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Partons and Gravitation: Some Puzzling Questions

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The rather general belief in the theory of gravitation according to which neutral massive bodies with zero electric and magnetic moments are surrounded by a null electromagnetic field is analyzed from a critical viewpoint. Beginning the analysis at an atomic level, it is not difficult to see that neutral atoms are surrounded by a nonnull electromagnetic field generated by their peripheral electrons and by their nuclei even though their overall charge is zero. The data emerging from recent deep inelastic e–p scattering experiments clearly indicate that nucleons are composed by a number of charged constituents, often called partons, in a highly dynamical behavior.

Consequently, nucleons and nuclei can also be a rather relevant source of electromagnetic field, in view of the presumed large number of partons, which is produced not only by their overall charges, but more properly by the charges of their individual constituents. Summing up the contributions from a large number of atoms, the possibility of a seizable electromagnetic field surrounding any neutral massive body emerges. Three assumptions, termed standard, weak, and strong according to which the energy–momentum tensor of the electromagnetic field generated by the matter constituents does not contribute, or partially contribute, or is entirely responsible of the gravitational field, are introduced. In order to assess the physical relevance of each of the above assumptions, a simple bound state model of the π^0 particle is introduced in terms of two charged valence partons in a ^1S state. Some models of the electromagnetic field produced by the π^0 charged constituents are derived as a ground for further extension to the case of nucleons, nuclei, and entire atoms. The gravitational field equations for the π^0 particle according to the standard assumption are recalled and the ones according to the weak and strong assumptions are introduced. The puzzling implications of our analysis clearly cast shadows on the standard assumption, leaving as possible alternative for an exact formulation a selection between the weak and the strong assumptions. Some implications of the latter assumptions are discussed; the restrictions for the exterior case are derived using the framework of the "already unified theory"; some inconsistency with the gravitational wave theory is briefly discussed; and it is emphasized that the strong assumption implies a fully geometrical unification of gravitational and electromagnetic fields since the gravitational field is identified with a particular form, or "mutation," of the electromagnetic field originated primarily in the nuclear, but also in the atomic structure. The admissibility of both the weak and the strong assumptions on the basis of our present knowledge is discussed and the feasibility of some experiments aiming at the proper selection as well as the ultimate physical assessment of the new assumptions is briefly analyzed.

1. INTRODUCTION

The gravitational field equations, in their most widely accepted form, are given by

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}(R - 2\lambda) = \frac{8\pi G}{c^4}(M_{\mu\nu} + T_{\mu\nu}), \tag{1.1}$$

where $R_{\mu\nu}$ is the Ricci tensor, $R = R_{\mu\mu}$ is the curvature invariant, $g_{\mu\nu}$ is the metric tensor, λ is the cosmological constant, and G is the gravitational constant.

Equations (1.1) essentially relate the Einstein tensor $G_{\mu\nu}$ to the energy-momentum tensor of matter $M_{\mu\nu}$ and of the electromagnetic field $T_{\mu\nu}$.

This theory ultimately considers the superposition of two fields, the gravitational and the electromagnetic fields, assumed physically disjoint in origin and structure. To emphasize its hybrid character, the theory, which is known under the name of Einstein–Maxwell theory, is sometimes also called "la théorie provisoire."

A vast effort has been devoted to a unified theory of the gravitational and the electromagnetic fields. Table I summarizes some of the most relevant efforts [1, 2]. Nevertheless, either because of lack of consistency with the experiments or because of basic conceptual drawbacks, no theory has achieved to ultimate goal and the problem of the unified theory remains still open.

TABLE I

Unified Field Theories [1, 2]

Non-Riemannian theories [1]	Weyl (1919) Einstein (1923) Eddington (1921) Infeld (1928) Einstein (1929) Eyrand (1926) Einstein (1942) Schrödinger (1943)
Five-dimensional theories [1]	Koluza–Klein (1921) Jordan–Thiry (1945–1948) Einstein–Mayer (1931) Veblen–Hoffmann (1931) Shouten–Van Danzig (1932) Pauli (1933) Einstein–Bergmann–Bargmann (1941)
"Already unified" theories [2]	Rainich (1925) Misner–Wheeler (1957)

It is the purpose of the present paper to investigate whether the results of some recent experiments on the structure of the nucleon can contribute to the problem of the unified field theory or to a deeper understanding of the origin of the gravitational field.

Recent experiments on inelastic electron–proton scattering [3] have shown that the nucleon is composed by a number of charged constituents, usually called the Feynman "partons" [4], in a highly dynamic behavior.

Although a complete theory as well as a proper identification of the charged constituents is far from being accomplished, the data emerging from those experiments have sufficient relevance from a gravitational viewpoint.

Nucleons can be thought, collectively, as the primary source of the gravitational field of any massive body since they contribute to the almost totality of its mass. If one ignores electromagnetic waves, electrons, and nuclear binding energy, at a very naive approximation any massive body can be conceived as an aggregate of nucleons. Consequently, a deeper understanding of the gravitational field of the nucleon, or more generally of the strongly interacting particles, will contribute to our knowledge of the gravitational field of any macroscopic object.

At the present stage of our knowledge, the gravitational field of a nucleon, say the neutron, considered from sufficiently large distances as a classical particle, is represented by the equations (assuming $\lambda = 0$ for simplicity)

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} M_{\mu\nu} . \tag{1.2}$$

The energy–momentum tensor $M_{\mu\nu}$ can be written

$$M_{\mu\nu} = c^2 d_n u_\mu u_\nu , \tag{1.3}$$

u_μ being the four-velocity of the particle and d_n its mass density.

At a nonrelativistic limit with $g_{00} = -1 - (2/c^2)\,\phi$ the field equations (1.2) reduce to the Poisson equation

$$\Delta\phi = 4\pi G d_n \tag{1.4}$$

from which we recover the usual expression of the potential of the gravitational field of the neutron

$$\phi = -G/R \int d_n \, dV = -G(M/R), \tag{1.5}$$

where M is the gravitational mass of the particle.

Clearly, the introduction of the "number" M in Eq. (1.5) is an expression of our ignorance on the structure of the particle. At the same time, it is a satisfactory expedient to eliminate any reference to it in the theory.

The data emerging from the inelastic e–p experiments are sufficient to motivate investigations on a deeper approach to the problem.

Consider again the neutron. It is a neutral particle. Nevertheless, it is composed by a number of charged constituents in highly dynamic behavior, as schematically represented in Fig. 1. Even though the total charge of the particle is zero, the electromagnetic field outside the particle (as well as inside it) is not zero since each constituent contributes to it in accordance with its charge and velocity. Consequently, we can say that the dynamic behavior of the charged constituents of the neutron produce an electromagnetic field outside the particle which, according to the Einstein–Maxwell theory should ultimately contribute to its gravitational field.

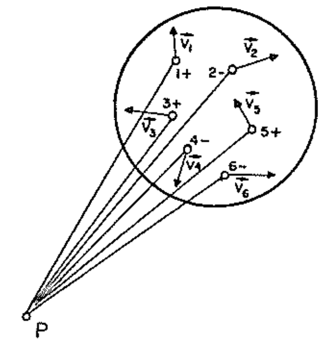

FIG. 1. A schematical picture of the neutron as a "gas" of charged partons in highly dynamica behavior. Even though the total charge is zero, the electromagnetic field at any point P outside the particle (as well inside it) will not be zero since each single constituent contributes to it in accordance with its charge and velocity.

The situation for the proton is conceptually equivalent even though the particle has an overall nonzero charge. A similar situation occurs for an entire atom on account of the highly dynamic behavior of the peripheral electrons.

Thus, the results of the recent inelastic e–p experiments, once extrapolated to a massive body according to the above approach, offer sufficient motivation to investigate one of the following assumptions.

WEAK ASSUMPTION. *The gravitational field of any massive body is* partially *due to the electromagnetic field of its charged basic constituents (partons and electrons).*

We shall write the field equations under the weak assumption

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}(R - 2\lambda) = \frac{8\pi G}{c^4} \left(T_{\mu\nu}^{\text{p.e.elm}} + M_{\mu\nu}^{\text{Res}}\right), \tag{1.6}$$

namely, the $M_{\mu\nu}$ tensor of Eq. (1.1) is now given by

$$M_{\mu\nu} = T^{\mathrm{p,e}}_{\mu\nu} + M^{\mathrm{Res}}_{\mu\nu}, \tag{1.7}$$

$T^{\mathrm{p,e,elm}}_{\mu\nu}$ being the energy–momentum tensor of the electromagnetic field generated by partons (from the nuclei of the body), electrons (from the peripheral electron clouds of the atoms) and source-free waves (if any), and $M^{\mathrm{Res}}_{\mu\nu}$ being the "residual" energy–momentum tensor of matter, namely the part of $M_{\mu\nu}$ which cannot be accounted for as of electromagnetic nature.

Viewed from an elementary particle viewpoint, the weak assumption may have one of its frameworks within parton models with high values of the rest mass (such as the quark–parton model), which cannot be interpreted as of entirely electromagnetic nature.

In the framework of the weak assumption, the gravitational field equations (1.2) of the neutron (in the absence of external electromagnetic fields) are replaced by

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} (T^{\mathrm{p}}_{\mu\nu} + M^{\mathrm{Res}}_{\mu\nu}), \tag{1.8}$$

where $T^{\mathrm{p}}_{\mu\nu}$ arises from the electromagnetic field of the partons and

$$M^{\mathrm{Res}}_{\mu\nu} = M_{\mu\nu} - T^{\mathrm{p}}_{\mu\nu}.$$

At a naive classical approximation, by knowing the charge of the partons, their number and their kinematical behavior inside the neutron, the term $T^{\mathrm{p}}_{\mu\nu}$ in (1.8) can be calculated, and then the knowledge of the residual term $M^{\mathrm{Res}}_{\mu\nu}$ should follow.

STRONG ASSUMPTION. *The gravitational field of any massive body is* entirely *due to the electromagnetic field of its basic constituents (partons and electrons).*

The field equations under the strong assumption then become

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}(R - 2\lambda) = \frac{8\pi G}{c^4} T^{\mathrm{p,e,elm}}_{\mu\nu}, \tag{1.9}$$

where $T^{\mathrm{p,e,elm}}_{\mu\nu}$ is the energy–momentum tensor of the electromagnetic field of partons and electrons, as well as of any sourcefree electromagnetic field, if any.

The strong assumption may have one of its frameworks within parton models with rest mass sensibly smaller than the nucleon mass and which can be accounted for as of primary electromagnetic nature, if one assumes that its strong (weak) interaction nature is not responsible for the phenomenology of the particle at large distances.

In the framework of the strong assumption the gravitational field equations (1.2) of the neutron become

$$R_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mathrm{p}}_{\mu\nu}, \tag{1.10}$$

namely

$$M_{\mu\nu} \equiv T^{\mathrm{p}}_{\mu\nu}, \tag{1.11}$$

where $T^{\mathrm{p}}_{\mu\nu}$ arises from the electromagnetic field of the partons, and all terms due to the mass disappear.

Again within the realm of a naive classical approximation, a sufficiently good knowledge of the charge of the constituents, their number and their kinematical behavior should allow the calculation of $T^{\mathrm{p}}_{\mu\nu}$.

Clearly, the strong assumption with corresponding field equation (1.9) represents a fully unified field theory, since the gravitational field is identified with a particular form (or "mutation") of the electromagnetic field originated in the nuclear as well as in the atomic structure.

The above two assumptions must be complemented by the underlying assumption of Eq. (1.1).

STANDARD ASSUMPTION. *The electromagnetic field generated by the basic charged constituents (partons and electrons) of any neutral massive body with zero electric and magnetic moments* does not *contribute to its gravitational field.*

The above assumption has its framework within theories in which the electromagnetic field produced by positively charged constituents cancels out with the field produced by the negatively charged constituents in the case of zero electric and magnetic moments in such a way that no measurable gravitational effect is produced outside neutral massive bodies.

This basic assumption is the implicit working ground of almost all gravitational theories. It is interesting to remark, however, that this assumption is questionable at an atomic level, even though it might be ultimately true at a subatomic level.

Consider in that respect an atom as a classical system composed of a positive charge at rest (the nucleus) and a cloud of orbiting negative charges (the peripheral electrons). Since opposite charges have nonequivalent dynamic behavior, a simple classical calculation shows a nonnull electromagnetic field outside the atom even though its total charge is zero. Consequently, there is no reason why such an electromagnetic field, once summed up over a large number of atoms, should not contribute to the gravitational field of a massive (neutral) body. This contribution is ultimately forbidden by the standard assumption, but is certainly in line with our weak or strong assumption.

The situation might be different at a subatomic level. Consider in that respect again the neutron as a classical particle at sufficiently large distances. Even though this particle is composed by charged constituents in highly dynamic behavior, at a first analysis the standard assumption might be true in this instance in view of the small size of the particle as well as the conceivable symmetry in the dynamic behavior of the positively and negatively charged constituents.

Nevertheless, as we shall show later on, even assuming a completely symmetric dynamic behavior between pairs of positively and negatively charged constituents, the overall electromagnetic field is not zero. Consequently, in view of the large number of charged constituents, nuclei might well be a relevant source of the electromagnetic field in addition to the field produced by the peripheral electron clouds.

The puzzling implication of the above approach clearly cast shadows on the standard assumption, leaving as possible alternative for an exact formulation a selection between the weak and the strong assumptions.

In the following sections we shall present a preliminary, mainly qualitative analysis of the weak and strong assumptions aiming at an evaluation of their physical relevance and consistency.

The major difficulty of our analysis lies on the divergent character of parton model theories, which are essentially quantum field theoretical in nature, and of gravitational theories, which are assumed classical in their most widely accepted form.

The lack at the moment of a fully established quantization of the theory of gravitation does not allow us a quantum field theoretical analysis of the electromagnetic field produced by the charged nucleon constituents.

This leads us, as a first evaluational step, into a "classical" approximation of a strongly interacting particle as a source of the gravitational field. We have, however, to emphasize that, even though this approximation refers only to the kinematical behavior of the charges of the basic constituents, and is mitigated by the fact that particles are considered from large distances, it ultimately remains rather questionable in nature.

Nevertheless, the standard, weak, and strong assumptions should hold at both classical and quantum mechanical levels. Consequently, the above approximation should not constitute a major drawback in a qualitative analysis of the problem.

2. The Parton Model

The deep inelastic e–p scattering experiments [3] have produced relevant information about the structure of the nucleon. It clearly emerges from those experiments that the nucleon is composed by a number of charged constituents (partons) in a highly dynamic state.

In order to reduce the complexity of the scattering analysis, the parton model puts particular emphasis on a very large (or infinite) momentum frame of Ref. [5], such as the e–p center of mass frame at high energies.

Indeed, when viewed from this frame of reference, the nucleon reduces to a thin disc on account of the Lorentz contraction and the proper motion of its constituents is substantially slowed down by time-dilation. Then the incident electron scatters incoherently from the individual partons.

Furthermore, if the interaction time is sensibly smaller than the lifetime of virtual states, the nucleon in the above frame of reference can be conceived as a "gas" of free partons.

The observed large value of the inelastic cross section is interpreted in terms of a point-like structure of the constituents since in this case there are no form factors to decrease the process at large momentum transfer.

