First Proposal of the Universal Speed of Light by Voigt in 1887

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Voigt discussed Doppler's effect on the basis of the universal speed of light and the invariance of the wave equation in 1887. He was very close to suggesting a conceptual framework for special relativity. Historical remarks and discussions are made on Voigt's paper and a translation of his short paper with modern notation is also included.

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I. Introduction

Voigt's 1887 paper 'ON DOPPLER'S PRINCIPLE' [1] is a very remarkable work. It is remarkable because it contains several original and fundamental ideas of modern physics:
(a) Voigt appears to be the first physicist to conceive of the idea of the universal speed of light.
(b) He appears to be the first physicist who postulated the invariance of a physics law, the wave equation in 'an elastic incompressible medium' (i.e., the aether or ether),

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0, \tag{I1}$$

and employed it to derive the Doppler effect.
(c) He showed that the Doppler shift of frequency is incompatible with Newtonian absolute time ($t' = t$) and is in harmony with a 'relative time'

$$t' = t - \frac{V_a x}{c^2}, \tag{I2}$$

which is the approximate relativistic time.
(d) He first derived a type of 4-dimensional space-time transformation

$$x' = x - V_a t, \quad y' = y\sqrt{1 - \frac{V_a^2}{c^2}}, \quad z' = z\sqrt{1 - \frac{V_a^2}{c^2}}, \quad t' = t - \frac{V_a x}{c^2}, \tag{I3}$$

which differs from the Lorentz transformations by an overall constant factor $\sqrt{1 - V_a^2/c^2}$. According to Voigt, the transformations (I3) are for the (absolute) rest frame $F(ct, x, y, z) = F(0)$ and the frame $F'(ct', x', y', z') = F'(V_a)$ which moves with a constant (absolute) velocity V_a in the aether. The existence of such an aether was taken for granted at that time.

It is regrettable that Voigt's original ideas were unnoticed and, hence, did not play a role in the vigorous development of special relativity, somewhat similar to Poincaré's work [2] in 1905. Lorentz, Poincaré, Einstein and others did not refer to Voigt's paper in their works. It appears that young Pauli was an early physicist to mention Voigt's transformation (I3) in his book '*Theory of Relativity*', [3] published in 1921, but no further comment was made.

If physicists had been imaginative enough in the 1880s, they might have recognized the potential of these ideas to open up a virgin soil of physics. It is understandable that around 1880, when the ideas of Newtonian absolute time dominated the whole physics, people simply dismissed Voigt's ideas as nonsense.

In modern language, Voigt's result (I3) is a conformal 4-dimensional transformation which leaves the following space-time interval in $F(ct, x, y, z)$ and $F'(ct', x', y', z')$ frames invariant:

$$\begin{aligned} ds^2 &= c^2dt^2 - dx^2 - dy^2 - dz^2 = g_{\mu\nu}(V_a)dx'^{\mu}dx'^{\nu} \\ &= \left(\frac{1}{1 - V_a^2/c^2}\right)[c^2dt'^2 - dx'^2 - dy'^2 - dz'^2], \\ g_{\mu\nu}(V_a) &= \left(\frac{1}{1 - V_a^2/c^2}, \frac{-1}{1 - V_a^2/c^2}, \frac{-1}{1 - V_a^2/c^2}, \frac{-1}{1 - V_a^2/c^2}\right). \end{aligned} \tag{I4}$$

The conformal 4-dimensional transformation is a class of transformations of coordinates (w, x, y, z) for which the invariant interval $dw'^2 - dx'^2 - dy'^2 - dz'^2$ is proportional though generally not equal to $dw^2 - dx^2 - dy^2 - dz^2$. If one sets $dw = cdt$ and $dw' = cdt'$, then it also leaves the speed of light c invariant [4]. Weinberg has commented that the physical relevance of the conformal transformations in 4-dimensional spacetime is not yet clear [5].

One might think that Voigt's transformations differ from the Lorentz transformations in the second order $(V_a/c)^2$ and, therefore, it is excluded by precision experiments such as the Doppler shift experiment with a laser. However, this turns out to be not the case. (See Appendix.) It can be shown that his transformation (I3) leads to an observable Doppler effect which is identical to that of special relativity and consistent with precision laser experiments. The reason for this surprising result is that Voigt 's transformations are endowed with a 4-dimensional symmetry, called conformal 4-dimensional symmetry. Voigt's conformal 4-dimensional transformation is a special case in which the proportional constant is a function of the (absolute) velocity V_a of the inertial frame $F'(V_a)$, as one can see in equation (I4).

One may wonder why Voigt obtained the transformation (I3) by using the invariance of the wave equation (I1). We usually expect that if one uses the invariance of a physical law such as the wave equation (I1) and the linearity of coordinate transformations, one should obtain the Lorentz transformation [6].

Voigt wrote down the linear transformations between the two frames $F(ct, x, y, z)$ and $F'(ct', x', y', z')$ which moves with a constant velocity as follows:

$$\begin{aligned} x' &= m_1x + n_1y + p_1z - \alpha t, \quad \text{etc.} \\ t' &= t - (a_0x + b_0y + c_0z). \end{aligned} \tag{I5}$$

However, he chose $\alpha = V_a$ and set the coefficient of t in the last equation to be 1. Thus, he effectively imposed some conditions so that he then has the obvious Galilean relation $x' = x - V_at$

for his transformation in the x-direction. The other three transformations for y', z' and t' are not intuitively obvious, as shown in (I3). However, the transformation properties of y', z' and t' are dictated by the invariance of the wave equation (I1).

It was shown by Poincaré in 1905 that the Maxwell equations are invariant under the transformations [2]

$$x' = \kappa\gamma(x - V_a t), \quad y' = \kappa y \quad z' = \kappa z, \quad t' = \kappa\gamma\left(t - \frac{V_a x}{c^2}\right), \tag{I6}$$
$$\gamma = \frac{1}{\sqrt{1 - V_a^2/c^2}},$$

where, κ is an arbitrary constant. The set of all these transformations forms the conformal 4-dimensional symmetry group. The Voigt transformation (I3) is just a special case of (I6) with $\kappa = \sqrt{1 - V_a^2/c^2}$. So, it is not surprising that one cannot detect the absolute motion of the $F'(V_a)$ frame in the Voigt transformation (I3) by using electromagnetic and optical experiments.

One may think that kinematic properties of particles (with non-zero rest masses) in high energy experiments may be able to test and exclude Voigt's transformation. But it turns out to be not so easy, as demonstrated in the Appendix.

