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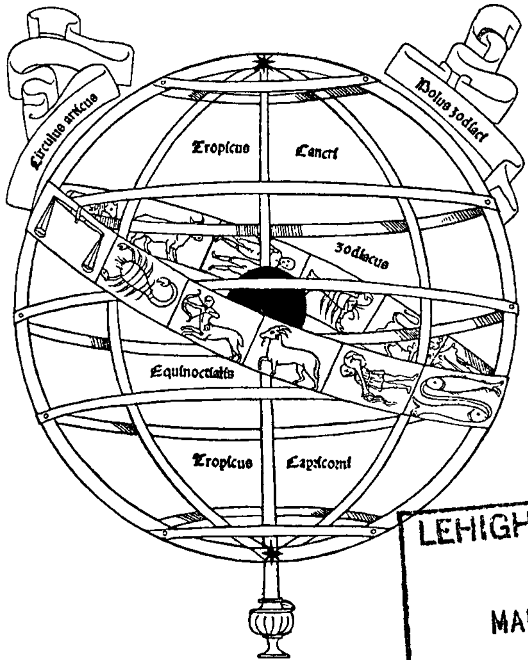

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LINES OF MATHEMATICAL THOUGHT IN THE ELECTRODYNAMICS OF AMPÈRE

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SUMMARY — Between 1820 and 1827 Ampère devoted the bulk of his research effort to the experimental, physical, chemical and mathematical aspects of electromagnetism, especially «electrodynamics» (his word), concerning the mutual action of two wires carrying current. This paper is concerned with the mathematical features.

Ampère progressed through an interesting sequence of stages of desimplification: from a trigonometric expression for the action between two straight infinitesimal elements through a differential expression of curved elements to a line integral for an element acting upon a wire in space. He also mathematicised his conception of a magnetic dipole as an electrical «solenoid» (his word again). This last idea was introduced in connection with his reductionistic view that magnetism was a special case of electricity; and in this connection he had to confront Poisson's completely different theory of a magnetic body. By means of a remarkable theorem on the transformation of areal integrals and their conversions into contour integrals, he achieved the required reduction of Poisson's theory to his own talk of solendoidal action.

1. Enter the physicist

> My three great things, the attractions, the projections, the areas.
>
> A.-M. Ampère, undated manuscript note [1]

Between 1820 and 1827 André-Marie Ampère (1775-1836) devoted

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[1] Ampère's *Nachlass* is kept in the archives of the *Académie des Sciences* (Paris); this note is in *chemise* (that is, file) 172. This quotation is published in J. Joubert (ed. for *La Société Française de Physique*), *Mémoires sur l'électrodynamique*, 2 vols., Paris, Gauthier-Villars, vol. II, p. 37; in later footnotes this edition is cited as 'Joubert'.

the majority of his research effort to the the study of the interaction between a current-carrying electrical wire and a magnet, and especially the action between wires. The first topic was called 'electromagnetism' by Oersted, soon after his announcement of the effect in 1820;[2] the second was named 'electrodynamics' by Ampère himself, after his rapid discovery of it upon hearing of Oersted's work. From then on for the next seven years Ampère worked intensively on (chiefly) electrodynamics, from the theoretical, experimental, chemical and mathematical aspects. This paper is concerned with the last of these aspects,[3] which (as so often in the practise of the supposed 'history' of science) has been either seriously neglected or poorly handled.[4] Even the other aspects have been examined in historical detail only in recent years.[5]

One of the most daunting features of the story facing the historian is the extraordinary chaos of Ampère's publications. He wrote one book (which itself appeared in two different forms), and around 60 papers and notes, many of which came out more than once in revised printings, yielding around 100 publications in all. Thus the chronology of his work is fine-grained, and hard to establish. The task is not aided

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[2] H. C. OERSTED, *Neuere electro-magnetische Versuche*, «J. Chem. Phys.», XXIX, 1820, pp. 364-369; also in *Scientific Papers*, 3 vols., Copenhagen, ed. by K. Meyer 1920, vol. II, pp. 219-223. French translations appeared in «Bibl. univ.», XV, 1820, pp. 137-141; and in «J. Phys. Chim. Hist. Nat.», XCI, 1820, pp. 78-80. For his first paper, see footnote 8 below.

[3] For a rather more extended account, and set in the context of all mathematical developments in France at the time, see ch. 14 of my *Convolutions in French Mathematics, 1800-1840. From the Calculus and Mechanics to Mathematical Analysis and Mathematical Physics*, 3 vols., Basel, Birkhäuser and Berlin (DDR), Deutscher Verlag der Wissenschaften 1990: cited hereafter as *Convolutions*. A short summary of the mathematical aspects can be found in R. REIFF, A. J. W. SOMMERFELD, *Standpunkt der Fernwirkung. Die Elementargesetze*, in *Enc. der math. Wiss.*, vol. V, sec. 2, 3-62 (article V, 12, dated 1912).

