On a Simple Formulation of the General Fundamental Law of Electric Action

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Editor's Note: An English translation of Wilhelm Weber's 1869 paper "Ueber einen einfachen Ausspruch des allgemeinen Grundgesetzes der elektrischen Wirkung", [Web69].

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By Wilhelm Weber[1,2,3]

In these *Annalen* 1848, Vol. 73, p. 193 and following,[4] where I have given an excerpt from my first treatise on "Electrodynamic Measurements,"[5] I have added on p. 229[6,7] that the expression given in that treatise for the general fundamental law of electrical action could be simplified by specifying the expression of the *potential* instead of the expression of the *force*, that is, the function of the coordinates x, y, z, whose negative partial differential coefficients with respect to x, y, z, correspond to the *components of the force* parallel to these coordinates. If we denote by e, e' two electric particles, by r their distance from one another, and by c a certain constant, then the expression of the *force* was[8]

$$\frac{ee'}{r^2}\left(1-\frac{1}{c^2}\frac{dr^2}{dt^2}+\frac{2r}{c^2}\frac{d^2r}{dt^2}\right) ,$$

while the expression of the *potential* was[9,10]

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[1] [Web69].

[2] Translated by H. Härtel, haertel@astrophysik.uni-kiel.de and http://www.astrophysik.uni-kiel.de/~hhaertel/index_e.htm. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis.

[3] The Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by W. E. Weber are represented by [Note by WEW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[4] [Note by AKTA:] [Web48] with English translation in [Web52], [Web66] and [Web19].

[5] [Note by AKTA:] [Web46] with a partial French translation in [Web87] and a complete English translation in [Web07].

[6] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 245.

[7] [Note by AKTA:] [Web48, p. 229 of Weber's original 1848 paper and p. 245 of Vol. III of Weber's *Werke*]. See also [Web52, p. 520], [Web66, p. 520] and [Web19, p. 42].

[8] [Note by AKTA:] The expression dr^2/dt^2 should be understood as $(dr/dt)^2$.

[9] [Note by WEW:] Independently of this, Neumann in his investigations on the principles of electrodynamics started from the hypothetical formula for the potential of electrical masses as $(ee'/r)\left(1+\frac{1}{c^2}\frac{dr^2}{dt^2}\right)$.

[10] [Note by AKTA:] Let V represent Weber's potential energy:

$$V=\frac{ee'}{r}\left[1-\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right] .$$

Neumann's Lagrangian energy W, on the other hand, is expressed nowadays as:

$$W=\frac{ee'}{r}\left[1+\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right] .$$

$$\frac{ee'}{r}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right) \ .$$

The latter statement of the law can now be put into words in the following way, whereby the *physical meaning* of the law and the dependence of the *stimulation of motion*[11] on the *existing motion* emerge more clearly, namely:

> Between every two electrical particles there is partly mutual motion, partly stimulation to mutual motion. If one calls the following values, namely that of mutual motion when no stimulation takes place, and that of mutual stimulation when no motion takes place, *limit values*, then the fraction missing from one limit value is always represented by an equal fraction of the other limit value.

The *latter* of the two limit values is the well-known *electrostatic potential* ee'/r, while the *former* limit value is always the same, namely the *value of a mutual motion with the velocity* c, which can be represented by ac^2. — If there is now a mutual motion between e and e' with the velocity $dr/dt < c$, the value of which is $= a\frac{dr^2}{dt^2}$, and therefore the following fraction is missing at the first limit value

$$\frac{ac^2 - a\frac{dr^2}{dt^2}}{ac^2} = 1 - \frac{1}{c^2}\frac{dr^2}{dt^2} \ ,$$

then this missing fraction is represented by an equal fraction of the other limit value ee'/r, i.e. by $(ee'/r)(1 - [1/c^2][dr^2/dt^2])$, which is the general expression of the *potential*, as given above.

Neumann presented this expression in 1868 when he introduced the Lagrangian and Hamiltonian formulations of Weber's electrodynamics, [Neu68], with English translation in [Neu20].

Weber's potential energy $V = (ee'/r)[1 - (1/c^2)(dr/dt)^2]$ differs from Neumann's Lagrangian energy $W = (ee'/r)[1 + (1/c^2)(dr/dt)^2]$ in the sign in front of the square of the relative velocity dr/dt.

For a system of two particles interacting through Weber's force, the sum of V with the kinetic energy T of the particles is a constant in time, namely, $E = T + V =$ constant in time. The Lagrangian L and the Hamiltonian H of this system are given by, respectively, $L = T - W$ and $H = E = T + V$. We discussed this topic in [Ass94, Section 3.5: Lagrangian and Hamiltonian Formulations of Weber's Electrodynamics].

[11] [Note by AKTA:] In German: *Bewegungsanregung*. This expression can also be translated as "excitation of motion".