Voigt did not discuss the physical meaning of his 'relative time' in (I2) and the universal speed of light in his 1887 paper. Presumably, his understanding of these 'new ideas' was no better than that of Lorentz. As Dyson pointed out that 'when the great innovation appears, it will almost certainly be in a muddled, incomplete and confusing form. To the discoverer himself it will be only half-understood; to everybody else it will be a mystery. For any speculation which does not at first glance look crazy, there is no hope.' He explained that 'the reason why new concepts in any branch of science are hard to grasp is always the same; contemporary scientists try to picture the new concept in terms of ideas which existed before' [7]. This explains why Voigt imposed certain conditions on the coefficients of the linear transformations (I5) rather than letting the invariance of the wave equation (I1) to exercise its full power to guide his theory.

II. Historical remarks on Voigt's 1887 paper

II-1. H. A. Lorentz and Voigt

The correspondence between H. A. Lorentz and W. Voigt began in March 1883. In a letter Lorentz criticized a calculation which occured in one of Voigt's papers on crystal physics. In the years after 1883, there were only a few letters between the two physicists, but the number of letters increased as the years went by. Voigt met Lorentz for the first time on July 25, 1897 when he visited Göttingen.[1] From that day on, the content of the letters became more and more personal, as the two physicists became friends in the course of time.

After Michelson's experiment in 1881, Lorentz started to work on an explanation of the absence of any effect due to the motion of the earth though the ether. In 1886, he wrote a paper in which he criticised Michelson's experiment. He dismissed Michelson's experimental result being doubtful of its accuracy. Michelson was persuaded by Thomson and others to repeat the experiment and he did so with Morley, again reporting in 1887 that no effect had been found. In

[1]See Deutsches Museum München, Archives, HS 5523b

spring 1888, Lorentz sent some of his work on Michelson's experiment to Voigt. In March 1888, Lorentz sent another letter to Voigt. He read some of Voigt's calculations concerning the Michelson experiment and wanted to correct his own earlier remarks on Michelson's experiment in his letter. He asked Voigt for his opinion concerning the new calculations.[2] Voigt's paper '*Über das Doppler'sche Princip*' was not mentioned. But if one reads the correspondence between Lorentz and Voigt carefully, it becomes obvious that Lorentz did not know about the existence of Voigt's 1887 paper in 1888.

Lorentz's theory of the ether, the Lorentz transformations or the Michelson experiment did not play a role at all in the correspondence with Voigt in the years between 1889 and 1908, and after 1908. The reason might be the fact that Voigt was occupied by other things and that he did not have time to pay much attention to Lorentz's papers on these subjects.

However, one interesting letter from Lorentz to Voigt from July, 1908, can be found.[3]

> H. A. Lorentz to W. Voigt Leiden, July 30, 1908
>
> Dear friend,
>
> I would like to thank you very much for sending me your paper on Doppler's principle together with your enclosed remarks. I really regret that your paper has escaped my notice. I can only explain it by the fact, that many lectures kept me back from reading everything, while I was already glad to be able to work a little bit.
> Of course I will not miss the first opportunity to mention, that the concerned transformation and the introduction of a local time[4] has been your idea.
>
> Sincerely,
>
> Your H. A. Lorentz

As Lorentz had promised in his letter to Voigt, his book '*The Theory of Electrons*' [8] (the first edition of which had been published in 1909) contained in a footnote the statement:

> 'In a paper "Über das Doppler'sche Princip", published in 1887 (Gött. Nachr., p. 41) and which to my regret has escaped my notice all these years, Voigt has applied to equations of the form (6) (§3 of this book) a transformation equivalent to the formulae (287) and (288). The idea of the transformations used above (and in §44) might therefore have been borrowed from Voigt and the proof that it does not alter the form of the equations for the free ether is contained in his paper.'

Since Lorentz's book was based on his lectures at Columbia University in 1906, one might think that Lorentz knew Voigt's paper and mentioned his regret in the lectures [8]. However, based on their correspondences around that time, Lorentz's statement in the footnote was included in his book after July 1908.

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[2]See Deutsches Museum München, Archives, HS 5520

[3]See Deutsches Museum München, Archives, HS 5549 (Translation by the authors)

[4]It had been Lorentz's idea to distinguish between *true time* t, and a so-called *local time* t'.

II-2. Emil Wiechert and Voigt

We refer now to a paper by W. Schröder, published in the *Archive for History of Exact Sciences*, which contained detailed information about the relations between Lorentz, Voigt and Wiechert [9]. On November 28, 1911 Emil Wiechert asked Lorentz for a list of all of his works concerning the theory of relativity, because he was working on his paper '*Relativitätsprinzip und Aether*' [10]. Apparently it was important for Lorentz to mention Voigt's transformation in his response to Emil Wiechert, and to mention Voigt's priority in the development of the relativity transformation. In his response to Lorentz' letter, Wiechert mentioned the fact that the coefficient of t is one in Voigt's transformations and emphasized that it was an important progress which Lorentz made, namely, to dismiss the factor 1 in the transformation for the x-direction. In his response to Wiechert's letter, Lorentz mentioned that he originally considered the t' not to have any physical meaning at all. Lorentz claimed that Einstein was the first who discovered that the t' plays the role of the physical quantity 'time' [11].

II-3. Hermann Minkowski and Voigt

L. Pyenson suggested that Minkowski might have known Voigt's paper of 1887 since 1889. Minkowski remarked in a letter [12] to David Hilbert from June 19, 1889 deprecatingly that he had read an essay by Voigt for a jubilee [sic] of the University of Göttingen and that 'it was inconceivable that anyone would develop mathematical equations only with the hope that someone might later demonstrate their utility' [13]. Minkowski refered to the 150th anniversary of the University of Göttingen which was celebrated in 1887. If one considers that Voigt did not have other papers in 1887 which are of this nature, as mentioned by Minkowski, Pyenson's suggestion appears to be tenable.

A. Pais mentions that, many years later, Minkowski tried to draw attention to Voigt's 1887 paper in a physics meeting in 1908 [14]. Voigt made a modest remark without referring to his own suggestions (i.e., invariance of wave equation and universal constancy of the speed of light) made more than 20 years earlier. In the *Physikalische Zeitschrift* [15], one can read in the protocol of the discussion:

> Minkowski: '[...] I would like to mention that, historically, the transformations which play a role in the principle of relativity, have been first investigated mathematically by Voigt in 1887. Using these transformations, Voigt drew already in those days some conclusions concerning Doppler's principle.'
>
> Voigt: 'Mr Minkowski has one of my old works in mind. It is about some applications of Doppler's principle which occur in special cases, not because of the electromagnetic but because of the elastic theory of light. Already then some results were found which later were obtained from the electromagnetic theory.'[5]

II-4. Max Born and Voigt

There is no indication that Max Born had read Voigt's 1887 paper, even though Born was a physicist at Göttingen university. In his well known book '*Die Relativitätstheorie Einsteins*' [16] he mentions Voigt's transformations merely in a footnote:

[5]Translation by the authors.

