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On the Homocentric Spheres of Eudoxus

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"In tale dubbio io seguirò l'usato metodo, di non decidere intorno a ciò che più non è possibile sapere;" G.V. Schiaparelli, 1877.

a. Introduction

In 1877, Schiaparelli published a classic essay on the homocentric spheres of Eudoxus. In the years that followed, it became the standard, definitive historical reconstruction of Eudoxian planetary theory. The purpose of this paper is to show that the two texts on which Schiaparelli based his reconstruction do not lead in an unequivocal way to this interpretation, and that they actually accommodate alternative and equally plausible interpretations that possess a clear astronomical superiority compared to Schiaparelli's. One should not mistake all of this for a call to reject Schiaparelli's interpretation in favor of the new one. In particular, the alternative interpretation does not recommend itself as a historically more plausible basis for reconstructing Eudoxus's and Callippus's planetary theories merely because of its astronomical advantages. It does, however, suggest that the exclusivity traditionally awarded to Schiaparelli's reconstruction can no longer be maintained, and that the little historical evidence we do possess does not enable us to make a justifiable choice between the available alternatives.

b. Schiaparelli's Reconstruction of the Eudoxian System

Our current knowledge of Eudoxus's homocentric spheres relies primarily upon two sources. His own writings have not survived.[1] The earliest known reference to Eudoxus's astronomical work is a short passage in Aristotle's *Metaphysics*. Coming from a contemporary of Eudoxus, and one who is generally regarded as our most reliable source on ancient Greek thought, the importance of this reference is clear. Here is how Aristotle described Eudoxus's arrangement:

[1] For a survey of the sources, ancient and modern, on the homocentric system devised by Eudoxus, see E. Maula, "Eudoxus Encircled," *Ajatus*, **33** (1971): 201–253, esp, pp. 206–211.

> Eudoxus assumed that the sun and moon are moved by three spheres in each case; the first of these is that of the fixed stars, the second moves about the circle which passes through the middle of the signs of the zodiac, while the third moves about a circle latitudinally inclined to the zodiac circle; and, of the oblique circles, that in which the moon moves has a greater latitudinal inclination than that in which the sun moves. The planets are moved by four spheres in each case; the first and second of these are the same as for the sun and moon, the first being the sphere of the fixed stars which carries all the spheres with it, and the second, next in order to it, being the sphere about the circle through the middle of the signs of the zodiac which is common to all the planets; the third is, in all cases, a sphere with its poles on the circle through the middle of the signs; the fourth moves about a circle inclined to the middle circle (the equator) of the third sphere; the poles of the third sphere are different for all the planets except Aphrodite and Hermes, but for these two the poles are the same[2]

Concentrating first on the theory of the five planets, it is not difficult to understand the role of the first two spheres in Aristotle's description. Obviously, the first - henceforth "stellar sphere" - accounts for the diurnal motion common to all the heavenly components. The second - henceforth "ecliptic sphere" - creates a component of motion confined to the plane of the ecliptic for the five planets as well as for the sun and moon. As for the third and fourth spheres, Aristotle's text is less helpful. All he relates about them is that the poles of the third rest on diametrically opposed points of the ecliptic, and that the poles of the fourth are fixed inside the third at an unspecified inclination to the third sphere's poles. Schiaparelli's classical reconstruction of the Eudoxian scheme is not derived directly from Aristotle's Fourth Century BC passage, but from the commentary of Simplicius, written about 900 years later.[3] Simplicius was more specific about the particular function of the third and fourth spheres:

> And, as Eudemus related in the second book of his astronomical history, and Sosigenes also who herein drew upon Eudemus, Eudoxus of Cnidos was the first of the Greeks to concern himself with hypotheses of this sort, Plato having, as Sosigenes says, set it as a problem to all earnest students of this subject to find what are the uniform and ordered movements by the assumption of which the phenomena in relation to the movements of the planets can be saved. ... The third sphere, which has its poles on the great circle of the second sphere passing through the middle of the signs of the zodiac, and which turns from south to north and from north to south, will carry round with it the fourth sphere which also has the planet attached to it, and will moreover be the cause of the planet's movement in latitude. But not the third sphere only; for, so far as it was on the

[2] Aristotle, *Metaphysics*, 1073b 17 - 1074a 15, in Sir Thomas L. Heath, *Greek Astronomy*, (London: J.M. Dent & Sons LTD., 1932), pp. 65–66.

[3] Simplicius based his account mainly on the Second Century AD writer Sosigenes, whose own work is not based directly on Eudoxus, but on Eudemus, Aristotle's student who flourished during the Fourth Century BC. The works of both Sosigenes and Eudemus have been lost (see Maula, note 1, p. 210).

third sphere (by itself), the planet would actually have arrived at the poles of the zodiac circle, and would have come near to the poles of the universe; but, as things are, the fourth sphere, which turns about the poles of the inclined circle carrying the planet and rotates in the opposite sense to the third, i.e. from east to west, but in the same period, will prevent any considerable divergence (on the part of the planet) from the zodiac circle, and will cause the planet to describe about this same zodiac circle the curve called by Eudoxus the *hippopede*(horse-fetter), so that the breadth of the curve will be the (maximum) amount of the apparent deviation of the planet in latitude, a view for which Eudoxus has been attacked.[4]

The geometrical arrangement of spheres 2, 3, and 4 that Simplicius described has been interpreted by Schiaparelli as illustrated in Fig. 1 below. The two inner spheres rotate at equal speeds in opposite directions, so that the planet's motion is the superposition of the two rotations. The trace projected by the moving planet on the surface of the ecliptic sphere (and on the stellar sphere, as long as the ecliptic sphere remains stationary) is the *hippopede*, literally meaning horse-fetter, shown in Fig. 2. Its long axis of symmetry coincides with the ecliptic - the vertical great circles in Figs. 1 and 2. The hippopede's projection on the horizontal equatorial plane of the ecliptic sphere (perpendicular to the plane of the paper) is a circle. Consequently, the hippopede is also the intersection of the ecliptic sphere with a cylinder whose base diameter is equal to the difference between the radius of the ecliptic sphere and the radius of either of the bounding latitude circles at the top and bottom of the hippopede. The central angle of these latitudes measured from the equatorial plane is equal to the inclination between the axes of spheres 3 and 4, which is also equal to the angle at

