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Physics in Perspective

Carl Neumann's Contributions to Electrodynamics

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I examine the publications of Carl Neumann (1832–1925) on electrodynamics, which constitute a major part of his work and which illuminate his approach to mathematical physics. I show how Neumann contributed to physics at an important stage in its development and how his work led to a polemic with Hermann Helmholtz (1821–1894). Neumann advanced and extended the ideas of the Königsberg school of mathematical physics. His investigations were aimed at founding a mathematically exact physical theory of electrodynamics, following the approach of Carl G.J. Jacobi (1804–1851) on the foundation of a physical theory as outlined in Jacobi's lectures on analytical mechanics. Neumann's work also shows how he clung to principles that impeded him in appreciating and developing new ideas such as those on field theory that were proposed by Michael Faraday (1791–1867) and James Clerk Maxwell (1831–1879).

Key words: Herman Helmholtz; Heinrich Hertz; Carl G.J. Jacobi; James Clerk Maxwell; Carl Gottfried Neumann; Franz Ernst Neumann; Wilhelm Eduard Weber; electrodynamics; potential theory; mathematical physics; theoretical physics.

Introduction

Carl Gottfried Neumann (1832–1925) was a prominent mathematical physicist who contributed to potential theory, electrodynamics, analytical mechanics, and hydrodynamics during the last third of the nineteenth and the first decades of the twentieth century, a period that witnessed fundamental changes in physics. In the history of electrodynamics, this period was characterized by conflicting theories, principally those of Wilhelm Eduard Weber (1804–1891), Hermann Helmholtz (1821–1894), and James Clark Maxwell (1831–1879). Neumann sought an exact mathematical foundation of electrodynamics, which he expected to find in light of his earlier and important results in potential theory. In the end, however, his research program failed, and his contributions to electrodynamics had little influence in terms of the physical results he obtained, but not in regard to methodological aspects of mathematical and theoretical physics.

I first sketch Neumann's personal and scientific background and his methodological views on mathematical physics. I then discuss his research on electrodynamics in the late 1860s and 1870s in its historical context and show how his resulting dispute with

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Helmholtz revealed fundamental differences in their basic assumptions on electrodynamics. Finally, I discuss his further work on electrodynamics at the end of the nineteenth and beginning of the twentieth century and offer an evaluation of his contributions.

Biographical Sketch, Scientific Background, and Methodological Views

Carl Gottfried Neumann, born on May 7, 1832, was the son of Franz Ernst Neumann (1798–1895), Professor of Mineralogy and Physics at the University of Königsberg. After his primary and secondary education in Königsberg, he entered the University of Königsberg, where he attended the famous physical-mathematical seminar that had been founded by his father and Carl Gustav Jacob Jacobi (1804–1851) in 1834.[1] Many of the students who attended it were appointed later to professorships in German universities and disseminated its innovative program of joining mathematical and physical research, thus contributing significantly to the institutionalization of mathematical physics and to the emergence of theoretical physics as a discipline.

Problems in mathematical physics were a central focus in Neumann's scientific work. He regarded the application of mathematics to physics, astronomy, and related disciplines as an indispensable part of mathematical research and as a fertile source of new knowledge in mathematics and physics.[2] In his doctoral dissertation of 1856, which was supervised by the analyst Friedrich Julius Richelot (1808–1875), he solved a specific problem in mechanics by applying the theory of hyperelliptic integrals to it. Two years later, in his Habilitation thesis (*Habilitationsschrift*) at the University of Halle, which was supervised by the mathematician Heinrich Eduard Heine (1821–1881), he treated mathematically the rotation of the plane of polarization of light by magnetism (the Faraday effect). He also investigated mathematical problems that were not directly related to physics. These included studies on Bernhard Riemann's theory of Abelian integrals, which he published in 1865 in an influential book that introduced many mathematicians to Riemann's work on multivalued functions.[3] His work on potential theory occupied a middle ground, since it dealt with problems in both pure mathematics and physics.* He used potential theory to treat various problems in physics, for instance by employing series expansions in terms of special functions. Gaps remained, however. Already in 1853, Helmholtz had pointed out the necessity of elaborating potential theory and improving its mathematical methods in the treatment of physical problems. Thus, in considering the distribution of electrical currents in three-dimensional conductors, Helmholtz complained that the available mathematical methods permitted a complete solution "only in a few of the easiest cases."[4] The difficulties were similar to those that arose in the case of the distribution of electrical charges on the surface of a conductor.

Neumann (figure 1) worked hard to improve the mathematical methods used in potential theory. In 1861, he solved the Dirichlet problem in the plane by introducing

* I will analyze Neumann's contributions to potential theory in detail in another article that may be considered as the mathematical counterpart to this article.