[4] On the general neglect of the history of mathematics in the history of science, see my *Does History of Science Treat of the History of Science? The Case of Mathematics*, «History of Science», XXVIII, 1990, to appear, pp. 149-173.

[5] The most significant source to date is C. BLONDEL, *A.-M. Ampère et la création de l'électrodynamique*, Paris 1982. On the very early stages of Ampère's work, see also her *Sur les premières recherches du formule électrodynamique par Ampère (octobre 1820)*, «Rev. hist. sci.», XXXI, 1978, pp. 53-65; and compare it with L. P. WILLIAMS, *What were Ampère's earliest Discoveries in Electrodynamics?*, «Isis», LXXIV, 1983, pp. 492-508. On the experimental side, especially recommended is S. DEVONS, *The Search for Electromagnetic Action*, «Phys. Teacher», XVI, 1978, pp. 625-631; see also footnote 12.

For the religious and aetherian side, see K. L. CANEVA, *Ampère, the Etherians, and the Oersted Connexion*, «Brit. J. Hist. Sci.», XIII, 1980, pp. 121-138. E. T. WHITTAKER, *History of the Theories of Aether and Electricity. The Classical Theories*, London, Nelson 1951 has a limited amount of information, set in the broader context of aetherian theories and electricity.

by Ampère's own reminiscences, which are sometimes demonstrably inaccurate in dates and occasionally inconsistent in details even when they were written soon after the (alleged) events. I shall not handle these matters here, but cite only a first main paper in which a principal result was published.[6] I shall also not refer to the many pertinent manuscripts, which are kept in his *Nachlass* in the archives of the *Académie des Sciences.* [7]

A few preliminary points will be recorded here. Firstly, while the initial reaction to Oersted's original announcement in 1820 was far more than normal for a scientific paper – for example, four translations into French were quickly made [8] – the rush to research was nothing like as extensive as is often thought. Previously the normal view had been that electricity and magnetism were separate classes of phenomena, and therefore Oersted's discovery was regarded by some scientists as an anomaly or maybe a mistake. In particular, in Paris, then the scientific capital of the world, only Ampère made an *extended* commitment to study it (although a few others contributed very usefully from time to time). Further, he was a respected but *not* a major figure in Paris at the time: 45 years old, professor at the *École Polytechnique*, inspector in the *Université* system, member of the *Académie des Sciences* because of his papers in mathematics, renowned eccentric – and with no previous training in physics, or in any experimental science, to his credit.[9]

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[6] Joubert contains a number of his main writings, and also some by other authors. Otherwise there is a small selection of translations into English of certain passages in R. A. S. TRICKER, *Early Electrodynamics* [...], Oxford, Pergamon 1965; however, the original texts are messed around and preceded by an unreliable introduction. I note also here Ampère's own anthological volume of articles: A.-M. AMPÈRE (éd.), *Recueil d'observations électrodynamiques* [...], Paris, Crochard 1823. In later footnotes this book is cited as *Recueil.*

[7] See principally chs. 156-206 bis.

[8] H. C. OERSTED, *Experimenta circa effectum conflictus electrici in acum magneticam*, Copenhagen 1820; also in «J. Chem. Phys.», XXIX, 1820, pp. 275-281; and *Scientific Papers*, ed. by K. Meyer, 3 vols., 1920, Copenhagen, vol. II, pp. 214-218. The French translations were in «Ann. chim. phys.», XIV, 1820, pp. 417-425; «J. phys. chim. hist. nat.», XCI, 1820, pp. 72-76; «Bibl. univ.», XIV, 1820, pp. 274-284; «Ann. gén. sci. phys.», 1820, pp. 259-264; and JOUBERT, *op. cit.*, vol. I, pp. 1-6.

[9] The best general biographies of Ampère are these obituaries: C. A. SAINTE-BEUVE, M.P.E. Littré, *Illustrations scientifiques. III. M. Ampère*, «Rev. deux mondes», IX, 1837, pp. 389-439 (with a reduced version in Ampère, *Essai sur la philosphie des sciences*, Paris, Bachelier 1843, II^e partie, pp. I-XCVI); F. V. E. ARAGO, *'Ampère'*, in *Biographie universelle ancienne et moderne*, nouvelle édition, vol. I, 1843, pp. 596-613; and D. F. J. ARAGO, *Ampère*, in his *Oeuvres com-*

It seems that Ampère's stimulus was largely religious, in that he saw the effect as a pantheistic manifestation of God in the aether. The history of electricity and magnetism contains a number of such figures, both before and after Ampère.[10] Perhaps in the same optimistic spirit, he came to regard his theory as «a truth founded on incontestable proofs».[11]

