relativity. One should say that the relation (D.4) together with the relation (D.2) derived from the Lorentz transformations, leads to the experimental prediction (D.3). Now suppose the decay process, the clock ticking, and the aging process are the same. Then it could be argued that the result (D.4) suggests that the aging of the twin brothers in F(earth) and F'(rocket ship) are the same, when they are brought back together both have the same age, *provided that the time intervals of accelerations are negligible and the ages are measured according to the conditions related in (D.4).* For example, the twins may express their ages in terms of the mean lifetime of the particles decaying at rest relative to them. This appears to be the qualitative argument of Dingle.[6] It must be stressed that the equality in the relation (D.4), i.e., the twins have the "same age," cannot be observed by a single observer, until they are brought back together with negligible effects due to acceleration.

Clearly, the results (D.1) and (D.2) are just another way of saying that two observers in different frames comparing two time intervals will arrive at different conclusions depending on how the intervals are measured. This is one of the most basic traits of relativity theory. One cannot use the lifetime dilatation experiment, which gives a relationship between measurements made by a single observer, to rule out result (D.4) because *it refers to measurements made by two different observers.* Similarly, to design an experiment to rule out (D.4) and confirm (D.2) (or vice versa) is impossible. If one reflects for a moment, one can see that both (D.2) and (D.4) are correct for different conditions of measurement within the conceptual framework of special relativity. This is the so-called "paradox"—the heart of the problem which is the source of a long controversy.[6] So far, all experiments support the first postulate of relativity that two inertial frames F and F' are equivalent and symmetrical as long as their relative velocity is constant. However, in the final analysis, it can be asserted unequivocally that, logically, there is absolutely no paradox in relativity theory.

As a result of this analysis, it appears reasonable to conclude that (i) the relativity (or the reciprocal relation of the two particles' lifetimes) can only be broken by taking into account the acceleration[7] of one of the particles and (ii) the numerical difference between the two lifetimes must be determined by taking the effects of linear accelerations into account. All physicists appear to agree with the conclusion (i), but not (ii).[8] Thus, a direct experimental test of these different views is warranted.

Db. A Direct Experiment on the interpretations of the "Twin Paradox" by Using Twin Particles

Since special relativity has been tested by hundreds of experiments, one might think that there is no point in doing one more experiment to test it. However, this experiment involving constant-linear-acceleration is necessary. The reason is that it really does not test special relativity. Rather, it tests various interpretations of the "twin paradox." Furthermore, it tests the transformations for linearly accelerated frames and gives clues to the understanding of physics in non-inertial frames, as discussed in chapter 23.

An idealized experimental setup for testing the "paradox" of the twins by using identical particles is as follows:

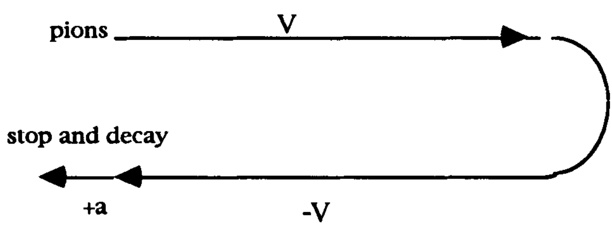

Fig. D.1 An idealized experimental arrangement to illustrate the test of the "twin paradox"

To be specific for comparisons, let us first consider an idealized trip (as shown in Fig. D.1). We have two twin brothers, T_1 and T_2, who were born at the same time in the earth laboratory and have the same life expectancy. (The following discussions hold also for two identical clocks with the same 'life expectancy.') Let us omit the initial acceleration and suppose T_1 (travelling twin) moves with a constant velocity V over a very long distance L_0. He then turns 180^o and travels with the same speed V over the same distance. Then twin T_1 decelerates and stops within a certain time interval. There are two ways to check whether he is absolutely younger: One is to measure his age when he returns and one is to measure his remaining lifetime, and to compare with his stay-at-home brother T_2.

Now suppose one replaces the two twin brothers by two "identical groups" of identical pions in an inertial laboratory. One has the following clear analogous experimental situation:

A bunch of identical pions are created in a high-energy laboratory. They are separated into two groups, denoted by $G(\pi_1)$ and $G(\pi_2)$, and both groups move with a constant velocity V. The first group $G(\pi_1)$ (representing the travelling twin) moves with a constant velocity V over a suitable distance L_0, as measured from the earth laboratory frame F. Then it undergoes an acceleration which reverses its velocity, so that it returns with a constant velocity $V' = -V$. After it travels a distance L_0, it is decelerated (by a field) within a certain time to zero velocity. When the group $G(\pi_1)$ stops, one counts the number of pions left in the group and deduces the mean lifetime $\tau(\pi_1)$.