Fig. 1. Carl Gottfried Neumann (1832–1925) at about age 40. Courtesy of the Sächsische Akademie der Wissenschaften zu Leipzig.

a logarithmic potential (a term he coined),[5] and in 1870 he solved the Dirichlet problem* more generally by introducing his method of the arithmetic mean.[6] He returned to this topic from time to time in succeeding decades, supplementing his treatment of potential theory with new mathematical proofs and results.

Neumann's researches enabled him to advance rapidly up the academic scale. In 1858, with the completion of his Habilitation thesis, he qualified as a lecturer (*Privatdozent*) at the University of Halle, and in 1863 was appointed as extraordinary (*ausserordentlicher*) professor there. That same year, however, he was appointed as ordinary (*ordentlicher*) or full professor at the University of Basel. Two years later, he moved again at the same academic level to the University of Tübingen, and in yet another three years, in 1868, he transferred to the University of Leipzig where he retired in 1911. In his inaugural lectures both at Tübingen and at Leipzig, he elaborated his methodological views on mathematical physics and also made some general remarks on electrodynamics.

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* The Dirichlet problem or the first boundary-value problem of potential theory is as follows: Given the values of a function on the boundary of a region in space or in a plane, find a function f that satisfies $\Delta f = 0$ in this region and that takes on those boundary values.

Neumann's methodological views on the proper structure of a physical theory were influenced primarily by Jacobi's as based on the latter's work in analytical mechanics. Shortly after his appointment at Leipzig in 1868, Neumann became acquainted with Jacobi's ideas, which Jacobi developed after his move from Königsberg to Berlin in 1844. Neumann's Leipzig colleague Wilhelm Scheibner had taken notes of Jacobi's lectures on analytical mechanics in Berlin in the winter semester of 1847–1848,[7] and Neumann now studied and partly transcribed them. He characterized Jacobi's lectures as constituting an exceptionally penetrating critique of the foundations of mechanics.[8] Now, like Jacobi, Neumann took his goal to be to start from some unanalyzable basic assumptions or principles and to deduce general theorems from them, in this way constructing a physical theory of the field under consideration. His goal thus was to establish a physical theory by employing an axiomatic approach like that found in geometry. He stressed, however, what he considered to be a fundamental difference between mathematical axioms and physical principles: He claimed that the latter, which form the basis of a physical theory, can never be described as true or probably true; they always embody some uncertainty or arbitrariness.[9]

To Neumann, a physical theory thus should be derived deductively from a few basic unanalyzable principles. Empirical facts also should be deduced from these basic principles in a mathematically correct way. Conversely, an important objective of physical research consists in exposing empirical facts and formulating them as physical principles. The task of mathematical physics then consists in formulating these principles in a mathematically manageable manner, drawing logically correct conclusions from them, and presenting these conclusions in a form that can be tested by experiments.[10]

Neumann's Early Work on Electrodynamics

Neumann began his researches on electrodynamics in the 1860s. The main problems at that time were to describe quantitatively the new phenomena that had been discovered, to improve the methods of their measurement, and to seek a general theoretical explanation of them that would encompass both contact and spatially-separated effects. One approach was to adopt the field concept that Faraday had proposed and Maxwell had formulated mathematically. Another was to assume action-at-a-distance forces. By adopting the latter approach, Franz Ernst Neumann had deduced the electrodynamic potential for the interaction of two linear currents in 1845, from which he also derived the electrodynamic force that two closed circuits exert on each other. Wilhelm Eduard Weber (1804–1891), also in 1845, had proposed his fundamental law describing the electrodynamic force acting between two current elements. Weber's law marked the culmination of a search for a fundamental law of electrodynamics to which André-Marie Ampère (1775–1836), Hermann Günther Grassmann (1809–1877), and Jean Baptiste Biot (1774–1862) and Félix Savart (1791–1841) had made substantial contributions. A peculiarity of Weber's law was that it was a function not only of the distance between the two current elements but also of the relative velocity and acceleration of their constituent charges.

A further problem involved the explanation of electrical conduction. Following the ideas of the German psychophysicist Gustav Theodor Fechner (1801–1887), Weber

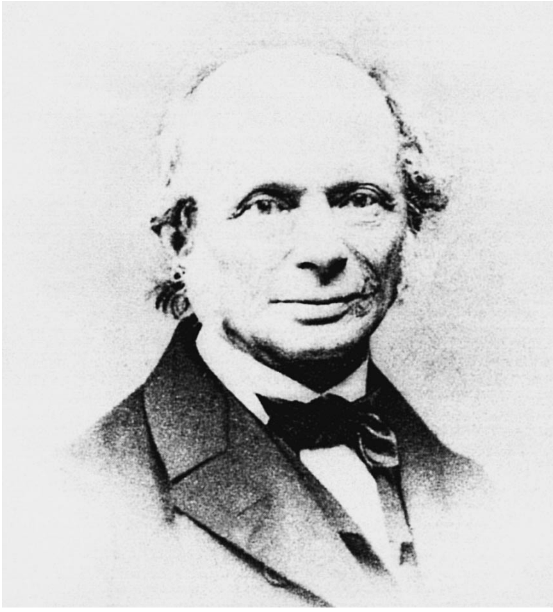

Fig. 2. Wilhelm Eduard Weber (1804-1891), professor at the University of Leipzig (1843–1849) and one of the founders of the Sächsische Akademie der Wissenschaften in 1846. Courtesy of the Sächsische Akademie der Wissenschaften zu Leipzig.