The scattering, schematically represented in Fig. 2, occurs through the intermediate emission of a virtual photon which interacts with a free structurless parton, leaving all the other partons undisturbed. The cross section is then a sum over all single electron–parton interactions and can be written [5]

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4(\frac{1}{2}\theta)} [W_2 \cos^2(\tfrac{1}{2}\theta) + 2W_1 \sin^2(\tfrac{1}{2}\theta)], \tag{2.1}$$

where W_1 and W_2 are inelastic structure functions depending on the (laboratory frame) invariants

$$\nu = E - E' = q \cdot P/M, \qquad Q^2 = -q^2 = 4EE' \sin^2(\tfrac{1}{2}\theta), \tag{2.2}$$

where, according to the notation of Fig. 2, $q = k - k'$ is the momentum transfer of the electrons, P is the initial momentum of the proton, E and E' are the energies

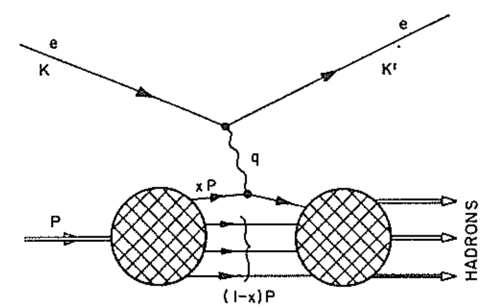

Fig. 2. The e–p deep inelastic scattering in the parton model.

of the incoming and outgoing electrons, θ is the scattering angle, and M is the proton mass.

Using the absorption cross sections σ_t and σ_l for transverse and longitudinal photons, for $\frac{1}{2}\theta \ll 1$ Eq. (II.1) can be written

$$\frac{d\sigma}{d\Omega\, dE'} \cong \frac{\alpha^2}{4E^2 \sin^4(\frac{1}{2}\theta)}\, W_2(q^2, \nu) \left[1 + \left(\frac{\sigma_t}{\sigma_t + \sigma_l}\right) \frac{\nu^2}{2EE'}\right]. \tag{2.3}$$

In the infinite momentum frame the transverse momentum of the ith parton can be ignored and its longitudinal momentum p_i can be considered to be a fraction x_i of the proton momentum; i.e., $\mathbf{p}_i = x_i \mathbf{P}$. Then the inelastic function W_2 can be written

$$\begin{aligned} W_2(q^2, \nu) &= \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N \int_0^1 dx\, f_N(x)\, \delta\left(\nu - \frac{Q^2}{2xM}\right) \\ &= \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N \frac{Q^2}{2M\nu^2} f_N\left(\frac{Q^2}{2M\nu}\right), \end{aligned} \tag{2.4}$$

where $P(N)$ is the probability of finding a configuration of N parton, Q_i is the charge of the ith parton, $\langle \sum_i Q_i^2 \rangle_N$ is an average in such configuration and $f_N(x)$ is the probability of finding in such configuration a parton with longitudinal fraction x of the proton momentum.

Above the resonance region and for $-q^2 > 1$ BeV^2 the inelastic structure functions W_1 and νW_2 scale [3] (namely, they become function of a single variable alone), a result theoretically anticipated by Bjorken.

Apparently,[1] the parton model is able to interpret this experimental behavior since from (2.4)

$$\nu W_2 = F(\omega), \tag{2.5}$$

with $\omega = Q^2/2M\nu$. This is one of the major achievements of the theory in its present formulation.

For more details on the parton model and related topic see, for instance, references [6] and quoted papers.

Even though the parton model has many appealing features, it is at the moment far from being well established primarily because of lack of ultimate identification of partons with physically known particles.

One of the first possibilities which has been investigated is to identify partons with quarks [5, 7]. This identification, however, has various conceptual drawbacks,

[1] It is unknown at the moment whether at energies much higher than the ones presently available the scaling behavior will still hold, or other effects will dominate, such as effects due to a fundamental length $L \ll 10^{-13}$ cm.

some of which are due to the still open question of the experimental detection of the quarks. Nevertheless, it cannot be ruled out by presently available data.

According to the original quark model, the nucleon is made up by three quarks. The prediction of this naive quark model, however, is in contradiction with experiments. To overcome this difficulty, Kuti and Weisskoff [7] have investigated a quark model in which the nucleon is made up of three "valence" quarks which determine its quantum numbers and a sea of quark–antiquark pairs (also called the "core") with the quantum numbers of the vacuum. Quantitative agreements with present experimental data is reached by adding uncharged "gluons."

The model predicts that the binding between quarks is weak compared to the momentum transfer of present experimental range. This implies the possibility of producing free quarks in deep inelastic e–p scatterings contrary to the present experimental evidence. Nevertheless, the model might be still valid if some presently unknown law forbids the quarks to leave the nucleon-bound state except in quark–antiquark pairs or in triplets.

In order to attempt an assessment of the present situation for the identification of the parton basic characteristics, we can introduce the following remarks:

(1) *Parton charge.* A relevant effort has been devoted to the identification of the parton charge [6]. Essentially, two types of models exist at the moment: model with fractionally charged constituents, such as the quark–parton models [5, 7], and models with integrally charged constituents, such as models with several fundamental triplets [8]. There is evidence according to which present inclusive experiments below the threshold of "charmed" particles cannot rule out some model with integrally charged partons [9].[2] In the present paper we shall assume that partons have integral charge even though our results can be extended to arbitrary charges.

(2) *Parton spin.* The Callan–Gross sum rule [10] allows a direct test for the spin of the partons. Present evidence on the small value of the longitudinal structure function seems to indicate that the majority of the partons have spin 1/2. Nevertheless, models with spin 0 parton are being considered, too, in which case the transverse structure function vanishes. The latter models are related to the pion cloud surrounding the nucleon (the π-partons) and are complemented with a σ-parton for contribution to neutrino reaction via the axial-vector current. It is unlikely, however, that there are more σ-partons than π-partons in the nucleon [11]. In the framework of our analysis we shall assume that partons have spin 1/2, although spin 0 models will also be considered for simplicity. Clearly, the nonzero

[2] At a quantum mechanical level, the fluctuations due to charge-exchange processes ask for a differentiation between instantaneous and average charge. The charge which is referred to in actual parton model calculation is the instantaneous charge, in view of the free configuration of partons at ∞-momentum transfer.

spin of the charged partons implies a further contribution to their electromagnetic field due to the presence of intrinsic magnetic moments.

(3) *Parton rest mass.* Two different types of theories exist for the parton mass: models for which the parton mass is of the order of (or higher than) the nucleon mass, such as for some quark–parton models, and models for which the parton mass is sensibly smaller than the nucleon mass. Clearly, the value of the parton mass has primary relevance from a gravitational viewpoint since it might ultimately affect, as pointed out in the introduction, the selection among the weak, strong, and the standard assumption. Gravitational models of the nucleon with parton mass substantially smaller than the nucleon mass should have a stronger electromagnetic field due to the higher dynamical behavior of the constituents, as well as smaller "residual terms" due to the smaller rest mass. For the purpose of carrying the strong assumption to its extreme consequences, we shall consider in the present paper parton models with the smallest allowable rest mass. Implications for larger values of the parton mass will be considered, too.

(4) *Number of constituents.* As pointed out earlier, the most promising parton models assume a fixed number of basic constituents (the valence partons) together with an undetermined number of parton–antiparton pairs. Those pairs can ultimately be interpreted as the quanta of the underlying strong field, and apparently they do not play a major role for the structure functions and for scaling. For our evaluational purposes, we shall consider only the valence partons. This is ultimately the basic step for our "classical approximation" of the kinematic behavior of the charges of the nucleon constituents, as indicated in the Introduction. It must be emphasized, however, that the sea of parton–antiparton pairs might produce a substantial contribution to the electromagnetic field surrounding the nucleon. Consequently, if the strong assumption acquires full physical relevance in the framework of the valence parton approximation, it should have an even deeper effectiveness in a complete theory.

(5) *Parton isospin and hypercharge.* Various models exist for the assignment of those further quantum numbers, such as models for which the basic fields carry a representation of SU (2) × SU (2) × Y [11] or SU (3) × SU (3). Those quantum numbers, apparently, have no bearing on the electromagnetic field at large distances produced by the nucleon constituents, and, consequently, we shall not indulge at the moment on their assignment.

As it clearly emerges from the above remarks, we are at the moment far from a well established dynamic model on the structure of the nucleon [12].

Because of the complexity of the nucleon structure, we are led toward the investigations on a strongly interacting particle with a simpler and more established structure.

In the framework of our analysis, the best alternative is clearly the π^0 meson. Indeed, the π^0 has an overall zero charge (which facilitates a gravitational analysis) and ultimately plays a major role in the nucleon structure itself [13]. Consequently, if we analyze the strong, weak, and standard assumptions in the framework of the π^0, our findings can be reasonably considered extendable to the nucleon [14].

The π^0 particle has a rather well-established structure as a bound state of a (valence) pair of parton–antiparton (plus a sea of the quanta of the underlying strong field).

If one assumes that the partons are (bare) charged particles of spin $\frac{1}{2}$, the parton-aniparton bound state in a ^1S state can clearly represent three basic features of the π^0, namely the overall zero charge, the spin zero and the negative space parity. In addition, the model is also able to represent the positive charge parity of π^0 and gives rise to a zero magnetic moment, a zero electric dipole moment (the state being stationary) and a zero quadrupole moment for the ground state (the total angular momentum being smaller than 1).

The remainder basic feature of the π^0, such as the rest energy (134.975 MeV), the mean life (0.83×10^{-16} sec) and the charge radius (< 1F), can be recovered in terms of a parton–antiparton bound state in the framework of both nonrelativistic and relativistic formulations [15].

Again, two types of theories are in principle possible for what concerns the rest energy of the constituents, namely theories with parton rest energy bigger or smaller than the π^0 rest energy.

The parton potential can be assumed to be a combination of the Coulomb potential with short range potentials of the type familiar in nuclear physics [16].

$$V(r)\, U(r) = -\frac{q^2}{r}\, U(r) + V_{sr}(r)\, U(r) + \int V(r', r)\, U(r')\, d\mathbf{r}, \qquad (2.6)$$

where q is the parton charge and $r = |\mathbf{r}|$. $V_{sr}(r)$ is a short range potential such as, for instance, the Hulthén potential

$$V_{sr}(r) = -V_0 \frac{e^{-br}}{1 - e^{-br}}, \qquad (2.7)$$

where b^{-1} is the range.

From the data on the rest energy, the mean life, and the charge radius of π^0, indications can be then derived on the lowest allowable rest energy of the partons. A simple model is discussed in Appendix A.

Two remarks are in order at this point. First of all, it must be emphasized that from the present experimental data no conclusion can be drawn on this key feature of the pion constituents, namely their rest energy. As for the nucleon, both theories with parton rest energy greater or smaller than the pion rest energy are in principle allowable and compatible with scattering data.

Secondly, both types of theories are at the moment equally affected by a serious conceptual difficulty, namely the proper identification of the constituents with physically known particles. Indeed, quarks are still a question mark at the moment, despite a rather long and costly search [17], and similarly, we do not know at the moment a particle with strong interacting behavior and rest energy smaller than the π^0 rest energy (for more details see Appendix A).

According to our opinion, an investigation from a gravitational viewpoint might throw some light on this issue which has basic relevance for the ultimate physical identification of the nucleon constituents.

3. On the Electromagnetic Field of π^0

From the previous section and Appendix A, we can schematically conceive the π^0 in the valence parton approximation as a ^1S bound state of two structurless constituents carrying the same rest energy, the same average kinetic energy, the same spin, the same magnetic moment, and opposite charges.

Let us consider at the moment some models for the electromagnetic field produced at large distances by the π^0 charged constituents in a purely classical frameworks and in a flat space.

Since the partons have the same average kinetic energy, the simplest classical approximation on the kinematic behavior of the charges of the constituents is the configuration in which both partons rotate in the same circular orbit of radius $R = b^{-1}$ at diametrically opposite positions with spin orientation perpendicular

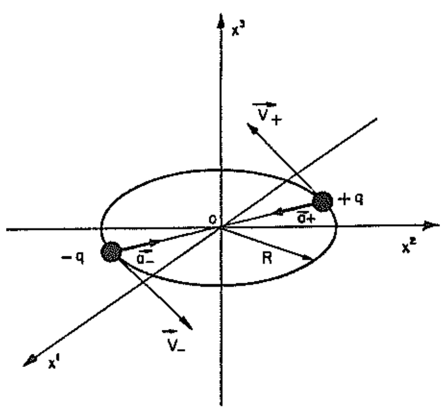

Fig. 3. A classical view of the kinematical behavior of the charges of the π^0 constituents in the approximation of two valence partons.

to the plane of the orbit (Fig. 3). Incidentally, this turns out to be one of the most symmetrical configurations in the kinematic behavior of the charges.

Let P be any point $x^\mu \equiv (x_0\,; \mathbf{x})$ of a Minkowski space with (space) origin at the symmetry center of the $(+q, -q)$ system with $|\mathbf{x}| \gg R$ and let the (x_1, x_2)-plane coincide with the plane of the orbit. We shall assume the metric $(-1; +1, +1, +1)$.

Consider the light Cone C with vertex at $P(x)$ as in Fig. 4. Let λ_n, $n = +(-)$ be the world line of the parton with $+q(-q)$ charge and let Q_{nm}, $n = +, -$, $m =$ Ret, Adv, be the four intersections of $\lambda \pm$ with C.

Four types of electromagnetic fields produced by the parton charges can be distinguished at $P(x)$ in correspondence with the four possible positions Q_{nm} of the charges.

Let

$$y^\mu_{nm} \equiv (y^0_{nm}\,; \mathbf{y}_{nm}); \qquad n = +, -; \qquad m = \text{Ret, Adv}, \tag{3.1}$$

be the four-vector position corresponding to Q_{nm}. Then the four-velocities of the partons can be represented by

$$v^\mu_{nm} = \frac{dy^\mu_{nm}}{d\tau} \equiv (\gamma c;\, \gamma \mathbf{v}_{nm})$$
$$\mathbf{v}_{+m} = -\mathbf{v}_{-m}\,; \qquad |\mathbf{v}_{nm}| = v = \omega R = \text{const.}; \qquad n = +, -; \qquad m = \text{Ret, Adv}. \tag{3.2}$$

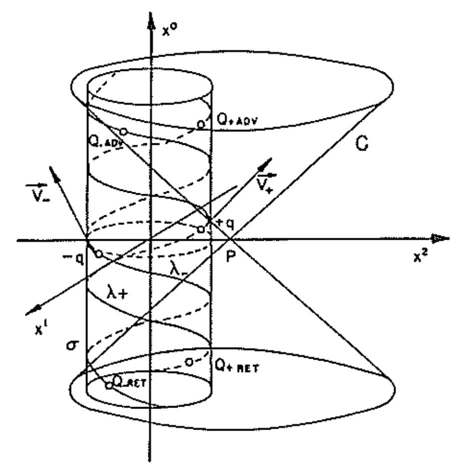

Fig. 4. Four types of electromagnetic fields produced by the charges of the π^0 parton model of Fig. 3 can be distinguished at any point P outside the system. They are determined by the intersections of the world lines $\lambda_\pm$ of the partons with the light cone C with vertex at P.