On the other hand, the second group $G(\pi_2)$ is allowed to move with the constant velocity without any acceleration or deceleration. One can measure its mean lifetime decaying in flight with the velocity V. As discussed in section 11b, after the effect due to motion is taken into account, the result can lead to the "rest lifetime" $\tau_0(\pi_2)$ which is the same as the lifetime of the pions produced at rest in the laboratory. Therefore, $\tau_0(\pi_2)$ corresponds to the lifetime of the stay-at-home twin because these pions in the second group $G(\pi_2)$ are neither accelerated nor decelerated. With the help of $\tau_0(\pi_2)$, one can calculate the number of pions left in the $G(\pi_2)$, if it is produced at rest, at the time when the group $G(\pi_1)$ stops.

Note that, in this experiment, one compares the lifetimes of both groups, $G(\pi_1)$ and $G(\pi_2)$, when they are at rest in the earth laboratory, after the traveler group of pions has returned. This measurement is free from the reciprocal relations of the lifetime dilatation when they have constant relative motion.

One can perform experiments with different distance L_o and/or the acceleration. The result can test the conventional relation

$$\tau(\pi_1) - \tau_0(\pi_2) = \frac{2L_o}{V}(1 - \sqrt{1-V^2/c^2}) > 0 . \tag{D.6}$$

provided twin's lifetime (or clocks' time) and particles' mean lifetime have the same physical property.[9]

In order to test the "twin paradox", one should choose particles with suitable lifetimes, vary the distance L_o of the particle moving with a constant velocity and L_o should be sufficiently large, so that the difference in (D.6) can be detected. In view of these considerations, the muon with a longer lifetime ($c\tau \approx 6.6\times10^4$ cm) is more suitable than the pion ($c\tau \approx 780$ cm) for such an experiment.

Actually, it is not necessary for the angle between **V** and **V'** to be 180^o. All that is needed is for the traveler group to have the experience of acceleration. This property could simplify the experiment. A much more simplified version of the "twin paradox" experiment is to do just half of the round-trip as follows:

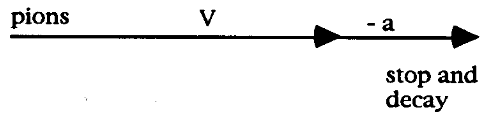

Fig. D.2 An idealized half-trip test of the "twin paradox" by using twin particles

The traveler twin can be represented the pions in Fig. D.2 because they travel a certain distance L_o and are decelerated by a field to stop and decay. In this case their age difference will be just half of that in (D.6):

$$\tau(\pi_1) - \tau_o(\pi_2) = \frac{L_o}{V}\left(1 - \sqrt{1-V^2/c^2}\right) > 0 . \qquad \text{(D.7)}$$

This is the simplest experiment to test the "twin paradox."[10]

The effect on lifetime due to the acceleration "a" of charged particles in a potential field should be investigated. The experimental results can also test another two views:

$$\text{"Naive view": } \tau(\pi_1) - \tau_o(\pi_2) = 0, \qquad \text{(D.8)}$$

$$\text{"Noninertial view": } \tau(\pi_1) - \tau_o(\pi_2) = f(L_o,V,a). \qquad \text{(D.9)}$$

The function $f(L_o,V,a)$ can be calculated if one has accelerated transformations, as discussed in chapter 23.

It must be stressed that this type of experiment tests only various interpretations of the theory of special relativity regarding the "twin paradox" or the "clock paradox," but not the theory itself. In other words, even if, say, (D.9) is confirmed by future experiments, this does not invalidate Einstein's theory of special relativity because it involves assumptions related to accelerations of reference frames.