(figure 2) adopted a dualistic approach that assumed the existence of two electrical fluids of opposite charge whose equal and opposite flow in a conductor constituted an electrical current. Others advanced a unitary approach that assumed the existence of a single, positively-charged electrical fluid with the negative charges being bound to ponderable particles in matter. Neumann, who was mostly familiar with work in Germany, adopted Weber's dualistic approach when he began his investigations on electrodynamics.

Neumann published the results of his first researches on electrodynamics in 1868 in a Festschrift that was dedicated to the University of Bonn on the 50th anniversary of its foundation.[11] Five years later, he developed his work further in a treatise entitled *The Electrical Forces. Part 1*, whose subtitle was *The Theories developed by A. Ampère, F. Neumann, W. Weber, and G. Kirchhoff presented and enlarged*.[12] In accordance with his methodological views on the structure of a physical theory, Neumann carefully pointed out the basic assumptions or principles on which a theory of electrodynamics could be based, and his discussion revealed how much he had struggled to formulate them. He analyzed various basic principles and their implications and grappled with objections to them, mostly ones that had been raised by Helmholtz. His goal was to present a systematic treatment of electrodynamics and to stimulate its further development. The first part of his projected two-part monograph was based on the work of Ampère and F. Neumann.

In accordance with his methodological views and mathematical approach, Neumann paid a great deal of attention to determining the exact validity of each electrodynamic law or principle and the logical relationships among them. He divided them into four groups, the first of which consisted of those laws or principles to which he ascribed the highest degree of certainty. He noted that a special difficulty was to create a uniform nomenclature in domains that had been treated at different times from various points of view.[13] To further his goal of establishing a mathematically exact theory of electrodynamics, he differentiated among the notions of axiom, principle, and hypothesis to stress their different characters and to mimic Euclid's venerable axiomatic method. He argued that a theory based upon axioms, principles, or hypotheses unavoidably involves a degree of uncertainty.[14] Its confirmation rested, first, on its internal consistency, which was equivalent mathematically to the impossibility of deriving two contradictory statements from it, and second, on the agreement of the statements deduced from its hypotheses with known experimental results.

Neumann's first group of basic assumptions included the "axiom of living forces," that is, the principle of conservation of energy, and James Prescott Joule's law of the "electrodynamic production of heat." His second group consisted of F. Neumann's two integral laws of electrodynamics,* his third of three statements on fundamental properties of ponderomotive and electromotive forces, and his fourth of Ampère's two hypotheses on the electrodynamic force between two current elements, that is, that it acts along the line connecting them, and that the total force exerted by a closed circuit acts perpendicularly on a current element.[15] To Neumann, Ampère's two hypotheses were the most questionable ones, but he saw no reason to eliminate them. In fact, he reinforced his confidence in them in a separate article in 1874,[16] where he deduced Ampère's law of electrostatics and Ampère's law of electromotive force without using these two hypotheses. He replaced them with a hypothesis on three basic properties of the ponderomotive and electromotive forces and was able to deduce qualitative descriptions of some known effects from it. He was convinced that his theory of electrodynamics was highly credible.[17]

Both in his monograph of 1873 and in his article of 1874, Neumann emphasized the logical relationships among his principles or hypotheses and deduced from them an elementary electrodynamic law that specified the two components of the force acting from a current element in one conductor to a point in a second conductor during an element of time *dt*. His law completed his system of principles, and with it he believed that he had attained his objective of showing that his system as a whole offered an explanation of electrodynamic phenomena. His principles of electrodynamics exhibited a parallelism between electromotive and ponderomotive phenomena. The fundamental structure of his theory thus seemed to be clear, but he was unable to deduce important new physical results from it.

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* These two laws describe, respectively, the ponderomotive work done and the electromotive force exerted by two closed circuits acting on each other. Both laws were based on the electrodynamic potential between two currents that F. Neumann had introduced in 1845.

Neumann's Debate with Helmholtz

Some of Neumann's contemporaries disputed his assumptions and conclusions. Neumann's analysis had revealed certain obscurities that could have been resolved logically by other theories of electrodynamics, but the current state of knowledge was insufficient to make a choice among them. The resulting debates centered on deriving conclusions from the various theories of electrodynamics that could be verified experimentally and hence used to either confirm or falsify them.