> 'It is historically interesting to note that the formula for the transformation in a moving frame, which we call nowadays Lorentz transformation [...], was already mentioned by Voigt in 1877 [sic] in a dissertation, which was still based on the elastic ether theory of the light.'[6]

The missing factor in Voigt's transformation is not mentioned, and the year of publication is incorrect. Furthermore, Born's statement that "the formula for the transformation in a moving frame ...... was still based on the elastic ether theory of the light" missed the main idea of Voigt. Rather, Voigt's formula for the transformation in a moving frame ...... was based on the INVARIANCE OF THE WAVE EQUATIONS for the oscillations of an elastic incompressible medium. One can conclude that Born had not read Voigt's paper carefully or even did not read it when he published his book. Born, who studied in Göttingen under Voigt, does often mention his teacher in his autobiography [17], but not his transformations. However, Born's remarks shine some light on Prof. W. Voigt and the research at Göttingen university in the beginning of the twentieth century:

> '[...] We studied the papers of Lorentz, Poincaré and others about the difficulties, which confronted the theories concerning the electromagnetic ether because of Michelson's well known experiment. The experiment showed that the motion of the earth through the ether did not produce an ether wind, as the common sense and all the theories predicted, which were considered to be valid at that time. The existence of the ether was taken for granted at that time, and some scientists claimed that its properties would be better known than those of matter. Ether was the mostly used word in Voigt's lectures about optics. And now all experiments which should detect the existence of an ether led to a negative result. This was exciting, and I had the wish to concentrate myself on this field of research. Minkowski must have felt the same [...]'[7]

II-5. Arnold Sommerfeld and Waldemar Brückel

A. Sommerfeld also knew about Voigt's transformations. In 1943, the German engineer W. Brückel corresponded with Arnold Sommerfeld. Sommerfeld argues in a lette[8] to Brückel from September 1943 against Brückel's attempt to disprove the theory of relativity. Voigt's transformations are mentioned as a 'casual result' of Voigt's calculations. In his detailed response,[9] Brüeckel justifies his belief in the existence of the ether, referring to Voigt's transformations which make the theory of relativity dispensable.

In the following translation of Voigt's paper, we use some modern notations, e.g., c for the speed of light; Voigt used ω and κ for the speed of light and the velocity of the $F'(V_a)$ frame respectively. Also his notation $q = \sqrt{1 - V_a^2/c^2}$ was replaced by $1/\gamma_a$.

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[6]Translation by the authors.

[7]Translation by the authors.

[8]Deutsches Museum München, Sommerfeld Archives, NL 89, 015

[9]Deutsches Museum München, Sommerfeld Archives, HS 1977-28/A, 44

III. 'On Doppler's principle' by Voigt

The differential equations for the oscillations of an elastic incompressible medium are, as is generally known,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u,$$
$$\frac{\partial^2 v}{\partial t^2} = c^2 \Delta v, \tag{1}$$
$$\frac{\partial^2 w}{\partial t^2} = c^2 \Delta w,$$

in which c denotes the propagation velocity of the oscillations or, more precisely, the propagation velocity of plane waves with constant amplitude. It is assumed that u, v, w satisfy the relation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1'}$$

Let $u = U$, $v = V$, $w = W$ be solutions of these equations, which on a given surface $f(\bar{x}, \bar{y}, \bar{z}) = 0$ take on the time-dependent values $\bar{U}$, $\bar{V}$, $\bar{W}$. Then one can say that the functions U, V, W represent the physical law according to which the surface $f = 0$ shines.

If one exchanges in U, V, W, respectively,

$$\begin{aligned} x &\quad \text{with} \quad \xi = xm_1 + yn_1 + zp_1 - \alpha t, \\ y &\quad \text{with} \quad \eta = xm_2 + yn_2 + zp_2 - \beta t, \\ z &\quad \text{with} \quad \zeta = xm_3 + yn_3 + zp_3 - \gamma t, \\ t &\quad \text{with} \quad \tau = t - (a_0 x + b_0 y + c_0 z), \end{aligned} \tag{2}$$

and denotes the functions thus obtained as (U), (V), (W), respectively, then $u = (U)$, $v = (V)$, $w = (W)$ satisfy the equations in (1) as well.

For example, the first of these equations turns out to be:

$$\begin{aligned} \frac{\partial^2 (U)}{\partial \tau^2}(1 - c^2(a_0^2 + b_0^2 + c_0^2)) = c^2 \Bigg[& \frac{\partial^2 (U)}{\partial \xi^2}\left(m_1^2 + n_1^2 + p_1^2 - \frac{\alpha^2}{c^2}\right) \\ & + \frac{\partial^2 (U)}{\partial \eta^2}\left(m_2^2 + n_2^2 + p_2^2 - \frac{\beta^2}{c^2}\right) + \frac{\partial^2 (U)}{\partial \zeta^2}\left(m_3^2 + n_3^2 + p_3^2 - \frac{\gamma^2}{c^2}\right) \\ & + 2\frac{\partial^2 (U)}{\partial \eta \partial \zeta}\left(m_2 m_3 + n_2 n_3 + p_2 p_3 - \frac{\beta\gamma}{c^2}\right) \\ & + 2\frac{\partial^2 (U)}{\partial \zeta \partial \xi}\left(m_3 m_1 + n_3 n_1 + p_3 p_1 - \frac{\gamma\alpha}{c^2}\right) \\ & + 2\frac{\partial^2 (U)}{\partial \xi \partial \eta}\left(m_1 m_2 + n_1 n_2 + p_1 p_2 - \frac{\alpha\beta}{c^2}\right) \\ & - 2\frac{\partial^2 (U)}{\partial \tau \partial \xi}\left(a_0 m_1 + b_0 n_1 + c_0 p_1 - \frac{\alpha}{c^2}\right) \end{aligned}$$

$$-2\frac{\partial^2(U)}{\partial\tau\partial\eta}\left(a_0m_2+b_0n_2+c_0p_2-\frac{\beta}{c^2}\right)$$
$$-2\frac{\partial^2(U)}{\partial\tau\partial\zeta}\left(a_0m_3+b_0n_3+c_0p_3-\frac{\gamma}{c^2}\right)\Bigg]$$

