Resuming Ampère's experimental investigation of the validity of Newton's third law in electrodynamics

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ABSTRACT. In 1822, Ampère published the final form of his law for the electrodynamic force between current elements. Since then, it has been said that he carried out all his work in electrodynamics assuming both Newton's 3rd law and the absence of elementary torques as 'a priori' truths. However, we have noticed that this kind of analysis is valid only when one considers just the first four experimental cases of equilibrium, as found in his "Theory of Electrodynamic Phenomena, Uniquely Derived from Experience" (1826). Here in this paper, we show that Ampère treated the third law as a consequence of the experiments reported in the last part of his main work — independent of the expression adopted for the force — combined with the Ockham's razor, his actual 'a priori' assumption.

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1 Introduction

In 2017, the first edition of the main work of André-Marie Ampère (1775–1836), namely the *Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience* [1], hereafter called *Théorie*, has celebrated its 190th year of publication. As shown by Assis and Chaib in [2, pp. 326–332], although the edition of 1826 (our Ref. [3]) was first printed, it was actually a revised edition. For a complete English translation of the *Théorie*, see Refs. [4, 5]. Although the unity of electric current intensity is named 'ampere', the true accomplishment of Ampère's works in electrodynamics remains unknown to most physicists, unfortunately. Worse, the history behind Ampère's electrodynamics appears distorted in most textbooks [6, 7], his rich work being typically put aside in favor of an unfinished law relating an electric current to the magnetic field $\vec{B}$ created by it, the so-called *Ampère's law.*[1] This law has no relation with Ampère's original works and concepts. Actually, *Ampère deduced the form and the values of the parameters of an* ***electrodynamic*** *force* between two infinitesimal portions of wires conducting constant electric currents. In the *Théorie*, this force appears as [3, p. 32]:

$$\frac{i\,i'\,ds\,ds'}{r^n}\,(\sin\theta\,\sin\theta'\,\cos\omega + k\,\cos\theta\,\cos\theta')\;. \tag{1}$$

Ampère named the interacting agents $i\,ds$ and $i'\,ds'$ the *current elements* [8]. Here, r is the distance between the midpoints of the current elements. Also, $i\,ds$ and $i'\,ds'$ form with the straight line that joins those points angles θ and θ', respectively. Those angles may pertain to different planes with an angle ω between them. The constants n and k were found experimentally to be 2 and $-1/2$, respectively. In modern notation, using the International System of Units (SI), this element of force reads [2, p. 29]:

$$d^2\vec{F}^{A}_{I'ds'\text{ on }Ids} = -\,\frac{\mu_0}{4\pi}\,I\,I'\,\frac{\hat{r}}{r^2}\,\left[2\,(d\vec{s}\cdot d\vec{s}\,') - 3\,(\hat{r}\cdot d\vec{s})\,(\hat{r}\cdot d\vec{s}\,')\right]\;, \tag{2}$$

where μ_0 is the permeability of free space, I and I' are current intensities (in SI units), and $\hat{r}$ is the unit vector pointing from the middle point of $I'\,ds'$ to that of $I\,ds$.

The interchange of $I'\,ds'$ and $I\,ds$ in Eq. (2) directly shows that

$$d^2\vec{F}^{A}_{I'ds'\text{ on }Ids} = -d^2\vec{F}^{A}_{Ids\text{ on }I'ds'}\;, \tag{3}$$

which means that this model of electrodynamical interaction obeys Newton's third law *along the straight line that joins the current elements.* This property of Ampère's force has received criticism from some researchers. For instance, Ivor Grattan-Guinness wrote in his work "Convolutions in French Mathematics" [9, p. 956.]:

> Ampère's loyalty to Newtonian principles unfortunately forbad him from following Poinsot (§ 6.2.1) and admitting couples in his theory.

[1]Namely, $\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0\,I$, where I is the intensity of the electric current (in SI units) flowing through the inner part of the closed path C.

When one admits an interaction where the fundamental force is perpendicular to the line that joins the interacting elements, this particular configuration form an *elementary force couple* or an *elementary torque.* Darrigol, as well, pointed out that the "absence of elementary torque" in Ampère's force law was an "unwarranted hypothesis" [10, p. 26]. Also, E. T. Whittaker remarked in 1910 that this lack of elementary couple is the "weakness of Ampère's work" [11, p. 91].

In fact, these opinions are not new. In 1845, H. G. Grassmann (1809–1877) called the Newton's third law in electrodynamics "an arbitrary assumption" [12, p. 202]. However, J. C. Maxwell (1831–1879) disagreed to these opinions when he compared Ampère's force with Grassmann's force, as well as with two other propositions (our emphasis) [13, art. 527, p. 161]:

> Of these four different assumptions **that of Ampère is undoubtedly the best**, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them.

In the next sections, based upon Ampère's original discussions and investigations, it will be shown the arguments that probably influenced Maxwell to endorse the third law as a fundamental characteristic of an electrodynamic force.

2 The four cases of equilibrium and Newton's third law

Before presenting the main discussion on the validity of Newton's third law in electrodynamics, let us present the final experiments suggested by Ampère to deduce his force law, as reproduced by other researchers [2, Secs. 10.4 and 10.5], and its relation to the action-reaction principle (along the line joining the two interacting elements). Maxwell didn't hide his admiration for Ampère's work. Right after the above-quoted statement he wrote [13, art. 528, p. 162]

> The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seem as if it had leaped, full grow and full armed, from the brain of the 'Newton of electricity.' It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and **which must always remain the cardinal formula of electro-dynamics**.

Maxwell chose the word *cardinal*, above, to highlight the role of Ampère's force not only for its importance but mainly as a *guide* to further researches in this area. In fact, soon before Maxwell's main works, some scientists began to propose different models in view to unify the electrostatic and electrodynamic phenomena into a unique expression of force. We may mention C. F. Gauss (1777–1855), G. F. B. Riemann (1826–1866) and W. E. Weber (1804–1891) as examples of Maxwell's contemporaries who presented different expressions describing the elementary interaction between two electrically charged objects that are at relative rest or relative motion [13, art. 502, p. 146]. Within this context, we understand that Maxwell is alerting his colleagues that such expressions *would have to be reduced to Ampère's force.*

Which else were the experiments presented by Ampère in his *Théorie* that led him to the "formula from which all the phenomena may be deduced"? Indeed, how Newton's third law is related to them?

2.1 *The case of equilibrium of the sinuous wire and the vectorial sum of current elements*

Ampère begins his reasoning in his main book by proposing that the elemental agent of electrodynamic force consists in the product of an infinitesimal portion of a conductor wire (ds) by the constant current intensity (i) through it. After this *current element* hypothesis, the mathematical form of Eq. (1) and the values of the parameters n and k were determined by the *equilibrium experimental method*, as devised by Ampère himself [2, 14].[2] He presented four 'cases of equilibrium' in order to deduce the expression of his force. We remark that he developed many other equilibrium experiments, some of them being the first alternative to arrive at his force.