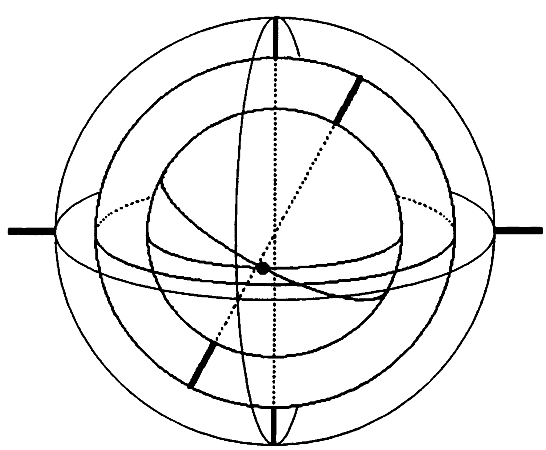

Figure 1

[4] Simplicius, *In Aristotelis de Caelo Commentaria*, edited by I.L. Heiberg, (1894), pp. 488: 18–24; 496: 23–497:6, trans. by Heath, *Greek Astronomy*, pp. 67–68.

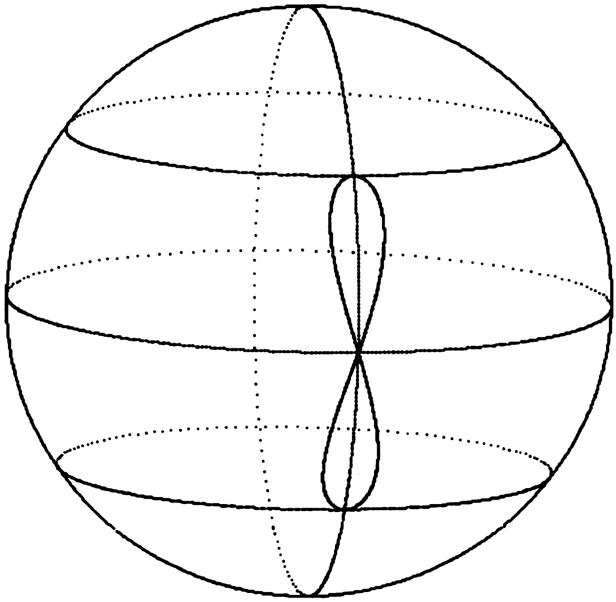

Figure 2

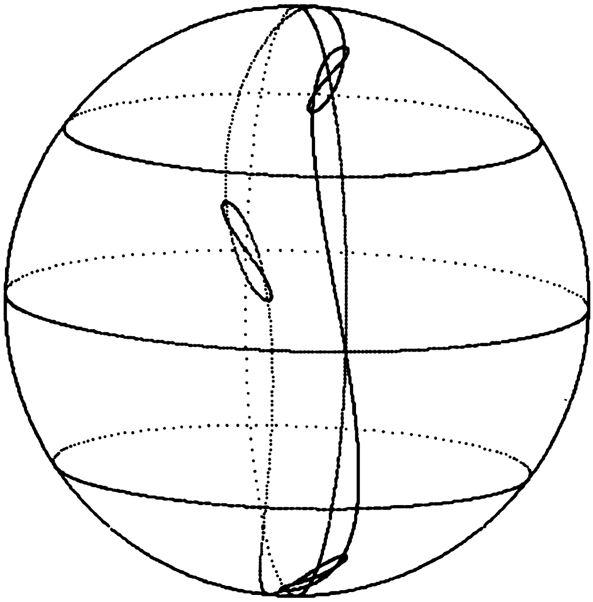

Figure 3

which the hippopede intersects itself. The inclination between the axes of spheres 3 and 4 (set to 35°in Fig. 2) is therefore the single variable parameter that fully determines the shape of the hippopede - the larger the angle, the longer and wider the hippopede. Combination of the hippopedal motion generated by the rotations of spheres 3 and 4 with a simultaneous rotation of the ecliptic sphere, projects on the stellar sphere (not shown in Fig. 1) the motion of the planet relative to the fixed stars. The trace of this motion circuits the stellar sphere while wobbling slightly above and below the projection of the ecliptic, which marks the center of the zodiacal belt. Most importantly, if the speeds of spheres 3 and 4 are properly chosen, the general circuital motion of the planet around the ecliptic is interrupted at fixed intervals by retrograde motion. An example of this is shown in Fig. 3, where spheres 3 and 4 that create the hippopede of Fig. 2 are made to revolve exactly three times as fast as the ecliptic sphere.

The superposition of the diurnal rotation of the stellar sphere over this motion generates the full motion of a planet relative to the horizon of the earthbound observer. Each of the planets in this scheme has its own set of four spheres, the inner two of which create a hippopede. The differences between the resulting planetary paths stem from the different size of the hippopede that pertains to each of them and from the ratio between the rotation period of spheres 3 and 4 and the rotation period of the ecliptic sphere of each planet. Thus did Schiaparelli turn the hippopede into the retrogression generator, and the core of the Eudoxian planetary theory.

c. An Aristotelian Alternative, and a Restriction by Simplicius

There exists, however, another way of interpreting the texts of Aristotle and Simplicius. This alternative will be arrived at here by first reading Aristotle's text as a blueprint for the construction of a general geometrical model, which will then be further constrained by reference to Simplicius's text. Aristotle explicitly specified the following details about the four spheres of the Eudoxian scheme:

1. The first sphere is the sphere of the fixed stars, and its motion is imparted to all three remaining spheres. Note that Aristotle's text does not say that this sphere rotates with the period of the stars' diurnal motion. We ascribe to it the diurnal period because this makes the system work properly.
2. The second sphere rotates in the plane of the ecliptic. Here, too, Aristotle refrained from specifying its speed, and we ascribe to it the mean ecliptic period for each particular planet to make the system work properly. This point will become particularly relevant later, upon examination of Simplicius's problematic discussion of the speeds of the second and third spheres in the theory of the moon.
3. The poles of the third sphere are fixed to diametrically opposed points on the ecliptic.
4. The poles of the fourth sphere are inclined by an unspecified angle relative to the poles of the third sphere.
5. As with the first two spheres, Aristotle wrote nothing about the speeds of rotation of the two inner spheres. Here, as in Schiaparelli's interpretation, the assignment of specific speeds is guided by the assumption that the system's primary purpose was to account for planetary retrograde motion in terms of uniform homocentric rotations.[5]

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[5] Bernard Goldstein has recently observed that in the accounts of astronomical knowledge from the 4th, 3rd, and 2nd centuries BC, there is no term corresponding to the words "retrograde motion." (B.R. Goldstein, "Saving the Phenomena: The Background to Ptolemy's Planetary

6. Finally, nowhere in Aristotle's text is there any indication that the planet must be affixed to the equator of the fourth sphere, nor is any such requirement necessitated by the basic program of reducing the motion of the heavens to uniform homocentric rotations. Indeed, the majority of stars on the stellar sphere are placed away from its equator.