and, since

$$\frac{\partial^2(U)}{\partial\tau^2}=c^2\left(\frac{\partial^2(U)}{\partial\xi^2}+\frac{\partial^2(U)}{\partial\eta^2}+\frac{\partial^2(U)}{\partial\zeta^2}\right)$$

must be true, this equation is satisfied if the following new equations hold as well:

$$\begin{aligned}1-c^2(a_0^2+b_0^2+c_0^2)&=m_1^2+n_1^2+p_1^2-\frac{\alpha^2}{c^2}\\&=m_2^2+n_2^2+p_2^2-\frac{\beta^2}{c^2}\\&=m_3^2+n_3^2+p_3^2-\frac{\gamma^2}{c^2},\end{aligned}\tag{3}$$

$$\begin{aligned}\frac{\beta\gamma}{c^2}&=m_2m_3+n_2n_3+p_2p_3,\\\frac{\gamma\alpha}{c^2}&=m_3m_1+n_3n_1+p_3p_1,\\\frac{\alpha\beta}{c^2}&=m_1m_2+n_1n_2+p_1p_2,\end{aligned}\tag{4}$$

$$\begin{aligned}\frac{\alpha}{c^2}&=a_0m_1+b_0n^1+c_0p_1,\\\frac{\beta}{c^2}&=a_0m_2+b_0n_2+c_0P_2,\\\frac{\gamma}{c^2}&=a_0m_3+b_0n_3+c_0p_3.\end{aligned}\tag{5}$$

Suppose α, β and γ are given, then we have 12 available constants, three of which are arbitrary. The most convenient way to obtain a solution is to use temporarily a coordinate system X_1, Y_1, Z_1, for which β and γ in equations (2) vanish and $\alpha=V_a$; i.e., a system, whose direction cosines of the X_1 axis with respect to X, Y, Z are proportional to α, β, and γ.

Furthermore, we set

$$m_h^2+n_h^2+p_h^2=q_h^2,\quad m_h/q_h=\mu_h,\quad n_h/q_h=\nu_h,\quad p_h/q_h=\pi_h,$$
$$a_0^2+b_0^2+c_0^2=d_0^2,\quad a_0/d_0=\mu,\quad b_0/d_0=\nu,\quad c_0/d_0=\pi,$$

then μ, ν, π are the direction cosines of four directions with respect to the coordinate system X_1, Y_1, Z_1, which we will denote as δ_1, δ_2, δ_3, δ. Introducing these parameters, our equations (3), (4) and (5) become

$$1-c^2d_0^2=q_1^2-\frac{V_a^2}{c^2}=q_2^2=q_3^2,\tag{3'}$$
$$\mu_2\mu_3+\nu_2\nu_3+\pi_2\pi_3=\mu_3\mu_1+\nu_3\nu_1+\pi_3\pi_1=\mu_1\mu_2+\nu_1\nu_2+\pi_1\pi_2=0,$$

$$\text{implying } \cos(\delta_2,\delta_3)=\cos(\delta_3,\delta_1)=\cos(\delta_1,\delta_2)=0, \tag{4'}$$

$$\mu\mu_1+\nu\nu_1+\pi\pi_1=\frac{V_a}{c^2 q_1 d_0},\quad \mu\mu_2+\nu\nu_2+\pi\pi_2=\mu\mu_3+\nu\nu_3+\pi\pi_3=0,$$

$$\text{implying } \cos(\delta,\delta_1)=\frac{V_a}{c^2 q_1 d_0},\quad \cos(\delta,\delta_2)=\cos(\delta,\delta_3)=0. \tag{5'}$$

According to (4') the three directions δ_1, δ_2, δ_3 are perpendicular to each other, according to (5') δ_1 and δ have the same directions. Therefore, we must have

$$\mu=\mu_1,\quad \nu=\nu_1,\quad \pi=\pi_1 \quad\text{and}\quad \frac{V_a}{c^2 q_1 d_0}=1. \tag{6}$$

Inserting this into (3') determines d_0 and q_1, q_2, q_3. Because only positive signs make sense here, we get first of all

$$q_1=1 \text{ or } \frac{V_a}{c},$$
$$d_0=\frac{V_a}{c^2} \text{ or } \frac{1}{c}.$$

I will only use the first solution, because the second solution is not interesting.[10] It follows from the first solution that:

$$d=\frac{V_a}{c^2},\ q_1=1,\ q_2=q_3=\sqrt{1-\frac{V_a^2}{c^2}}=\frac{1}{\gamma_a}. \tag{7}$$

According to this we can write the equations (2) as follows:

$$\begin{aligned}
\xi_1&=x_1\mu_1+y_1\nu_1+z_1\pi_1-V_at=a_1-V_at,\\
\eta_1&=(x_1\mu_2+y_1\nu_2+z_1\pi_2)\frac{1}{\gamma_a}=b_1\frac{1}{\gamma_a},\\
\zeta_1&=(x_1\mu_3+y_1\nu_3+z_1\pi_3)\frac{1}{\gamma_a}=c_1\frac{1}{\gamma_a},\\
\tau&=t-\frac{V_a}{c^2}(\mu_1x+\nu_1y+\pi_1z)=t-\frac{V_aa_1}{c^2},
\end{aligned} \tag{8}$$

where μ_h, ν_h, π_h do not have to satisfy any other conditions than those which come from the fact that they are the direction cosines of three directions which are normal to each other but otherwise arbitrary.

Therefore a_1, b_1, c_1 can be considered to be the coordinates of the point x_1, y_1, z_1 with respect to a coordinate system ABC whose axes point into the same directions as δ_1, δ_2, δ_3.

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[10] From the second solution follows $q_2=q_3=0$, therefore m_2, n_2, p_2, m_3, n_3, p_3 vanish as well, and therefore $\zeta=\eta=0$.

Any such system μ_h, ν_h, π_h yields a solution (U), (V), (W) from the given U, V, W. If on a surface $f(x, y, z) = 0$, U, V, W take on the given values $\bar{U}$, $\bar{V}$, $\bar{W}$, then on the surface $(f) = f(\bar{\xi}, \bar{\eta}, \bar{\zeta}) = 0$, (U), (V), (W) take on the values $(\bar{U})$, $(\bar{V})$, $(\bar{W})$ which can be derived from them. Because of the values of ξ_1, η_1, ζ_1 the surface has the property that it is moving with a constant velocity V_a parallel to the direction δ_1 or A which is given by the direction cosines μ_1, ν_1, π_1. Therefore the solutions (U), (V), (W) yield the physical laws, according to which certain moving surfaces are shining, if they fulfil the additional condition

$$\frac{\partial(U)}{\partial x} + \frac{\partial(V)}{\partial y} + \frac{\partial(W)}{\partial z} = 0.$$

The two surfaces $f = 0$ and $(f) = 0$ are in their form identical only if $1/\gamma_a = 1$, i.e., if V_a is so small compared to c that V_a^2 can be neglected compared to c^2. If this is the case, then they differ only in their position relative to the coordinate axes. By a suitable choice of the arbitrary constants and the functions U, V and W, one can obtain special cases that are easily visualized. By a transformation of coordinates, one can obtain more general cases in which the shift of the surface is not only parallel to the A-axis but points into an arbitrary direction.