Similarly, the four accelerations can be written[3]

$$a^{\mu}_{nm} = \frac{dv^{\mu}_{nm}}{d\tau} \equiv \left[\frac{\gamma^4}{c}(\mathbf{v}\cdot\mathbf{a})_{nm}\,;\,\gamma^2\mathbf{a}_{nm} + \frac{\gamma^4}{c^2}\mathbf{v}_{nm}(\mathbf{v}\cdot\mathbf{a})_{nm}\right];$$
$$\mathbf{a}_{+m} = -\mathbf{a}_{-m}\,;\qquad |\,\mathbf{a}_{nm}\,| = a = \omega^2 R = \text{const.};\tag{3.3}$$
$$n = +, -;\qquad m = \text{Ret, Adv},$$

where $(\mathbf{v}\cdot\mathbf{a})_{nm} = 0$ for the system of Fig. 3.

The four-vector distance $D^{\alpha}_{nm} = x^{\alpha} - y^{\alpha}_{nm}$ from the point $P(x)$ to the charge position Q^{α}_{nm} can be written

$$D^{\alpha}_{nm} \equiv (\epsilon_m\,|\,\mathbf{D}_{nm}\,|;\,\mathbf{D}_{nm}) = (\epsilon_m D_n\,;\,\mathbf{D}_{nm});$$
$$D^{2}_{nm} = 0;\qquad |\,\mathbf{D}_{n\,\text{adv}}\,| = |\,\mathbf{D}_{n\,\text{ret}}\,| = D_n\,;\tag{3.4}$$
$$n = +, -;\qquad m = \text{Ret, Adv};$$

where

$$\epsilon_m = \begin{cases} +1 & \text{for } m = \text{Ret},\\ -1 & \text{for } m = \text{Adv}.\end{cases}\tag{3.5}$$

Let also introduce the notation

$$d_{nm} = D_{nm}\cdot v_{nm} = -\epsilon_m \gamma c D_n\left(1 - \epsilon_m\frac{\mathbf{D}_{nm}\cdot\mathbf{v}_{nm}}{D_n c}\right).\tag{3.6}$$

One of the most general expressions of the current density for a parton of charge q, mass m, and magnetic moment

$$\boldsymbol{\mu} = g\,\frac{q}{2mc}\,\mathbf{s}\tag{3.7}$$

is given by [18]

$$j^{\alpha}(x) = qc^2\int_{-\infty}^{+\infty} d\tau\,\Big\{v^{\alpha}(\tau)\left[1 + \frac{a^{\mu}(\tau)}{c^2}(x_{\mu} - y_{\mu}(\tau))\right]f_1[(x-y)^2]$$
$$- \omega^{\alpha\mu}(\tau)(x_{\mu} - y_{\mu}(\tau))\,f_2[(x-y)^2]\Big\}\,\delta[v\cdot(x-y)],\tag{3.8}$$

where f_1 and f_2 are form factors, and $\omega^{\alpha\mu}$ is a relativistic generalization of the intrinsic angular velocity $\boldsymbol{\alpha}$ arising from $\mathbf{S} = I\boldsymbol{\alpha}$, I being the moment of inertia of the particle.

[3] Recall that the usual relations for circular motion $V = \omega R$, $a = \omega \times \mathbf{V}$ are also valid in a relativistic framework. Then $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$ are constants in our scheme.

If we assume that partons are point-like, then this implies the usual substitution

$$f(D^2)\,\delta(d) \to \delta^4(D)/c\tag{3.9}$$

and the current density of one particle in the absence of magnetic moment ($\omega^{\mu\nu} = 0$) is given by the expression

$$j^{\alpha}(x) = qc\int_{-\infty}^{+\infty} v^{\alpha}(\tau)\,\delta[x - y(\tau)]\,d\tau.\tag{3.10}$$

The four-vector potential at $P(x)$ is

$$A_m{}^{\alpha}(x) = \frac{4\pi}{c}\int G_m(x - x')\,j^{\alpha}(x')\,d^4x'$$
$$= 4\pi q\int_{-\infty}^{+\infty} G_m(x - z)\,v^{\alpha}\,d\tau,\qquad m = \text{Ret, Adv},\tag{3.11}$$

where the Green functions are

$$G_m(x) = \frac{1}{4\pi r}\,\delta(r - \epsilon_m ct).\tag{3.12}$$

In this way we obtain the familiar form of the Liénard–Wiechert potential [19]

$$A_m{}^{\alpha}(x) = -q\,\frac{v_m{}^{\alpha}}{d_m}.\tag{3.13}$$

Under the above assumptions of point-like structure of the partons and absence of magnetic moments (spin zero), the most general form of the potential of the $(+q, -q)$-system at $P(x)$ is given by[4]

$${}_{q}A^{\mu}_{\pi^0}(x) = -q\sum_{nm}\epsilon_n\epsilon_m C_{nm}\frac{v^{\mu}_{nm}}{d_{nm}}$$
$$= -q\left\{\left[C_{+\text{Ret}}\frac{v^{\mu}_{+\text{Ret}}}{d_{+\text{Ret}}} - C_{+\text{Adv}}\frac{v^{\mu}_{+\text{Adv}}}{d_{+\text{Adv}}}\right]\right.$$
$$\left. - \left[C_{-\text{Ret}}\frac{v^{\mu}_{-\text{Ret}}}{d_{-\text{Ret}}} - C_{-\text{Adv}}\frac{v^{\mu}_{-\text{Adv}}}{d_{-\text{Adv}}}\right]\right\} = \sum_{nm} C_{nm}A^{\mu}_{nm}(x),\tag{3.14}$$

where

$$\epsilon_n = \begin{cases} -1 & \text{for positive charge},\\ +1 & \text{for negative charge};\end{cases}\tag{3.15}$$

[4] We shall assume no summation convention for the indices n, m.

C_{nm} are arbitrary constants satisfying the relations

$$C_{+\mathrm{Ret}} + C_{+\mathrm{Adv}} = 1, \qquad C_{-\mathrm{Ret}} + C_{-\mathrm{Adv}} = 1, \tag{3.16}$$

and

$$A^{\mu}_{nm}(x) = -q\epsilon_n\epsilon_m \frac{v^{\mu}_{nm}}{d_{nm}}. \tag{3.17}$$

The electromagnetic field at $P(x)$ originating from (3.14) is given by

$$_qF^{\alpha\beta}(x) = \sum_{nm} C_{nm}\,{}_qF^{\alpha\beta}_{nm}(x), \tag{3.18}$$

with

$$_qF^{\alpha\beta}_{nm}(x) = q\epsilon_n\epsilon_m \left\{ \frac{c^2[D^\alpha, v^\beta]_{nm}}{d^3_{nm}} - \frac{[D^\alpha, a^\beta]_{nm}}{d^2_{nm}} + \frac{D_{nm}\cdot a_{nm}}{d^3_{nm}} [D^\alpha, v^\beta]_{nm} \right\}, \tag{3.19}$$

where we have used the notation

$$[D^\alpha, v^\beta]_{nm} = D^\alpha_{nm} v^\beta_{nm} - D^\beta_{nm} v^\alpha_{nm}. \tag{3.20}$$

Let us identify the $1/D^2$ and $1/D$ terms in (3.19) as follows

$$_qF^{\alpha\beta}_{nm,1/D^2}(x) = q\frac{c^2\epsilon_n\epsilon_m}{d^3_{nm}} [D^\alpha, v^\beta]_{nm}, \tag{3.21}$$

$$_qF^{\alpha\beta}_{nm,1/D}(x) = q\epsilon_n\epsilon_m \left\{ \frac{D_{nm}\cdot a_{nm}}{d^3_{nm}} [D^\alpha, v^\beta]_{nm} - \frac{1}{d^2_{nm}} [D^\alpha, a^\beta]_{nm} \right\}. \tag{3.22}$$

The $(+q, -q)$ quantum mechanical bound system is in its ground state and consequently do not lose energy through radiation.

Our problem is to reach a classical representation of such a system where the zero-component of the four-momentum of the overall electromagnetic field of the isolated $(+q, -q)$-system

$$P^\mu_{\pi^0} = 1/c \int T^{\mu\alpha}_{\pi^0}\, d\sigma_\alpha, \tag{3.23}$$

is conserved, i.e.,

$$\frac{dP^0_{\pi^0}}{d\tau} = 0. \tag{3.24}$$

In (3.23) $T^{\alpha\beta}_{\pi^0}$ is the energy–momentum tensor of the electromagnetic field defined, as usual, by

$$T^{\alpha\beta}_{\pi^0} = \frac{1}{4\pi}\left(F^{\alpha\mu}_{\pi^0} F^{\beta}_{\pi^0\mu} + \tfrac{1}{4} g^{\alpha\beta} F^{\rho\sigma}_{\pi^0} F_{\pi^0\rho\sigma}\right). \tag{3.25}$$

To satisfy restriction (3.24), we can first consider the "complete absorber" assumption; namely, we can assume that each parton in the π^0 system absorbs all the radiation emitted by the other parton.

This essentially implies that at large distances only the velocity field (3.21) is present, since the entire radiation field (3.22) is trapped into the π^0 system.

We can, thus, introduce the first two models of the electro-magnetic field of the π^0 system (in the absence of the parton magnetic moments) as follows

$$_{1q}F^{\alpha\beta}_{\pi^0} = qc^2 \sum_n \frac{\epsilon_n}{d^3_{n\mathrm{Ret}}} [D^\alpha, v^\beta]_{n\mathrm{Ret}}, \tag{3.26}$$

$$_{2q}F^{\alpha\beta}_{\pi^0} = qc^2 \sum_{nm} \frac{\epsilon_n\epsilon_m C_{nm}}{d^3_{nm}} [D^\alpha, v^\beta]_{nm}, \tag{3.27}$$

where in conformity with (3.16) one can assume, for instance, $C_{nm} = \frac{1}{2}$ by giving equal weight to the advanced and retarded potentials.

For more details on the above fields see Appendix B.

If one wants to preserve the radiation field at large distance, then there is only one expression for the electromagnetic field of the $(+q, -q)$-system satisfying restriction (3.24), given by the whole field (3.18) with $C_{nm} = \frac{1}{2}$ which we rewrite as follows

$$_{3q}F^{\alpha\beta}_{\pi^0} = \tfrac{1}{2} \sum_{nm} \epsilon_m\, {}_qF^{\alpha\beta}_{nm} \tag{3.28}$$

The proof that the above field has zero energy rate of radiation is given in Appendix C.

We shall not enter at the moment on a critical analysis of the above models in relation to causality. Clearly, if the preservation of causality is demanded in the strict sense, together with the restriction of zero energy loss for radiation, then the only admissible model is (3.26).

The energy–momentum tensor of model (3.26), using (3.25), is given by

$$\begin{aligned} _{1q}T^{\alpha\beta}_{\pi^0} = \frac{q^2c^4}{4\pi} \sum_{nn'} \Big\{ & \frac{1}{d_n^6} [c^2 D_n^\alpha D_n^\beta + (D\cdot v)_n \{D^\alpha, v^\beta\}_n \\ & - \frac{(1-\delta_{nn'})}{d_n^3 d^3_{n'}} [(D_n\cdot v_{n'})\{D^\alpha_{n'}, v_n^\beta\} - \tfrac{1}{2}(v_n\cdot v_{n'})\{D_n^\alpha, D^\beta_{n'}\} \\ & - \tfrac{1}{2}(D_n\cdot D_{n'})\{v_n^\alpha, v^\beta_{n'}\}] - \tfrac{1}{2} g^{\alpha\beta} \frac{(D_n\cdot v_n)^2}{d_n^6} \\ & - g^{\alpha\beta} \frac{(1-\delta_{nn'})}{d_n^3 d^3_{n'}} [(D_n\cdot D_{n'})(v_n\cdot v_{n'}) - (D_n\cdot v_{n'})(D_{n'}\cdot v_n)] \Big\}, \end{aligned} \tag{3.29}$$

where

$$\{A^\alpha, B^\beta\} = A^\alpha B^\beta + A^\beta B^\alpha. \tag{3.30}$$

In the same way, the energy–momentum tensors for models (3.27) and (3.28) can be obtained. For an approximate expression of (3.29) see Appendix B.

Let us now extend models (3.26), (3.27), and (3.28) to the case of partons with nonzero spin, namely to include in the electromagnetic field the contribution from the magnetic moments of the constituents.

In general, the current density of a spinning charge is made up from three contributions: 1) The motion of the charge center; 2) the Thomas precession of the particle rest frame R relative to the lab frame; and 3) The rotation of the charge within R.

Expression (3.8) for the current density satisfies the above requirements. Indeed, the δ-function in (3.8) allows the use of the expression [20]

$$v^\alpha a \cdot (x - y)/c^2 = -\eta^{\alpha\mu}(x_\mu - y_\mu), \tag{3.31}$$

with

$$\eta^{\alpha\mu} = -(1/c^2)[v^\alpha, a^\mu]. \tag{3.32}$$

Then, Eq (3.8) can be rewritten

$$j^\alpha(x) = qc^2 \int_{-\infty}^{+\infty} d\tau \, \{[v^\alpha - \eta^{\alpha\mu}(x_\mu - y_\mu)] f_1[(x-y)^2] - \omega^{\alpha\mu}(x_\mu - y_\mu) f_2[(x-y)^2]\} \, \delta[v \cdot (x - y)], \tag{3.33}$$

where $\eta^{\alpha\mu}$ is a relativistic generalization of the angular velocity of the Thomas precision

$$\eta_T = -\frac{1}{c^2} \frac{\gamma^2}{\gamma^2 + 1} \mathbf{v} \times \mathbf{a}, \tag{3.34}$$

and $\omega^{\alpha\mu}$ is a relativistic generalization of the intrinsic rotational motion of the charge as in the expression

$$\mu^{\alpha\beta} = g \frac{q}{2mc} S^{\alpha\beta} = g \frac{qI}{2mc} \omega^{\alpha\beta}. \tag{3.35}$$

The considered models (3.26), (3.27), and (3.28) arise only from the first term of the current density (3.33), namely they represent only the contribution from the motion of the charge centers. The desired supplementary expression for the electromagnetic field should arise from the terms in $\eta^{\alpha\beta}$ and $\omega^{\alpha\beta}$ in (3.33).

Let us recall that a dipole moment can be represented by the antisymmetric tensor $\mu^{\alpha\beta}$ defined in terms of a magnetic moment $\mathbf{m}$ and an electric moment $\mathbf{e}$ as follows

$$\mu_{ij} = \epsilon_{ijk} m^k; \qquad \mu^{0k} = e^k, \qquad i, j, k = 1, 2, 3. \tag{3.36}$$

If we have a magnetic moment at rest, then $\mathbf{m} = \boldsymbol{\mu}$ and no electric moment appears. If we have a magnetic moment in motion, then the $\mu^{\alpha\beta}$ tensor can be calculated from the corresponding value at rest and generally an electric moment appears.