Keeping these points in mind, the arrangement depicted in Fig. 4 below is just as consistent with Aristotle's text as Schiaparelli's. Note that this arrangement possesses two variable inclination parameters as opposed to the single one that characterizes Schiaparelli's interpretation, namely, the inclination between the axes of spheres 3 and 4, and the inclination, or latitude, of the planet relative to the pole of sphere 4.

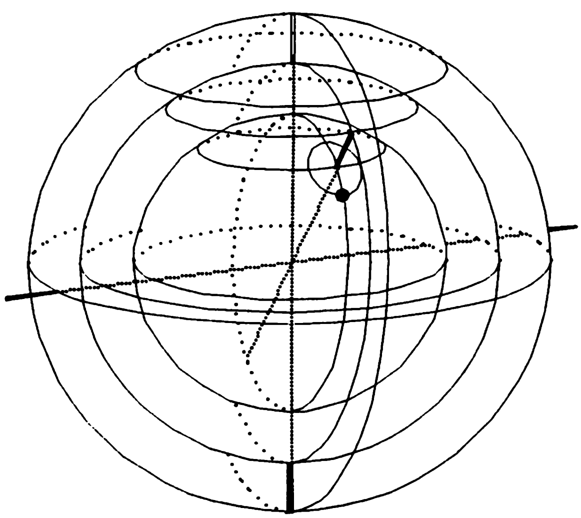

Figure 4

Theory," *Journal for the History of Astronomy*, **28** (1997): 1–12, esp. pp. 3–4). However, as a working hypothesis, the assumption that Eudoxus and Callippus designed their systems primarily to account for retrograde motion provides a natural explanation for the association of four or five spheres with each individual planet, and Goldstein suggested no plausible alternative reason for their existence. One could conjecture that since already Plato knew that both Mercury and Venus move sometimes faster and sometimes slower than the sun, the purpose of the Eudoxian arrangement was to give each planet a variable speed. This, however, can be crudely done with just three spheres per planet, and makes Callippus's use of five spheres for each of Mars, Venus, and Mercury quite mystifying. Furthermore, aside from Aristotle's short, general account, practically nothing has remained of the original treatises on the subject of planetary motion from the time of Eudoxus, Callippus, and their immediate successors. This weakens the impact of Goldstein's philological argument from silence, although it does not warrant the rejection of the *possibility* that indeed, retrograde planetary motion was not recognized by Eudoxus and his contemporaries, and that the Eudoxian homocentric system was meant to address entirely different concerns. If anything, this further emphasizes the need for a general suspension of judgement along side with the development of several possible historical reconstructions.

This arrangement may be anachronistically described as a spherical version of the co-planar deferent and epicycle arrangement that became the staple of Ptolemy's planetary theory. When the two inner spheres spin at equal speeds (relative to the ecliptic sphere) in opposite directions, the planet traces a closed loop (qualitatively illustrated in Fig. 5). Unlike the plane version, however, the spherical counterpart is sensitive to switching the sizes of the deferent and the epicycle as shown in Figs. 6 and 7. In other words, the non-commutative nature of the two inclinations actually increases the flexibility of this arrangement. In addition to the two inclinations, this arrangement can make good use of a third degree of freedom because the initial inclination of the axis of sphere 3 relative to the ecliptic sphere's axis need not be confined to the plane of the ecliptic. This defines a phase angle (Fig. 8), which orients the loop's long axis of symmetry along a great circle other than the ecliptic, while keeping the loop's center of symmetry at the point on the ecliptic where the pole of sphere 3 is attached (Fig. 9).[6] This loop replaces the hippopede of Schiaparelli's interpretation as the retrogression generator. As Fig. 10 shows, the combination of motion along this loop with ecliptic rotation generates planetary traces around the ecliptic that are markedly different from those generated by the hippopede.

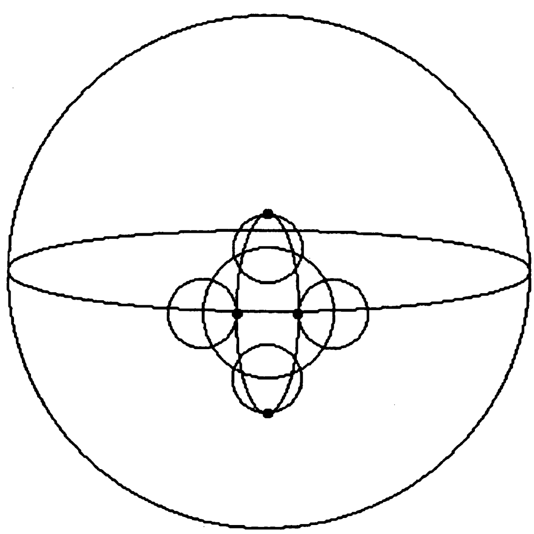

Figure 5

[6] Similarly, the initial position of the planet need not be confined to the bottom of the latitude circle drawn around the pole of sphere 4. There are actually two degrees of freedom here, but they are degenerate to the extent that both rotate the resulting loop's axis of symmetry relative to the projection of the ecliptic. Because sphere 4 rotates twice as fast relative to sphere 3 as the latter rotates relative to the ecliptic sphere (sphere 2), the phase angle of the fourth sphere must be twice the phase angle of the third sphere to effect the same rotation of the loop's axis of symmetry relative to the ecliptic. The starting position of the planet on the loop will not, in general, be the same in both cases, but for the purpose of investigating the trace of retrogression this is inconsequential.