We follow up the special case that the three directions δ_1, δ_2, δ_3 point in the directions of the three coordinate axes X_1, Y_1, Z_1, i.e.

$$\begin{aligned} &\mu_1 = \nu_2 = \pi_3 = 1, \\ &\mu_2 = \mu_3 = \nu_1 = \nu_3 = \pi_1 = \pi_2 = 0. \end{aligned} \tag{9}$$

Then we obtain simply, and formally identical with (8):

$$\begin{aligned} \xi_1 &= x_1 - V_a t, \\ \eta_1 &= y_1 \frac{1}{\gamma_a}, \\ \zeta_1 &= z_1 \frac{1}{\gamma_a}, \\ \tau &= t - \frac{V_a x_1}{c^2}, \end{aligned} \tag{10}$$

where $1/\gamma_a = \sqrt{1 - \frac{V_a^2}{c^2}}$.

In this case the condition (1') has the form

$$\left(1 - \frac{1}{\gamma_a}\right) \frac{\partial(U)}{\partial \xi} = \frac{V_a}{c^2} \frac{\partial(U)}{\partial \tau},$$

which can be exchanged without any difficulties with

$$\left(1 - \frac{1}{\gamma_a}\right) \frac{\partial U}{\partial x} = \frac{V_a}{c^2} \frac{U}{\partial \tau}, \tag{10'}$$

This means that in U the arguments x and t are only allowed to appear in the combination $(1 - 1/\gamma_a)t + V_a x/c^2$ or not at all. The latter is the case if $U = 0$, i.e., if the propagated oscillations are normal to the direction of translation of the shining surface everywhere. If one goes from the special coordinate system X_1, Y_1, Z_1 to the more general X, Y, Z which is connected to the other by the relations

$$\begin{aligned} x_1 &= x\alpha_1 + y\beta_1 + z\gamma_1, \\ y_1 &= x\alpha_2 + y\beta_2 + z\gamma_2, \\ z_1 &= x\alpha_3 + y\beta_3 + z\gamma_3, \end{aligned} \tag{11}$$

one obtains finally

$$\begin{aligned} \xi &= x\frac{1}{\gamma_a} + (x\alpha_1 + y\beta_1 + z\gamma_1)\alpha_1\left(1 - \frac{1}{\gamma_a}\right) - V_a\alpha_1 t, \\ \eta &= y\frac{1}{\gamma_a} + (x\alpha_1 + y\beta_1 + z\gamma_1)\beta_1\left(1 - \frac{1}{\gamma_a}\right) - V_a\beta_1 t, \\ \zeta &= z\frac{1}{\gamma_a} + (x\alpha_1 + y\beta_1 + z\gamma_1)\gamma_1\left(1 - \frac{1}{\gamma_a}\right) - V_a\gamma_1 t, \\ \tau &= t - \frac{V_a}{c^2}(x\alpha_1 + y\beta_1 + z\gamma_1). \end{aligned} \tag{12}$$

This is the general form (2) from which we started, but with constants which are completely determined by V_a, α_1, β_1, γ_1. It contains what we normally call the 'Doppler principle', if it is valid.

If we can neglect V_a^2 against c^2, then we have $1/\gamma_a = 1$ and obtain very straightforwardly

$$\begin{aligned} \xi &= x - V_a\alpha_1 t, \\ \eta &= y - V_a\beta_1 t, \\ \zeta &= z - V_a\gamma_1 t, \\ \tau &= t - \frac{V_a}{c^2}(x\alpha_1 + y\beta_1 + z\gamma_1). \end{aligned} \tag{13}$$

Here the condition (1') has the form

$$0 = \frac{V_a}{c^2}\frac{\partial}{\partial t}(U\alpha_1 + V\beta_1 + W\gamma_1), \tag{13'}$$

and has, with the above approximation, to be fulfilled as far as the terms to first order in V_a/c. If in addition to the shining surface the observer is also moving, for example with the constant velocity V_a' in a direction given by the direction cosines α', β', γ', then the shifts u, v, w are only relative to a coordinate system X', Y', Z' which is moving with the observer. Hence in (12) or (13) x has to be exchanged with $x' + V_a'\alpha' t$, y with $y' + V_a'\beta' t$, z with $z' + V_a'\gamma' t$. We are now going to look at some applications of what we have found.

[[1]] Suppose a plane parallel to the YZ plane is being excited to oscillate according to the law

$$\overline{W} = A \sin \frac{2\pi t}{T},$$

then the motion propagating in positive X-direction is given by

$$W = A \sin \frac{2\pi}{T} \left(t - \frac{x}{c} \right).$$

If we make in here the substitution according to (10) we get:

$$(W) = A \sin \frac{2\pi}{T} \left(1 + \frac{V_a}{c} \right) \left(t - \frac{x}{c} \right).$$

This yields for $x = V_a t$:

$$(\overline{W}) = A \sin \frac{2\pi t}{T} \left(1 - \frac{V_a^2}{c^2} \right) = A \sin \frac{2\pi t}{T'}. \tag{14'}$$

Therefore, we have an oscillating (shining) plane with an oscillation period $T' = T/(1 - V_a^2/c_2)$ (differing from T only in a second order term). The transmitted oscillation can be written as

$$(W) = A \sin \frac{2\pi}{T'(1 - V_a/c)} \left(t - \frac{x}{c} \right). \tag{14}$$

Thus the propagated wave has a period of oscillation which is decreased by a factor of $(1 - V_a/c)/1$. If the observer is also moving, we have:

$$\begin{aligned}(W') &= A \sin \frac{2\pi}{T'(1 - V_a/c)} \left(t - \frac{x' + V_a' t}{c} \right) \\ &= A \sin 2\pi \left(t \frac{c + V_a'}{T'(c - V_a)} - \frac{x'}{T'(c - V_a)} \right).\end{aligned}$$

This formula yields the ‘Doppler principle’ for plane waves. However it is not generally valid, but requires a wave plane with a constant amplitude throughout.