In our case the term $\eta^{\alpha\beta}$ from the Thomas precession produces an electric dipole moment in the rest frame R of the parton, proportional to the acceleration, and the motion of the particle contributes to both its magnetic and electric moments according to a moment tensor $\mu^{\alpha\beta}$ which, in terms of (3.36) is characterized by [18]

$$\mathbf{m} = \mu_0[\boldsymbol{\alpha} - \tfrac{1}{2}(\gamma^3/c^2)\, \mathbf{v} \times \mathbf{a}], \tag{3.37}$$

$$\mathbf{e} = \mu_0[(1/c)\, \mathbf{v} \times \boldsymbol{\alpha} + \tfrac{1}{2}(\gamma^3/c)\, \mathbf{a}], \tag{3.38}$$

where

$$\mu_0 = g \frac{qI}{2mc} \tag{3.39}$$

and $\mu_0 \boldsymbol{\alpha} = \boldsymbol{\mu}$.

If we assume a point charge q as well as a point dipole moment $\mu^{\alpha\beta}$, then with the substitution (3.9) and in the absence of pair creation, the current density can be written

$$j^\alpha(x) = qc \int_{-\infty}^{+\infty} v^\alpha(\tau) \, \delta[x - y(\tau)] \, d\tau + c^2 \frac{\partial}{\partial x^\beta} \int_{-\infty}^{+\infty} \mu^{\alpha\beta}(\tau) \, \delta[x - y(\tau)] \, d\tau; \tag{3.40}$$

the four-vector potential at $P(x)$ is then

$$A_m{}^\alpha(x) = -q \frac{v_m{}^\alpha}{d_m} - \frac{c}{d_m} \left[\frac{d}{d\tau} \frac{\mu^{\alpha\beta} D_\beta}{d}\right]_{\tau=\tau_m} \tag{3.41}$$

and can be written

$$A_m{}^\alpha(x) = {}_qA_m{}^\alpha(x) + {}_\mu A_m{}^\alpha(x), \tag{3.42}$$

where

$${}_qA_m{}^\alpha(x) = -q \frac{v_m{}^\alpha}{d_m} \tag{3.43}$$

$${}_\mu A_m{}^\alpha(x) = \left[c^3 \frac{\mu^{\alpha\rho} D_\rho}{d^3} + c(D \cdot a) \frac{\mu^{\alpha\rho} D_\rho}{d^3} - c \frac{\dot{\mu}^{\alpha\rho} D_\rho}{d^2}\right]_{\tau=\tau_m}. \tag{3.44}$$

In deriving the above formula we took into account the relation

$$\mu^{\alpha\rho} v_\rho = 0, \tag{3.45}$$

which represents the vanishing of the electric dipole moment in the rest system of the particle.

Since the term (3.43) is identical to (3.13), the presence of a magnetic moment produces the term (3.44) in the potential of the field.

Dropping the subscript m, the electromagnetic field produced by potential (3.44) is composed of terms of various order in $1/D$ which we write

$$ {}_\mu F^{\alpha\beta} = {}_\mu F^{\alpha\beta}_{1/D^3} + {}_\mu F^{\alpha\beta}_{1/D^2} + {}_\mu F^{\alpha\beta}_{1/D} \tag{3.46}$$

where[5]

$$ {}_\mu F^{\alpha\beta}_{1/D^3} = \frac{2c^3}{d^3}\,\mu^{\alpha\beta} - \frac{3c^5}{d^5}\,(\mu^{\alpha\rho}D^\beta - \mu^{\beta\rho}D^\alpha)\,D_\rho\,; \tag{3.47}$$

$$\begin{aligned} {}_\mu F^{\alpha\beta}_{1/D^2} = &-\frac{2c}{d^2}\,\dot\mu^{\alpha\beta} + \frac{2c}{d^3}\,(D\cdot a)\,\mu^{\alpha\beta} - \left(\frac{6c^3}{d^4} + \frac{6c^3}{d^5}\,(D\cdot a)\right)(\mu^{\alpha\rho}D^\beta - \mu^{\beta\rho}D^\alpha)\,D_\rho \\ &+ \frac{3c^3}{d^4}\,(\dot\mu^{\alpha\rho}D^\beta - \dot\mu^{\beta\rho}D^\alpha)\,D_\rho + \frac{c}{d^3}\,(\mu^{\alpha\rho}a^\beta - \mu^{\beta\rho}a^\alpha)\,D_\rho \\ &+ \frac{c}{d^3}\,(\dot\mu^{\alpha\rho}D^\beta - \dot\mu^{\beta\rho}D^\alpha)\,v_\rho + \frac{2c}{d^3}\,(\dot\mu^{\alpha\rho}v^\beta - \dot\mu^{\beta\rho}v^\alpha)\,D_\rho\,, \end{aligned} \tag{3.48}$$

$$\begin{aligned} {}_\mu F^{\alpha\beta}_{1/D} = &\left[\frac{c(a\cdot D)}{d^4} - \frac{3c(D\cdot a)^2}{d^5}\right](\mu^{\alpha\rho}D^\beta - \mu^{\beta\rho}D^\alpha)\,D_\rho \\ &+ \frac{3c(D\cdot a)}{d^4}\,(\dot\mu^{\alpha\rho}D^\beta - \dot\mu^{\beta\rho}D^\alpha)\,D_\rho - \frac{c}{d^3}\,(\ddot\mu^{\alpha\rho}D^\beta - \ddot\mu^{\beta\rho}D^\alpha)\,D_\rho\,. \end{aligned} \tag{3.49}$$

In our π^0 model we have two magnetic moments of the same magnitude as represented in Fig. 5. Under the above assumptions of point dipole and no pair creation, the most general form of the contribution to the potential of the $(+q, -q)$-system due to the magnetic moments $\pm\mu$ of the constituents is given by

$$ {}_\mu A^\alpha_{\pi^0}(x) = \sum_{nm} \epsilon_m C_{nm}\,{}_\mu A^\alpha_{nm}(x), \tag{3.50}$$

where ${}_\mu A^\alpha_{nm}(x)$ is given by (3.44), ϵ_m is defined by (3.5); C_{nm} satisfies relations (3.16) and the index $n = +, -$ now refers to $\pm\mu$.

For an explicit form of the electromagnetic field we must again satisfy restriction (3.24) on zero energy rate of radiation.

The "complete absorber" assumption leads to the lack of terms in $1/D$ outside the $(+q, -q)$ system. Ignoring at large distance the terms in $1/D^3$, we obtain the

[5] Let us recall that for the field (3.19) produced by the motion of the charge centers the acceleration appears only in the $1/D$ terms. This is no longer the case for the field (3.46) produced by the motion of the magnetic moments since the acceleration appears in the same definition of the dipole tensor $\mu^{\alpha\beta}$ through (3.37) and (3.38). The radiation energy rate, however, in the latter case is a function only of the $1/D$ terms (3.49), while the $1/D^2$ terms (3.48) behaves as true Coulomb terms.

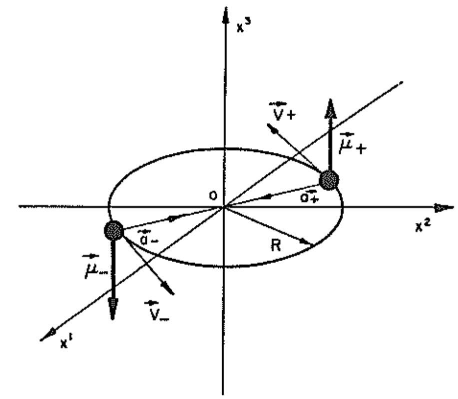

FIG. 5. A classical view of the kinematical behavior of the magnetic moments of the π^0 constituents in the approximation of two valence partons. The view is a complement of Fig. 3 on the behavior of the parton charges.

following two models for the electromagnetic field of the $(+q, -q)$-system due to the magnetic moments of the constituents

$$ {}_{1\mu}F^{\alpha\beta}_{\pi^0}(x) = \sum_n {}_\mu F^{\alpha\beta}_{n\mathrm{Ret},1/D^2}(x), \tag{3.51}$$

$$ {}_{2\mu}F^{\alpha\beta}_{\pi^0}(x) = \sum_{nm} \epsilon_m C_{nm}\,{}_\mu F^{\alpha\beta}_{nm,1/D^2}(x), \tag{3.52}$$

where $\mu F^{\alpha\beta}_{nm,1/D^2}(x)$ is defined by (3.48). For more details see Appendix B.

A third model can be introduced as for model (3.28) by considering a superposition of the full fields in their retarded and advanced form with equal weights $C_{nm} = \frac{1}{2}$

$$ {}_{3\mu}F^{\alpha\beta}_{\pi^0}(x) = \tfrac{1}{2}\sum_{nm} \epsilon_m\,{}_\mu F^{\alpha\beta}_{nm}(x), \tag{3.53}$$

where ${}_\mu F^{\alpha\beta}_{nm}(x)$ is now given by (3.46). As the reader can verify with lengthly but simple calculations, the above model satisfies restriction (3.24).

By considering the contributions from both the point charges and the point magnetic moments, our models for the full electromagnetic field of the π^0 are given by

$$ {}_kF^{\alpha\beta}_{\pi^0}(x) = {}_{kq}F^{\alpha\beta}_{\pi^0}(x) + {}_{k\mu}F^{\alpha\beta}_{\pi^0}(x), \qquad k = 1, 2, 3. \tag{3.54}$$

The corresponding energy momentum tensors ${}_kT^{\alpha\beta}_{\pi^0}$, $k = 1, 2, 3$, can be then calculated using (3.25).

We must emphasize that out of the three models (3.54) only the model for $k = 3$

is compatible with the classical Maxwell equations in view of the "complete absorber" assumption used for models $k = 1, 2$ which implies the dissappearance of the $1/D$ terms in the field.

Various generalizations of the above models can be introduced.

First of all, the models can be generalized to a number of partons $N > 2$ through expressions of the type

$$ {}_kF_N^{\alpha\beta} = \sum_{1j}^{N} {}_kF_j^{\alpha\beta}, \tag{3.55}$$

where ${}_kF_j^{\alpha\beta}$ is the field of the jth parton and $k = 1, 2, 3$ represents the type of considered model. This extension has conceptual relevance for the construction of models of the electromagnetic field of π^0 with more than two valence partons, or of $\pi^\pm$. Similarly, formula (3.55) can be used to construct models for nucleons as well as for nuclei, up to the construction of models for an entire atom, if one assumes that N represents the total number of both peripheral electrons and nuclear valence partons.

A second meaningful generalization can be introduced by extending the field to a curved space formulation [21].

Similarly, generalizations of the above models to elliptical orbits or to arbitrary orbits within a sphere of radius R can be done, but they are of no conceptual relevance at the moment.

Furthermore, the models can be generalized to the case of nontrivial form factors in the current (3.8) or (3.33). Nevertheless, this would contradict the present belief that partons have a point-like structure.

Finally, let us recall that the proper framework for a rigorous calculation of the parton electromagnetic field is within a fully quantized theory.[6] In principle, this might have some relevance for the same problem of a quantized theory of gravitation.

4. On the Gravitational Field of π^0

According to the present stage of our knowledge, the gravitational field of π^0 can be represented by the equations (for $\lambda = 0$)[7]

$$R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} M_{\alpha\beta}^{\pi^0}, \tag{4.1}$$

[6] Let us note that from the viewpoint of quantum field theory, the $(+q, -q)$ system has many similarities with the positronium. Indeed, apart some intrinsic differences related to size, mean life, and short range phenomenology, both systems have the same type of electromagnetic field at large distances.

[7] We shall denote partial differentiation with commas and covariant differentiation with semicolons. $g_{\alpha\beta}$ shall now denote the metric tensor of a Riemannian space.

where $M_{\pi^0}^{\mu\nu} = c^2 d_{\pi^0} u^\mu u^\nu$, u^α is the four-velocity of the particle and d_{π^0} its mass density.

In the nonrelativistic limit with $g_{00} = -1 - 2\phi/c^2\,\pi^0$, we have $M_{0\pi^0}^0 = -d_{\pi^0}c^2$, $R_0{}^0 = -(4\pi G/c^2)\, d_{\pi^0}$ and using known expressions for R_0, Eqs. (4.1) reduce to the Poisson equation

$$\Delta\phi_{\pi^0} = 4\pi G d_{\pi^0}. \tag{4.2}$$

The potential ϕ_{π^0} of the gravitational field is then

$$\phi_{\pi^0} = -\frac{G}{D}\int d_{\pi^0}\, dV = -G\,\frac{m_{\pi^0}}{D}, \tag{4.3}$$

where m_{π^0} is the gravitational mass of π^0.

If we have a test particle of mass m within the field of π^0, then the Newton law is given by

$$F = m\,\frac{\partial\phi_{\pi^0}}{\partial D} = -G\,\frac{m m_{\pi^0}}{D^2}. \tag{4.4}$$

The above formulation belongs to the framework of what we have termed the standard assumption. Namely, it holds under the assumption that the electromagnetic field produced by the charged π^0 constituents does not contribute to its gravitational field.

In the previous section we have introduced some models for the energy–momentum tensor $T_{\pi^0}^{\alpha\beta}$ of the electromagnetic field produced by the π^0 constituents. According to Einstein's theory of gravitation, the energy–momentum tensor of any electromagnetic field acts as a source of the gravitational field.

As a consequence of that, we are led toward the investigations of two possible alternatives which we have termed the weak and the strong assumption, referring as to whether the tensor $T_{\pi^0}^{\alpha\beta}$ is partially or entirely responsible for the gravitational field of the particle.

Consider the structure model of π^0 according to Fig. 4. Let σ be the four-tube which surrounds the world lines of the charged constituents as well as their current and mass densities. With respect to σ we can differentiate two regions: the interior region D_i and the exterior region D_e.

Since the π^0 particle is fully contained in the interior region D_i, in the exterior region D_e only the electromagnetic field (3.54) is present with energy–momentum tensor satisfying the Maxwell equations

$$T_{\pi^0;\beta}^{\alpha\beta} = 0, \tag{4.5}$$

$$T_{\pi^0\alpha\beta;j} + T_{\pi^0\beta j;\alpha} + T_{\pi^0 j\alpha;\beta} = 0, \tag{4.6}$$

the four-vector current of the field being zero outside σ.

The equations for the gravitational field of π^0 in the exterior region D_e for both the weak and the strong assumptions are given by (for $\lambda = 0$)

$$G_{\alpha\beta} = R_{\alpha\beta} = \frac{8\pi G}{c^4} T^{\pi^0}_{\alpha\beta}, \tag{4.7}$$

since $R = 0$ in view of the traceless character of $T^{\alpha\beta}_{\pi^0}$. The Ricci identities

$$G^{\alpha\beta}{}_{;\beta} = 0 \tag{4.8}$$

are then identically satisfied in view of (4.5). Taking into account the way according to which the field (3.54) was constructed, Eq. (4.8) or (4.5) ensure the conservation of energy and momentum through usual procedures.

Equations (4.7) should be compared with the equations for the same region D_e for the standard assumption

$$G_{\alpha\beta} = 0. \tag{4.9}$$

Since the gravitational field of π^0 is certainly weak, the usual linear approximation should constitute a good approximation for our case.