Neumann assumed that the electrodynamic potential propagates with a finite velocity and that Weber's hypothesis, that the force between two current elements depends on the relative velocity and acceleration of their constituent charges, was correct. This feature of his theory prompted objections by Helmholtz, who already in his famous memoir on the conservation of energy (*Kraft*) in 1847 had pointed out that conservation of energy would be violated by velocity-dependent forces.[18] Now, in 1870, Helmholtz presented his own theory of electrodynamics,[19] which challenged both Weber's and F. Neumann's theories and paved the way for new theoretical insights and experimental tests.[20] As Jed Buchwald has argued, Helmholtz's theory assumed an interaction between states of energy that represented definite physical states of the charged or current-carrying entities.[21] His theory included elements of Weber's atomistic theory and of Faraday and Maxwell's field theory but differed completely from both in its basic ideas. "In the face of [such] conflicting theories," Helmholtz wrote, he preferred "to remain as close as possible to the base of facts and to leave undetermined those parts of the theory that until now cannot be considered as corroborated by experiments."[22] He recognized that the various theories might be distinguished by their differing treatments of open circuits. He therefore proposed a general electrodynamic potential that embodied the known laws of induction and also applied to open circuits. His expression for the electrodynamic potential p between two current elements, which depended only on a single parameter k, was as follows:

$$p = -\frac{1}{4}\frac{ij}{r}\left[(1+k)\cos(D\xi, D\sigma) + (1-k)\cos(r, D\xi)\cos(r, D\sigma)\right],$$

where $D\xi$ and $D\sigma$ are current elements of intensity i and j, respectively, r is the distance between them, and $(D\xi, D\sigma)$, $(r, D\xi)$, and $(r, D\sigma)$ are the angles between the specified elements.[23] F. Neumann's electrodynamic potential corresponded to the case of $k = 1$, Weber's to that of $k = -1$, and Maxwell's law of electromagnetic induction to $k = 0$, although this was not quite correct.[24] These expressions all agreed in the case of closed circuits but differed substantially in the case of open circuits.

Helmholtz discussed the fundamental differences between F. Neumann's and Weber's action-at-a-distance theories and Faraday and Maxwell's field theory and showed that both were consistent with experiment. He also noted that both could account for experiments on the motion of electricity in dielectrics, the velocity of electrical currents, and the like. He pointed out, however, that Weber's theory violated conservation of energy and permitted charged particles to attain an infinite velocity in a finite amount of time owing to his assumption of velocity and acceleration-dependent forces.[25] This also constituted Helmholtz's reason for criticizing Neumann's theory of electrodynamics.

Neumann and Weber and some of their followers responded to Helmholtz's criticisms, in some cases polemically, and rejected them. Weber, in particular, asserted that his theory was compatible with conservation of energy and discussed the motion of two charged particles resulting from their interaction. To do so, however, he had to investigate the motion of the particles at the molecular level,[26] which was inaccessible to experiment, so he argued that his theory could serve only as a guide to some future theory of such molecular motions, as assumed for instance in Ampère's theory of molecular currents. As Weber stated:

> without knowledge and strict consideration of the *molecular forces restricted to molecular distances* that undoubtedly come into play in molecular motions, no exact *quantitative* validity can be ascribed to the derived results, but only a *qualitative* validity within certain limits, which have meaning only for a first investigation of the topic.[27]

Weber concluded that for the time being the validity or invalidity of Helmholtz's criticisms could not be proven experimentally.

This was the context in which Carl Neumann attempted to generalize Weber's law and introduce his own electrodynamic potential. Instead of an inverse $1/r$ dependence, he assumed a general function $\phi(r)$ that went over to the limit $1/r$ at sufficiently large r but took on much larger values at very small r. Already in 1863, in his investigation of the Faraday effect for his Habilitation thesis at the University of Halle, Neumann had appealed to an analogy to the theory of capillarity and had applied this generalized version of Weber's law to the interaction between an electrical particle and an aether particle.[28] Now, in 1871, he argued similarly that "Newton's [inverse-square] law [as seen in the theory of capillarity and elasticity or more generally as manifested in the forces of cohesion and adhesion] … has to undergo a certain modification for *very small* distances *r*."[29] Neumann took this modification to hold for Weber's law as well and applied it to electrical conduction. That did not mean, however, that his picture of atomic forces and particle dimensions was more precise than Helmholtz's. Helmholtz thus was able to present counterarguments, justifying his theory, but he failed to convince his opponents. In the end, the debate ebbed away in the late 1870s, leaving various important questions unresolved.