[[2]] Suppose the same plane is being excited to oscillate according to the law:

$$\overline{W} = A \exp \left[(\mu y + \nu z) \frac{2\pi}{Tc} \right] \sin \frac{2\pi t}{T},$$

which is similar to what occurs when a wave with an originally constant amplitude has been transmitted through a prism made of an absorbing material. Then the transmitted wave is

$$W = A \exp \left[(\mu y + \nu z) \frac{2\pi}{Tc} \right] \sin \frac{2\pi t}{T} \left(t - \frac{x\sigma}{c} \right),$$

where $\sigma = \sqrt{1+\mu^2+\nu^2}$.
If one substitutes according to (10) and sets $\sqrt{1-V_a^2/c^2} = 1/\gamma_a$, one obtains:

$$(W) = A\exp\left[\frac{(\mu y+\nu z)\frac{1}{\gamma_a}2\pi}{Tc}\right]\sin\frac{2\pi t}{T}\left[t\left(1+\frac{V_a\sigma}{c}\right)-x\left(\frac{\sigma}{c}+\frac{V_a}{c^2}\right)\right].$$

This yields for $x = V_a t$, if one writes $\mu\gamma_a = \mu'$, $\nu\gamma_a = \nu'$:

$$(\overline{W}) = A\exp\left[\frac{2\pi(\mu' y+\nu' z)}{T'c}\right]\sin\frac{2\pi t}{T'},$$

where again $T' = T/(1-V_a^2/c^2)$. Therefore, we have a plane which is oscillating and moving at the same time; the propagated displacement can be written as follows:

$$(W) = A\exp\left[\frac{2\pi(\mu' y+\nu' z)}{T'c}\right]\sin\left[\frac{2\pi}{T'}\left(t\frac{1+V_a\sigma/c}{1-V_a^2/c^2}-x\frac{\sigma/c+V_a/c^2}{1-V_a^2/c^2}\right)\right], \tag{15}$$

where now

$$\sigma = \sqrt{1+(\mu'^2+\nu'^2)\left(\frac{1}{\gamma_a}\right)^2}.$$

We notice that laws hold here that are quite different from those given by the ‘Doppler principle’, even if we restrict ourselves to the first approximation and neglect V_a^2/c^2 compared to 1.

[[3]] If the shining surface is a very small sphere of the radius R, which is oscillating around the X-axis according to the law for the rotation angle

$$\overline{\psi} = A\sin\frac{2\pi t}{T},$$

then the propagated rotations ψ at distance $r = \sqrt{x^2+y^2+z^2}$ from the center of the sphere are given by[11] :

$$\begin{aligned}\psi &= \frac{R^3 A}{r^3}\left[\sin\frac{2\pi}{T}\left(t-\frac{r-R}{c}\right)+\frac{2\pi(r-R)}{Tc}\cos\frac{2\pi}{T}\left(t-\frac{r-R}{c}\right)\right]\\ &= \frac{R^3 A}{r^3}\sqrt{1+\left(\frac{2\pi(r-R)}{Tc}\right)^2}\cos\frac{2\pi}{T}\left(t-\frac{r-R}{c}-\eta\right),\end{aligned} \tag{16}$$

where

$$\frac{2\pi(r-R)}{Tc} = \text{ctg}\frac{2\pi\eta}{T}.$$

[11] W. Voigt, Crelles Journ. Vol. 89, 298.

Thus for $r = R$ and $\eta = 0$, we have $\eta = T/4$ if r is very large compared to the wavelength Tc. The transmitted displacements result from ψ as follows:

$$U = 0, \quad V = -\psi z, \quad W = \psi y.$$

We set:

$$U = 0, \quad V = MC, \quad W = NC.$$

If one substitutes the values ξ, η, ζ for $x,\ y, z$ according to (10), then the periodic part C becomes:

$$(C) = \cos\frac{2\pi}{T}\left[t - \frac{V_a x}{c^2} - \frac{1}{c}\left(\sqrt{(x - V_a t)^2 + y^2 + z^2} - R\right) - (\eta)\right] \tag{17}$$

if

$$\mathrm{ctg}\frac{2\pi(\eta)}{T} = \frac{2\pi}{Tc}\left(\sqrt{(x - V_a t)^2 + y^2 + z^2} - R\right).$$

For $(x - V_a t)^2 + y^2 + z^2 = R^2$ (i.e. on the surface of a sphere which is displaced parallel to the X-axis with velocity V_a) this becomes

$$(\overline{C}) = \sin\frac{2\pi}{T}\left[t\left(1 - \frac{V_a^2}{c^2}\right) - \frac{V_a}{c^2}\sqrt{R^2 - y^2 - z^2}\right].$$

We assumed that V_a^2/c^2 and $V_a R/c^2$ are of the second order. Hence we get

$$(\overline{C}) = \sin\frac{2\pi t}{T}.$$

(M) and (N) have the same value as if the small sphere were oscillating around the equilibrium position $x_0 = V_a t$ reached at the time t. (U), (V), (W) therefore denote the motion caused by a rotating shining point that moves with constant speed V_a parallel to the rotation axis.

The propagated wave planes are characterized by the value (17) for (C), which can be written in the relative coordinates with respect to the shining point $\xi = x - V_a t$, $y = \eta$, $z = \zeta$, neglecting V_a^2/c^2 compared to 1 and for $r \gg Tc$:

$$(C) = \cos\frac{2\pi}{T}\left[t - \frac{V_a \xi}{c^2} - \frac{1}{c}\left(\sqrt{\xi^2 + \eta^2 + \zeta^2} - R\right)\right].$$

Therefore the wave surfaces are spheres, but not around the shining point but rather around a position which lies a distance of V_a/c times the radii of the wave surfaces away from the shining point in the direction which is opposite to that of the motion.

Since the direction of a light source as perceived by an observer at rest is given by the normal to the wave surface, such an observer would see the shining point at a position where it was located at an earlier time r/c. In other words: If his radius vector r had an angle ϕ with respect to the direction of motion, he would observe an 'Aberration' of the magnitude $(r/c)\sin\phi$ in the direction opposite to the direction in which the point is moving. According to the above, the transmitted amplitudes (M) and (N) at position x, y, z at time t have such values, as if the shining point had been all the time at a position which it only reached at time t. However the wave surface in x, y, z, has a form as if the shining point had remained at rest at the position it had reached at time $[t - r/c]$. In that sense wave surface and amplitude do not belong together as in the case of a point at rest. The amplitude depends on the current position, the form of the wave plane depends on a position the shining point has left behind. We obtain a strange result: It a light source with a constant intensity is located at a distance r away from an observer at time t (and moving in the manner described above), this observer would see the source in a distance away from him when the source was located at time r/c, but the intensity he would see, would be the one of its actual position (i.e., r) (which can be either larger or smaller).