The experiment named *case of equilibrium of the sinuous wire*, as seen in [2, p. 91] and illustrated in Fig. 1, was the first equilibrium experiment developed by Ampère, despite being presented as the second one in the *Théorie*. This case of equilibrium consists in a mobile circuit (FGHI and CDIF) between two vertical conductors (RS and PQ). One of these vertical conductors is straight and the other has a sinuous (or zigzag) form in such a way that both parts of the circuit begin and end at the same height, as indicated in Fig. 1 (a). The currents flow in the same direction in RS and PQ. The current in GH, however, flows in the

[2]These *cases of equilibrium* were also named 'null method' (see Refs. [15, p. 101], [13, art. 503, p. 147], and [10, p. 11]) or 'null experiment' [16, p. 25].

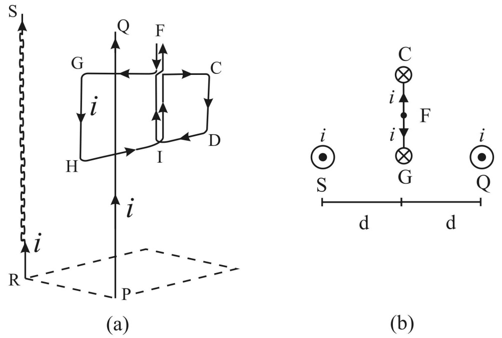


Figure 1: Schema of the *case of equilibrium of the sinuous wire*, as seen in [2, p. 93].

opposite direction, as seen in Fig. 1 (b). The current CDIF balances the Earth's magnetic action over the current FGHI and nullifies the torque. This entire mobile circuit was named *astatic coil* in [2, p. 90].

When the *astatic coil* is pulled out from its rest position and then released, we observe that GH oscillates around the equilibrium vertical line, which is exactly in the middle between RS and PQ. Then, both actions on the current GH have the same modulus, even taking into account the geometric difference between RS and PQ. From that experimental result, Ampère concluded that two current elements "[...] will exert, at any case, precisely the same action which corresponds to their resultant itself" [17, p. 226]. This means that Ampère *used this experimental result to infer that the current elements are vector quantities*, in modern words.

Since many of Ampère's contemporaries have not accepted the assumption of current elements, it is natural that they also rejected their vectorial sum. The skepticism on the validity of the current element and its properties appears in a letter addressed in 1822 to Baron Jean-Frédéric Maurice (1775-1851) — a mathematics teacher and Ampère's friend [18, p. 924]:

> It's not so much the contention of mind to comprehend the actions of currents that disgusts me in this theory, than the great number of all unwarranted assumptions, the abuse of consideration of small infinitesimals on which one can says whatever one

> wants, and the mixing of some dynamic ideas whose introduction is not sufficiently motivated neither their influence is characterized significantly.

As another example, for J. B. Biot (1774-1862) the *case of equilibrium of the sinuous wire*, as well as the deflection of the needle in Ørsted's experiment, would not be a consequence of a sum of interactions between *elementary currents*, but "[...] only a compound result of the unknown distribution of the **elementary magnetism** [*molecular magnetism*, from the original], when we attribute the magnetic action of the wires to such an action" (our emphasis) [19, p. 376], [20, p. 771]. On the other hand, nowadays most textbooks assume that current elements are vector quantities without further discussion.

Returning to Ampère, he took the result found in the *case of equilibrium of the sinuous wire* to decompose the current element on a three-dimensional system. If we do that, assuming that the force is inversely proportional to the distance r, we can rewrite the interaction between $i'\,d\vec{s}\,'$ and $i\,d\vec{s}$ as:

$$\frac{i\,i'\,ds\,ds'}{r^n}\Big(\sin\theta\,\sin\theta'\,\cos\omega + j\,\sin\theta\,\sin\theta'\,\sin\omega + k\,\cos\theta\,\cos\theta' \\ + l\,\cos\theta\,\sin\theta' + m\,\sin\theta\,\cos\theta'\,\cos\omega + p\,\sin\theta\,\cos\theta'\,\sin\omega\Big)\,, \quad (4)$$

where j, k, l, m and p are unknown parameters reflecting proportions with respect to the first term.

2.2 The case of equilibrium of anti-parallel currents and the opposite actions

In the *case of equilibrium of anti-parallel currents*, see Fig. 2, the currents in d'O and dO flow in opposite directions. The conductor below these two currents make n loops in order to multiply the action of the electric current i' [22, p. 15]. As the *astatic coil* stands in any position in which it is released, Ampère deduced that the actions from AB over d'O and dO have the same modulus but opposite directions (see [2, pp. 94–100] for a detailed analysis).

In Fig. 3, Ampère took into account a consequence that comes from the *case of equilibrium of anti-parallel currents* to formulate a theorem in order to nullify the terms multiplying j, l, m and p [5, p. 357]:

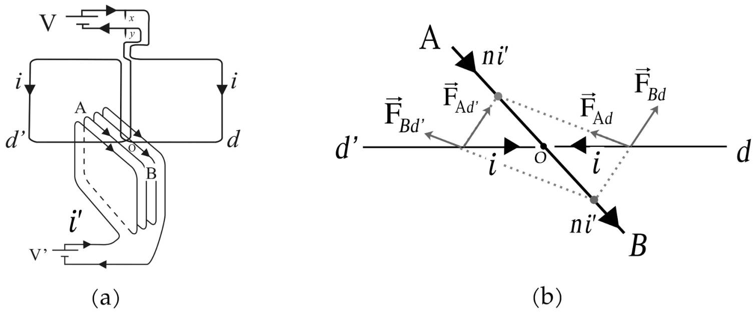


Figure 2: (a) Schema of the *Théorie*'s experimental apparatus to perform the *Case of equilibrium of anti-parallel currents* [21]. (b) Ampère's interpretation of the action from AO and OB over d'O and dO.

> [...] and the theorem which I will establish, namely: that an infinitely small portion of current exerts no action on another infinitesimal portion of a current which is situated in a plane which passes through its midpoint and which is perpendicular to its direction. In fact, the two halves of the first element produce equal actions on the second, the one attractive and the other repulsive, because the current tends to approach the common perpendicular in one of these halves and to move away from it in the other. The two equal forces form an angle which tends to two right angles according as the element tends to zero. Their resultant is therefore infinitesimal in relation to these forces and in consequence it can be neglected in the calculations.

Then, he arrived at the general form of the electrodynamic force as given in Eq. (1).

The theorem assumes the *case of equilibrium of anti-parallel currents* as a result from the configuration of forces shown in Fig. 2 (b). It implies the admission *from the beginning* that the elemental forces are directed along the line joining the elements, obeying the action-and-reaction principle. **Thus, the deduction of the general form of Ampère's force, as we see, lies on the validity of Newton's third law**.

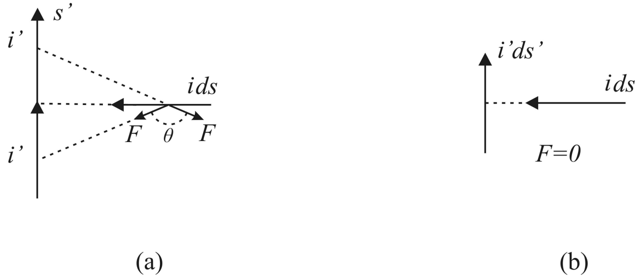


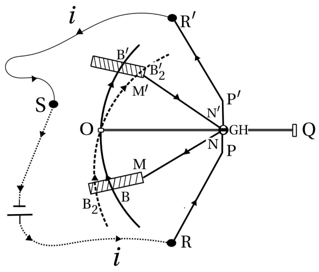
Figure 3: Ampère's theorem which was used to justify the null action between orthogonal current elements. (a) The force components of both halves of the wire tend to stay opposite each other when s' tends to ds'. (b) In the limit, there will be a null resultant.