After finding the electrodynamical effect, Ampère had to decide how the apparently disparate fields of electricity and magnetism were related. Partly because of his finding of an effect between wires alone, he decided that magnetism had to be a particular kind of electricity; so he had to develop theories for electrodynamics, electromagnetism *and* magnetism. In a clever further move, he focussed upon 'cases of equilibrium', where no motion was taking place between the wires; he presumably realised that in this new area it would be too difficult to determine laws according to which the motion was occurring.[12] He felt sure that the action between elements of wire lay along the line joining them, and obeyed some power law of that distance: a main aim of his mathematicisation was to find it. His hope was to find an inverse square law, as Coulomb had determined for electricity and magnetism; and for this reason and especially because of the mentality of 'hypotheses non fingo' which hung over so many post-Newtonian scientists, Ampère referred to Newton sometimes and often wrote of the phenomena in a phenomenological vein; but his aetherian knickers showed from time to time, especially in his (pretty incoherent) interpretation of the electric current as created by 'the series of decompositions and recomposi-

plètes, ed. by J.-A. Barral, vol. II, Paris, Gide, pp. 1-116. In addition, much of his personality emerges from the (incomplete) edition of *Correspondance du grand Ampère*, éd. par L. Launay, 3 vols. consecutively paginated, Paris, Gauthier-Villars 1936-43. His mathematical work is treated in my *Convolutions*, esp. chs. 3, 4 and 11 *passim*; see also footnote 29.

[10] On the history before Ampère see especially J. L. HEILBRON, *Electricity in the 17th and 18th Centuries*, Berkeley and Los Angeles, University of California Press 1979; and R. W. HOME, *Introduction*, in *Aepinus's Essay on the Theory of Electricity and Magnetism*, ed. by R. W. Home, trans. by P. J. Connor, Princeton, University Press 1979, pp. 1-224.

[11] Quoted from *Lettre de Mr. Ampère à Mr. Erman* [...], «Bibl. univ.», XVII, 1821, pp. 183-191; also in «J. phys. chim. hist. nat.», XCII, 1821, pp. 304-309; his *Recueil*, pp. 113-120; and his *Correspondance* (footnote 9), pp. 914-918.

[12] For more detailed discussions of this situation, see also J. R. HOFMANN, *Ampère, electrodynamics and experimental evidence*, «Osiris», II, 1987, pp. 45-76, and his *Ampère's invention of equilibrium apparatus: a response to experimental anomaly*, «Brit. J Hist. Sci.», XX (1987), pp. 319-351.

tions of the fluid formed by the reunion of the two electricities' in the aether.[13]

2. Ampère's trigonometric form, 1821

The first case in which Ampère treated electrodynamics mathematically came in a paper of 1820;[14] it was revised in a volume of 1823,[15] from which Figure 1 is taken. Two infinitesimal elements of wire AG

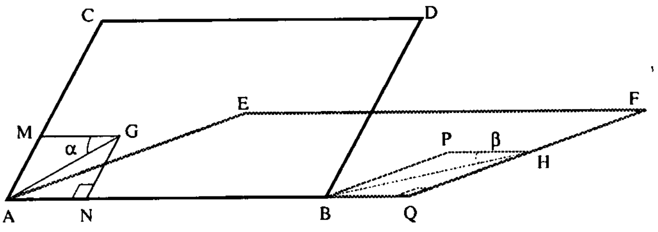

Fig.1 - Ampère: two straight elements *AG* and *BH* (1820)

and BH lay at angles α and β to the line AB, of length r, joining their ends. Assuming that the forces of electrodynamical action could be composed and decomposed as in mechanics, he took the cumulative action to contain four components. Those between AM and BQ and between AN and BP were held to be zero, because the lines were at right angles; those between AM and BP and between AN and BQ did contribute, the first involving the angle γ between the planes and the second a reducing factor k due to the collinearity of the directions. Thus, if the 'intensities' of the currents in the wires were i and i', then the action was given by

$$ii'(\sin\alpha\ \sin\beta\ \sin\gamma + k\ \cos\alpha\ \cos\beta)/r^2, \tag{2.1}$$

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[13] See, for example, the *Extrait d'une lettre de M. Ampère à M. le Professeur de la Rive (15 mai 1821)*, «Bibl. univ.», XVII, 1821, pp. 192-194 (also in *Recueil*, cit., pp. 121-124); and the letter of 1823 to P. Erman, transcribed in my *Convolutions*, cit., document 20.6.

[14] A.-M. Ampère (with help from F. Gillet de Laumont), *Note sur les expériences électro-dynamiques de MM. Oersted, Ampère et Arago* [...], «Ann. mines», V, 1820, pp. 535-558; also published as a pamphlet, Paris, Huzard 1821; and in *Recueil*, cit., pp. 69-92. Part is to be found in Joubert, *op. cit.*, vol. I, pp. 136-140.