Assume that the metric tensor has the form

$$g_{\alpha\beta} = g^0_{\alpha\beta} + h_{\alpha\beta}, \tag{4.10}$$

where $g^0_{\alpha\beta}$ is the metric tensor of the (flat) Minkowski space and $h_{\alpha\beta}$ are small quantities. Then the field equations (4.7) reduce to

$$\Box h_{\alpha\beta} = \frac{16\pi G}{c^4} T^{\pi^0}_{\alpha\beta}, \tag{4.11}$$

with solutions given by

$$h_{\alpha\beta}(\mathbf{x}, t) = -\frac{4G}{c^4} \int \frac{T^{\pi^0}_{\alpha\beta}(\mathbf{x}', t - r/c)}{r} d\mathbf{x}', \tag{4.12}$$

where $r = |\mathbf{x} - \mathbf{x}'|$.

With a further approximation, if we neglect the terms with time derivatives, Eqs. (4.11) reduce to

$$\Delta h_{00} = \frac{16\pi G}{c^4} T^{\pi^0}_{00}, \qquad \Delta h_{ij} = \Delta h_{i0} = 0, \tag{4.13}$$

then

$$i, j = 1, 2, 3,$$

$$h_{00} = -\frac{4G}{c^4} \int \frac{T^{\pi^0}_{00}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}. \tag{4.14}$$

By putting $h_0{}^0 = (4/c^2)\,\phi_{\pi^0}$ we finally have

$$\phi_{\pi^0} \cong -\frac{G}{Dc^2} \int T^{\pi^0 0}{}_0\, dV = -G\frac{\overline{m}_{\pi^0}}{D}, \tag{4.15}$$

with

$$\overline{m}_{\pi^0} = \frac{1}{c^2} \int T^{\pi^0 0}{}_0\, dV. \tag{4.16}$$

Taking into account the asymptotic conditions, the line element becomes

$$ds^2 = \left(1 + \frac{4}{c^2}\phi\right) c^2\, dt^2 - d\mathbf{x}^2. \tag{4.17}$$

Let us recall that the equations corresponding to (4.11) for the standard assumption are given by

$$\Box h_{\alpha\beta} = 0, \tag{4.18}$$

which is the well known basis for the gravitational wave theory.

Let us consider now the interior problem in the framework of the weak assumption. This assumption ultimately implies that the π^0 charged constituents have a rest mass m_p which cannot be accounted for as of electromagnetic nature, and, consequently, it contributes to the gravitational field of the particle.

In this case a simple model for the energy–momentum tensor is given by

$$M^{\pi^0,\text{weak}}_{\alpha\beta} = T^{\pi^0}_{\alpha\beta} + M^{\pi^0,\text{Res}}_{\alpha\beta}, \tag{4.19}$$

where $T^{\pi^0}_{\alpha\beta}$ is the energy–momentum tensor again of the electromagnetic fields (3.54) satisfying the Maxwell equations (4.6) and

$$T^{\pi^0\alpha\beta}{}_{;\beta} = -\frac{1}{c} F^{\pi^0\alpha\beta} j^{\pi^0}_\beta, \tag{4.20}$$

$j^{\pi^0}_\beta$ js the total four-current of the π^0 constituents; $M^{\pi^0,\text{Res}}_{\alpha\beta}$ is the "residual term" which can be written [22]

$$M^{\pi^0,\text{Res}}_{\alpha\beta} = \mu_{\text{Res}} u_\alpha u_\beta, \tag{4.21}$$

u_α is the four-velocity of the π^0 symmetry center, and μ_{Res} is the mass density of the π^0 constituents which by assumption is such that

$$m^{\pi^0}_{\text{Res}} = 2m_p = \int \mu_{\text{Res}}\, dV < m^{\pi^0}_{\text{Grav}}. \tag{4.22}$$

The field equations for the interior region D_i under the weak assumption assumption are given by (for $\lambda = 0$)

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} M_{\alpha\beta}^{\pi^0,\text{Weak}}. \tag{4.23}$$

The Ricci identities (4.8) now imply the conditions

$$M^{\pi^0,\text{weak}\,\alpha\beta}{}_{;\beta} = 0,$$

which give rise to the equation of conservation of the "residual mass"

$$(\mu_{\text{Res}} u^\alpha)_{;\alpha} = 0. \tag{4.24}$$

This is compatible with Eqs. (4.20) since in our case $j^\mu \propto u^\mu$.
The equation of conservation of the total charge follows from the usual property

$$j^{\pi^0\alpha}{}_{;\alpha} = 0 \tag{4.25}$$

A first generalization of model (4.23) can be investigated by considering instead the residual term (4.21) expressions of the type

$${}_kM_{\alpha\beta}^{\pi^0,\text{Res}} = \sum_{nm} \epsilon_m C_{nm}^k \mu_{nm} v_{nm\alpha} v_{nm\beta}\,, \tag{4.26}$$

where $n = +, -$; $m = \text{Ret, Adv}$; μ_{nm} is the mass density of the (n, m) — parton; $v_{nm}\alpha$ is given by (3.2); ϵ_m is defined by (3.5); the index $k = 1, 2, 3$ depends on the type of field (3.54) chosen for the construction of $T_{\alpha\beta}^{\pi^0}$; and C_{nm}^k is given by

$$\left.\begin{aligned} C_{n\text{Ret}}^k &= 1 \\ C_{n\text{Adv}} &= 0 \end{aligned}\right. \quad \text{for} \quad k = 1, \quad m = +, -, \tag{4.27}$$

$$C_{nm}^k = \tfrac{1}{2} \quad \text{for} \quad k = 2, 3; \quad n = +, -; \quad m = \text{Ret, Adv}. \tag{4.28}$$

However, the Ricci identities (4.8) can now be written

$$\Big(\sum_{nm} \epsilon_m C_{nm}^k \mu_{nm} v_{nm}^\alpha v_{nm}^\beta\Big)_{;\beta} = (1/c)\, F_{\pi^0}^{\alpha\beta,\pi^0} j_\beta\,, \tag{4.29}$$

and they do not give rise to a standard form of conservation of the "residual mass" such as (4.24) without the inclusion of reaction terms.

Model (4.26) for the "residual term" is here indicated mainly in relation to the cases $k = 2, 3$ of models (3.54) in order to keep a parallelism in the simultaneous use of both retarded and advanced terms.

Finally, let us consider the interior problem in the framework of the strong assumption. This assumption essentially implies one of the following hypotheses on the rest mass m_p of the π^0 constituents:

(1) m_p can be entirely accounted for as of electromagnetic nature;
(2) m_p cannot be accounted for as of electromagnetic nature, but it does not constitute a source of gravitational field;
(3) m_p can be partially accounted for as of electromagnetic nature, but the "residual mass" does not act as a source of gravitational field.

We must emphasize that hypotheses (2) and (3) are not in agreement with the most widely accepted assumption according to which mass or energy of any origin acts as a source of the gravitational field. Therefore, according to this assumption, the energy–momentum tensor of short range fields (such as in weak and strong interactions) cannot be ignored in the gravitational field equations. Our strong assumption constitutes an a priori exclusion of such an approach since it ultimately implies that the electromagnetic interactions alone are entirely responsible for the gravitational phenomenology, while the weak and strong interactions do not constitute a source of the gravitational field.

Clearly, the latter hypothesis is the most suggestive from a parton model viewpoint and has a framework within theories for which the parton rest mass m_p is partially of strong interaction nature, and as such it does not contribute to the phenomenology of the particle at large distance (according to our strong assumption), and partially of electromagnetic nature.

Independent of the chosen hypothesis the strong assumption ultimately implies that the volume integral (4.16) represents the entire gravitational mass of π^0.

In order to write the field equations under the strong assumption for the interior region D_i we have to recall that now the Ricci identities (4.8) fail in general to be valid in view of the Maxwell equations (4.20).

A first model can be investigated by assuming that in the interior region

$$F_{\pi^0}{}^{\alpha\beta} j_{\pi^0\beta} = 0, \tag{4.30}$$

such as, for instance, when the overall current $j_{\pi^0\beta}$ (as the overall charge) is identically or approximately zero in view of the extremely small distance between the charged constituents.

In this case the field equations either with an exact or an approximate meaning are again given by (4.7) and any major differentiation between the exterior and the interior problem disappears.

Further models can be investigated by recalling that the overall electromagnetic fields (3.54) of π^0 were constructed as a linear combination of the fields of the individual constituents without any reaction term due to the small distance involved in the system.

Since we consider π^0 as an isolated system, its charged constituents influence each other by exchanging energy and momentum. This gives rise to a supplementary term in the construction of the energy-momentum tensor $T_{\alpha\beta}$ in such a way that the equations

$$T^{\alpha\beta}{}_{;\beta} = 0 \tag{4.31}$$

hold with the meaning of conservation laws instead of that of Maxwell equations for the interior region.

Those supplementary or reaction terms supposedly should not primarily contribute to the phenomenology of the field at large distances, and for that reason they have been ignored in our analysis for the exterior problem. Furthermore, they should be of entire electromagnetic nature and as such they should not contradict the strong assumption.

With an open mind for the above remarks we shall write the field equations for the interior problem under the strong assumption as follows:

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4}\left(T^{\pi^0}_{\alpha\beta} + T^{\text{Rea}}_{\alpha\beta}\right) \tag{4.32}$$

with the conditions

$$T^{\pi^0\alpha\beta}{}_{;\beta} = -T^{\text{Rea}\,\alpha\beta}{}_{;\beta} \tag{4.33}$$

in such a way that the Ricci identities (4.8) are satisfied.

Equations (4.32) can give rise to various models. An interesting model occurs if one assumes that the reaction term is proportional to the curvature scalar R according to

$$\frac{8\pi G}{c^4} T^{\text{Rea}}_{\alpha\beta} = -\tfrac{1}{4} g_{\alpha\beta} R. \tag{4.34}$$

In this case the equations (4.32) reduce to [23]

$$R_{\alpha\beta} - \tfrac{1}{4} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T^{\pi^0}_{\alpha\beta}, \tag{4.35}$$

namely, they reduce to a form in which both members are traceless, with the properties

$$\frac{c^5}{32\pi G}\frac{\partial R}{\partial x_\beta} = F_{\pi^0 \beta\alpha} j^{\alpha}_{\pi^0}. \tag{4.36}$$

In the above model the curvature scalar is constant everywhere $j^{\alpha}_{\pi^0} = 0$. Furthermore, since (4.36) implies the relation [23]

$$\frac{\partial R}{\partial x_\beta}\frac{dx_\beta}{d\tau} = 0. \tag{4.37}$$

The curvature scalar is also constant on the world lines of the π^0 constituents. This is the model which historically is related to the cosmological constant. Indeed, putting $\lambda = \tfrac{1}{4} R_0$, where R_0 is the constant curvature scalar, Eqs. (4.35) can be written

$$R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta}(R - 2\lambda) = \frac{8\pi G}{c^4} T^{\pi^0}_{\alpha\beta}. \tag{4.38}$$

Clearly, the rigorous treatment of the interior problem requires a careful handling of the world line singularities. This will be done elsewhere.

A few comments are now in order. The relevent property arising from the previous analysis is that for the exterior problem the weak and the strong assumptions lose any differentiation giving rise to the same field equations (4.7).

A first implication of the theory is a lack of full compatibility with the costumary gravitational wave theory. Indeed, if the energy-momentum tensor $T^{\pi^0}_{\alpha\beta}$ in (4.7) is not ignorable, namely if the electromagnetic mass (4.16) is of the same order of magnitude of the gravitational mass of π^0, then the basic equations (4.18) for the gravitational wave theory cannot hold even in first approximation, the correct equations being given by (4.11).

Obviously, the new equations (4.11) admit gravitational waves, but only as solutions of the associated homogeneous equations. Indeed, the general solutions of Eqs. (4.11) are, as usual, a superposition $h'_{\alpha\beta} + h''_{\alpha\beta}$ where $h'_{\alpha\beta}$ are special solutions of the inhomogeneous equations and $h''_{\alpha\beta}$ are the general solutions of the homogeneous equations (4.18). The full implications of this new framework for the gravitational wave theory will be investigated elsewhere.

A second implication of the theory is that if the electromagnetic mass (4.16) can account entirely for the gravitational mass of π^0, then the door is open to a geometric unification of the gravitational and electromagnetic field. More particularly, the theory would imply the identification of the gravitational field of π^0 with a particular form or "mutation" of the electromagnetic field originated in the π^0 structure.

The term "mutation" is here used in the sense that, on account of the high orbital velocity of the constituents, the distribution in space of the lines of force of the $(+q, -q)$ system is considerably different than the corresponding distribution for the case of stationary charges. In Figs. 6a and 6b we schematically represent the "mutation" of the lines of force for the case of the electric field of the system. A similar "mutation" occurs also for the lines of force of the magnetic field. The action along the lines of force obviously propagates with velocity c, but its orientation in space depends on the local composition with an orbital velocity $v \approx c$.

The term "mutation" is here introduced also in relation to a possible breakdown of quantum electrodynamics under the bound state conditions at extremely small distances of the π^0 constituents. This is also reflected in the breakdown of Maxwell

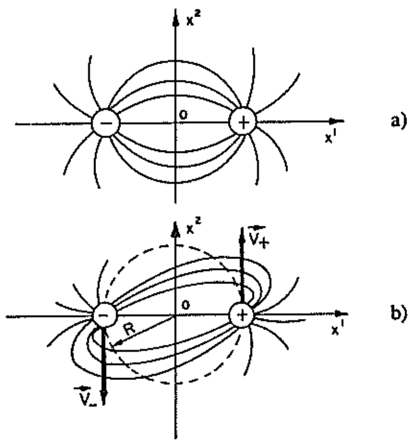

FIG. 6. a) The customary distribution of the lines of force for two opposite charges; b) the "mutation" of the same lines of force due to high orbital velocity of the charges.

equations for models $k = 1, 2$ of (3.54) for the electromagnetic field produced by the π^0 constituents under the restriction of zero energy loss through radiation.

Furthermore, if the strong assumption holds true, then its implications would be rather deep and not restricted only to the framework of a fully unified theory.

Indeed, since we have a well established knowledge on the quantization of the electromagnetic field, the strong assumption might contribute to the same problem of a quantized theory of gravitation.

Similarly, the strong assumption might have implications also in the framework of the elementary particle theory, and, more precisely, it might contribute to the ultimate identification of the nucleon constituents. For instance, the hypothesis of structureless constituents might acquire more physical weight and the search for the nucleon constituents might be shifted from the heavy quarks (the "Yeti particles" [17]) to lighter constituents.

The geometrical implications of the above findings, as well as a preliminary extension to the case of a massive body, will be discussed in the next section.

Conceivably, there should not be any contradiction of the theory for both the weak and the strong assumption with our present experimental knowledge, since both assumptions give rise to perfectly acceptable gravitational models, the ultimate nature of the geometry of space being independent of the origin of the energy–momentum tensor.

The singular nature of the charges involved in the model should not constitute a conceptual drawback even though they open various problems of proper formula-

tion. Indeed, it is an experimental fact that any massive body is ultimately a large collection of charges at both an atomic and a nuclear level and our formulations should eventually comply with such a reality.

With an open mind for further specific investigations, we can thus say that, at least in principle, both weak and strong assumptions are admissible on the ground of our present knowledge.