2.3 The cases of equilibrium of the nonexistence of tangential forces and of the law of similarity

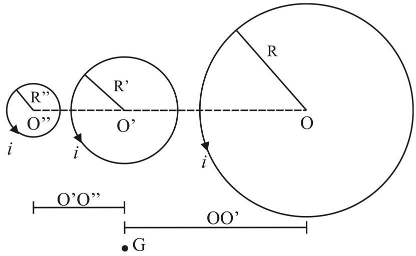
Let us continue with the description of the two subsequent cases in order to complete the deduction. This will be helpful in our discussion of Bertrand's analysis, in the last section. In his *Théorie*, Ampère presented the third case of equilibrium named as the *case of equilibrium of the nonexistence of tangential forces* in [2, p. 182], as illustrated in Fig. 4 (a).

This experiment consists in a circuit RPP'R'S — which forms a closed circuit with the pile — and a mobile conductor arched with centre on GH whose extremities are floating on mercury. The mercury fills the two cavities M and M'. A thin bar OQ is suspended by a fulcrum on GH. This bar, in turn, suspends the mobile arc in O with the help of a counterbalance in Q. If the arc has its position on the line BOB' — which means that its centre is on GH — then it will not move. But, if the arc is over the line B_2OB_2', it will slide from its position. That is, a torque arises on the lever OGH. Since this movement only occurs when the arc centre is not on GH and the corresponding angle can assume distinct values, Ampère concluded that *the closed circuit does not exert any tangential force on the arc's current elements.*

If we name as $\vec{B}$ the contribution of a closed circuit to its mutual interaction with a current element $i'\,d\vec{s}\,'$, the result of the *case of equi-*



(a)



(b)

Figure 4: (a) Schema of *Théorie*'s *case of equilibrium of the nonexistence of tangential forces.* (b) Schema of *Théorie*'s *case of equilibrium of the law of similarity* [2, p. 187].

librium of the nonexistence of tangential forces may be represented as

$$d\vec{F} = i'\, d\vec{s}\,' \times \vec{B} \tag{5}$$

or, when the parameters assume the values $k = -1/2$ and $n = 2$,

$$d\vec{F} = I' d\vec{s}\,' \times \left(\frac{\mu_0}{4\pi} I \oint_C \frac{d\vec{s} \times \vec{r}}{r^3} \right) \,, \tag{6}$$

which are the same expressions used nowadays to model the force of a magnetic field over a current element. And Ampère was the first to obtain it! We remark that neither Eq. (5) nor Eq. (6) are Ampère's force, but a special case of his fundamental expression, our Eq. (2), valid when a *closed circuit* interacts with an element of another circuit.

Before analyzing this particular case, Ampère had to find the values of n and k. So, he developed Eq. (1) with these unknown parameters and confronted the resulting expression with this third experimental result, concluding that

$$k = \frac{1-n}{2} \,. \tag{7}$$

Ampère then resorted to the *case of equilibrium of the law of similarity* in order to derive the value of n [2, p. 185].

In Fig. 4 (b), we have three circular circuits on the same plane. As the numerator in Eq. (1) carries a product of two linear spatial quantities, Ampère remarked that if the distance force's dependence $1/r^n$ has a positive power and $n = 2$, the circuit O' will sense the same action from both circuits O" and O when

$$\frac{R}{R'} = \frac{R'}{R''} = \frac{OO'}{O'O''} \ . \tag{8}$$

This result means that if the circuit O' is mobile — fixed as an *astatic* pendulum — it will always return to its equilibrium position. Once he obtained $n = 2$, he deduced from Eq. (7) that $k = -1/2$, so his force becomes

$$\frac{i\,i'\,ds\,ds'}{r^2}\left(\sin\theta\,\sin\theta'\,\cos\omega - \frac{1}{2}\,\cos\theta\,\cos\theta'\right) . \tag{9}$$

On commenting about this massive work, Tricker wrote a beautiful quote [16, p. 36]:

> In the theory of gravitation, Newton was already provided with the knowledge of a range of the phenomena, mainly through the medium of Kepler's law. Ampère had to discover the laws as well provide the theory, and thus do the work of Tycho Brahe, Kepler and Newton rolled into one.

For a detailed discussion on the six years during which Ampère developed his experiments and theory on electrodynamic interactions, see [2].

3 Ampère's philosophical defence of Newton's third law

It is necessary to comprehend Ampère's epistemological basis before understanding his relation with Ockham's razor and Newton's 3rd law. When he started to develop his assumptions, he faced the choice between two theories. The first theory assumes the existence of poles and currents as elementary agents to model three distinct sets of phenomena, namely magnetostatic (pole-pole interaction), electromagnetic (pole-current interaction), and electrodynamics (current-current interaction). The second theory could fully describe those phenomena assuming the existence of a single kind of interaction between one kind of elementary agents. Ampère himself explained this to his son in an exciting letter from September 1820 [23, L. 590, p. 562]:

> Since I heard for the first time about the beautiful discovery of Mr. Oersted, a professor at Copenhagen, on the action of a galvanic current on the magnetic needle, I have thought on it constantly, I do nothing but write a great theory on these phenomena and about all those already known on the magnet and perform experiments indicated by this theory, all of which were successful and they have introduced me to so many new facts.

On the other hand, following *Oersted's Experiment*, all scientists who started to work on this subject, including Biot, M. Faraday (1791-1867), and Oersted himself, adopted several assumptions that imply in an *elementary torque* between a real elemental pole and a wire conducting an electric current. Ampère rejected this model and remarked that [5, p. 414]

> [...] The demonstration on which I rely results above all from the fact that my theory explains with a single principle three sorts of actions that all the associated phenomena prove are due to one common cause, and this cannot be done otherwise. In Sweden, Germany and England it has been thought possible to explain the phenomena by the interaction of two magnets as determined by Coulomb; the experiments which produce continuous rotational motion are manifestly at variance with this idea. In France, those who have not adopted my theory, are obliged to regard the three kinds of action which I have brought under one law, as three kinds of phenomena absolutely independent from one another. It should be remarked, in this context, that one can deduce from the law proposed by M. Biot for the interaction of an element of a conducting wire and that of what he termed a magnetic molecule, the law that Coulomb established for the action of two magnets if one accepts that one of these magnets is composed of small electric currents, like those which I have suggested; but then how can it be objected that the other is not likewise composed, thereby accepting all of my point of view?
>
> Moreover, though M. Biot determined the value and direction of the force when an element of conducting wire acts on each particle of a magnet and defined this as the elementary force, it is clear that a force cannot be regarded as truly elementary which manifests itself in the action of two elements which are not of the same nature, or which does not act along the straight line which joins the two points between which it is exerted.

It is quite important to remark that both Ampère's experimental procedures and discoveries were disqualified by much of the academic community [10, p. 14] [2, pp. 63–64]. Also, his theoretical assumptions and opinions were not accepted right from the start. Paradoxically, he was attacked by other French scientists as an anti-Newtonian, and, at the same time, he was rejected by foreign academics exactly for being too Newtonian.