[15] A. M. Ampère, *Notes sur cet exposé [...] des nouvelles expériences [...]*, in *Recueil*, cit., pp. 207-236; also in Joubert, *op. cit.*, vol. I, pp. 244-269.

where the (Newtonian) inverse square law was assumed. At first he thought that experiment suggested that $k = 0$; but by 1822 he had come to realise his mistake.

3. Ampère's differential form, 1822

Ampère's presentation of 1823 contained not only (2.1) but also this extension to curved infinitesimal elements Mm and M′m′, of arc lengths s and s' respectively, as illustrated in Figure 2. The argument was already in a paper of the previous year:[16] the figure on the left is his, the other is my attempt to clarify his geometry. Allowing for increase or decrease in values of the pertinent variables,

$$\cos\alpha = dr/ds, \ \cos\beta = -dr/ds' \ \text{(so that } \sin\beta \ d\beta' ds = d^2r/ds \ ds'\text{)}, \quad (3.1)$$

while

$$\angle \mathrm{MM'm} = \tan \mathrm{MM'm} = \mathrm{me/eM'} = \mathrm{me/MM'} = ds \ \sin\alpha/r. \quad (3.2)$$

Thus the trigonometric term from (2.1) became

$$-(r \ d^2r/ds \ ds' + k \ dr/ds \ dr/ds'), \quad (3.3)$$

so that the term itself, when modified to allow for an inverse n-th law for distance, took the form

$$-ii' \mathrm{r}^{1-k-n}(d(r^k \, dr/ds')/ds'), \text{ or } -ii' r^{1-k-n}d(r^k d' r), \quad (3.4)$$

where he used «d'» to indicate differentiation along s'.

Ampère now related (3.4) to a «case of equilibrium» that he had found. If B′M′ were taken as belonging to a fixed horizontal circular wire of radius a and BM was part of a wire of arbitrary shape but with ends located along the vertical axis of the circle, then in fact this wire did not rotate about this axis; thus the moment of the putative rotatory action, as given by (the appropriate version of) (3.4), would be zero. Working in cylindrical polar coordinates (z, u, t) for M with primes for M′, he found that (3.4) became

$$-ii' \ ar^{1-n-k} \ d' t' d(r^{k-1}u \ \sin(t-t')), \quad (3.5)$$

[16] A. M. Ampère, *Mémoire sur [...] la formule* for two infinitesimal wires, «Ann. chim. phys.», XX, 1822, pp. 398-419; also in *Recueil*, cit., pp. 293-318; Joubert, *op. cit.*, vol. I, pp. 270-289; and his *Mémoires sur l'électromagnétisme et l'électrodynamique*, Paris, Gauthier-Villars 1921, pp. 75-104.

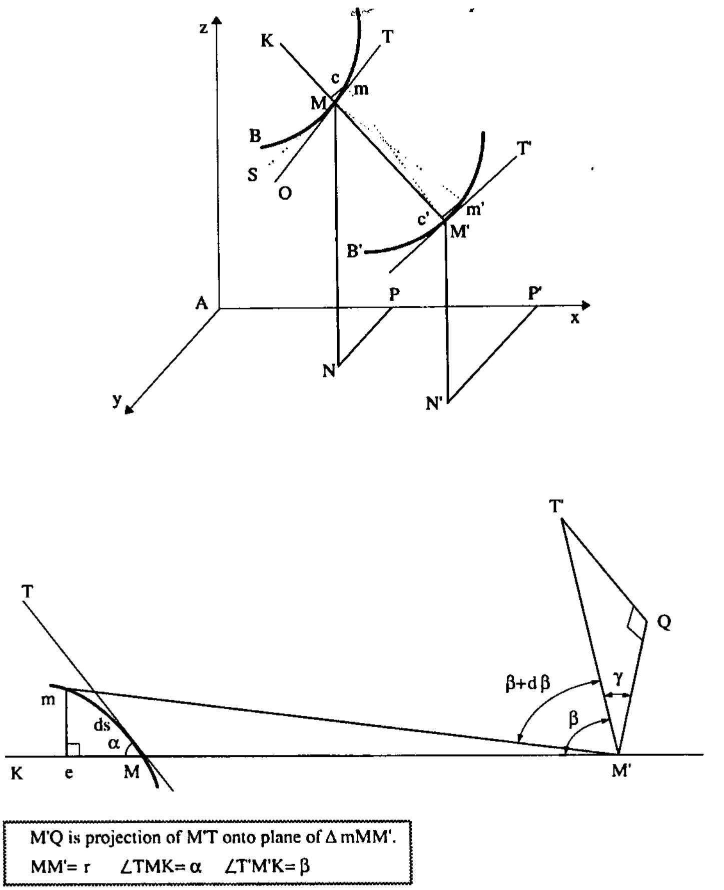

Fig.2 - Ampère: two curved wires *BMT* and *B'M'T'* (1823)

and after passage-work that the moment was given by the expression

$$-ii'\,\mathrm{d}'t'\,r^{n-k}u\ \sin(t'-t)d(r^{k-1}\,u\sin(t'-t)). \qquad (3.6)$$