The Helmholtz-Neumann debate might seem somewhat surprising, since both used potential theory, and Helmholtz's approach corresponded closely to Neumann's methodological views on the structure of a physical theory. Thus, Helmholtz assumed the validity of conservation of energy and derived a general formula for the electrodynamic potential that covered "the entire experimentally known area of electrodynamics … with one and the same relatively simple mathematical expression."[30] He noted with satisfaction that Neumann had required many hypotheses to reach a similar result. Neumann stuck to his guns, however, although he acknowledged that conservation of energy was a basic principle of physics,[31] and he admitted that Helmholtz's theory appeared to have a simpler structure. One important reason, in general, that Neumann opposed Helmholtz's approach was that Helmholtz's states of energy and their interaction made the explanation of many electrodynamic phenomena more complicated than explanations based upon Weber's or Maxwell's theories.[32]

There also were other difficulties. There was no crucial experiment involving open circuits, for instance, that could decide between the competing theories of electrodynamics. Also, an unanswered question was whether certain phenomena could be explained only by appealing to microscopic considerations, as in the case of Weber's and Neumann's theories, or if a macroscopic point of view was sufficient. Thus, Neumann pointed out in an article of 1878 that the differential equation describing the motion of electricity in conductors could be derived from macroscopic and microscopic considerations.[33] He also insisted on the necessity of distinguishing between far and near effects. But he could not convince himself of the validity of either the macroscopic or microscopic points of view. Rather, he said, "*I believe that each of them contains some seeds of truth, and I therefore studied both thoroughly*."[34] His debate with Helmholtz had revealed no contradiction with his theory, and he ignored completely Maxwell's field theory.

This reflected Neumann's methodological views on mathematical physics. His objective was to analyze the logical relationships among the hypotheses of his theory of electrodynamics and on their basis to construct a mathematically exact theory that was free of internal contradictions. He acknowledged the importance of experiments in verifying or falsifying a theory, but as a mathematical physicist he ignored experimental research as not being within his province. This probably explains why he turned away from electrodynamics in the 1880s. By then electrodynamic theory had reached a stage where further progress depended either on increasing the certainty of its assumptions or on carrying out experiments that would force fundamental revisions in it. The earlier discussions on the different approaches to electrodynamics, their theoretical assumptions, and the laws deduced from them had indicated some possible experimental tests, and to Neumann it was sufficient that he had contributed to this development. Helmholtz (figure 3), by contrast, regarded theory as inseparable from experiment and hence always sought to explain experimental results theoretically. He used mathematics and acknowledged its importance, but he did not take mathematics as a model for theoretical physics.[35] Thus, it is not surprising that Helmholtz encouraged experiments to test the various theories of electrodynamics in his laboratory in Berlin in the 1870s, some of which were carried out by visiting scientists. Helmholtz's own theory of electrodynamics soon fell by the wayside, but his general stance on the role of theory in physics proved to be much more suitable than Neumann's for accommodating new experimental data.[36] Neumann's strong mathematical orientation led to inflexibility in his attitude toward the basic assumptions in his theory of electrodynamics. As we now will see, this also had a negative influence on his subsequent researches in electrodynamics.

Neumann's Return to Electrodynamics

In the early 1890s, Neumann refocused his researches on electrodynamics. Progress in mathematics, especially improved methods for treating potential theory, prompted him to reexamine the relationship of mathematics to physics when applied to physical phenomena. He did not aim at a fundamental reconstruction of physical theory, but in a book he published in 1893 he treated some special cases in electrodynamics and other areas of physics,[37] demonstrating the power of the improved mathematical methods in

Fig. 3. Hermann Helmholtz (1821–1894) at about the time of his debate with Carl Neumann. *Source*: Hermann Helnholtz, *Wissenschaftliche Abhandlungen*. Erster Band (Leipzig: Johann Ambrosius Barth, 1882), frontispiece.

calculating the potential functions for these special cases. Difficulties arose in his calculations, but they did not stem from new physical ideas but from the specific configurations he assumed for electric-charge or magnetic distributions. He thus followed the traditional approach in mathematical physics, using physical problems to present mathematical results. He emphasized the mathematical character of his work in the first chapter of his book by summarizing the mathematical theorems he required and by discussing related geometrical problems in an appendix to it.

Two features of Neumann's book are especially noteworthy, since they show that he had not changed his methodological views on mathematical physics. First, he analyzed thoroughly the analogy between electrodynamics and hydrodynamics that Helmholtz had discussed in a paper of 1858 and that Gustav Kirchhoff (1824–1887), Ludwig Boltzmann (1844–1906), Carl Victor Riecke (1845–1915), and others subsequently investigated. He concluded that this analogy had no deep physical meaning that could serve as a common basis for the two fields. The analogy arose instead for mathematical reasons, that is, the continuous functions occurring in hydrodynamics could be constructed electrodynamically, meaning that an arbitrary function and its derivatives on a domain G are continuous and could be represented in G as a potential of a system "partly of magnetic and partly of electrical" origin. Neumann presented proofs of several theorems like this one in the eighth chapter of his book.