According to Hofmann [24, p. 433]:

> In England, Germany and Holland, this sympathetic state of affairs did not exist, and the scientific journals were much more reserved, if not hostile, in their appraisal of Ampère's theory. The published record thus should not make us fail to notice that even in France there was considerable opposition alive during the early 1820's.

Caneva indicated that several researchers attested that Ampère's theory was "immediately and widely accepted by his French contemporaries" [25], but he remarked that this assertion disagrees with the facts (see the section "Most Scientists Against Ampère" in [2, pp. 197–225]).

As Williams pointed out, there was a far deeper epistemological reason motivating the rejection of Ampère's theory, and it is the common point among all his opponents (Newtonians or not) [26, p. 145]:

> [...] In short, it was to be considered the foundation of a new theory of matter. This was one of the reasons why Ampère's theory of electrodynamics was not immediately and universally accepted. To accept it meant to accept as well a theory of the ultimate structure of matter itself.

Being Ampère right, that would mean there was no need to assume the *physical* existence of elementary poles or dipoles because all magnetic effects would be due to electric currents. Also, there would be no need to assume a medium which intermediates electrodynamic interactions to express the force. The attacks on the third law, as well as on other aspects of Ampère's theory, is a consequence of the rejection of these properties.

On arguing that it is redundant to remain postulating the physical existence of magnetic poles, Ampère reveals his philosophical posture (our emphasis) [27, p. 60]:

> [...] **It is this habit of multiplying, as it is said, the entities without necessity** which during some time made one admits in physics a luminous fluid distinct from the fluid to which one attributed the phenomena of heat; it is this habit which leads one to suppose up to now two magnetic fluids different from the two electric fluids, although it has been shown that electricity, by moving around the particles of the magnetized bodies in the same way as it moves in the voltaic conductor and, consequently, exerting in this way the same action, should necessarily produce effects completely identical to the effects which are attributed to what is called *molecules of austral fluid and of boreal fluid.*

Every theory is a relation constructed between a set of phenomena and a set of hypotheses. The latter is a subjective set that involves analogy, abstraction, and human intuition. Then, there is an epistemological principle which states that if one diminishes the number of assumptions and abstractions of a theory and increases its phenomenological basis, more accurate will be the model and its predictions. This is known as *Ockham's razor*, whose principles put "emphasis on eliminating superfluous entities contributed to a more empiricist and less inflationary ontology" [28].[3]

In fact, Ampère followed this epistemological path since the beginning of his researches in electrodynamics. As Hofmann pointed out "Ampère's 'noumenal world' is thus distinguished from the phenomenal world by eliminating the personal aspects of subjective human perceptions" [30, p. 149]. To do so, he announced a line of reasoning which agrees with that epistemology (our emphasis) [31, pp. 313 and 315]:

> When first I wanted to find the causes of the new phenomena discovered by M. OErsted, I reflected that since **the order in which two facts are discovered in no way affects any conclusions which can be drawn from the analogies they present**, it might, before we knew that a magnetized needle points constantly from South to North, have first been known that a magnetized needle has the property of being influenced by an electric current into a position perpendicular to the current, in such a way that the austral pole of the magnet is carried to the left of the current, and it could then have subsequently been discovered that

[3] *"What can be accounted for by fewer assumptions is explained in vain by more"* [29].

> the extremity of the needle which is carried to the left of the current points constantly towards the North: would not the simplest idea, and the one which would immediately occur to anyone who wanted to explain the constant direction from South to North, be to postulate an electric current in the Earth [...]?
>
> [...]
>
> Now, if electric currents are the cause of the directive action of the Earth, then electric currents could also cause the action of one magnet on another magnet. It therefore follows that a magnet could be regarded as an assembly of electric currents [...] I simulated this arrangement as much as possible by bending a conducting wire in a spiral.

It seems that Ampère had considered this reasoning just as customary as logical among his contemporaries. However, we understand that he enunciated a method to minimize the arbitrariness in the abstract analogies and assumptions to be adopted. When Ampère searched a conclusion that is not affected by "the order in which two facts are discovered", and took "the simplest idea", he tried to diminish the subjectivities brought by the historical influence. The new analogies should, of course, be discarded if not validated by experimental research, but this was not the case concerning the experiments performed by Ampère with conducting wires and the conclusions he drew from them.

We can find a resembling reasoning in the "Rules of Reasoning in philosophy", as stated by Newton in his *Principia* (Book 3) [32]. There at p. 384 of this book, the first and second rules assert that " *We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances* [...] *Therefore to the same natural effects we must, as far as possible, assign the same causes*". Ampère adopted this procedure from the very beginning of his researches, as well as of his main work. He clarifies that, more than mathematical principles, Newton provided a *method* — the "new highway", the "road" — as a guide to move forward [5, pp. 342–343]:

> The new era in the history of science marked by the works of Newton, is not only the age of man's most important discoveries in the causes of natural phenomena, it is also the age in which the human spirit has opened a new highway into the sciences which have natural phenomena as their object of study. [...]
>
> To observe first the facts, varying the conditions as much as possible, to accompany this with precise measurement, in order to

> deduce general laws based solely on experience, and to deduce therefrom, independently of all hypothesis regarding the nature of the forces which produce the phenomena, the mathematical value of these forces, that is to say, to derive the formula which represents them, such was the road which Newton followed. [...] Guided by the principles of Newtonian philosophy, I have reduced the phenomenon observed by M. Oerstedt, as has been done for all similar natural phenomena, to forces acting along a straight line joining the two particles between which the actions are exerted.

Therefore, it was the method that led him not to *extrapolate* hypotheses that motivated Ampère not to discard the validity of the third law at the beginning. As he stated in [33, p. 374], he saw no reason to *a priori* admit another kind of action "which nature offers no other example", "even when this force is only transmitted from one of the material particles to the other by means of an interposed fluid" [5, p. 417].

As we can also see in the last quote, he did not use this method to discuss the existence of an *aether*. And even if Ampère believed on a medium that mediates the interaction, as discussed in [25] and [34], the overture of the *Théorie* does not contradict that belief. As a matter of fact, if we observe the Ampère's way of reasoning from the beginning, we do not see that overture as a compromise to the 'French Newtonians'. It is more coherent to interpret that Ampère starts his main work pointing out that he *did follow* Newton's philosophy *unlike* those 'newtonians', such as Biot, who received several criticisms in the next sections in *Théorie* [2, p. 232]. The overture can be a provocation rather than a compromise, but it also needs not to be either.

Then, Ampère's reasoning is in a sense diametrically opposite to an "arbitrary assumption" that Grassmann impeached to him. Nine years after Ampère's death, Grassmann reasoned against Ampère's electrodynamics accusing it of mathematical complexity, lack of coherence with intuition, and of arbitrarily assuming Newton's 3rd law. After these statements, he pointed out [12, p. 202–203]:

> Ampère was obliged, therefore, in order to obtain his formula, to use an arbitrary assumption together with the experimental results. The assumption used for this purpose is, at first glance, very simple and natural, consisting in the supposition that two infinitely small circuit elements exert force on each other along the straight line connecting their mid-points, either of attraction or repulsion. [...]

> Without making any arbitrary assumption of my own, therefore, I propose to eliminate the arbitrary factor in the Ampère hypothesis [...].