Therefore the required action between the element of the circle and the rotatable wire was given by the *d*-integral of (3.6); and this was zero. Ampère now appealed to a theorem in the calculus of variations: for the line integral involved to be zero, the differential had to be exact with respect to *d*. A necessary condition was provided by equating the powers of *r* there, yielding a relation between *k* and *n*:

$$n + 2k = 1. \qquad (3.7)$$

Should $n = 2$, as hoped, then $k = -1/2$ instead of the mistaken earlier value 0. Still better would be a second equation relating n and k; then the matter could (re)solved mathematically. This was soon to come.

In a brief addition Ampère made a significant further finding. Taking the component (given by $\cos\beta$) of the action along ds' and d-integrating along the wire BT, he found an action I given by.

$$I = 1/2 \ ii' r^{2k} d's' \ \cos^2\beta + C \ (C \text{ a constant}). \qquad (3.8)$$

If the circuit was closed, then $\beta = 0$, so that «the resultant of all the actions exerted by a closed circuit upon a small portion of the other conductor is always perpendicular to the direction of this little portion».[17] The ramifications of this result will be taken up in section 5.

4. Savary on elements and circles, 1823

As was mentioned in section 1, the new phenomena did not attract a large body of Parisian interest, and Ampère's supporters were rather minor figures there. But in 1823 an important paper was written by Felix Savary, then 25 years old and a graduate of the *École Polytechnique*.[18] He considered in Figure 3 the action between the element ds of the inclined circle at the origin O of coordinates and an isolated vertical element ds' at A'. Proceeding in Ampère's way with a theory using n and k, he calculated the components of action between circle and element. He then modified the results for the case when the radius a of the circle was very tiny, and the integrated them for a horizontal annulus (not shown in Figure 3) composed of such tiny circles.

The purpose of this analysis was as follows. Firstly, as was mentioned in section 1, Ampère had to create an electrical theory of magnets. He had originally imagined a magnet to consist of concentric rings of electricity; but his friend (and indeed from about 1822 his lodger) Fresnel outlined in 1821 a theory of «particulate currents» in which a magnetic dipole was construed as a tiny electrical helix with its axis

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[17] A.-M. Ampère, *Extrait d'une note additionnelle* to (footnote 16), «Ann. chim. phys.», XX, 1822, 2, pp. 419-421; also in *Receuil*, cit., pp. 316-318; Joubert, *op. cit.*, vol. I, pp. 287-289; and *Mémoires*, cit., (cf. footnote 16), pp. 105-110.

[18] F. Savary, *Mémoire sur l'application du calcul aux phénomènes électrodynamiques*, «J. phys. chim. hist. nat.», XCVI, 1823, pp. 1-25; also published as a pamphlet, Paris, Bachelier 1823; and in Joubert, *op. cit.*, vol. I, pp. 338-370.

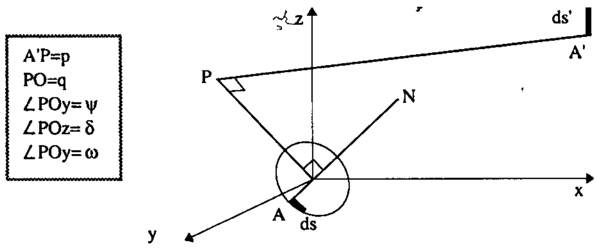

Fig.3 - Savary: circle *AN* and element *A'* (1823)

lying along the (local) line of magnetisation, and a magnet regarded as an assembly of such helixes:[19] Savary's treatment of the tiny circles was a representation of this theory. Secondly, in recent unpublished work known in Paris, Gay-Lussac and Welter had magnetised an iron annulus by winding an electrical wire around it and passing current, and had found that nevertheless this annulus exercised no action on an electrical element while unbroken.

Savary's calculations represented this situation mathematically. He determined the components of action when the element was parallel to the axis of the annulus, perpendicular and pointing to it, and at right angles to these two directions; and the expressions were either zero (because of the configuration) or took the form

$$-(kn+1)\, c_p \int_0^{2\pi} \sin^2\varphi / r^{n\times 3} d\varphi \qquad (4.1)$$

where c, p and φ were known parameters pertaining to the situation. Now the experiment suggested that this expression should be zero; and since the integrand was positive, the only possibility was that

$$kn + 1 = 0. \qquad (4.2)$$

When taken with Ampère's equation (3.6), two pairs of solution for n and k emerged:

$$k = -1/2,\ n = 2;\ \text{and}\ k = 1,\ \text{n} = -1. \qquad (4.3)$$

Since the action between collinear elements was known to be repulsive, the second case was excluded; and the first gave the Newtonian value –2 for which Ampère had hoped.