Second, Neumann realized that electrodynamics could not be based only on the axioms or principles of mechanics. "The effects of heat," he noted, could not be explained by "purely *mechanical* principles, which could not either be applied to the theory of *electricity* and *magnetism*." He concluded that finding the "true and simple principles of electrodynamics and magnetism … is still a long way off, and they will not be of a purely mechanical nature."[38] Thus, while many German-speaking theoretical physicists such as Boltzmann, Hermann Ebert (1861–1913), Woldemar Voigt (1850–1919), and Arnold Sommerfeld (1868–1951) attempted to formulate Maxwell's theory on the basis of mechanical principles,[39] Neumann emphasized the necessity of broadening this framework. He himself, however, did not pursue this insight, since he shunned experimental research, the results of which were required for uncovering the basic principles of electrodynamics. In particular, he did not reexamine his position on Weber's theory and his own basic assumptions, and hence he did not search for a better understanding of electrodynamics based on the work of Helmholtz, Heinrich Hertz (1857–1894), and others.

Neumann's methodological views on mathematical physics thus embodied a fundamental ambivalence. On the one hand, physical phenomena served as stimuli for his mathematical investigations, which enabled him to identify mathematical gaps in the formulation of theoretical concepts. On the other hand, his stance prevented him from accepting new ideas that failed to meet his strong mathematical criteria for a general theoretical foundation of physical phenomena.

In 1898 Neumann published the second volume of his treatise on electrical forces.[40] Its specific title, *On the Investigations of Hermann von Helmholtz in his Early and Recent Works*, indicated its focus. He remained dissatisfied with Helmholtz's explanations of electrical and magnetic phenomena. His objective now was to analyze critically Helmholtz's theories and to investigate "those universal principles by which electrical phenomena are connected with those of gravitation and heat and are bound together in a unified whole."[41] He thus envisioned searching for a unified theory based on a few general physical principles.

As indicated in the title of his book, Neumann distinguished between the work that Helmholtz published between 1870–1875 and between 1892–1894. He characterized the former as resting on Helmholtz's assumption of instantaneous action-at-a-distance forces and his rejection of Ampère's law, the latter as resting on Helmholtz's assumption of short-range forces along the lines that had been proposed by Faraday, Maxwell, and Hertz, as well as by Joseph Fourier (1768–1830) in his theory of heat. He appreciated the value of both approaches and insofar as possible wished to complete them.

Neumann's analysis of Helmholtz's early work occupied around 300 pages in his book and adhered to his style of mathematical deduction. He discussed his old dispute with Helmholtz but offered no new insights into it or fundamentally new results, only some refinements of old ones. He then turned, in the second part of his book, to those "ideas that Faraday, Maxwell, [Oliver] Heaviside [1850–1925], [John Henry] Poynting [1852–1914], Hertz, and others had introduced into science."[42] He presented their ideas in some detail but undervalued their influence, probably because of his strong focus on Helmholtz's work and potential theory. However, he did mention two articles of Hertz of 1890 in which Hertz (figure 4) extended Maxwell's theory to include the motion of charged particles, and he stressed that field theory avoided action-at-a-distance forces, assuming that the aether filled all space and that changes in its electrical and magnetic states were caused by the action of short-range forces. But he also criticized some assumptions that Hertz had made in describing electrodynamic effects, although he admitted that he may have misinterpreted Hertz's ideas.[43] In general, Neumann judged the value of Hertz's theory from his own mathematical and potential-theoretical point of view, which made it difficult for him to understand and appreciate its importance. He stuck to his old approach and underrated field theory as lying "far away" from the mainstream development of electrodynamics.[44] He did not appreciate Hertz's systematic and clear presentation of Maxwell's theory and Hertz's extension of it to the motion of charged particles.

Neumann spoke instead of what he considered to be the remarkable progress in electrodynamics owing to Helmholtz's use of the principle of least action, which he called an attempt "to bring the isolated formulas, hypotheses, and theorems in Hertz's articles into a unified point of view."[45] The principle of least action allowed Neumann to return to analytical mechanics and was consonant with his attempt to found electrodynamics on formal mathematical and unanalyzable basic principles. He saw Helmholtz's use of the principle of least action and the specific minimal principle Helmholtz had derived from it as a great advance over the assumption of action-at-a-distance forces, since it permitted the unification of disparate physical phenomena. In the end, however, Helmholtz's ideas were based on potential theory, and when Neumann attempted to apply Helmholtz's minimal principle to magnetostatics and related phenomena, as well as to electrodynamics, he encountered difficulties in reproducing Ampère's and F. Neumann's formulas. He therefore concluded that Helmholtz's use of the principle of least action was an insufficient basis for a theory of electrodynamics. He believed, however, that it should be investigated further, since he felt that it might be possible to unify electrodynamics, gravitational theory, and the theory of heat by using such a general principle.[46]

Neumann displayed his tension between adhering to old ideas and reacting to new ones in his discussion of Weber's law. On the one hand, he defended it. On the other hand, he characterized it as being incomplete, since it did not describe all electrodynamic effects and thus required supplementary assumptions. That meant that for each problematic case one had to decide whether it arose from a failure of Weber's law or from one or more of the supplementary assumptions. Neumann was forced to conclude that the validity of Weber's law *per se* could not be determined. Nevertheless, he praised it, since it had had such great influence on the development of electrodynamics.