So, Grassmann considered the electrodynamic interaction as an elementary torque, and his force takes the form (SI units):

$$d^2\vec{F}^G_{Ids\ \text{on}\ I'ds'} = I'd\vec{s}\,' \times \left(\frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}\right)\ . \tag{10}$$

For this force, except in a few particular configurations, one has

$$d^2\vec{F}^G_{Ids\ \text{on}\ I'ds'} \neq -d^2\vec{F}^G_{I'ds'\ \text{on}\ Ids}\ . \tag{11}$$

For instance, in Fig 3 (b) the vertical current element $i'\,d\vec{s}\,'$ acts on the current element $i\,d\vec{s}$, but the inverse action (i.e., an action from $i\,d\vec{s}$ on $i'\,d\vec{s}\,'$) **does not occur**.

However, Eq. (10) is similar to Eq. (6) and those equations present the same results on a closed circuit acting upon an element of current. The difference of interpretation of phenomena appears when we compare a part of a circuit acting upon another part of the same closed circuit, as in the discussion of Ampère's bridge [35] and Ampère's motor [36]. Moreover, their differences will greatly reflect on the choice of how to construct an expression for an elementary force between charged objects in relative motion.

But there was not an experimental confrontation. As a matter of fact, Grassmann explained that "It will therefore be my task to derive the new explanation, and that of experienced physicist to test it experimentally" [12, p. 202].

Clearly, Grassmann adopted another epistemological point of view on how to construct a theory, in which both intuition and mathematical abstraction have priority over the experimental phenomena and induction. We do not condemn Grassmann for this epistemological option, but it becomes important to explicit that Grassmann employed an "against arbitrariness" principle to disqualify Ampère's force without an epistemological discussion explaining why to implement the elementary torque is 'less arbitrary', and without a phenomenon to support his choice.[4]

[4] We must recall that Ampère's force "deduces all the phenomena", and Grassmann presented a kind of elementary action "which nature offers no other example". Then, only if one considered the elementary torque "very simple and natural", the third law would seem to be an "arbitrary assumption". Therefore, Grassmann criticism can be applied to himself.

Moreover, he discarded the third law without a discussion about the *experiments* that Ampère performed to defend this principle of mechanics, as will be explained in the next section.

It seems that Grassmann used this argument to get rid of the responsibility of taking the *burden of proof.* In fact, he removes Newton's 3rd law from his model and introduces the elementary torque based only on his intuition, and therefore, *in an arbitrary way.* Since Ampère's main work was not fully translated until the XXI century, it is possible that Grassmann assertion about "Ampère's arbitrariness" — or other similar assertions from Ampère's opponents — might have been the starting point to come to know about Ampère's work. The historical consequences from those echoes created a noise that has confused many researchers.

On the other hand, Ampère built a strong argument based upon both philosophy and experiments to defend the third law. So, the claim by Grassmann (and others) *that Ampère was arbitrary is false.*

4 Ampère's experimental defence of Newton's third law

In the 1820s, mechanics widely tested Newton's third law, and most scientists acknowledged its general validity. It seems reasonable to assume that a fundamental law of physics works in several fields until falsified. If so, it also appears reasonable to attribute the *burden of proof* to those who wish to discard the action-and-reaction law.

However, the validity of Ampère's theory was so criticized that he felt necessary not only a philosophical defence of Newton's 3rd law but also a direct experimental one. Yet, *no one* of the researchers that scrutinized Ampère's original works have brought this topic to the open [2, 9, 10, 11, 15, 16, 24, 30, 37, 38]. Concerning this, Ampère made the following comment after presenting those experiments and the conclusions extracted from them [5, p. 448]:

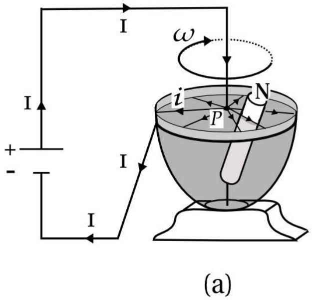
> I confess that this experimental proof of a principle which is nothing else but a necessary consequence of the first laws of mechanics appears to me completely useless, as it should have been clear to all the physicists who considered this principle one of the foundations of science. I would not have made this observation, if it had not been assumed [by others] that the mutual action of one element of a conducting wire and of a magnetic molecule, consisted in a primitive couple composed of two forces equal and parallel

> without being directly opposed, by virtue of which a portion of current which is located inside a magnet might move it; [this] supposition is contrary to the principle which is being discussed here, and is denied by the previous experiment [...].

As this quote reveals, though Ampère had no need to take to himself the *burden of proof*, he embraced it. Thus he showed the validity of the action-and-reaction along the straight line that joins the current elements — far from arbitrariness — as a direct conclusion extracted from the experiments that follow.

4.1 Motion of a magnet and the case of equilibrium of the coil above the mercury

At the beginning of his experimental defence of Newton's 3rd law, Ampère resumes his analysis of Faraday's electromagnetic experiment (1821) of *magnetic continuous rotation* [39]. In this experiment a thin magnet bar immersed in a bath of mercury rotates continuously around a vertical wire, despite the friction with the liquid, as illustrated in Fig. 5.[5] Ampère was the first to realize that the main action that moves the magnet pole comes from the electric currents flowing radially in the mercury — as indicated by i in Fig. 5 (a) — than those in the vertical wire, indicated by I [5, pp. 444–445].



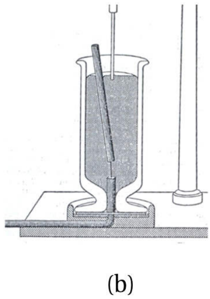


Figure 5: (a) Magnet pole (North) in continuous circular motion (ω). (b) Cut of Faraday's original figure [39].

We propose a variation of Faraday's experiment to confirm that the mercury is the main source of action on the magnet, causing its circular

[5] A video reproducing this phenomenon can be seen at `http://www.youtube.com/watch?v=Myy9tPs7H58`.

motion, Fig. 6. We hold the magnet on a mobile vertical wire using an insulating tape.

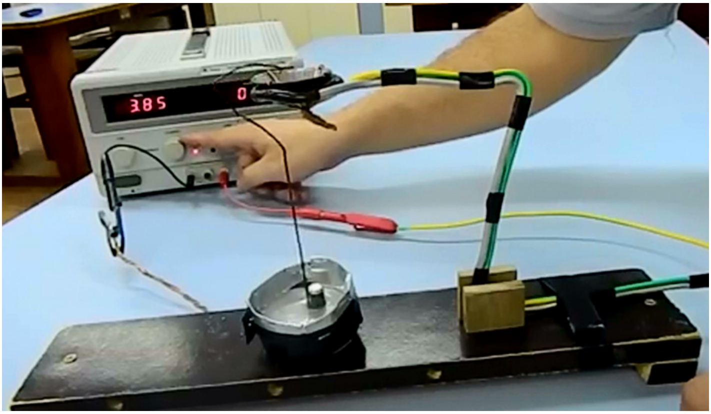


Figure 6: An electromagnetic variation of Faraday's experiment.

We activated the current source observing that the magnet (and the wire) presented a movement around the wire's contact spot on the mercury.[6] So, if the action of the electric current flowing in the vertical wire would be the main cause of the motion of the magnet, and being the third law invalid, one could obtain the same motion with only a vertical wire, without mercury, contrarily to our experiment.[7] At this point, for the sake of the discussion, the least we can conclude is that the mercury is the main source of action on the magnet, causing its circular motion in all those experiments.