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3. For example, Einstein (in 1916), Møller and Ta-You Wu, see ref. 8 below.
4. The time dilatation experiments have been carried out by using unstable particles in linear and in circular motions (e.g., muon experiment in a storage ring). Some authors use experimental results involving circular motion to support the conventional interpretation of the "twin paradox". This is not satisfactory because there is clearly no relativity (and, hence, no paradox) between a twin at rest and a twin in circular motion. (The reasonis that a frame in circular motion involves acceleration and cannot be described by special relativity, as discussed in chapter 25.) See, for example, D. Halliday and R. Resnick, *Fundamentals of Physics* (third edition, John Wiley, New York, 1988) pp. 960-961. Some physicists used the time dilatation in constant linear motion to support the conventional interpretation. See, for example, F. S. Crawford, Jr., Nature, 179, 35 (1957).
5. The crucial asymmetry of the twin's ages is usually explained as follows: The traveler twin feels an acceleration, whereas his twin on the earth does not. Logically, since the asymmetry of the twin is solely caused by the acceleration, one must calculate explicitly the asymmetry in their age due to accelerations. Most people did not do this, except Møller, Wu, and Lee, who derived a transformation for linearly accelerated frame and did the calculations (ref. 8).
6. There was a raging controversy concerning the "twin paradox" around 1958: Some people, e.g., H. Dingle, believed that the travelling twin will have the same age as his stay-at-home twin after he returns to the earth. (See Selected Reprints by American Institute of Physics in ref. 2.) If one takes the "twin-particle paradox" as an example, one could argue that the two particles have the same mean life (i.e., the relation in eq. (D.4)) when measured by two different observers at rest relative to each particle. However, this equality (D.4) cannot be shown by using the Lorentz

transformation. Rather, it follows directly from the first postulate of special relativity, namely, that all inertial frames are equivalent. In this sense, the result (D.4) is itself a basic postulate of special relativity and has been confirmed indirectly by many previous experiments which support relativity.

In this connection, we note that the "twin-particle paradox" differs from the "twin paradox" [or "clock paradox"] in two aspects: (i) There is no acceleration which reverses linear velocity involved in the measurement that was done in all previous experiment of lifetime dilatation involving LINEAR MOTION. (ii) the experimental asymmetry of their lifetimes [or the asymmetry of clocks' times in the Lorentz transformation] is completely due to the asymmetric conditions of measurements (i.e., $\Delta x=0$ or $\Delta x'=0$) and has nothing to do with the acceleration of particles or clocks for a certain period of time, as discussed in section 11b. If travelling twin's acceleration which reverses his velocity and the condition $\Delta x'=0$ can be shown to be equivalent, then the controversy would be clarified.

7. One might think that the previous experience of acceleration of one inertial frame F could break the symmetry between two inertial frames F and F'. If this were the case, then we would expect that the spectrum of a hydrogen atom (which has previously been accelerated by a_1) until at rest in a laboratory will be different from that of another hydrogen atom (which has a different previous acceleration $a_2 \neq a_1$) until at rest in the laboratory. But so far there seems to be no such difference being detected in any laboratory.

8. For a discussion of the "clock paradox" with the effect of acceleration taken into account (beyond the framework of special relativity) and calculated exactly on the basis of Møller's accelerated transformation, see Ta-You Wu and Y. C. Lee, Intern. J. Theor. Phys. 5, 307 (1972); Ta-You Wu, *Theoretical Physics*, vol. 4, *Theory of Relativity* (in Chinese, Lian Jing Publishing Co., Taipei, 1978) pp. 172–175; C. Møller, Dan. Mat. Fys. Medd. 20, No. 19 (1943), and *The Theory of Relativity* (Oxford, London, 1969) pp. 258–263. The correctness of the Wu-Lee's precise result can be tested experimentally by measuring a particle's mean decay lifetime under constant acceleration or deceleration. It is interesting to note that in his 1905 paper, Einstein first mentioned the following thought experiment: Suppose two synchronized clocks A and B are initially at the same position. Suppose clock A leaves and

moves along a closed path and returns. When clock A stops at the original position, it falls behind relative to clock B. However, Einstein changed his mind in 1916: He said that the logic of special relativity does not suffice for the explanation of this phenomenon since non-inertial frames are involved. See, for example, A. Pais, *Subtle is the Lord..., The Science and the Life of Albert Einstein* (Oxford Univ. Press, Oxford, 1982), p. 145. Today, at the dawn of the 21st century, most physicists believe the effects of accelerations can be neglected in the *quantitative* calculations in the "twin paradox" problem and follow Einstein's conclusion in 1905 rather than that in 1916. Such a difference can only be settled by experiments.

9. For the twins, one can measure their remaining lifetime τ(R, traveler) after the traveler twin returns. However, this cannot be done with the twin particles. When the traveler group $G(\pi_1)$ returns, one can only measure their mean lifetime, according to quantum mechanics and field theory.

10. Actually, this type of half-trip experiment has been carried out in many experiments in the early days to measure the mean lifetimes of particles decaying at rest. However, they were designed neither to test the "twin paradox" nor to detect effects due to acceleration, so that possible relevant effects were not explicitly investigated.

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