Fig. 4. Heinrich Hertz (1857–1894). *Source*: Heinrich Hertz, *Gesammelte Werke*. Band I. *Schriften Vermischten Inhalts* (Leipzig: Johann Ambrosius Barth, 1895), frontispiece.

Despite his intention to the contrary, Neumann thus did not achieve a general theory of electrodynamics in his two-part treatise on electrical forces of 1873 and 1898 based upon his methodological views and his analysis of Helmholtz's and Hertz's work. He did not explain why he had concentrated on their work, although he had a high appreciation of it. Nevertheless, this does not account for his almost completely ignoring the contributions of others to the development of Maxwell's theory. He did not mention, for example, Boltzmann's two-volume *Lectures on Maxwell's Theory of Electricity and Light* published in 1891 and 1893,[47] Kirchhoff's *Lectures on Electricity and Magnetism*, which were edited by Max Planck and published in 1891,[48] and August Föppl's *Introduction to Maxwell's Theory of Electricity* of 1894.[49] Föppl's book, in particular, had a strong influence on the reception of Maxwell's theory among German-

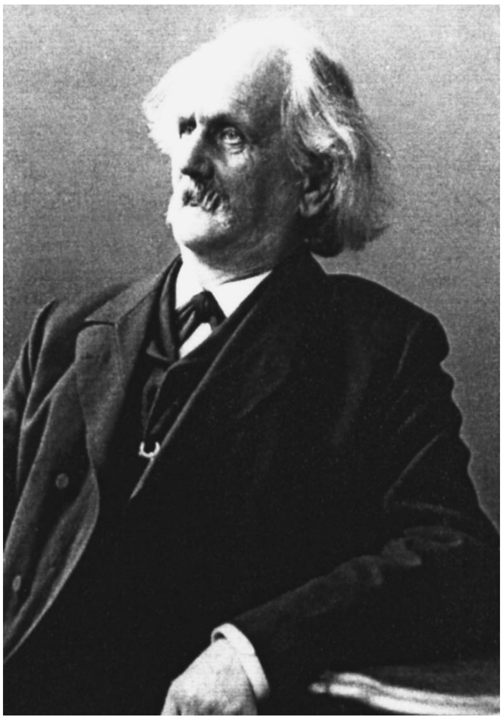

Fig. 5. Carl Gottfried Neumann (1832–1925) as professor at the University of Leipzig. Courtesy of the Bibliothek des Mathematischen Instituts, Universität Leipzig.

speaking physicists, and its revised edition that Max Abraham (1875–1922) published in 1904–1905 became a standard textbook in the field.[50] Neumann knew Föppl (1854–1924), since Föppl had lectured in Leipzig until the summer of 1894, but Neumann seems to have had no scientific interaction with him. Föppl did not mention Neumann in his reminiscences, but he did note his close association with Gustav Heinrich Wiedemann (1826–1899) and his son Eilhard (1852–1928).[51] Föppl and Neumann's scientific communication with each other no doubt was impeded because of their very different approaches to physics, and because until 1892 Föppl did not lecture at the University of Leipzig but instead at a trade school in Leipzig.

Neumann (figure 5) finally discussed Maxwell's theory in depth in three extensive publications in 1901–1904. In light of the great attention that others had paid to Maxwell's theory, Neumann decided at long last to examine it critically. He acknowledged that Hertz's extension of Maxwell's equations to moving bodies was an outstanding contribution to electrodynamics and that the resulting Maxwell-Hertz equations constituted the essence of Maxwell's theory.[52] He pointed out the assumptions that Hertz had made in extending Maxwell's equations, and he emphasized the analogy between Maxwell's theory and Fourier's theory of heat without, however, deciding on the validity of the former. He also noted possible modifications of the Maxwell-

Hertz equations by invoking mathematical transformations, and he explained the importance of coordinate systems in physics, a subject that he had discussed already in his inaugural lecture at the University of Leipzig in 1869, where he had argued for assuming an inertial coordinate system in classical mechanics and had introduced the Body Alpha as a fixed reference point, an idea that later was seen to constitute part of the prehistory of relativity theory. He recalled Hertz's remark that the fundamental equations of the electrodymanics of moving bodies (the Maxwell-Hertz equations) could be transformed "to any other arbitrary coordinate system" without changing their form,[53] or as he put it:

> Hertz's differential equations remain completely unchanged in their form, if they are transformed from one positive orthogonal coordinate system to another such coordinate system, irrespective of the relative motion of one system with respect to the other.[54]

That Neumann did not have the Lorentz transformation equations in mind here seems clear, since nowhere did he mention that time also underwent a transformation from one coordinate system to another. Moreover, in common with his contemporaries, Neumann accepted the existence of the aether as an all-pervasive medium at absolute rest.