Ampère's then performed Faraday's experiment and a variation in which the magnet is "covered with an insulating substance" [5, p. 444]. Both experimental results are represented in Fig. 7 which *shows the same phenomenon*, namely the rotation of the magnet around the spot `P`. Then, Ampère understood that the interaction of the currents crossing the magnet transversally — indicated in Fig. 7 (a) by PB and PB' — with the magnet itself have a null resultant. Therefore, Ampère concluded that *the action over the magnet that comes from the electrical currents*

[6] Our video is available at: `https://www.youtube.com/watch?v=bM5iBDuQaDU`.

[7] Otherwise, we would have found a continuous motor that neither Faraday nor Ampère did!

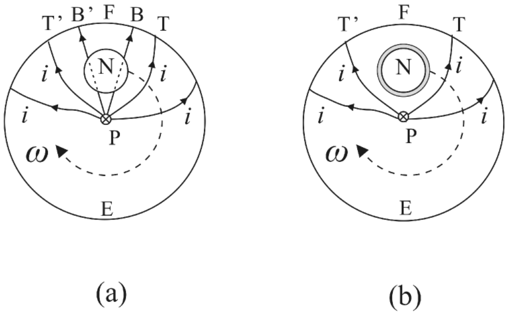


Figure 7: Currents and the motion of the magnet's pole around P: (a) *without* electric insulation; (b) "covered with an insulating substance".

flowing inside it are cancelled by a reaction from the magnet on these currents.

To understand this conclusion, let us trace a simple mechanical analogy. When someone try to nullify the force that a river exerts on a boat by paddling the water in a bathtub inside the boat, he finds that his effort is ineffective because internal actions cancel out and the motion of the boat remains the same.

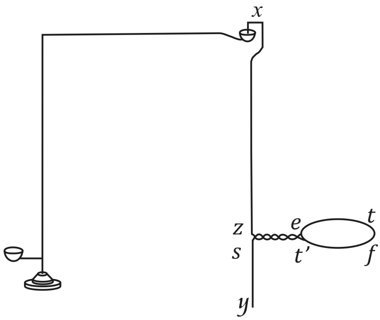


Figure 8: Ampère's illustration of a circuit to perform an electrodynamic experiment analogous to Faraday's experiment of the magnet's translational motion. This figure is an amelioration from *Théorie*'s figure 41 [2].

Maybe those procedures would have been enough to endorse Newton's 3rd law, but Ampère did not stop his investigation here. Considering the hypothetical case that the third law would be invalid, one can ask what would have been the experimental result. Would it be the same result? We need an answer independent of the force law in order *to deduce* the validity of the action-and-reaction. So, as Ampère did with all other electromagnetic phenomena, he conceived an analogous electrodynamic experiment to verify his expectations and deepen his knowledge.

A circuit's stand as shown in Fig. 8 is placed outside the same container used in the previous experiment. The mobile part of the circuit $xzetft'sy$ has two roles. The part $xzsy$ corresponds to the vertical wire of the previous experiment, in which its extremity y defines the spot P. The part corresponding to the coil $etft'$ acts as the 'magnet'.

Both Fig. 9 and Fig. 10 allow for a better comprehension of Ampère's quote in the next paragraph. The coil that will conduct the current i' is not on the mercury surface, but a few millimeters above. In Fig. 10 (a) the currents PB' and PB are below the coil, the currents PT' and PT are around the coil. The experimental result shows that the coil does not present any angular motion around P. We named the experiment in Fig. 10 (a) as the *case of equilibrium of the coil above the mercury*. In Fig. 10 (b), an insulating material was inserted below the coil's surface in order to stop the flow of the electric currents PB' and PB, and the coil presented an angular motion around P.

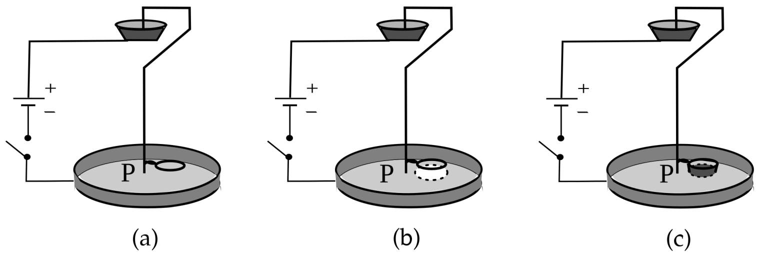


Figure 9: Illustration of the circuit to perform the electrodynamic experiment with the mobile part indicate in Fig. 8. (a) The electrodynamic coil is few millimeters above the mercury. (b) An insulating material interdicts the flow of the current below the circular surface of the coil. (c) Our proposal. The coil is fixed together with an electrical conductor below it.

Going on with Ampère's reasoning about his electrodynamic experiment, one reads [5, pp. 446–447]:

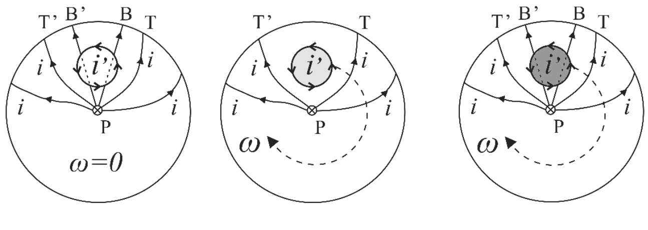


Figure 10: View from above of the mercury currents and the electrodynamic coil with its resulting movement around the spot `P`. (a) The *case of equilibrium of the coil above the mercury.* (b) Ampère's variation with an insulating interdicting the flow of the current i below the coil's circular surface. (c) Our proposal. The coil is fixed together with an electrical conductor below it. It is done in such a way that the conductor, the coil itself, and the segment of i below it belong to the same rigid body.

> [...] we suspend this mobile conductor in a manner such that the circle $etft'$ (figure 41) is very close to the mercury surface, and one sees that it rests immobile, by virtue of the equilibrium which is established between the forces exerted by the portions of the currents contained in the circle $etft'$ [i.e., the coil's current i' in Fig. 10], and those [forces] that are [exerted] by the currents and current portions outside this circle. But as soon as you remove the portions of the currents included in the space $etft'$ (figure 40), by inserting in the mercury below the circle $etft'$ (figure 41) a cylinder of insulating material whose base is such as to imitate that [base] which happens to the floating magnet, one sees it moving, like this magnet, in the direction AR.

Independently of the force law adopted to model the interactions, the results in Figs. 10 (a) and (b) lead us to three immediate conclusions. First, the action that moves the coil came from the radial electric currents in mercury. Second, in Fig. 10 (a) the currents i that flows below and around the coil act on i', canceling each other actions. Third, **the coil only moves when the actions of the currents around it are not canceled by the actions of the electric currents that flow just below it**.

