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Created, developed, and nurtured by Eric Weisstein at Wolfram Research

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Sister Celine's Method

A method for finding recurrence relations for hypergeometric polynomials directly from the series expansions of the polynomials. The method is effective and easily implemented, but usually slower than Zeilberger's algorithm. Given a sum $f(n) = \sum_k F(n, k)$, the method operates by finding a recurrence of the form

$$\sum_{i=0}^{I}\sum_{j=0}^{J} a_{i\,j}(n)\, F(n-j, k-i) = 0$$

by proceeding as follows (Petkovšek *et al.* 1996, p. 59):

1. Fix trial values of I and J.

2. Assume a recurrence formula of the above form where $a_{i\,j}(n)$ are to be solved for.

3. Divide each term of the assumed recurrence by $F(n, k)$ and reduce every ratio $F(n-j, k-i)/F(n, k)$ by simplifying the ratios of its constituent factorials so that only rational functions in n and k remain.

4. Put the resulting expression over a common denominator, then collect the numerator as a polynomial in k.

5. Solve the system of linear equations that results after setting the coefficients of each power of k in the numerator to 0 for the unknown coefficients $a_{i\,j}$.

6. If no solution results, start again with larger I or J.

Under suitable hypotheses, a "fundamental theorem" (Verbaten 1974, Wilf and Zeilberger 1992, Petkovšek *et al.* 1996) guarantees that this algorithm always succeeds for large enough I and J (which can be estimated in advance). The theorem also generalizes to multivariate sums and to q- and multi-q-sums (Wilf and Zeilberger 1992, Petkovšek *et al.* 1996).

SEE ALSO: Generalized Hypergeometric Function, Gosper's Algorithm, Hypergeometric Identity, Hypergeometric Series, Zeilberger's Algorithm

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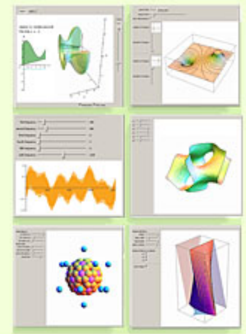

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