Neumann's final foray into electrodynamics exhibited no new physical ideas or methodological concepts and had little influence on physics. His analysis here too was dominated by its mathematical aspects, and he referred only selectively to recent progress in electrodynamics. He regarded Helmholtz's and Hertz's publications as embodying the most promising theoretical approaches to electrodynamics and thus based his attempts to improve its mathematical structure on them. Neumann's mathematical standards were high, however, and in that respect his final articles served to exhibit once again his methodological views on mathematical physics.

Conclusions

Neumann's lengthy articles and books on electrodynamics met with little response from his contemporaries. Thus, Gustav Wiedemann thoroughly discussed the "hypothetical views on the nature and effects of electricity"[55] in the final chapter of his book on the theory of electricity,[56] but he only mentioned Neumann's early ideas on potential theory and thermoelectricity, referring his readers to Neumann's original publications, since he said that he himself was incapable of summarizing them. Helmholtz seems to have been the only physicist who analyzed Neumann's early work on electrodynamics in detail. His subsequent dispute with Neumann, however, did not stimulate any later developments in physics.

The significance of Neumann's investigations rests on their methodical aspects. His picture of mathematical physics entailed the search for a mathematical foundation of a physical theory of electrodynamics, for a mathematically exact physical theory resting on established and unanalyzable basic principles or hypotheses. He also demonstrated the value of high mathematical standards in mathematics and physics. In this respect, his researches undoubtedly had a positive influence on mathematical physics.

To analyze each hypothesis of a given physical theory and its consequences inevitably improved that theory and supported the search for experiments to test it. Conversely, the mathematical analysis of physical problems served to display and stimulate new mathematical results, as was often the case in the nineteenth century. Neumann's investigations on potential theory, in particular, revealed the fruitfulness of treating physical problems mathematically. Kirchhoff, Riemann, and Karl von der Mühll (1841–1912) were among those who also contributed to the mathematical treatment of physical problems.

Neumann continued the ideas of the Königsberg school of mathematical physics in his work on electrodynamics, just as Jacobi had done in his work on analytical mechanics. Neumann, however, overestimated mathematical rigor as a criterion for the quality of a physical theory, which made it difficult for him to accept and incorporate new physical ideas into his researches, thus undermining their influence. Neumann's mathematical analyses of electrodynamics might have influenced further investigations had they been connected to experimental research; mathematical considerations alone did not suffice. He analyzed carefully the most important approaches to electrodynamics at the time, F. Neumann's, Weber's, Helmholtz's, Maxwell's, and Hertz's, and in each case identified problematic features of them, as he also did in his own theory. Had he paid close attention to experimental data, he might have tried to decide among the competing theories or to revise his own.

Neumann acknowledged the importance of experiments, but he was convinced that physical research should concentrate on its mathematical foundations. He expressed this view clearly, for example, in a letter to his colleague Otto Wiener (1862–1927) in 1902 in connection with an appointment to the chair of theoretical physics at the University of Leipzig. He wrote that he expected substantial progress in physics only after a long time and mainly by "working carefully and exactly through the known facts," which "indispensably necessitates a solid *mathematical education* and real *mathematical clarity*."[57]

In contrast to analytical mechanics, electrodynamics had not reached a stage of development that permitted a theoretical treatment of it in line with Neumann's methodological commitments. In this respect, his early work at the end of the 1860s and beginning of the 1870s was more important than his later work, since in his early work his mathematical treatment of physical problems and his methodological views on the mathematical structure of a physical theory served as a guide for the further development of mathematical physics. His later publications in the 1890s no longer could play this role. At the same time, however, his early mathematical investigations, especially on potential theory, were stimulated by his investigations of physical problems. Only by considering this reciprocal influence can we gain a fair appreciation of Neumann's contributions to the development of mathematical physics.

In the end, Neumann's emphasis on mathematics helped to foster a clear distinction between mathematical physics and theoretical physics. He was committed to the former, but most physicists rejected the former in favor of the latter.[58] Helmholtz, for example, attached great importance to a theoretical foundation of electrodynamics but always connected his theoretical work to experimental research. Neumann's work on electrodynamics, by contrast, exerted little influence on its further development. His contributions did not consist in an improvement of the principles of electrodynamics or in the achievement of particular results but mostly in his methodological insistence that electrodynamics should rest on a solid mathematical foundation. Thus, through his investigations on potential theory and electrodynamics, Neumann displayed the way in which mathematics can influence physics and physics mathematics. In this way, he contributed to the changing nature of mathematical physics and to the emergence of theoretical physics as a discipline during the second half of the nineteenth century.

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