To Ampère, the results represented in Figs. 10 (a) and (b) added to the results represented in Figs. 7 (a) and (b) were enough to conclude that there is no place for an electrodynamic force law in which the action originated from a rigid body on itself has a non-zero resultant (our emphasis) [5, pp. 447–448]:

> The identity of the action that one constantly observes between the movements of a mobile conductor and that of a magnet, in all cases that they are found in the same circumstances, does not permit any doubt, when one has done the preceding experiment, that **the magnet would also remain immobile**, when it is traversed by the portions of currents interior to the circle $etft'$, **if these portions could act on it**; and as one sees, on the contrary, that when it is not covered by an insulating material, and when the currents freely traverse it, it moves exactly as when it is [covered by an insulating material] and that no portions of currents can penetrate into the interior of this magnet, one has a direct proof of the principle which rests a part of the explanations that I have given, namely: **that the portions of currents which traverse the magnet do not act in any manner on it, because the forces which would result from their action on the currents proper to the magnet, or on those that one calls the magnetic molecules, by occurring between the particles of the same rigid body, are necessarily destroyed by an equal and opposite reaction**.

This conclusion is independent of the way of interaction (mediated or direct) or the form of the force law.

Someone more skeptical could argue that Grassmann's force is an action between current elements, and hence, to confront it, a complete analogy between the magnet and the coil would be invalid. Therefore, we propose a third experimental variation to discuss using only electrodynamics effects. In this experiment, the coil is fixed together with an electrical conductor below it, forming a rigid body. The conductor is also placed in a manner to allow the electric currents in the mercury flow through it, according to Fig. 9 (c) and Fig. 10 (c). This is the electrodynamic analogous to Fig. 7 (a).

Then, we perform the third variation as seen in Fig. 11. We used a small cup of aluminium fixed to the coil. In order to submerge the cup's bottom we put mercury within it. Due to the buoyant force, the mercury inside the fixed conductor is at the same level as the mercury outside. Then, we clearly saw an angular motion taking place around the spot P, as the reader can easily check.[8]

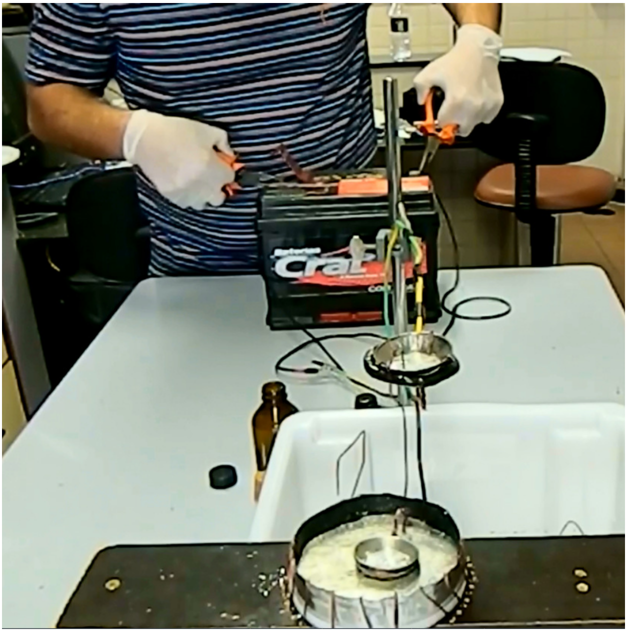


Figure 11: Our second proposal. The small aluminium cup is fixed to the coil.

With the results of our third experimental variation in hands, as illustrated in Fig. 10 (c), we endorse Ampère's reasoning. First, we compare our third result with the *case of equilibrium of the coil above the mercury*, our Fig. 10 (a): Both are electrodynamically equivalent, but they exhibited different mechanical effects. The fundamental difference between these experimental setups is the fact that the conductor below the coil and the coil itself both pertain to the same body (c) or do not (a).

Second, we observe that the angular motion around the spot P occurs when there is no currents such as PB and PB' below the coil, as seen in Fig. 10 (b), or when *both the coil and the portions of the currents PB and PB' right below the coil belong to the same rigid body*, our Fig. 10 (c).

[8]The video is available at: `https://youtu.be/BFrp0z9ePlw`.

From these comparisons, the explanation of the effect seen in our variation Fig. 10 (c) is that the portions of currents below the coil and the current i' in the coil wire cause no change on its motion due to actions that stem from themselves when they belong to the same rigid body. In other words, **the electrodynamic action originated from a rigid body on itself has a null resultant**. As we can see, this result is independent of the law of force and the conception of the interaction.

Ergo, the *simplest* conclusion — and not an arbitrary one — is that the third law has to be valid in the interactions between current elements, as Ampère already concluded.

4.2 Impossibility of the elementary torque

It seems that Ampère's logical reasoning was quite perspicacious. He noted that the elementary torque would not be invalidated by the above experiments, as long as this hypothesis admits that the action originated from a rigid body on itself has a null resultant in any case. Then, Ampère added another couple of experimental variations to the discussion in order to investigate this subject, as illustrated in Fig. 12.

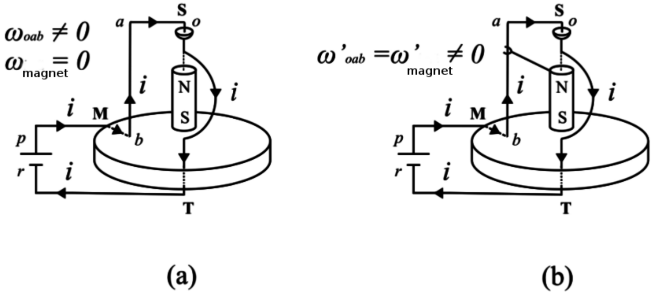
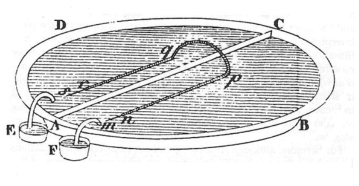


Figure 12: Schema of the variations from *Théorie*'s figure 13. M is a bowl with an axis TS and fulfilled with mercury. The fixed conductor T*o* is over the axis. The magnet NS is suspended by a thin thread, free to rotate around its axis. (a) The mobile circuit *oab* rotates with an angular velocity ω_{oab}. The magnet is at rest. (b) The magnet and the mobile circuit are fixed together and both rotates with $\omega'_{oab} \neq \omega_{oab}$.

There are two phenomena observed with the same experimental apparatus [5, p. 449]. First, when the mobile conductor (*oab*) turns about the axis TS and the magnet remains in its initial condition of motion, as seen in Fig. 12 (a). Second, when the magnet is fixed together with the mobile conductor — combined into a single rigid body — and they turn about the line TS, as indicated in Fig. 12 (b).

In the first situation, we can interpret that the magnet action (the torque $\vec{\Theta}$) rotates the mobile conductor. But, why is the magnet not rotated by the reaction (the torque $-\vec{\Theta}$) of the currents in the moving wire? The defenders of the elementary torque must admit that there's no such "strong" reaction. On the other hand, Ampère explains that the immobility of the magnet is due to the counter-torque exerted by the electric current in the stationary part of the closed circuit ($o\mathrm{T}rp\mathrm{M}$). In the second setup, however, the mobile wire and the magnet compose a single rigid body, so Ampère understood that the mobile part and the magnet nullify the torque from each other ($\vec{\Theta} - \vec{\Theta} = 0$), whereas their rotation comes from the reaction of the stationary part $o\mathrm{T}rp\mathrm{M}$. To the defenders of the elementary torque on the other hand, since they do not acknowledge the reaction of the mobile wire on the magnet, they must "assume that the connection of these two bodies into a system of invariable form, does not prevent the magnet to always act to impose on the mobile conductor the same torque" [5, p. 450]. That is, even when the magnet and the mobile part are fixed together, those defenders must admit that it is the action stemming from the magnet itself that moves it. Equally, in the phenomenon called *Ampère's motor* the defenders of the elementary couple have to admit the same *interpretation*: the magnet revolve about its own axis due to the actions originated from itself [36].[9]



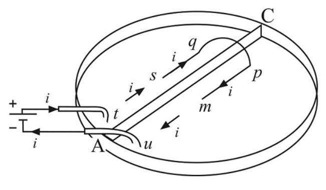


Figure 13: Ampère's bridge. (a) The original figure from the *Recueil d'Observations électro-dynamiques* [40]. (b) The schema from [2, p. 145].