[19] Fresnel's two manuscripts on electrodynamics were published in Joubert, *op. cit.*, vol. I, pp. 141-147; one of them is dated as of 1821. They survive in Ampère's *Nachlass*, cit., (cf. footnote 1), ch. 184.

Savary also gave Ampère's cause valuable support by exposing an error in a rival's research. Of the other Parisian scientists who worked (from time to time) on this new topic, the most significant was Jean-Baptiste Biot, one year Ampère's senior, who was assisted by Felix Savart (not to be confused with Ampère's Felix Savary!). They took the view that electricity and magnetism were separate types of phenomena but suceptible to interaction, and they studied exclusively properties of electromagnetism.[20] Unfortunately carelessness crept into their work: they had claimed that the effect of the action of a V-shaped wire upon a magnet was proportional to the angle $2i$ between the branches of the wire,[21] but Savary showed from their own theory that the function should be $\tan i$. Thus the correct 'Biot-Savart law', which they proposed later,[22] is as much due to Savary – to whom they did not refer, presenting the law as a supposition of their own!

5. Ampère's line-integral form, 1824

Savary's paper was an important inspiration for Ampère for reasons beyond (4.3) and the correction of Biot and Savart: he had calculated actions for open as well as closed wires, and found other results in magnetism (such as the relation between magnetic latitude and magnetic dip). Ampère wrote summaries of the paper (indeed, surviving letters in his *Nachlass* show that he partly rewrote the paper himself while it was in proof!);[23] and in a paper of 1824 he made important exten-

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[20] The work of Ampère and Biot is contrasted in T. Hashimoto, *Ampère vs. Biot: two Mathematizing Routes to Electromagnetic Theory*, «Hist. Scient.», 24, 1983, pp. 29-51.

[21] J. B. Biot, *Sur l'aimantation imprimé aux métaux par l'électricité en mouvement*, «J. sav.», 1821, pp. 222-235.

[22] J. B. Biot, F. Savart, *Sur l'aimantation imprimé aux métaux par l'électricité en mouvement*, in J. B. Biot, *Précis élémentaire de physique expérimentale*, Paris, Deterville 1824³, vol. II, pp. 704-774. Part is reprinted in Joubert, *op. cit.*, vol. I, pp. 80-127.

[23] As they contain extensive extracts from his writings, I attribute to Ampère these anonymous pieces: *Extrait des mémoires de M. Savary et de M. Demonferrand*, «Bull. Soc. Philom. Paris», 1823, pp. 61-63; and *Notice de deux mémoires [...] de MM. Savary et de Demonferrand*, «Bibl. univ.», XXII, 1823, pp. 259-264. See also his review with O. Rodrigues of Savary's paper in «Bull. gén. univ. ann. nouv. sci.», III, 1823, pp. 20-22. His correspondence with Savary may be consulted in the *Nachlass* (footnote 1), ch. 172; and in *Correspondance* (footnote 9), pp. 931-935.

J. B. F. Demonferrand wrote a paper on mathematical cases of electrodynamics at exactly the same time as Savary. Ampère found it less interesting, and it was not published. However, sions of its methods.[24]

Ampère introduced a new term: a collection of tiny circular currents, each one lying perpendicularelly to the curve through the centres, was called a «solenoid», from the Greek word σοληνοειδης, derived from σοληω, 'canal', and he went on to calculate actions between elements, solenoids and cylinders. The basic result followed from Figure 4, which showed an isolated element Ab of length *ds* carrying current *i* and acting on 'a system of closed currents' C with current *i*, of which mM was an element of length *ds*. Drawing on (3.5), he showed that the component along Ab of the cumulative action *F* was zero; thus *F* acted perpendicularedly to Ab. To specify its direction further he drew on (3.8) and some of Savary's results to cast the component *F* of the action in the plane bAG (where AG was not specified) in the form

$$F = -ii'ds' \int_c AN^2/2r^{n+1} d\varphi, \tag{5.1}$$

where $\varphi = \angle$ RAO and AN was the projection of AM on the plane bAG. After setting up a Cartesian system of axes, he converted this expression to

$$F = -ii'Dds' \sin\Psi', \tag{5.2}$$

where *D* was the resultant of the component actions corresponding to the integral of (5.1) for the axes of this system, and Ψ' the angle made with its projection onto bAG. The main point was that when bAG was perpendicular to both bA and the direction AD of the resultant action *D*, then $\Psi' = 0$; but then *F* was perpendicular to both the element Ab of wire and AD. However, the direction of AD was specified independently of Ab; thus, as the wire Ab varied in space, *F* changed its direction *only* in the plane normal to AD, which was therefore a characteristic of the system C and independent of Ab. He called this plane «the *director plane*» of the wire: later he called it the «directrix».