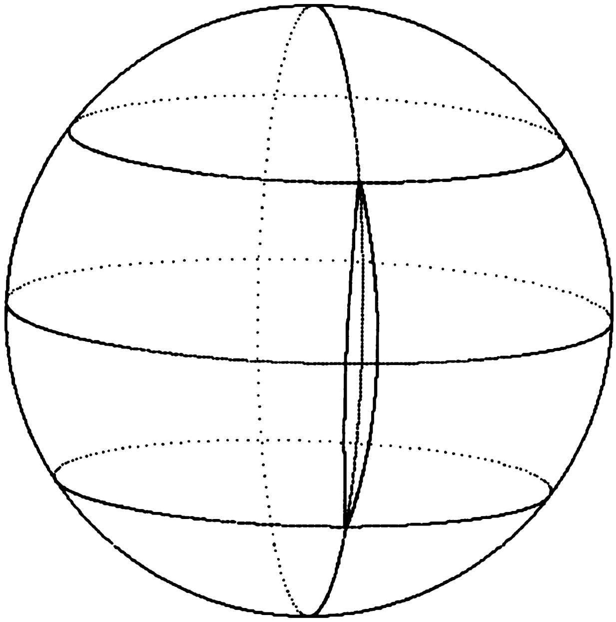

Figure 6

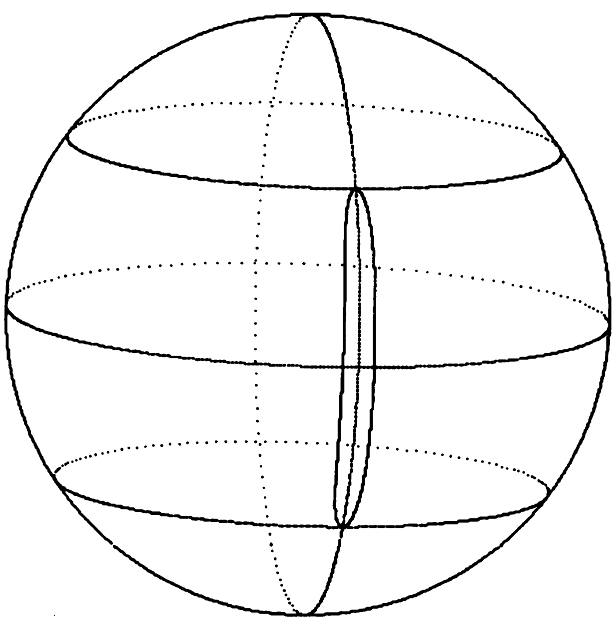

Figure 7

The loop in Fig. 6 is created when the axis of sphere 4 is inclined by 19° relative to the axis of sphere 3, and the planet is on latitude 16° measured from the pole of sphere 4. In Fig. 7 the two axes are inclined by 16°, while the planet rests on latitude 19° of sphere 4

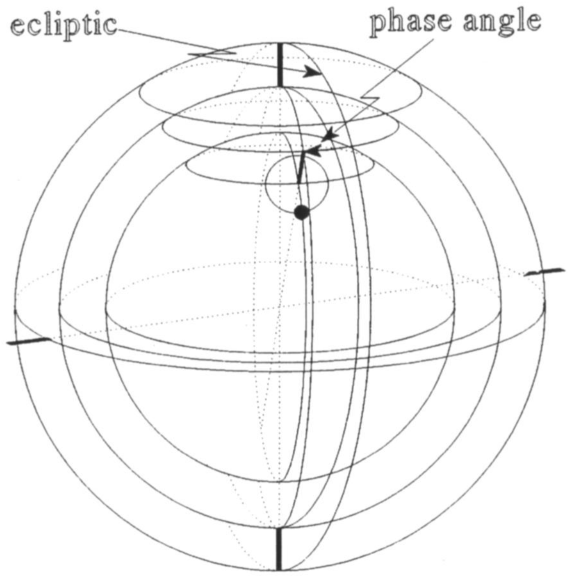

Figure 8

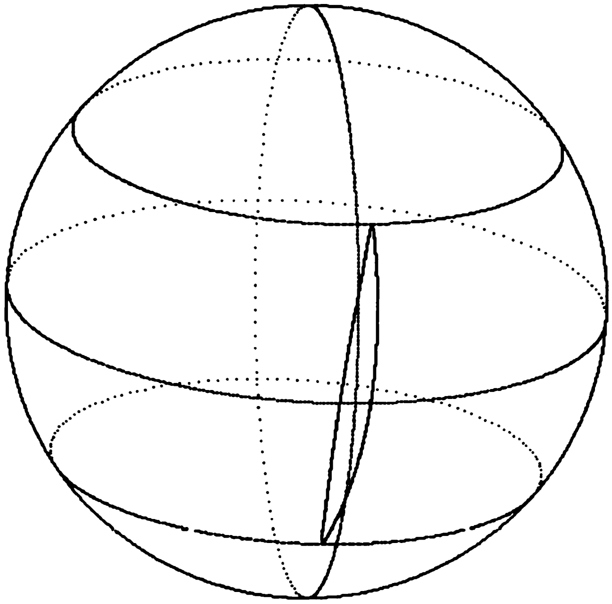

Figure 9

The phase angle determined by the starting position of the planet is also available in Schiaparelli's arrangement with the same effect, namely, it orients the hippopede's long axis of symmetry along a great circle that is inclined to the ecliptic, and intersects it at the poles of the third sphere. However, in marked distinction to the alternative retrogression generator, whose center of symmetry coincides with the pole of sphere 3,

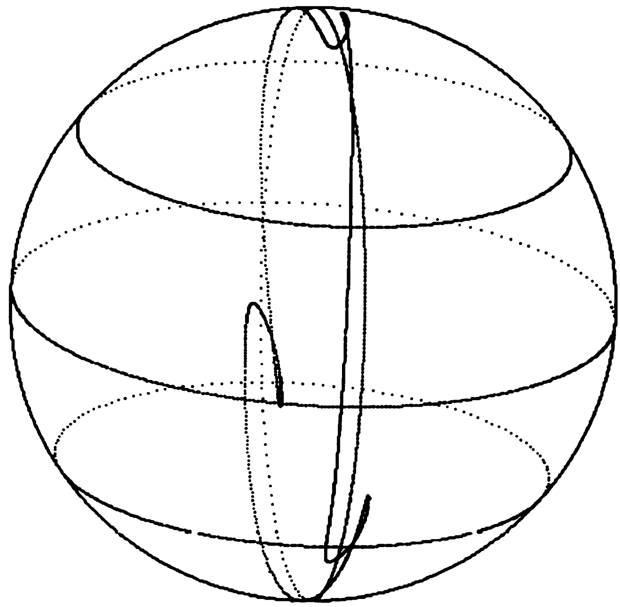

Figure 10

The loop in Fig. 9 is generated by the same inclinations that create the loop in Fig. 6, but lies along a great circle that intersects the ecliptic at a phase angle of 5°. Figure 10 shows the trace projected onto the stellar sphere by motion along the loop of Fig. 9 with a period three times shorter than the rotational period of the ecliptic sphere