Finally, we complete the discussion about the validity of the ele-

[9]The video is available at: `http://www.youtube.com/watch?v=KUDIKJ33Fvs`.

mentary torque with an *electrodynamic example*: the so-called *Ampère's bridge.* In this experiment, as indicated in Fig. 13, an insulator `AC` divides a container `DB` in two halves, and both parts are filled with mercury. The wire *mnpqrs* is added to the circuit whose extremities *m* and *s* are the only part without insulation. When we close the circuit (`F`*mnpqrs*`E`), the mobile part *mnpqrs* moves away from the contacts `F` and `E` whatever the direction of the current. At first, this phenomenon was foreseen and confirmed by Ampère just after getting the value $k = -1/2$, as already discussed in Sec. 2, which implies a collinear interaction between current elements, and that **was a counterintuitive result even to him** [2, pp. 144–147] !

Actually, in Ampère's electrodynamics, the motion of the wire (bridge) is due to its interaction with the rest of the circuit, mainly the currents *mu* and *ts*, see Fig. 13 (b). The same quantitative result can be found using Grassmann's force expression, our Eq. (10), only if one takes into account that *the action originated of the mobile conductor on itself has a non-zero resultant* [35, p. 434] !

Therefore, being or not a mediated action, once the elementary torque is assumed in a force law, at some experimental configurations one is obliged to interpret (and to admit) that the electrodynamic force from a rigid body on itself has a nonzero resultant. However, the *experimental results* in the previous subsection lead to the conclusion that the electrodynamic (or electromagnetic) action originated from a rigid body on itself has a null resultant, and this result does not depend on the adopted expression of force. Thus, the elementary torque hypothesis is in contradiction to the experimental conclusion. Ergo, *Ampère found no reason to abandon Newton's 3rd law.*

5 Further considerations: revisiting the four cases

Joseph Bertrand (1822–1900) was another scientist that defended Ampère's force law. He revisited the four cases of equilibrium in his article "Demonstrations of the theorems regarding the electrodynamic actions," our Ref. [41], in order to simplify the number of cases needed to deduce Ampère's force. He, at first, assumed the consequence of the *case of equilibrium of the nonexistence of tangential forces* as the following starting theorem [41, p. 297]:

> [...] **Theorem I** – The action from a closed current over a current element is always normal to the attracted element.

And he wrote a general formula for the electrodynamic force between $i\,ds$ and $i'\,ds'$, in modern notation, as $d^2\vec{F} = (i\,ds\,i'\,ds'\,T)\hat{r}$. That is, **he considered the elementary force directed along the line joining the two elements**. Then, he found that:

$$T = -\frac{\varphi_{(r)}}{r}\sin(\theta)\sin(\theta')\cos(\omega) - \frac{1}{2}\frac{\partial\varphi_{(r)}}{\partial r}\cos(\theta)\cos(\theta') \ , \qquad (12)$$

Next, we can use the *case of equilibrium of the law of similarity* to conclude that $\varphi_{(r)} = -1/r$ and express the force between current elements as

$$\frac{i\,i'\,ds\,ds'}{r^2}(\sin\theta\sin\theta'\cos\omega - \frac{1}{2}\cos\theta\cos\theta') \ . \qquad (13)$$

This means that we can deduce Ampère's force with only <u>two</u> cases of equilibrium and Newton's 3rd law.

From this result, Bertrand showed that the *vectorial sum of current elements* — which is an assumption largely accepted nowadays — is a theoretical consequence of the case of equilibrium of the nonexistence of tangential forces plus Newton's third law!

By knowing this, we can follow Bertrand's reasoning (our emphasis) [41, p. 301]:

> Let us suppose that Ampère — who has experimentally discovered the theorems I and III — **at first** had verified and announced the theorem I [the case of equilibrium of the nonexistence of tangential forces], and then only by reasoning — as we did — he deduced the theorem III [i.e., the vectorial sum of current elements]. Then he might have said: "If the action between two elements are along the straight line which joins them — as it seems true to me — then it is required that both sinuous conductor and straight conductor which follow the same direction, they will exercise the same action."
>
> Having experience afterwards confirmed this prediction [by the case of equilibrium of the sinuous wire] could it not be considered, rightly, as a very strong proof in favour of the hypothesis leading to it? **Would the order in which the truths were discovered and the time in which their mutual dependence were pointed out change anything as regards their probability even a little bit?**

6 Conclusion

We have seen that the *a priori* hypothesis assumed by Ampère — according to his own words — was that entities are not to be multiplied beyond necessity. His epistemic reasoning goes in line with *Ockham's Razor*. Then, his choice in favor of Newton's 3rd law did not come as a pre-established truth, but as a consequence of a method which agrees with his fundamental epistemological choice.

The French scientist made experiments guided him to conclude that, independently of the way of interaction and the force law to be adopted, the electrodynamic action originated from a rigid body on itself has a null resultant. Following this, he made some experimental configurations showing that, once the model of the elementary torque is adopted, one must embrace an interpretation that contradicts the first conclusion. This result led Ampère to conclude on the impossibility of the existence of the elementary torque. Then, Ampère's coherence with the experimental results led him to keep in his theory the action-and-reaction of the elements along the straight line which joins them.

Since those experiments were discussed in the end of his *Théorie*, one can no longer affirm that Ampère has extracted Newton's 3rd law either from the reasoning only or from an analogy with mechanics. On the contrary, the validity of this law in electrodynamics is the *simplest conclusion* — i.e., with less arbitrary assumptions — that one can take from those experiments and our experimental complements.

In summary, the philosophical defence of Newton's 3rd law developed by Ampère was highly coherent and his *experimental* defence, most importantly, remains untouchable. Therefore, the presence of this principle in Ampère's electrodynamic theory does not weaken it.

Finally, after Ampère's death the publication of his results in electrodynamics motivated the proposition of distinct force expressions using the concept of elements of current. Bertrand pointed out a common feature [41, p. 297]:

> The laws discovered by Ampère remained in science as a solid and indisputable basis, which supports with confidence even those who attempted to replace them with other laws. It is, indeed, by the complete accordance between their proposed principles with those of Ampère, in every case where verification is feasible, that it was thought possible to justify the new theories. [...]

As seen at the end of Sec. 1, even knowing those new propositions of electrodynamic force laws, Maxwell emphasized that Ampère's one "is undoubtedly the best" since it obeys Newton's 3rd law and "must always remain as the cardinal formula of electro-dynamics." The work developed here leads us to better understand the meaning of these words.

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