If e and e' indicate the *masses*[12] of the electric particles and α, β the *velocities of* e in the direction r and perpendicular to it, α', β' the same *velocities for* e', after which $\alpha - \alpha' = dr/dt$ is the relative velocity of both particles, then we have

$$\frac{1}{2}e\left(\alpha^2 + \beta^2\right) + \frac{1}{2}e'\left(\alpha'^2 + \beta'^2\right)$$

as the living force[13] or work belonging to the two particles, which expresses their motion, according to size, proportional to the moving masses and to the squares of their velocities. If we now set

$$\text{for } \alpha, \qquad \frac{e\alpha + e'\alpha'}{e + e'} + \frac{e'(\alpha - \alpha')}{e + e'} \ ,$$
$$\text{for } \alpha', \qquad \frac{e\alpha + e'\alpha'}{e + e'} + \frac{e(\alpha' - \alpha)}{e + e'} \ ,$$

and note that $\alpha - \alpha' = dr/dt$, then one can represent this living force or work of the two masses e and e' in the following *two parts*, namely

$$= \frac{1}{2}\frac{ee'}{e + e'} \cdot \frac{dr^2}{dt^2} + \frac{1}{2}\left(\frac{(e\alpha + e'\alpha')^2}{e + e'} + e\beta^2 + e'\beta'^2\right) \ .$$

The *former* may be called the *internal work*, the *latter* the *external work*, because for the *former* the knowledge of the particles e and e' and the increase or decrease of their distance from one another is sufficient, while for the *latter apart from the particles* e and e', a fixed coordinate system must be given in order to be able to observe and measure the velocities $(e\alpha + e'\alpha')/(e + e')$, β and β'.

It is now evident that this *internal work* $(1/2)[ee'/(e + e')] \cdot [dr^2/dt^2]$ is the exact value of the *mutual motion* of both particles, which was denoted above with $a[dr^2/dt^2]$, so that $a = (1/2)[ee'/(e + e')]$.

[12][Note by AKTA:] Weber is utilizing the symbols e and e' to indicate not only the values of the electric charges of the two particles, but also the values of their inertial masses. In 1871 he will represent the charges by e and e', while the inertial masses will be represented by ε and ε', [Web71] with English translation in [Web72].

[13][Note by AKTA:] In German: *lebendige Kraft*, or *vis viva* in Latin. Term coined by G. W. Leibniz (1646-1716).

Originally the *vis viva* of a body of mass m moving with velocity v relative to an inertial frame of reference was defined as mv^2, that is, twice the modern kinetic energy. For some years there was confusion in nomenclature and some authors called $mv^2/2$ by vis viva.

What Weber calls here the living force (*lebendige Kraft*) of a particle should be understood as the modern kinetic energy, namely, $mv^2/2$.

This *internal work* and the *potential* of the two particles e, e' at the distance r can have very different values, but if one value increases, the other decreases, and the increase and decrease are always in the same proportion. If the *potential* has decreased by ee'/r, the *internal work* has increased by $(1/2)[ee'/(e+e')]c^2 = ac^2$. If this *internal work*, which has taken the place of the vanished potential, is called the *work equivalent*[14] of that potential, the *work equivalent of an arbitrary potential* V results from the same relationship $= [rc^2/2(e+e')] \cdot V$.

The *existing internal work* and the *work equivalent of the existing potential* form together the *sum of the existing internal work values.* Understood in this way, the following simple formulation of our law results, namely:

> For two electrical particles e and e', at any distance from each other, the sum of the existing internal work values is always the same, equal to $(1/2)[ee'/(e+e')] \cdot c^2$.

Since the existing *internal work* is $(1/2)[ee'/(e+e')] \cdot [dr^2/t^2]$, the existing potential is V and *its work equivalent* is $[rc^2/2(e+e')] \cdot V$. Consequently, the *sum of the existing internal work values* is equal to

$$\frac{1}{2}\frac{ee'}{e+e'} \cdot \frac{dr^2}{dt^2} + \frac{rc^2}{2(e+e')} \cdot V = \frac{1}{2}\frac{ee'}{e+e'} \cdot c^2 \ ,$$

or, divided by the last term,

$$\frac{1}{c^2} \cdot \frac{dr^2}{dt^2} + \frac{r}{ee'} \cdot V = 1 \ ,$$

from which the *potential* $V = (ee'/r)(1 - [1/c^2][dr^2/dt^2])$ is obtained, as above.

The formulation of the law discussed here is only intended to show in the simplest way the dependence of two particles on each other in their motions, especially the dependence of their mutual stimulation on their existing motion. Quite different needs emerge when it comes to the task to find the complete mathematical development of all the consequences of this law in connection with the general principles of mechanics in the case of larger electrical masses connected in various ways with other bodies. For this task the principles of electrodynamics are to be brought into other forms, which was not the objective of this work.

[14] [Note by AKTA:] In German: *Arbeitsäquivalent*.

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