the hippopede's center of symmetry lies on the equator of sphere 3, 90° of latitude away from either pole. In Schiaparelli's arrangement, therefore, starting the planet from the intersection of the equators of spheres 3 and 4 (Fig. 1), but with a phase angle other than 90° results in moving the hippopede's center of symmetry away from the ecliptic. Consequently, the entire planetary trace would be displaced to one side of the ecliptic, instead of creating an equivalent of the ecliptically balanced trace generated by the skewed loop of the alternative arrangement (Fig. 10). Small phase angles would shift the entire path of the planet above or below the ecliptic without appreciable change in the form of the retrograde phases, while large phase angles would completely remove the planet's course from the zodiacal belt, which is clearly unacceptable. Note, therefore, that another tacit assumption is made in Schiaparelli's interpretation, namely, that the pole of the fourth sphere is inclined relative to the pole of the third along a great circle that intersects the projection of the ecliptic on sphere 3 at an angle of 90°. Any other orientation for the direction of inclination will move the center of the hippopede away from the ecliptic and will result in improper motion.

We have seen, then, that Aristotle's account of the Eudoxian system admits an alternative interpretation that satisfies its explicit specifications as fully as Schiaparelli's interpretation. The alternative interpretation possesses significantly increased flexibility, generates planetary motion that is markedly different from the motion created by Schiaparelli's interpretation, and thus far makes no use of a retrogression generator that can be described as a hippopede - a word that Aristotle does not mention. It also takes no account at all of Simplicius's commentary on Aristotle's text, to which we turn next.

The first point of interest in Simplicius's account is the observation on the planet's motion without a fourth sphere. Heath's translation has Simplicius saying that with three spheres only, the planet would arrive at the poles of the Zodiac. Simplicius's actual words, however, could as plausibly be translated as saying that the planet would move *toward the poles* of the Zodiac.[7] It is not clear from the text whether it indicates that the planet would always arrive at the poles of the Zodiac, or be displaced in their direction without necessarily arriving at them always.[8] The second possibility, which includes the first as an extreme case, is clarified in terms of the alternative interpretation as follows. It is, in fact, possible to create retrograde motion without a fourth sphere, by placing the planet on the third sphere at the point where the pole of the fourth sphere is attached. As the rotating third sphere is carried by the revolving ecliptic sphere, the combined motion will project a spiral path on the stellar sphere, providing that the third sphere rotates with a sufficiently shorter period than the ecliptic sphere. This means many retrogressions in the course of a single ecliptic revolution. If we wish to reduce the number of retrogressions per ecliptic period (an acute need in the cases of Mars and Venus) we need to increase the rotational period of the third sphere; but then we must compensate for the speed lost by the increased period of revolution. To do that we move the planet further away from the pole of the third sphere, or toward the poles of the ecliptic, to enlarge the circle it describes. The fastest motion that the planet can reach for a given period of revolution is always obtained by placing the planet on the equator of the third sphere. But since the third sphere's pole rests on the ecliptic, the equator of the third sphere necessarily passes through the poles of the ecliptic, and this would bring the planet right to the poles of the ecliptic. In short, the gist of Simplicius's observation is that without a fourth sphere the arrangement would create unacceptable departures from the Zodiacal belt toward the Zodiacal poles, while actual planetary motion never displays such departures. The passage is explained in a similar manner in the context of Schiaparelli's interpretation where the planet is always placed on the equator of the fourth sphere. Presumably, that is where it would be placed on the third sphere were the fourth sphere to be eliminated, with the outcome of carrying the planet all the way to the poles of the ecliptic. In both interpretations the fourth sphere constrains the latitudinal motion of the planet, preventing it from such unacceptable deviations. So far, then, neither interpretation can be plausibly ruled out on the basis of Simplicius's text.

A second point of interest involves Simplicius's explicit discussion of the rotational motion of the third and fourth spheres. Unlike Aristotle, who merely said that the two spheres rotate in inclined planes, Simplicius specified that they do so at equal speeds,

[7] Simplicius, *In Aristotelis de Caelo Commentaria*, edited by I.L. Heiberg, (1894), pp. 496:23-497:6. Schiaparelli translated this passage as saying that with the third sphere alone the planet would advance toward the pole of the Zodiac ("Le Sfere Omocentriche di Eudosso, di Callippo e di Aristotele," in *Scritti Sulla Storia della Astronomia Antica*, 3 vols., [Bologna: Nicola Zanchelli, Editore], vol. 2, p. 100).

[8] I thank Anthony Grafton and Reviel Netz for their help concerning the meaning and possible translations of Simplicius's passage.

but in opposite directions. Unfortunately, rather than clarifying Aristotle's passage, Simplicius mainly succeeded in being ambiguous, regarding both the direction and speed of rotation. Regarding first the direction, only when two rotations take place about parallel axes can they meaningfully be described as coincident or opposed, but the axes of the third and fourth spheres are generally inclined with respect to each other in the Eudoxian system. The standard way of getting round this difficulty has been to assume that Simplicius tried to imply that as long as the coincident rotational components are larger than the opposed ones, the rotations are to be considered in the same directions, and once the opposed components become dominant, the rotations are to be considered opposed. As a result, it has come to be accepted that the inclination angle between the axes of spheres 3 and 4 cannot exceed 90°, despite the fact that neither Simplicius nor Aristotle made this stipulation explicitly. The point is actually of no great consequence, because such angles occur only in the theory of Venus, whose full retrograde loop cannot be observed directly against an immediate background of fixed stars because of its proximity to the sun.[9] However, as a matter of principle there is another, more natural way of resolving Simplicius's ambiguous observation without the additional restriction on inclination angles. Rotations always take place about a pole that may be marked as "north" for the purpose of discussion, and their directions are best described relative to this pole, either westward, or eastward. Simplicius might have been trying to say that relative to their respective north poles the rotations of spheres 3 and 4 would always be opposed. Note that with this definition, if the two north poles are pointed 180°from each other, the spheres will appear to rotate in the same direction to an outside observer, but relative to their own poles they always remain opposed - one clockwise, the other counterclockwise. This interpretation of the passage is free of the ambiguities of the traditional way of understanding Simplicius's observation, and has the additional advantage of not imposing on the theory an additional restriction that is not indicated anywhere in the texts of Aristotle and Simplicius.[10]