The analysis reformed and extended the analysis (3.8) of 1822. From it Ampère reworked several of his own and Savary's special cases for electromagnetism, and also for magnets construed as collections of solenoids (of which he had also been aware in 1822).

Demonferrand described some of his results in the first textbook that was written on the new science: his *Manuel d'électricité dynamique*, Paris, Bachelier, 1823. Translations of this book were soon published in German (1824), Italian (1824, apparently: not found) and English (1827).

[24] A.-M. Ampère, *Extrait d'un mémoire sur les phénomènes électrodynamiques*, «Ann. chim. phys.», II, 26, 1824, pp. 134-161, 246-258; also in his *Précis de la théorie des phénomènes électrodynamiques*, Paris, Crochard and Bachelier, pp. 1-44.

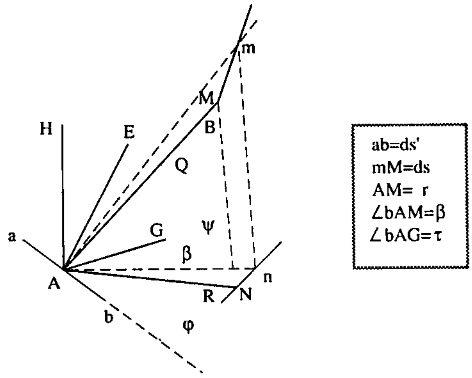

Fig.4 - Ampère: element *ab* and cylinder *BM* (1824)

6. Ampère's translation of Poisson's theory of magnetism, 1826

By 1826 Ampère had amassed a sufficient number of theoretical principles and mathematical results to attempt a definitive presentation of his ideas. In the event, the resulting paper was massive but rather a shambles, and even appeared in two slightly different versions: as a book with five notes, and the following year in the *Mémoires* of the *Académie des Sciences*, with the third note omitted (or, if this was the first version, not yet written).[25] Much of the mathematical material had already been rehearsed, though often in less detail; but one result was quite new, and of an unusual and remarkable character.

As an example of the modest interest taken in electrodynamics and electromagnetism in Paris mentioned in section 1, Siméon Dénis Poisson never wrote a word about it, although he expounded at enormous length on just about everything else in pure and applied mathematics. But in two papers published in 1826 he gave the first substantial mathematical theory of magnetism and magnetic bodies.[26] It was an extensive exercise in potential theory (including the first general divergence

[25] A.-M. Ampère, *Mémoire sur la théorie mathématique des phénomènes électro-dynamiques* [...], Paris, Méquignon-Marvis 1826; this version was reprinted in Joubert, vol. II, pp. 1-193. The *Académie* appearance was in «Mém. Acad. Sci.», VI, 1823 (publ. 1827), pp. 177-388; it was reprinted as a book (Paris, Hermann 1883) which itself was photoreprinted (Paris, Blanchard 1958).

[26] S. D. Poisson, *Mémoire* and *Second mémoire sur la théorie du magnétisme*, «Mém. Acad. Sci.», V, 1821-22, (publ. 1826), pp. 247-338 and 488-533.

theorem in mathematics), in which a magnet was viewed as a collection of isolated «magnetic elements» over the surface of which attraction or repulsion would be calculated.[27]

Poisson's theory had nothing to do with Ampère's electrical view of magnetism: indeed, in fairly typical style he worked in an implicit but snide remark on Ampère's position before saying that he was not going to tackle the subject anyway. But Ampère had to respond, by in effect translating the theory into his own talk of action between solenoids and solenoids. For the purpose he produced a beautiful result centered around Figure 5 (typically, developed in two different parts of his monograph).[28] The right hand shape was in space, its surface σ bounded by contour s, with an arbitrary element $d^2\sigma$ sitting at m; a second surface σ_1 (not shown), with the same contour, lay close to it. Similarly, an element $d^2\sigma'$ of a surface σ' (also not shown) was to be found at m.' Ampère took the Poissonian expression (E, say) of mag-

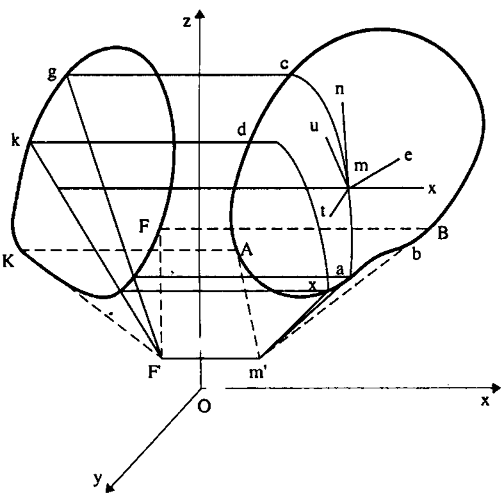

Fig.5 - Ampère: surface-contour theorem (1826)