The applicability of the above considerations to problems in the field of optics is restricted by the constraint (1'), which has led us to the formulas (10') and (13').

Such a restriction does not occur if we look at the corresponding problems in the field of acoustics of fluids, because the only condition for the propagated dilatation δ is

$$\frac{\partial^2 \delta}{\partial t^2} = c^2 \Delta \delta.$$

Therefore, if δ is given by the constraints along a given surface as an arbitrary function of time, the introduction of one of the substitutions (10), (12) or (13) always gives the transition from the action of a sound source at rest to a sound source in motion. If, for example, on a very small sphere of radius R, $\overline{\delta} = f(t)$ is given, then the propagated dilation is obtained as

$$\delta = \frac{R}{r} f\left(t - \frac{r - R}{c}\right).$$

The substitution (10) gives the influence of a translation of the 'sounding sphere' parallel to the X-axis. The discussion of the result is analogous to the one we considered in [[3]].

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V. Appendix

One may wonder: What are the basic postulates of Voigt's theory with the transformations (I3) for spacetime of inertial frames. Voigt did not discuss his postulates clearly and completely. It is interesting and instructive to compare basic postulates in Einstein's special relativity in his 1905 paper and Voigt's theory:

Einstein postulated the invariance of physical laws and the universal speed of light; while Voigt used the invariance of wave equation (I1) and the universal speed of light. The reason why Voigt obtained (I3) rather than the Lorentz transformations (I6) with $\kappa = 1$ is given in equation (I5) and the explanations following it. In other words, the invariance of the wave equation (I1) can only lead to a conformal transformation (I6) with an arbitrary constant κ. Voigt imposed some extra conditions to fix κ to be $\kappa = \sqrt{1 - V_a^2/c^2}$, as explained before. Thus it is inadequate to use the invariance of the wave equation (I1) as a basic postulate for Voigt's theory. The basic reason for this is that an inertial frame $F'(V_a)$ moving with a velocity V_a is not completely equivalent to an inertial frame F at rest due to the fact that the existence of an aether was taken for granted, as one can see from (I4) or the expression for a particle's momentum in equation (A6) below. The difference between F and $F'(V_a)$ must be explicitly postulated so that their transformations can be determined.

Suppose one makes the following two basic postulates:

(I) The laws of physics are conformal 4-dimensional invariant with the metric tensor,

$$g_{\mu\nu}(V_a) = \left(\frac{1}{1-V_a^2/c^2}, \frac{-1}{1-V_a^2/c^2}, \frac{-1}{1-V_a^2/c^2}, \frac{-1}{1-V_a^2/c^2}\right). \tag{A1}$$

(II) The speed of light is a universal constant.

These two postulates imply the invariant conformal 4-dimensional interval (I4), which is the law of motion of a free particle with a mass $m > 0$ (for $ds^2 > 0$) [18] and leads to the Voigt transformations (I3) unambiguously.

To complete the physical implications of the postulate (I), we must interpret the metric tensor $g_{\mu\nu}(V_a)$ associated with an inertial frame $F'(V_a)$ moving with an absolute velocity V_a in the aether as follows:

The spacetime coordinates x^μ is a contravariant vector and satisfies (I4). Thus, $\Delta t'$ of a clock or $\Delta x'$ of a rod at rest in $F'(V_a)$ and measured by the $F'(V_a)$ observer is inherently contracted by a factor $\sqrt{1-V_a^2/c^2}$ with respect to the corresponding Δt of a clock or Δx of a rod at rest in F and measured by the F observer,

$$\Delta x|_{\text{at rest in } F} = \frac{1}{\sqrt{1-V_a^2/c^2}}\Delta x'|_{\text{at rest in } F'(V_a)}, \tag{A2}$$

$$\Delta t|_{\text{at rest in } F} = \frac{1}{\sqrt{1-V_a^2/c^2}}\Delta t'|_{\text{at rest in } F'(V_a)}, \tag{A3}$$

The changes of $\Delta x'$ and $\Delta t'$ in (A2) and (A3) are absolute changes, not changes relative to some observers. For a covariant vector such as the wave 4-vector k_μ or the momentum p_μ, there is a similar interpretation. For example, the frequency $\omega' = k_0'c$ of a radiation emitted from a source at rest in $F'(V_a)$ and measured by the $F'(V_a)$ observer is related to the frequency $\omega = k_0 c$ of radiation emitted from an identical source (e.g., the same atom making the same transition) at rest in F and measured by the F observer,

$$\omega|_{\text{at rest in } F} = \sqrt{1-V_a^2/c^2}\;\omega'|_{\text{at rest in } F'(V_a)}. \tag{A4}$$

The reason for the difference between (A3) and (A4) is that a covariant vector k_μ' satisfies $g^{\mu\nu}(V_a)k_\mu' k_\nu' =$ invariant rather than (I4) which is for a contravariant vector.

It must be stressed that the relations in (A2) - (A4) refer to physical situations in which two different observers (one in F and one in $F'(V_a)$) measure two different objects or quantities. Their relations has nothing to do with the transformation (I3) which strictly refer to the situation in which two different observers measure the same object or the same quantity in spacetime.

Now, let us consider the kinematics of particles in Voigt's theory. The invariant action in a moving frame $F'(V_a)$ is assumed to be

$$S = -mc \int ds, \tag{A5}$$

which leads to the following covariant 4-momentum in the moving frame $F'(ct', x', y', z')$ [19]

$$p'_\mu = \frac{\partial S}{\partial x'^\mu} = \frac{1}{\sqrt{1 - V_a^2/c^2}} \frac{m v'_\mu}{\sqrt{1 - v'^2/c^2}}, \tag{A6}$$

where $v'_\mu = dx'_\mu/dt'$ (which is not a 4-vector) and $v'^2 = v'^2_x + v'^2_y + v'^2_z$. The 4-momentum p'_μ satisfies

$$g^{\mu\nu}(V_a) p'_\mu p'_\nu = m^2 c^2, \tag{A7}$$

Suppose one does high energy experiments in the moving frame $F'(V_a)$. One cannot use the conservation of 4-momentum in, say, a 2-body collision process $a(k'_1) + b(k'_2)$ to $c(p'_1) + d(p'_2)$ in the moving frame $F'(V_a)$

$$k'_{1\mu} + k'_{2\mu} = p'_{1\mu} + p'_{2\mu}, \tag{A8}$$

to detect the 'absolute velocity' V_a of the $F'(V_a)$ frame, because the constant factors $\sqrt{1 - V_a^2/c^2}$ are cancelled on both sides of the equation. Furthermore, one also cannot use the invariant relation (A7) to detect V_a because the factors $(1 - V_a^2/c^2)$ in $g^{\mu\nu}(V_a)$ cancel precisely the same factors $(1/\sqrt{1 - V_a^2/c^2})^2$ in $p'_\mu p'_\nu$.