Of greater consequence is Simplicius's statement that the spheres revolve with equal periods. Here again, the text leaves room for more than one interpretation. In Schiaparelli's interpretation, Simplicius's reference to equal periods in opposite directions does not describe the apparent motion of the two inner spheres, but provides a two-stage procedure by which to set the system in motion. First, spin the inner sphere. Then spin the outer one in the opposite direction but with the same period as the inner sphere and combine the two motions according to the rule that all rotations are communicated in-

[9] Venus itself, as opposed to its immediate stellar background, can be observed during nearly all of its retrograde phase. Therefore, given a good chronometer, a precise global mapping of the fixed stars on the stellar sphere, and a sufficiently accurate data base of stellar risings and settings, it is possible to reconstruct the form of Venus's retrograde path indirectly. However, as we shall see later on, it appears unlikely that in Eudoxus's time the Greeks possessed either a sufficiently accurate global star map or a sufficiently accurate table of risings and settings.

[10] Alternatively, one may see this in terms of procedural instructions that require none of the modern terminology for describing rotations: first the two poles must be aligned, then the spheres must be spun in opposite directions. Having done that, the poles of the oppositely spinning spheres may be inclined to any desired angle. This procedure is exactly equivalent to the interpretation given in the text above.

ward. This will produce the motion that generates Schiaparelli's hippopede, but it must be noted that Simplicius's text does not allude to any such procedure. Instead, it simply describes a system of two homocentric spheres, rotating at equal periods in opposite directions. This does not support Schiaparelli's reconstruction, where the inner sphere appears to stand still when its pole is aligned with the pole of the fourth sphere, or merely to wobble back and forth while its inclined pole orbits about the pole of the third sphere. For the two spheres to appear to rotate with equal periods in opposite directions as Simplicius's text indicates, one needs first to spin the inner sphere, and then to spin the outer one in the opposite direction at *twice* the period initially given to the inner sphere. As the two motions combine under the rule that all rotations are communicated inward, the speed of the inner sphere is reduced by one half, and the two spheres will appear to rotate in opposite directions with the same period, as Simplicius's text requires. With no further help from Simplicius, it is quite natural to take the text at face value as alluding to the actual appearance of two concentric spheres, rotating with equal periods in opposite directions. Such a motion will not generate the hippopede of Schiaparelli's reconstruction, but the loops of the alternative interpretation of Aristotle's account. Simplicius did not provide additional information with which one can make a sound choice between these two possibilities. Thus, the two alternatives that fit Aristotle's account because of its generality, also accommodate Simplicius's account by reason of its ambiguous nature.

Finally, Simplicius explicitly wrote that the combined motion of the third and fourth spheres produces what Eudoxus called the "hippopede." Nowhere did Simplicius identify this figure with the intersection of a sphere and a cylinder, or a sphere with two cones joined at the apex. Proclus, however, used the term to describe one of the so called "spiric sections" in his commentary on Euclid:

> Of the spiric sections one is interlaced like a horse's hobble, [ἵππου πέδη] another is broad in the middle and thins out at the sides, and another is elongated and has a narrow middle portion but broadens out at the two ends.[11]

A little later, Proclus commented on the origin of these curves:

> ...we have at once a notion of the most elementary kinds of surfaces, the plane and the spherical, though it is only through science and scientific reasoning that we discover the variety of surfaces that arise by mixture. What is remarkable about them is that from the circle there can often be generated a mixed surface. This is what we say happens in the case of the spiric surfaces, for they are thought of as generated by the revolution of a circle standing upright and turning about a fixed point that is not the center of the circle. Thus three kinds of spiric surface are generated, for the center lies either on the circumference, or inside, or outside the circle. If the center is on the circumference, the continuous spiric surface is generated; if within the circle, the interlaced spiric surface, and if outside, the

[11] Proclus, *A Commentary on the First Book of Euclid's Elements*, Translated by Glenn R. Morrow, (Princeton: Princeton University Press, 1992), p. 91 (112.5-112.8). See also, Heath, *Euclid's Elements*, Book I, (New York: Dover Publications, Inc., 1956), pp.162-163.

> open spiric surface. And the spiric sections are three in number corresponding to these different kinds of surface.[12]

On the strength of these passages, Heath declared:

> There is no doubt that Schiaparelli has restored, in his 'spherical lemniscate', the *hippopede* of Eudoxus, the fact being confirmed by the application of the same term *hippopede* (horse-fetter) by Proclus to a plane curve of similar shape formed by a plane section of an anchor-ring or *tore* internally and parallel to its axis.[13]

We know that Eudoxus, like Archytas his teacher, studied the intersections of spheres, cylinders, and cones, and there is no reason to doubt the possibility that Eudoxus recognized Schiaparelli's spherical lemniscate. At the same time, Heath may have been too hasty in giving this lemniscate such an exclusive hold on what Eudoxus might have called "hippopede." Note that what Proclus called a hippopede is different from Schiaparelli's spherical hippopede. The spiric section Proclus called by this name is a plane figure. It cannot be created by the plane projection of Schiaparelli's hippopede, and it cannot be transformed into Schiaparelli's hippopede by projecting it from a plane onto a sphere. Proclus's hippopede, then, refers to a family of curves that is distinct from the family of curves referred to by Simplicius as the hippopede of Eudoxus. The inevitable conclusion from this must be that by way of a rather loose, impressionistic analogy to an equestrian leg-cuff, the word "hippopede" describes a large variety of self-intersecting curves, that may lie on plane, spherical, and perhaps other surfaces. As such, the curve shown in Fig. 11 below is a perfectly legitimate hippopede. This hippopede, and others like it, are created by the alternative interpretation in the special case when the inclination between the axes of spheres 3 and 4 is equal to the latitudinal position of the planet measured from the pole of sphere 4.[14] They differ from the hippopedes of Schiaparelli's reconstruction in that they cannot be created by the intersection of a sphere with a cylinder. The two families of hippopedes are also distinguished by the tangent at the point of self contact, which exactly coincides with the long axis of symmetry in the case of the alternative hippopedes, while in the case of Schiaparelli's hippopedes this never occurs. A further distinction resides in the motion of the point that traces these hippopedes as the generating spheres revolve: it crosses over to the other side of the hippopede's

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[12] *Ibid.*, pp. 96–97(119.9–119.17).