It is interesting to remark that according to the model for the electromagnetic field of π^0 developed in Section 3 and Appendix B, the standard assumption holds true only for stationary charged components. Indeed, as it appears from Eqs. (B.9) and (B.10), if the velocity of the constituents is zero, then both the electric and magnetic fields vanish. Similarly, for $v = 0$, the energy densities (B.19) and (B.34) vanish too. The above findings seem to support the central idea of the present paper according to which the electromagnetic field of the individual charged constituents of matter might contribute to the gravitational field irrespectively of the value of the overall charge, electric and magnetic moments. More specifically, the gravitational mass of π^0 would be very close to zero if computed on the basis of the costumary approach, namely of its total electromagnetic data. Under the same approach, the gravitational mass of π^+ would be only of the order of 4.5 MeV. If, on the contrary the same masses are computed by considering the electromagnetic contribution of each individual charged constituent, then their values might be either close to (weak assumption) or identical to (strong assumption) the actual gravitational masses.

As a final comment we would like to remark that the weak assumption is implicitly used by various current models for charged masses and fluids, or stars acting as a source of electromagnetic waves. In all those models the electromagnetic tensor is usually considered disjoint from the matter tensor and only the overall charges of the bodies are considered in the theory. The weak assumption, however, implies a further step ahead in the theory, since it implies that not only the overall charges, but more properly the charges of each individual constituent of the body partially contribute to its gravitational field. This implies that, while the electromagnetic energy–momentum tensor is ignorable with respect to the matter tensor in most of the customary models, it is no longer so for the weak assumption.

5. Concluding Remarks. Some New Experiments?

Let us now assess our evaluation of the standard, weak and strong assumptions on the basis of the previously introduced models for π^0.

The data emerging from the recent deep inelastic e–p scattering experiments clearly indicates that nucleons are composed by a number of charged constituents in a highly dynamical behavior.

The analysis of Section 3 indicates the presence of a nonnull electromagnetic field outside the system of two charged constituents with opposite charges in one of the most symmetrical configurations between the dynamical behavior of charges of opposite sign.

Clearly, the extension of the above model to the case of a nonsymmetrical dynamical behavior will equally give rise to a nonnull electromagnetic field. Similarly, the extension of the model to the case of more than two valence constituents will also give rise to a nonnull electromagnetic field.

We can, thus, say that there is relevant evidence according to which nucleons and nuclei are surrounded by an electromagnetic field due not only to their overall charge, but more properly to the charge of their individual constituents.

If we add to the picture the peripheral electron clouds we have a further contribution to the electromagnetic field outside any neutral atom.

Consequently, there is relevant evidence for a sizeable electromagnetic field generated by any massive neutral body.

According to Einstein theory of gravitation, the energy–momentum tensor of any electromagnetic field acts as a source of the gravitational field.

The puzzling implications of the above findings clearly cast shadows on the standard assumption, leaving as possible alternative a selection between the weak and the strong assumption.

In the framework of our analysis we can, thus, say that the gravitational field equations of the exterior case for any massive neutral body (in the absence of source-free electromagnetic fields)

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta}(R - 2\lambda) = 0 \tag{5.1}$$

present some problematic aspects, having as possible alternative for the same exterior case the equations valid for both the weak and the strong assumptions

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta}(R - 2\lambda) = \frac{8\pi G}{c^4} T_{\alpha\beta}\,, \tag{5.2}$$

where $T_{\alpha\beta}$ is the energy–momentum tensor of the electromagnetic field produced by all charged constituents of the body, namely the electrons for the peripheral clouds of their atoms and the partons for their nuclear structure.

Furthermore, the results of the "already unified theory" [2] clearly apply for equations (5.2). We have, as a consequence, that the only admissible theories for the exterior case are the ones satisfying the following conditions

$$G_\alpha{}^\alpha = 0, \qquad G_0{}^0 \geqslant 0, \tag{5.3}$$

$$G_\mu{}^\alpha G_\alpha{}^\nu = \rho^2 \delta_\mu{}^\nu = \tfrac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \delta_\mu{}^\nu, \tag{5.4}$$

$$\oint \alpha_\mu \, dx^\mu = 2\pi n, \tag{5.5}$$

(5.6)

where $n = 1, 2, \ldots$;

$$\rho^2 = \frac{|\sigma \pi^2 G^2|}{c^8} T_{\alpha\beta} T^{\alpha\beta}, \tag{5.7}$$

$$\alpha_\sigma = (-g)^{1/2} (1/\rho^2)\, \epsilon_{\sigma\nu\lambda\mu} G^{\lambda\gamma;\mu} G_\gamma{}^\nu, \tag{5.8}$$

and the line of integration of (5.6) does not touch any null point [2].

The first condition (5.3) is a consequence of the traceless character of the energy–momentum tensor of any electromagnetic field. It implies that the only admissible theories for the exterior case are the ones for which the curvature scalar is proportional to the cosmological constant, $R = 4\lambda$, or $R = 0$ for $\lambda = 0$.

The second condition (5.4) follows from the fact that in Minkowski space $T_0{}^0 = (1/8\pi)(E^2 + H^2) > 0$ or, equivalently, for any time-like vector μ_α

$$T_{\alpha\beta} \mu^\alpha \mu^\beta \leqslant O. \tag{5.9}$$

The third condition (5.5) reflects the property that the square of $T_{\alpha\beta}$ is a multiple of the unit tensor, i.e.,

$$T_\mu{}^\alpha T_\alpha{}^\nu = \tfrac{1}{4} T_{\alpha\beta} T^{\alpha\beta} \delta_\mu{}^\nu = \sigma^2 \delta_\mu{}^\nu, \tag{5.10}$$

with

$$\sigma^2 = \mathrm{I}_1{}^2 + \mathrm{I}_2{}^2, \tag{5.11}$$

$$\mathrm{I}_1 = \tfrac{1}{2} F_{\alpha\beta} F^{\alpha\beta}, \qquad \mathrm{I}_2 = \tfrac{1}{2} F_{\alpha\beta} {}^*F^{\alpha\beta}, \tag{5.12}$$

$${}^*F_{\alpha\beta} = \tfrac{1}{2}(-g)^{1/2} \epsilon_{\alpha\beta\mu\nu} F^{\mu\nu}. \tag{5.13}$$

Conditions (5.3), (5.4), and (5.5) are sometimes called the Rainich algebraic conditions.

The last condition (5.6) is called the Rainich–Misner–Wheeler integral condition. It is derived using relations of the type

$$\alpha_\rho = (1/\sigma^2)(-g)^{1/2} \epsilon_{\rho\nu\lambda\mu} T^{\lambda\gamma;\mu} T_\gamma{}^\nu = \theta_{,\rho} = -(i/\sigma^2)\, \omega'_\mu{}^\nu{}_{;\nu} \omega_\rho{}'^\mu \omega'^2, \tag{5.14}$$

where

$$\omega_{\alpha\beta} = F_{\alpha\beta} + i^*F_{\alpha\beta}\,, \tag{5.15}$$

$$\omega'_{\alpha\beta} = \omega_{\alpha\beta} e^{-i\alpha}, \tag{5.16}$$

and ultimately follows from the property that the energy–momentum tensor, which can now be written

$$T_{\alpha\beta} = \omega_{\alpha\rho}\bar{\omega}^{\rho}{}_{\beta} \tag{5.17}$$

is invariant under duality rotations, namely transformations of the type (5.16). In (5.14) the quantity θ is also called the complexion.

Integral condition (5.6) holds for multiply connected spaces. For simply connected spaces it is substituted by the differential condition

$$\alpha_{\rho,\tau} - \alpha_{\tau,\rho} = 0. \tag{5.18}$$

Since in our case we deal with moving charged constituents we have a set of singular lines contained in a four-tube (see Fig. 4 for the parton–antiparton case). Consequently, integral condition (5.6) applies.

Furthermore, since we do not accomodate magnetic charges, the real part of the surface integral of $\omega_{\alpha\beta}$ around the set of line singularities must be zero. This in turn implies that the arguments of the surface integrals surrounding all singularities differ by a multiple of π, in which case one can adjust the constant phase of the theory to have all the integrals purely imaginary.

Let us recall that the framework of the "already unified theory" is to consider a four-dimensional region of a Riemannian space with given metric and then determine through conditions (5.3)–(5.6) a nonnull electromagnetic field whose energy–momentum tensor is proportional to the Einstein tensor.

Our framework is somewhat different then the above. Indeed, despite the problematic aspect of computing the electromagnetic field for a large number of charged constituents, we can assume in principle the electromagnetic field to be known, and then aim at the determination of the metric through conditions (5.3)–(5.6). Nevertheless, if the metric in the space surrounding a massive body is known, the approach of determining the electromagnetic field can be used too.

Independent of the followed approach, the gravitational field equations (5.2) with conditions (5.3)–(5.6) represent a fully geometric theory since all physical quantities can be represented in terms of the metric tensor and its derivatives.

We can, thus, say that the very existence of a "seizable" electromagnetic field outside any neutral or charged massive body has deep implications from a gravitational viewpoint.

Of course the theory basically depends on whether the "electromagnetic mass" [24]

$$m_{\text{elm}} = -\frac{1}{c^2}\int T^{00}\, dV \tag{5.19}$$

is of the same order of magnitude but smaller than the gravitational mass (weak assumption), or is identical to the gravitational mass (strong assumption).

In Appendix B we showed how for suitable values of the parameters involved, the electromagnetic mass of the considered π^0 model can well account for its entire rest mass. If this is the case, an extrapolation of the theory to nucleons, nuclei and atoms conceivably might imply the identity of m_{elm} with m_{gr} for any macroscopic body.

This will also be the case if the exterior field equations (5.2) are assumed as the fundamental equations for the geometry of space. Then the mass term (5.19) is entirely responsible for the gravitational field; the differentiation between the weak and the strong assumption lose any meaning; the "residual" masses of the matter constituents (if any) behave as "passive" terms since they would not act like sources of gravitational field; and a meaningful unification of the gravitational and electromagnetic field would be possible.

We have, however, to emphasize that to our opinion the exterior problem alone is insufficient for a full unification. It is precisely in the interior problem where the alternative between the weak and the strong assumptions acquires full physical weight.

The validity of the strong assumption basically depend on the role played by the weak and strong interactions in the characterization of the metric. Indeed, if those interactions contribute to the gravitational mass, then the weak assumption is admissible but the strong assumption is excluded a priori. More specifically, if one assumes the most widely accepted belief according to which the energy–momentum tensor of "any" field acts as a source of the gravitational field then the short range tensors due to weak and strong interactions will contribute to the exterior metric even though they do not appear in the exterior field equations, and, therefore, the electromagnetic interactions alone cannot account for the entire gravitational mass. This is due to the fact, as pointed out before, that in order to get a full characterization of the metric in the exterior region, the differential equations for the interior region must be considered too. We must, however, emphasize that at this point in time there is no ultimate theoretical or experimental evidence substantiating the assumption that short range interactions act as a source of the gravitational field. Therefore, an opposite approach, at least for completeness, must be investigated too. It is in this spirit that our strong assumption has been introduced.

Unfortunately, in view of the lack of well established data on the structure of the π^0 particle as well as of the nucleons, together with the limitations arising from a classical approximation of the problem, the above alternative cannot at the moment be settled on the ground of theoretical analysis alone.

It would be, thus, particularly appealing if some feasible experiment can be conceived aiming at the proper selection as well as the ultimate physical assessment of the new assumptions.

Again in this respect the elementary structure model for π^0 discussed so far might offer some basis for new experiments.

Essentially the considered π^0 model consists of two parts which can be considered either individually or jointly:

(1) The model of Fig. 3 consisting of two opposite charges rotating in the same circular orbit at diametrally opposite positions. In this case, from (B.20) the electromagnetic mass generated by the system is of the order of

$$ {}_q m_{\text{elm}} \cong N_1 \frac{q^2}{c^2 R} \beta^2 (1 - \beta^2)^2 \tag{5.20}$$

where N_1 is a numerical factor (e.g. $N_1 = 4$ for Eq. (B.20)).

(2) The model of Fig. 5 consisting of two magnetic moments in opposite orientation also rotating in the same circular orbit at diametrally opposite positions. In this case, from (B. 35) the related electromagnetic mass is of the order of

$$ {}_\mu m_{\text{elm}} \cong N_2 \frac{\mu^2}{c^4 R} \beta^2 (1 - \beta^2)^4, \tag{5.21}$$

where N_2 is also a numerical factor (e.g. $N_2 = 144$ for Eq. (B. 35)).

Since those models are purely classical they should be independent of the magnitude of the charges and magnetic moments and consequently they should hold at a macroscopic level, too.

The ultimate question we are facing is whether the models of Figs. 3 and 5 can be experimentally reproduced in a way suitable for measurements.

For $v = \omega R \ll c$, equations (5.20) and (5.21) can be rewritten

$$ {}_q m_{\text{elm}} \cong N_1 \frac{q^2}{c^4} \omega^2 R, \tag{5.22}$$

$$ {}_\mu m_{\text{elm}} \cong N_2 \frac{\mu^2}{c^6} \omega^2 R, \tag{5.23}$$

from which we see that the electromagnetic masses of the systems are proportional to the square of the charges or of the magnetic moment, to the square of the orbital angular velocity and to the radius R of the orbit. It is also interesting to note from (5.22) and (5.23) that for $\omega = 0$ namely for stationary charges, the electromagnetic masses ${}_q m_{\text{elm}}$ and ${}_\mu m_{\text{elm}}$ are zero.

The technical difficulties for experimental frameworks reproducing the models of Figs. 3 and 5 are, thus, represented by the need to reach sufficiently high values of the angular velocity for given q or μ and R, in such a way to have appreciable amount of the masses (5.22) or (5.23) to be detectable by gravity meters.

Even though the problem demands specific investigations, we would like to mention that, at least in principle, the above experiments are conceivable.

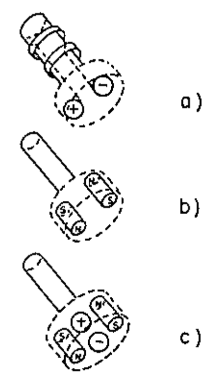

Fig. 7. Some "electromagnetic heads" for a conceivable experiment reproducing the framework of Figs. 3 and 5: a) The head incorporates two spheres of opposite charges. b) The case for two magnetic moments of the same magnitude and opposite orientation. c) The mixed case incorporating both charges and magnetic moments.

Consider, for instance, the "electromagnetic heads" of Fig. 7. In relation to the model of Fig. 3, we have in Fig. 7a a thin smooth disc of properly chosen non-conducting matrial incorporating conducting spheres which are connected to outside sources $\pm$s through brushes in such a way to reach two or more pair of opposite charges $\pm q$ at the same radial distance R and at diametrally opposite position. For the model of Fig. 5 we have in Fig. 7b also a thin smooth disc incorporating this time two or more pairs of solenoids properly connected with outside sources in such a way to have two or more magnetic moments μ with opposite orientation as shown in figure. Finally, in Fig. 7c we indicate the possibility of combining the system of Fig. 7a and 7b in a single unit. Various other configurations can be considered, too.

If the size and mass of the magnetic heads is not excessively high, the present technology now offers the possibility of putting them in rotation around their symmetry axis at speed of the order of 10^6 r.p.m. through the use of highly sophisticated turbines and gas bearings.[8]

A scheme for possible experiments is given in Fig. 8 where:

(1) *Electromagnetic head.* This is one of the heads previously described, precision balanced for high r.p.m.