One might think that the Doppler frequency shift derived by Voigt,

$$\omega = \omega'(1 - V_a/c), \tag{A9}$$

can be used to exclude Voigt's transformation (I3) because experimentally the shift can be measured to the second order $(V_a/c)^2$. Let us consider experiments designed to measure the Doppler shift of frequency (A9) carefully.

Suppose one performs the experiment in the $F'(V_a)$ frame and that the atoms are at rest in $F(0)$. In practice, one cannot know the frequency $\omega'|_{\text{at rest in } F}$ of the light emitted by the atoms as measured by observers in the $F(0)$ frame. One can only compare the shifted ω' with the unshifted quantities $\omega'|_{\text{at rest in } F'(V_a)}$ associated with the same kind of atoms at rest in the laboratory frame $F'(V_a)$. Since $F'(V_a)$ and $F(0)$ are not equivalent, the frequency of light emitted by atoms at rest in $F'(V_a)$ and measured by observers in $F'(V_a)$ are not the same as those emitted by the same kind of atoms at rest in $F(0)$ and measured in $F(0)$. Thus, one does not have the usual relations in special relativity, i.e. $\omega|_{\text{at rest in } F} = \omega'|_{\text{at rest in } F'}$. Rather, one has the relation in (A4). It follows from (A4) and (A9) that the observable Doppler frequency shift measured in the moving frame $F'(V_a)$ is given by:

$$\omega' = \omega'_0 \sqrt{\frac{1 + V_a/c}{1 - V_a/c}}, \qquad \omega'_0 = \omega'|_{\text{at rest in } F'(V_a)} \tag{A10}$$

which is exactly the same as that in special relativity. Therefore, we see that Voigt's theory is consistent with laser experiment of Doppler shift. One can verify that Voigt's theory is also consistent with the Michelson-Morley experiment. and the Fizeau experiment.

It appears that somehow the conformal 4-dimensional symmetry protects Voigt's theory by hiding the effect due to the 'absolute' velocity of the $F'(V_a)$ frame. Thus it seems as if there is a 'conspiracy,' so that the effect due to the 'absolute' velocity are cancelled to all orders in V_a/c. However, we stress that such a 'conspiracy' is not dynamical (i.e., not due to interactions between matter and the aether as Poincare believed [20]). Rather, the conspiracy is 'kinematical', namely, it is due to the inherent property of the conformal 4-dimensional symmetry of Voigt's theory.

Note: Voigt's transformations (I3) were discussed by A. G. Gluckman (Am. J. Phys. **36**, 226 (1968)). He stated that the Voigt transformation (I3) "does not form a group" and that "the Voigt transformation would yield $mv^2/2$ but it would also be impossible to arrive at the result $E = mc^2$." These staatcments are incorrect because of inappropriate interpretations and formulations. Within the conceptual framework of Voigt, an inertial frame $F(V_a)$ is associated with an absolute velocity V_a, where $c > V_a \geq 0$. The transformation (I3) is only a special case because one of the frame is at rest, $V_a = 0$. In order to see the group properties, one must consider the transformation between two moving frames in general. The Voigt transformation (I3) implies that the trnasformation between two moving frames $F'(\beta')$ and $F''(\beta'')$, where $\beta' = V'_a/c$ and $\beta'' = V''_a/c$, is given by

$$w^{*\prime} = \Gamma'(w^{*\prime\prime} + B'w^{*\prime\prime}),$$
$$x^{*\prime} = \Gamma'(x^{*\prime\prime} + B'x^{*\prime\prime}, \quad y^{*\prime} = y^{*\prime\prime}, \quad z^{*\prime} = z^{*\prime\prime},$$

where

$$B' = \frac{\beta'' - \beta'}{1 - \beta''\beta'}, \quad \Gamma' = \frac{1}{\sqrt{1 - B'^2}}$$
$$(w^{*\prime}, x^{*\prime}, y^{*\prime}, z^{*\prime}) = \frac{1}{\sqrt{1 - \beta'^2}}(w', x', y', z'),$$
$$(w^{*\prime\prime}, x^{*\prime\prime}, y^{*\prime\prime}, z^{*\prime\prime}) = \frac{1}{\sqrt{1 - \beta''^2}}(w'', x'', y'', z'').$$

One can show that the set of the above Voigt transformations forms the "Voigt group", i.e., a 4-dimensional conformal group with 2 parameters, V'_a and V''_a, which characterize the absolute motion of inertial frames F' and F'' respectively in the aether. One should employ covariant formulation with the invariant interval ds given in (I4) to write down the "invariant action" for, say, a charged particle moving in the electromagnetic potential A_u:

$$S = \int \left[-mcds - \frac{e}{c}A'_\mu dx'^\mu - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu}\sqrt{-det g_{\alpha\beta}}d^4z' \right],$$
$$F'_{\mu\nu} = \partial'_\nu - \partial'_\nu A'_\mu, \quad \partial'_\nu = \partial/\partial x'^\nu,$$

in a general frame $F'(V'_0)$. The energy of a free particle (when $A'_\mu = 0$) is given by

$$E'_0 = cp'_0 = \frac{1}{\sqrt{1 - (V'_a/c)^2}} \frac{mc^2}{\sqrt{1 - (v'/c)^2}},$$

where ν' is the velocity of the particle as measured in the $F'(V_a')$ frame. Thus, for the special case $V_a' = v' = 0$ one has the result for the energy $E_0 = mc^2$.

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[11] Poincaré's role in the understanding of the time t' in the Lorentz transformation was usually ignored. However, some physicists did pay attention to Poincaré's contribution in this aspect. For example, Feynman said in his public lecture at Cornell University: "Einstein realized, and Poincare too, that the only possible way in which a person moving and a person standing still could measure the speed (of light) to be the same was that their sense of time and their sense of space are not the same, that the clocks inside the space ship are ticking at a different speed from those on the ground, and so forth." This appears to be a more balanced view. R. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, MA 1965). p. 92. See also J. P. Hsu, *Einstein's Relativity and Beyond - New Symmetry Approaches* (World Scientific, Singapore/New Jersey, 2000) p. 56 and pp. 72-74.

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on Doppler's principle and on his space and time transformation around July, 1908. As a result, Minkowski appreciated Voigt's original idea and made an effort to draw attention to Voigt's 1887 paper in the physics meeting in September 1908.

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