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[27] POISSON, *Mémoire*, pt. 2, in a section entitled *Simplification of the preceding formulae*, which shows that he had not grasped the full significance of his new result. On the history of potential theory see especially M. BACHARACH, *Abriss der Geschichte der Potentialtheorie*, Würzburg, Thein 1883; also I. TODHUNTER, *A history of the mathematical theories of attraction and figure of the earth* [...], 2 vols., London, MacMillan 1873 (repr. New York, Dover 1962).

[28] AMPÈRE (cf. footnote 24), book version, pp. 139-147, 188-195.

netic action between m and m and mapped the surface σ onto the plane OYZ, giving the shape on the left side of the Figure. By transforming the variables appropriately, he found that E was converted into a version of his integrand of (5.1), which expressed the action between an element and an electrical line; therefore the basic link between Poisson's theory and his own electrical conception of magnetism was established. Various integrations were effected to develop the connections between the two theories: surface to neighbouring surface (so that a dipole became a tiny solenoid), surface to element, and finally surfaces to surfaces. He also emulated Savary by deriving the relationship between magnetic latitude and magnetic dip.

This was the final point of achievement for Ampère. In his last main paper he reviewed the three main theories of electromagnetism: that involving the traditional form of magnetism, with its magnetic fluids, such as Poisson used; his own electrical view; and the position held by his rival Biot, in which electricity and magnetism were separate types of phenomena but between which a primitive interaction could take place.[29] Part of his dismissal of this approach was very strange: Biot had talked of a couple-like turning effect between magnet and wire,[30] but for some reason Ampère held that couples were «directly contrary to the principles of Dynamics». His own researches in mathematics had included aspects of dynamics;[31] was his supposed Newtonianism (inverse-square law, and all that) so pervasive that he could not appreciate the fundamental contribution to statics made by his contemporary Poinsot[32] in introducing the theory of the couple?

7. Concluding remarks

The modest degree of interest shown in Paris to Oersted's discovery did not substantially increase because of Ampère's own efforts. After

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[29] A.-M. Ampère, *Mémoire sur l'action mutuelle d'un conducteur voltaique et d'un aimant*, «Mém. Acad. Belgique», IV, 1827, pp. 3-70; also in Joubert, vol. II, pp. 224-274.

[30] Biot (cf. footnote 21).

[31] See especially A.-M. Ampère, *Démonstration générale du principe des vitesses virtuelles*, «J. École Polyt.», (I) VI, 1806, 13, pp. 247-269; and *Mémoir e sur quelques nouvelles propriétés des axes permanens* [...], Paris, Bachelier 1823 (also in «Mém. Acad. Sci.», V, 1821-22, (publ. 1826), pp. 86-153).

[32] L. Poinsot, *Eléments de statique*, Paris, Calixte Volland 1803. Further editions came out from time to time (the 4th. in 1824) up to the 12th in 1877.

the paper of 1827 just noted he himself largely gave up electrodynamics and turned to the philosophy of science.[33] As a result of his researches he had gained in 1824 a chair in physics at the *Collège de France* (where, curiously, the other professor for physics was Biot); but within a few years he even switched his teaching there from the old to the new interest. A most distinguished group of mathematicians was still active in Paris, and a fine new generation was emerging; but of them all only the newcomers Joseph Liouville and Jean Duhamel paid any attention to his results, and then only to a very limited extent.[34]

The whole history of electrodynamics and electromagnetism is a surprisingly slow tale – after all, over half a century of industrialisation of society was to pass before house lighting was to be introduced – and a lack of interest is evident in Ampère's Paris. (His last main paper, described in the last section, was published by the Belgian Academy.) The initiative was to pass to Britain (especially Faraday, of course, and then Kelvin) and to Germany (Fechner, W. Weber, F. Neumann) as the next stages were – slowly – to develop.[35]

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[33] See especially AMPÈRE, *Essai* (cf. footnote 9).

[34] See my *Convolutions*, sect. 17.5.4.

[35] The history of this mathematicisation between Ampère and Maxwell is not well studied. Among useful general sources, see WHITTAKER (cf. footnote 5); K. L. CANEVA, *From Galvanism to electrodynamics: the transformation of German physics and its social context*, «Hist. stud. phys. sci.», IX, 1978, pp. 63-159; and several articles in G. CANTOR, M. J. S. HODGE (eds.), *Conceptions of ether* [...], Cambridge, University Press, 1981.