(2) *Power head.* This might be a small, specially designed gas turbine running on gas lubricated bearings, such as, for instance, foil bearings.

[8] The author would like to acknowledge an interesting correspondence on this subject with E. B. Arwas, Mechanical Technology Incorporated, Latham, NY.

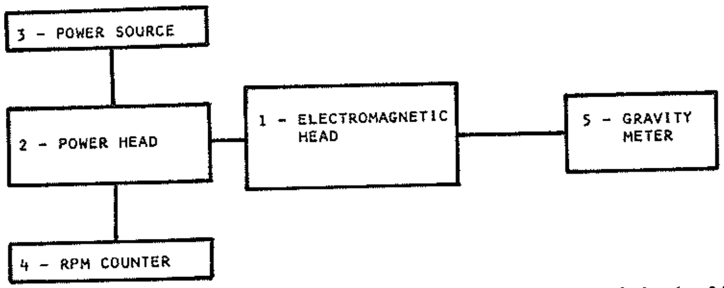

FIG. 8. A scheme for a conceivable experiment to put the electromagnetic heads of Fig. 7 in high rotation along their symmetry axis—The electromagnetic head (1) is incorporated in the shaft of a high r.p.m. gas turbine (2) served by a cylinder or compressor (3) of a filtered gas; the r.p.m. are measured by an optical probe (4); and the gravitational field produced by the electromagnetic heads in high rotational motion is detected by a highly sensitive gravity meter (5).

(3) *Power source.* It can be either a cylinder or a compressor operating on a specially filtered gas, such as air.

(4) *RPM counter.* Customarily, for measurements of high r.p.m. an optical probe is recommended. A region of the electromagnetic head is treated to have different reflective properties than the untreated region. The probe then senses a pulse at each revolution. The pulse is fed to an electronic speed counter which registers the rotational speed.

(5) *Gravity meter.* This should be a highly sensitive device able to detect the difference in the gravitational field between the stationary heads and the same in full rotation or between rotating heads with and without electromagnetic fields.

Clearly the feasibility of the experiments, after the computation of formulae more accurate than (B.22) and (B.23), depends on the existence of sufficiently accurate gravity meters in relation to the highest electromagnetic masses which can be reached by the present technology.

Without entering at the moment in an extensive analysis of the experiment, let us conclude with a few remarks. The "classical" set up of Fig. 8 cannot reproduce the "quantum mechanical" system of the π^0 constituents, namely a bound state of charged constituents with zero energy loss through radiation. Therefore, in the computation of the electromagnetic mass for the classical set up, the full fields (3.19) and (3.46) must be used instead of fields (3.54). This will leads to electromagnetic masses higher than (5.22) and (5.23). There is no doubt, however, that the expected effect will require a highly sophisticated technology to be reliably measured. Despite the above difficulties, the proposed experiment remains quite attractive. Irrespective of our findings, the Einstein–Maxwell theory predicts a modification of the gravitational field due to highly rotating charges or magnetic moments and it is important to verify with an experiment such a theoretical prediction. Once this is done, the experiment will undoubtedly contribute to the lines of investigation of the present paper.

APPENDIX A. A SIMPLE BOUND STATE MODEL FOR π^0

Let us represent the partons carrying the potential (2.6) with the symbols $\epsilon^\pm$ and their ^1S bound state with the notation (ϵ^+, ϵ^-). Then we can symbolically write in our valence parton approximation

$$\pi^0 = (\epsilon^+, \epsilon^-). \tag{A.1}$$

In view of the technical difficulties presented by the nonlocal term in (2.6) we can reduce it in first approximation to a velocity-dependent potential [16] of the form $(\rho - 1)\, p^2/m_0$, where ρ is a parameter to be determined and m_0 is the rest mass of the $\epsilon^\pm$ partons.

Using the potential (2.7) for the short range term, the potential for the $\epsilon^\pm$ partons can be written

$$V(r) = -\frac{q^2}{r} - V_0 \frac{e^{-br}}{1 - e^{-br}} + (\rho - 1)\frac{p^2}{2m_0}. \tag{A.2}$$

A key feature for the above velocity-dependent potential is that it allows the introduction of the so-called effective mass

$$m' = m_0/\rho. \tag{A.3}$$

For $\rho \ll 1$, a fully relativistic behavior for the "true" mass m_0 can be well approximated with a nonrelativistic behavior of the much higher effective mass m' [25].

In order to be bounded in a sphere of (charge) radius b^{-1}, the $\epsilon^\pm$ partons should possess a de Broglie wavelength $\lambda\!\!\!^{-}_\epsilon$ of the order of b^{-1}. By putting

$$\lambda\!\!\!^{-}_\epsilon = (K_1 b)^{-1}, \tag{A.4}$$

where K_1 is a small number (e.g. $K_1 \approx 2/\pi$ for the deuteron), we can write

$$E_K = mc^2 = K_1 \hbar bc = \frac{\hbar^2 b^2}{2m'} = \frac{p^2}{2m'}, \tag{A.5}$$

then

$$\rho = 2K_1^2 \frac{E_0}{E_K}, \qquad m' = \frac{\hbar b}{2K_1 c}. \tag{A.6}$$

If we assume $K_1 \cong 2^{-1/2}$, when $E_K \to E_0$, $\rho \to 1$, the velocity dependent potential tends to zero and we approach a fully nonrelativistic framework with local potential.

By neglecting spin-dependent contributions, we can, thus, represent the (ϵ^+, ϵ^-) bound state with the Schrödinger equation in the effective mass m', whose radial part (for $l = 0$) is

$$\left\{\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{m'}{\hbar}[E - V'(r)]\right\} U(r) = 0, \quad \text{(A.7)}$$

where $V'(r)$ is the potential for the effective particle, namely the potential (A.2) less the velocity-dependent term.

If we ignore the Coulomb potential, the solution of equation (A.7) is straightforward. By putting $U(r) = R(r)/r$, $w = br$, $A = m'E/\hbar^2b^2$ and $B = m'V_0/\hbar^2b^2$, equation (A.7) becomes

$$\left(\frac{d^2}{dw^2} + A + B\frac{e^{-w}}{1 - e^{-w}}\right) R(w) = 0. \quad \text{(A.8)}$$

With the substitutions $R(w) = \exp(-|A|^{1/2} w)\, S(w)$ and $x = 1 - \exp(-w)$ we finally get the equation

$$\left[x(1 - x)\frac{d^2}{dx^2} - (2|A|^{1/2} + 1)\frac{d}{dx} + B\right] S(x) = 0, \quad \text{(A.9)}$$

with the boundary conditions

$$S(0) = 0; \quad \lim_{w\to\infty} e^{-|A|^{1/2}w} S(w) = 0, \quad \text{(A.10)}$$

and $S(1)$ finite for $A = 0$.

The solutions of equation (A.9) are given by the Jacobian polynomials

$$G_n(x) = \sum_{1}^{n}{}_k (-1)^{k-1}\binom{n-1}{k-1}\binom{n + k + 2|A|^{1/2} - 1}{k} x^k. \quad \text{(A.11)}$$

The quantization condition for the binding energy reads[9]

$$BE_n = -E_n = \frac{\hbar^2b^2}{4m'}\left(\frac{m'V_0}{\hbar^2b^2n} - n\right)^2 \qquad n = 1, 2, 3,\ldots, \quad \text{(A.12)}$$

[9] We would like to emphasize that, in view of the low value of the rest energy of the ϵ-particles, there would be little meaning (if any) to search for an association of the exited states of (A.12) with other known mesons. In other words, the approach implies a considerable differentiation from the quark model in the sense that while in the latter model all mesons are bound states of two (valence) quarks, in the former approach the number of (valence) partons increases with the increase of the rest energy of the particle. For instance the $\pi^\pm$ particle would be a bound state of three ϵ partons.

and the normalized eigenfunction for the ground state ($n = 1$) is

$$U_1(r) = \left[b\frac{\Gamma(2|A_1|^{1/2} + 3)}{\Gamma(3)\,\Gamma(2|A_1|^{1/2})}\right]^{1/2} e^{-|A_1|^{1/2}br}\frac{1 - e^{-br}}{r}. \quad \text{(A.13)}$$

By putting

$$V_0 = K_2\frac{\hbar^2b^2}{m'} = 2K_1K_2\hbar bc = 2K_2E_K, \quad \text{(A.14)}$$

where K_2 is also an unknown parameter, the binding energy of the (ϵ^+, ϵ^-) bound state in the ground state is

$$BE_1 = \tfrac{1}{2}K_1(K_2 - 1)^2\,\hbar bc = \frac{(K_2 - 1)^2}{4K_2} V_0, \quad \text{(A.15)}$$

from which we see that for $BE_1 \ll V_0$, $K_2 \approx 1$.

From the rest energy of π^0 we can write a first relation in the two unknown parameters K_1 and K_2, namely

$$E_{\pi^0} = 2K_1[1 - (K_2 - 1)^2]\,\hbar bc. \quad \text{(A.16)}$$

A second relation can be obtained from the mean life of π^0. Let us remark that the rigorous calculation of the mean life of the (ϵ^+, ϵ^-)-system requires a fully relativistic treatment and the computation of transition matrix elements. For a preliminary analysis, we can approximate the mean life of the (ϵ^+, ϵ^-)-system with the known formula

$$\tau^{-1} = 4\pi\lambda\!\!\!^{-2}\,|U(0)|^2\frac{\alpha^2E_K}{\hbar}. \quad \text{(A.17)}$$

By putting $\lambda\!\!\!^{-}_\epsilon = (K_1b)^{-1}$ and by calculating $U(0)$ in the ground state from (A.13) we have

$$\tau^{-1} = \frac{\pi}{4}\frac{(K_2 + 3)(K_2^2 - 1)}{K_1^2}\alpha^2bc. \quad \text{(A.18)}$$

The solution of Eqs. (A.16) and (A.18), assuming $b^{-1} = 10^{-13}$ cm (a rather well established value for the π^0 charge radius [26]) gives the values

$$K_1 = 3.8 \times 10^{-1}, \qquad K_2 = 1 + 4.3 \times 10^{-9}. \quad \text{(A.19)}$$

Then the total energy of the $\epsilon^\pm$ parton in the (ϵ^+, ϵ^-) bound state is of the order of 68 MeV which can be assumed as the desired upper bound for the rest energy of the constituents.

The considered $\pi^0 = (\epsilon^+, \epsilon^-)$ model can, thus, recover all basic features of π^0, such as charge, spin, space and charge parity, rest energy and mean life. However, the model should not be taken seriously since it lacks the identification of the constituents with physically known particles in a way compatible with the π^0 strong phenomenology.

At the present stage of our knowledge, no strongly interacting particle with rest energy smaller than the π^0 rest energy is known.[10]

Nevertheless, the model has been discussed in order to show that π^0 models with rest energy of the constituents smaller than the π^0 rest energy are technically admissible and cannot be ruled out by present experimental knowledge. This point is not emphasized in the existing literature on the subject, by and large dominated by the equally unknown quarks, while it has central relevance from a gravitational viewpoint.

APPENDIX B. Some Approximate Expressions

Let us consider model (3.26) for the electromagnetic field of the π^0 particle in Minkowski space. The electric and magnetic field can be explicitly written

$$ {}_{1q}\mathbf{E}_{\pi^0} = \frac{q}{j^2}\left[\frac{D_-}{S_-^3}\left(\frac{\mathbf{D}_-}{D_-} - \frac{\mathbf{v}_-}{c}\right) - \frac{D_+}{S_+^3}\left(\frac{\mathbf{D}_+}{D_+} - \frac{\mathbf{v}_+}{c}\right)\right], \tag{B.1} $$

$$ {}_{1q}\mathbf{H}_{\pi^0} = \frac{q}{j^2 c}\left[\frac{\mathbf{D}_+ \times \mathbf{v}_+}{S_+^3} - \frac{\mathbf{D}_- \times \mathbf{v}_-}{S_-^3}\right], \tag{B.2} $$

[10] Clearly, the only known massive charged particles with rest energy smaller than 68 MV and able to fit the considered $\pi^0 = (\epsilon^+, \epsilon^-)$ model are the electrons. At the present stage of our knowledge, however, electrons must be rejected as proper identification of the ϵ-particles since they cannot account for the strong phenomenology of π^0, even though they can account for its weak and electromagnetic behavior as well as for all its basic features such as charge, spin, space, and charge parity; rest energy, and mean life.

Nevertheless, the possibility that electromagnetic interactions have the same strength of the weak and strong interactions at very small distances (or at very high energy) is already under investigation. This implies the possibility of identifying the ϵ-particle with electrons only when they leave the (ϵ^+, ϵ^-)-bound state, i.e., $\epsilon^\pm \xrightarrow[\text{free behavior}]{} e^\pm$, more than the actual identification $\epsilon^\pm \equiv e^\pm$. In other words, we do not know at the moment whether bare electrons, in the very high dynamical behavior at very small distances, are subjected to a "mutation" of their field as well as of their phenomenology, giving rise to an overall state with strong interacting behavior.

After all, if the strong assumption on the origin of the gravitational field proves to have full physical relevance, the search for light particles as candidates for parton identification should deserve serious consideration.

where

$$ S_\pm = -\frac{d_\pm}{\gamma c} = D_\pm - \frac{\mathbf{D}_\pm \cdot \mathbf{v}_\pm}{c} \tag{B.3} $$

and we have dropped the index Ret.

We shall now search for some approximate expressions of relations (B.1) and (B.2). Since the distance D is much greater than the radius R of the orbit, a first approximation can be obtained by putting

$$ \mathbf{D}_+ // \mathbf{D}_- ; \qquad \mathbf{D}_+ \cong \mathbf{D}_- = \mathbf{D}, \tag{B.4} $$

where $\mathbf{D}$ is the vector distance from P to the origin.

Assuming $\mathbf{v} = \mathbf{v}_- = -\mathbf{v}_+$, relations (B.1) and (B.2) can be written

$$ {}_{1q}\mathbf{E}_{\pi^0} \cong \frac{q}{j^2}\left[\mathbf{D}\left(\frac{1}{S_-^3} - \frac{1}{S_+^3}\right) - \frac{\mathbf{v}}{c} D\left(\frac{1}{S_-^3} + \frac{1}{S_+^3}\right)\right], \tag{B.5} $$

$$ {}_{1q}\mathbf{H}_{\pi^0} \cong \frac{q}{j^2 c}\, \mathbf{D} \times \mathbf{v}\left(\frac{1}{S_-^3} + \frac{1}{S_+^3}\right). \tag{B.6} $$

Consider a point P in the (x_1, x_2)-plane of the orbit and let $\gamma(t)(\delta(t))$ be the angle between $\mathbf{D}_+$ and $\mathbf{v}_+$ ($\mathbf{D}_-$ and $\mathbf{v}_-$). Then in view of (B.4), we have

$$ \gamma(t) \cong 180^\circ - \delta(t) \cong \omega t, \tag{B.7} $$

where ω is the orbital angular velocity of the partons. We can, thus, write

$$ S_\pm \cong D(1 \pm \beta \cos \omega t), \tag{B.8} $$

and relations (B.5) and (B.6) become

$$ {}_{1q}\mathbf{E}_{\pi^0} \cong \frac{q}{\gamma^2 D^2}\left\{\frac{\mathbf{D}}{D}\left[\frac{1}{(1-\beta\cos\omega t)^3} - \frac{1}{(1+\beta\cos\omega t)^3}\right] - \frac{\mathbf{v}}{c}\left[\frac{1}{(1-\beta\cos\omega t)^3} + \frac{1}{(1+\beta\cos\omega t)^3}\right]\right\}, \tag{B.9} $$

$$ {}_{1q}\mathbf{H}_{\pi^0} \cong \frac{q}{\gamma^2 D^2}\frac{\mathbf{D}}{D} \times \frac{\mathbf{v}}{c}\left[\frac{1}{(1-\beta\cos\omega t)^3} + \frac{1}{(1+\beta\cos\omega t)^3}\right], \tag{B.10} $$

from which we can also write

$$ {}_{1q}\mathbf{H}_{\pi^0} \cong \frac{\mathbf{D}}{D} \times {}_{1q}\mathbf{E}_{\pi^0}. \tag{B.11} $$

If the point P is on the x_3-axis then

$$ S_+ = S_- \cong D, \tag{B.12} $$

and relations (B.5) and (B.6) become

$$ {}_{1q}\mathbf{E}_{\pi^0} \cong -\frac{2q}{\gamma^2 D^2}\frac{\mathbf{V}}{c}, \tag{B.13}$$

$$ {}_{1q}\mathbf{H}_{\pi^0} \cong -\frac{2q}{\gamma^2 D^2}\frac{\mathbf{D}}{D}\times\frac{\mathbf{V}}{c}, \tag{B.14}$$

and again (B.11) holds.