[13] Heath, *Aristarchus of Samos*, p. 207.

[14] This will be recognized as the spherical case of the Tusi couple. As F.J. Ragep noted, there seems to be no reason to doubt that al-Tusi was both supported and inspired by Aristotle's text in the general venture of considering homocentric alternatives to certain Ptolemaic arrangements, but it does not necessarily follow from this that al-Tusi constructed his own model as a specific interpretation of Aristotle's passage (F.J. Ragep, *Nassir al-Din al-Tusi's Memoir on Astronomy (al-Tadhkira fi cilm al-hay'a)*, 2 vols., (New York: Springer-Verlag, 1993), p. 33). Indeed, al-Tusi's failure to notice that in the spherical case the Tusi couple traces a self-intersecting loop and not merely linear oscillatory motion along a great arc suggests that he conceived of the spherical version as an extension of the co-planar case where the resulting motion is an oscillation along a straight line without any latitudinal deviations.

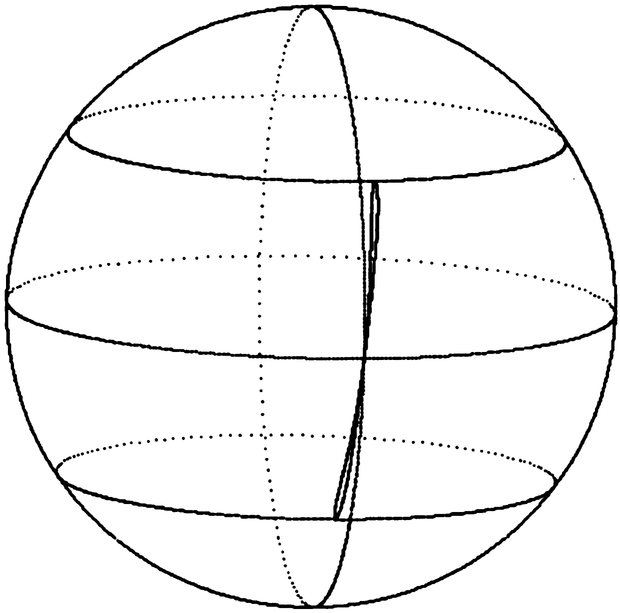


Figure 11

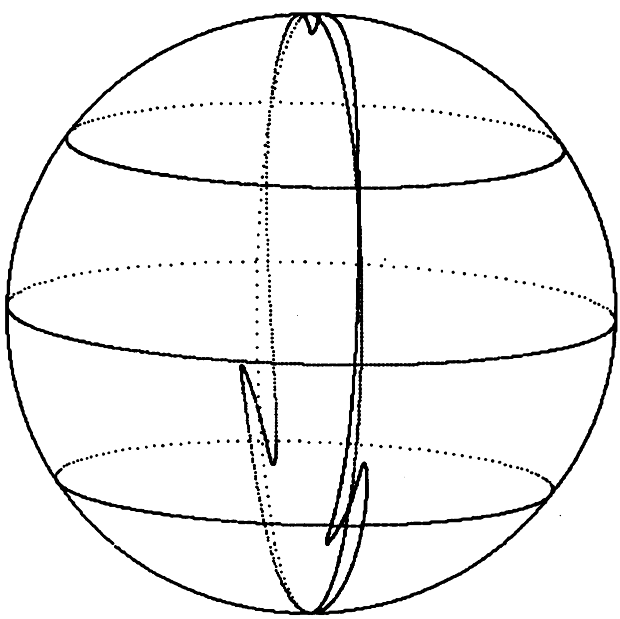
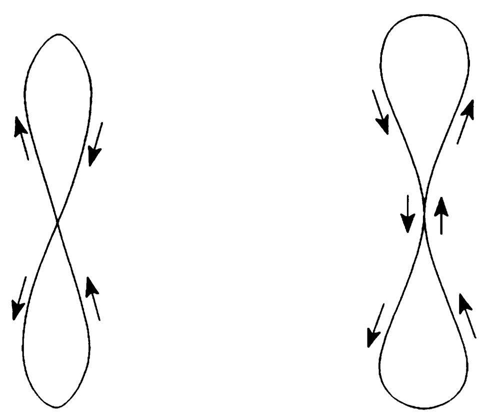


Figure 12

Figure 11 shows the curve created by the alternative interpretation, with sphere 4 inclined by 17.5° to sphere 3, with the planet on latitude 17.5° measured from the pole of sphere 4, and with a phase angle of 5°. Figure 12 shows the trace of the resulting planetary motion with an ecliptic period three times longer than the synodic period long axis of symmetry at the point of self-intersection in Schiaparelli's case (Fig. 13a); in the alternative case, it swings back from the intersection point, and crosses the axis of symmetry only at the two extreme turning points of the loop (Fig. 13b). That both types of hippopedes are equally likely to have been considered by Eudoxus appears to be as plausible a conclusion as any to draw from the testimony of Simplicius.[15]

To recapitulate, we have seen that with the word *hippopede*, we may use Simplicius's testimony to restrict the alternative interpretation of Aristotle's text to the specific case where the two inclinations are always equal. We have also seen that this does not suffice to reject the alternative interpretation in favor of Schiaparelli's. Indeed, Simplicius's text cannot be read as a geometrical blueprint containing a set of instructions for the construction of Schiaparelli's arrangement to the exclusion of any alternative. Quite to the contrary, Simplicius's text is rather ambiguous at several crucial points, and it is only a reverse-interpretation guided by a preconceived notion of *hippopede* as the intersection of a sphere and a cylinder that leads to our understanding of this text in the exclusive terms of Schiaparelli's reconstruction. It then takes another 900 year backward implication to impose this restricted interpretation on Aristotle's testimony. The last step is justifiable only on the assumption that Simplicius's description of the Eudoxian system is more authoritative than Aristotle's. It is true that Simplicius supplied more information than



a *b*

Fig. 13. The motion of the tracing point in Schiaparelli's reconstruction (a) and in the alternative reconstruction (b)

[15] O. Neugebauer reportedly pointed out that the figure of 8 traced by the spherical Tusi couple cannot be considered a hippopede because the tangent at the point of self-intersection is 0° (F.J. Ragep, *Nassir al-Din al-Tusi's Memoir on Astronomy (al-Tadhkira fi cilm al-hay'a)*, 2 vols., (New York: Springer-Verlag, 1993), p. 455 (note)). This, however, presupposes the exclusivity of Schiaparelli's reconstruction without independent justification on the basis of the historical record.