Approximate expressions of the energy–momentum tensor ${}_{1q}T^{\alpha\beta}_{\pi^0}$ can be calculated accordingly. For the case of a point P on the x_3-axis, since

$$ {}_{1q}F^{\rho\sigma}_{\pi^0}\,{}_{1q}F_{\pi^0\rho\sigma}\,|_{x^3} \cong 0, \tag{B.15}$$

we have

$$ {}_{1q}T^{\alpha\beta}_{\pi^0}|_{x^3} \cong \frac{1}{4\pi}\,{}_{1q}F^{\alpha\mu}_{\pi^0}\,{}_{1q}F^{\beta}_{\pi^0\mu} \tag{B.16}$$

from which the energy–density is

$$ {}_{1q}T^{00}_{\pi^0}|_{x^3} \cong -\frac{q^2\beta^2}{\pi\gamma^4 D^4} \tag{B.17}$$

The total energy produced by the fields (B.1) and (B.2) is given by

$$ {}_{1q}\overset{T}{\epsilon}_{\pi^0} = -\int {}_{1q}T^{00}_{\pi^0}\,dV = \frac{1}{8\pi}\int [{}_{1q}\mathbf{E}^2_{\pi^0} + {}_{1q}\mathbf{H}^2_{\pi^0}]\,dV \tag{B.18}$$

and is a constant since model (3.26) satisfies restriction (3.24) on the zero energy rate of radiation.

The rigorous calculation of (B.18) is rather involved. A simple approximate expression can be obtained by approximating the energy–density with (B.17) everywhere in three-space. Then the total energy contained outside a sphere or radius R (where R is the radius of the orbit) is given by

$$ {}_{1q}\overset{T}{\epsilon}_{\pi^0} \cong -\int {}_{1q}T^{00}_{\pi^0}|_{x^3}\,dV = \frac{4q^2\beta^2}{\gamma^4}\int_R^\infty \frac{dD}{D^2} = \frac{4q^2\beta^2}{\gamma^4 R}. \tag{B.19}$$

Consequently, we can say that an approximate expression for the "electromagnetic mass of the π^0 particle according to the model of Fig. 3 and in the absence of magnetic moments (spin zero partons) is given by

$$ {}_{1q}m_{\pi^0} = -\frac{1}{c^2}\int {}_{1q}T^{00}_{\pi^0}\,dV \cong \frac{4\beta^2 q^2}{\gamma^4 c^2 R}. \tag{B.20}$$

We obtain in this way an expression which is similar to the one for the electromagnetic mass of a single particle of charge q and radius R, apart from the factor $\beta^2(1-\beta^2)^2$ incorporating the orbital velocity of the constituents.

Presumably, the full expression (B.18) for the total energy is proportional to the approximate expression (B.19) through a numerical factor. Similarly, the computation of quantity (B.19) outside a thin disc of radius R (instead of a sphere of radius R) gives rise to the same expression with a different numerical factor.

It is interesting to remark that even without the contribution from magnetic moments, expression (B.20) alone is able to recover the gravitational mass of π^0 for suitable values of the charge q of the constituents, their orbital velocity v, and the radius R of the orbit.

Let us consider now the π^0 model of Fig. 5 in which we have only two magnetic moments of the same magnitude and opposite orientation moving along the same circular orbit of radius R at diametrally opposite positions. The expressions (3.37) and (3.38) for the magnetic and electric moments of the individual constituents, recalling that $\mathbf{a} = \boldsymbol{\omega}\times\mathbf{v}$, can be written

$$ \mathbf{m}_\pm = \mu_0[\boldsymbol{\alpha}_\pm - \tfrac{1}{2}j^3\beta^2\boldsymbol{\omega}_\pm], \tag{B.21}$$

$$ \mathbf{e}_\pm = \frac{\mu_0}{c}\mathbf{v}_\pm \times [\boldsymbol{\alpha}_\pm - \tfrac{1}{2}j^3\boldsymbol{\omega}_\pm], \tag{B.22}$$

where $\boldsymbol{\alpha}_\pm$ and $\boldsymbol{\omega}_\pm$ are the intrinsic and orbital angular velocities, respectively. By putting

$$ \begin{aligned} \boldsymbol{\alpha} &= \boldsymbol{\alpha}_- = -\boldsymbol{\alpha}_+; & \boldsymbol{\omega} &= \boldsymbol{\omega}_- = \boldsymbol{\omega}_+; \\ \mathbf{v} &= \mathbf{v}_- = -\mathbf{v}_+; & \mathbf{a} &= \mathbf{a}_- = -\mathbf{a}_+, \end{aligned} \tag{B.23}$$

we can write

$$ \mathbf{m}_\pm = \mu_0[\mp\boldsymbol{\alpha} - \tfrac{1}{2}\gamma^3\beta^2\boldsymbol{\omega}], \tag{B.24}$$

$$ \mathbf{e}_\pm = \frac{\mu_0}{c}\mathbf{v}\times[\boldsymbol{\alpha} \pm \tfrac{1}{2}\gamma^3\boldsymbol{\omega}]. \tag{B.25}$$

The dipole tensor $\mu^{\alpha\beta}_\pm$ is then defined by (3.36).

Assuming $D_\alpha = D_{-\alpha} \cong D_{+\alpha}$, model (3.51) can be explicitly written

$$ \begin{aligned} {}_{1\mu}F^{\alpha\beta}_{\pi^0} = &-2c\left(\frac{\dot\mu^{\alpha\beta}_-}{d_-^2} + \frac{\dot\mu^{\alpha\beta}_+}{d_+^2}\right) + 2c(D\cdot a)\left(\frac{\mu^{\alpha\beta}_-}{d_-^3} - \frac{\mu^{\alpha\beta}_+}{d_+^3}\right) \\ &- 6c^3\left[\left(\frac{\mu^{\alpha\rho}_-}{d_-^4} + \frac{\mu^{\alpha\rho}_+}{d_+^4}\right)D_\rho D^\beta - \left(\frac{\mu^{\beta\rho}_-}{d_-^4} + \frac{\mu^{\beta\rho}_+}{d_+^4}\right)D_\rho D^\alpha\right] \\ &- 6c^3(D\cdot a)\left[\left(\frac{\mu^{\alpha\rho}_-}{d_-^5} + \frac{\mu^{\alpha\rho}_+}{d_+^5}\right)D_\rho D^\beta - \left(\frac{\mu^{\beta\rho}_-}{d_-^5} - \frac{\mu^{\beta\rho}_+}{d_+^5}\right)D_\rho D^\alpha\right] \end{aligned}$$

$$+ 3c^3 \left[\left(\frac{\dot{\mu}_-^{\alpha\rho}}{d_-^4} + \frac{\dot{\mu}_+^{\alpha\rho}}{d_+^4} \right) D_\rho D^\beta - \left(\frac{\dot{\mu}_-^{\beta\rho}}{d_-^4} + \frac{\dot{\mu}_+^{\beta\rho}}{d_+^4} \right) D_\rho D^\alpha \right]$$
$$+ c \left[\left(\frac{\mu_-^{\alpha\rho}}{d_-^3} a_-^\beta + \frac{\mu_+^{\alpha\rho}}{d_+^3} a_+^\beta \right) D_\rho - \left(\frac{\mu_-^{\beta\rho}}{d_-^3} a_-^\alpha + \frac{\mu_+^{\beta\rho}}{d_+^3} a_+^\alpha \right) D_\rho \right]$$
$$+ c \left[\left(\frac{\dot{\mu}_-^{\alpha\rho}}{d_-^3} v_{-\rho} + \frac{\dot{\mu}_+^{\alpha\rho}}{d_+^3} v_{+\rho} \right) D^\beta - \left(\frac{\dot{\mu}_-^{\beta\rho}}{d_-^3} v_{-\rho} + \frac{\dot{\mu}_+^{\beta\rho}}{d_+^3} v_{+\rho} \right) D^\alpha \right]$$
$$+ 2c \left[\left(\frac{\dot{\mu}_-^{\alpha\rho}}{d_-^3} v_-^\beta + \frac{\dot{\mu}_+^{\alpha\rho}}{d_+^3} v_+^\beta \right) D_\rho - \left(\frac{\dot{\mu}_-^{\beta\rho}}{d_-^3} v_-^\alpha + \frac{\dot{\mu}_+^{\beta\rho}}{d_+^3} v_+^\alpha \right) D_\rho \right]. \tag{B.26}$$

The energy-momentum tensor ${}_{1\mu}T_{\pi^0}^{\alpha\beta}$ can be then calculated accordingly. The total energy is again

$${}_{1\mu}\epsilon_{\pi^0} = - \int {}_{1\mu}T_{\pi^0}^{00}\, dV \tag{B.27}$$

and now represents the contribution from the magnetic moments alone.

To reach an approximate expression of (B.27) let us consider the field (B.26) on the x^3 axis. Ignoring the contribution from $\dot{\mu}^{\alpha\beta}$, we can write

$${}_{1\mu}F_{\pi^0}^{\alpha\beta}|_{x^3} \cong - \frac{\sigma}{\gamma^4 D^4 c} \left[(\mu_-^{\alpha\rho} + \mu_+^{\alpha\rho}) D_\rho D^\beta - (\mu_-^{\beta\rho} + \mu_+^{\beta\rho}) D_\rho D^\alpha \right]. \tag{B.28}$$

Again in this case

$${}_{1\mu}F_{\pi^0}^{\alpha\beta}\, {}_{1\mu}F_{\pi^0\alpha\beta} \cong 0, \tag{B.29}$$

and we have

$$\begin{aligned} {}_{1\mu}T_{\pi^0}^{\alpha\beta}|_{x^3} &\cong \frac{1}{4\pi}\, {}_{1\mu}F_{\pi^0}^{\alpha\rho}\, {}_{1\mu}F_{\pi^0\rho}^{\beta} \\ &\cong - \frac{q}{\pi j^8 D^8 c^2} (\bar{\mu}^{\sigma\rho} D_\rho \bar{\mu}_{\sigma j} D^j) D^\alpha D^\beta, \end{aligned} \tag{B.30}$$

where

$$\bar{\mu}^{\alpha\beta} = \mu_-^{\alpha\beta} + \mu_+^{\alpha\beta}. \tag{B.31}$$

Consequently,

$${}_{1\mu}T_{\pi^0}^{00}|_{x^3} \cong - \frac{q}{\pi \gamma^8 D^4 c^2} \bar{\mathbf{e}}^2 = - \frac{36\beta^2 |\mu|^2}{\pi \gamma^8 D^4 c^2}, \tag{B.32}$$

since

$$\bar{\mathbf{e}}^2 = (\mathbf{e}_- + \mathbf{e}_+)^2 = 4 \frac{\mu_0^2}{c^2} (\mathbf{v} \times \boldsymbol{\alpha})^2 = 4\beta^2 |\mu|^2. \tag{B.33}$$

Indicating with μ the magnitude of the parton magnetic moment we can write for the total energy

$${}_{1\mu}\epsilon_{\pi^0} \cong - \int {}_{1\mu}T_{\pi^0}^{00}|_{x^3}\, dV = \frac{144\beta^2\mu^2}{\gamma^8 c^2} \int_R^\infty \frac{dD}{D^2} = \frac{144\beta^2\mu^2}{\gamma^8 c^2 R} \tag{B.34}$$

an expression which is rather similar to the corresponding one for the charge contribution.

The "electromagnetic mass" due to the magnetic moments is then

$${}_{1\mu}m_{\pi^0} \cong - \frac{1}{c^2} \int {}_{1\mu}T_{\pi^0}^{00}|_{x^3}\, dV \cong \frac{144\beta^2\mu^2}{\gamma^8 c^4 R}. \tag{B.35}$$

Again it is interesting to note that also the above formula alone is able to recover the gravitational (or inertial rest) mass of π^0 for suitable values of the magnetic moment μ of the π^0 constituents, their orbital velocity v and the radius R of the orbit.

Clearly, in our full model inclusive of the charge and magnetic moment contributions, the electromagnetic mass is given by

$$m_{\pi^0} = - \frac{1}{c^2} \int {}_kT_{\pi^0}^{00}\, dV, \tag{B.36}$$

where ${}_kT_{\pi^0}^{00}$ is the energy density for one of the full electromagnetic field (3.54), and it includes both terms (B.20) and (B.35) together with mixed terms.

Since the expressions (B.20) and (B.35) are individually able to recover the rest mass of π^0, we can conclude by saying that the considered model is able to interpret the gravitational mass of the π^0 system as of entirely electromagnetic nature. This, however, is not necessarily the case in view of the lack of established data on the π^0 structure as well as the used "classical approximation," leaving in this way an unsettled alternative between the weak and the strong assumptions.

APPENDIX C. ON THE ENERGY RATE OF RADIATION

As is known [27], the energy rate of radiation (in Minkowski space) is proportional to the integral

$$I = \int T^{\alpha\beta} v_\alpha n_\beta\, d\sigma = \int F^{\alpha\mu} v_\alpha F_\mu{}^\beta n_\beta\, d\sigma, \tag{C.1}$$

where σ is a two-sphere proportional to the time axis and n_β is the unit normal to σ and to v_α.

Assume a reference frame for which

$$v^{\alpha}_{nm} = v^{\alpha} \equiv (\gamma c; 0)$$
$$|\mathbf{D}_{+}| = |D_{-}| = D. \tag{C.2}$$

Then after simple but tedious calculations one can show that

$${}_{3q}F^{\alpha\mu}_{\pi^0} v_{\alpha}\, {}_{3q}F^{\beta}_{\pi^0\mu} n_{\beta} = 0. \tag{C.3}$$

Since the integral (C.1) is invariant, we have $I = 0$ in any frame. Let us remark that the above result would not necessarily be true for $C_{nm} \neq \frac{1}{2}$.

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