Aristotle, but as we shall see in the next section, it does not follow from this that his information is necessarily more dependable than Aristotle's.

d. How Central is the Role of the Hippopede in Eudoxus's Homocentric Astronomy?

It is remarkable that the hippopede, which in the hands of Simplicius and Schiaparelli has become the hallmark of Eudoxus's planetary theory, was not mentioned at all by Aristotle. This could have several explanations, all of them both plausible and tentative owing to the general lack of source material that typifies the historical analysis of early Greek thought. Aristotle's own contribution to the Eudoxian theory was in adding connecting spheres between the four-fold sets of spheres for consecutive planets, so as to turn the entire scheme into a single, mechanically continuous system. For his purpose it sufficed to note that the motion of each planet involved separate rotational components that had to be counteracted by the intervening spheres before the next planetary motion could begin.[16] We may add to this the suggestion that while Aristotle ranked mathematics highly as a subject for cultivation, mathematical detail does not feature prominently in his work. Therefore, we should not marvel to find that he avoided the details of Eudoxus's inherently mathematical construction. This depiction of Aristotle as not actively pursuing the details of mathematical astronomy and merely taking over without modification (albeit with some expressed doubts) the improved Eudoxian model of Callippus is quite common, and need not be further developed here.[17] It may be

[16] It has often been noted that Aristotle used superfluous spheres in his construction by using a diurnal outer sphere for each individual planet, despite the fact that this motion is automatically transferred from the previous planet whose distinctive rotational components have been canceled by three counteracting spheres. To the best of my knowledge, however, it has not been noted that from a purely geometrical standpoint, Aristotle actually used two superfluous spheres for each planet. He could use two spheres to cancel out the rotations of the hippopede-forming spheres 3 and 4. The next counteracting sphere, with axis aligned with the axis of the ecliptic sphere, should then be made to rotate such that the sum of its rotation and that of the previous planet's ecliptic sphere would amount to the ecliptic period of the next planet. Only two extra spheres are then needed to produce the hippopede (or alternative) of the next planet. In short, Saturn would have four spheres, then Jupiter would follow with three counteracting spheres plus two more spheres, as opposed to Aristotle's three plus four. Following this for the rest of the planets, the sun, and the moon would yield a total of 31 spheres as opposed to Aristotle's 43. Speculations on the reasons for the presence of unnecessary spheres in Aristotle's scheme are outside the scope of the present discussion.

[17] See e.g. Michael J. Crowe, *Theories of the World from Antiquity to the Copernican Revolution*, (New York: Dover Publications, Inc., 1990), pp. 26–27. J.L.E. Dreyer, *A History of Astronomy from Thales to Kepler*, 2nd edition, (Dover Publications, Inc., 1953; originally published in 1906), pp. 112–114. Thomas S. Kuhn, *The Copernican Revolution*, (Cambridge, Mass.: Harvard University Press, 1957), p. 80. David C. Lindberg, *The Beginnings of Western Science*, (Chicago: The University of Chicago Press, 1992), pp. 95–96. G.E.R. Lloyd, *Early Greek Science: Thales to Aristotle*, (New York: W.W. Norton & Company, 1970), pp. 92–94. J.D North, *The Fontana History of Astronomy and Cosmology*, (London: The Fontana Press, 1994), pp. 82–84. S. Toulmin

submitted, therefore, that Aristotle showed only marginal interest in the detailed description of planetary motion and that he studied planetary astronomy primarily to evaluate its place in larger cosmological and theological contexts.

These contexts, however, require none of the details Aristotle supplied regarding the planes of revolution of the various spheres, and should have led to a non-specific description that could accommodate the homocentric schemes of Al-Bitruji, Amico, and Fracastoro in addition to Schiaparelli's reconstruction and the alternatives suggested in the previous sections.[18] Each of these homocentric schemes could provide the basis for the theological exercise of counting the number of overseeing deities, and each could be unified into a grand mechanical scheme by means of interconnecting counter-spheres. Al-Bitruji's scheme, however, has rotations relative to the ecliptic sphere about poles that are far removed from the ecliptic, which explicitly violates Aristotle's description. The planetary schemes of both Amico and Fracastoro require more than four spheres with mutual inclinations that differ from those described by Aristotle, and this too violates his account of the Eudoxian and Callippic systems. It appears, then, that Aristotle's description of the Eudoxian scheme outlines a restricted class of homocentric arrangements with a degree of geometrical specificity that exceeds the requirements of his theological and cosmological motivations. His neglect to mention the hippopede, then, cannot be written off as stemming from a general lack of interest in the specifics of the Eudoxian scheme. So the question arises again, why did Aristotle fail to mention the hippopede? After all, if Simplicius's overt emphasis on the hippopede is justified, then Aristotle's account appears like a description of Kepler's system without mentioning ellipses.

For all we know, Aristotle may be guilty of such an oversight, but there also exists another possibility. It is generally acknowledged that Eudoxus's scheme represents the first sophisticated Greek attempt to reduce planetary motion to circular components. It is also generally admitted that this attempt is further constrained by the desire to achieve the reduction while maintaining perfect spherical symmetry and uniformity of motion in the heavens.[19] Both Aristotle's and Simplicius's descriptions of Eudoxus's system clearly preserve these two fundamental requirements, but Simplicius's account is more restrictive than Aristotle's - perhaps too much so. It may be the case that Aristotle did not mention the hippopede because it represented only a limited class of Eudoxian systems, and because for his cosmological discussion Aristotle considered it important to outline their shared principles rather than their particular differences. He therefore wrote an

and J. Goodfield, *The Fabric of the Heavens: The Development of Astronomy and Dynamics*, (New York: Harper & Row, Publishers, 1965), pp. 106–107.

[18] Al-Bitruji, *On the Principles of Astronomy*, Translated and Analyzed by Bernard R. Goldstein, (New Haven: Yale University Press, 1971). Noel Swerdlow, "Aristotelian Planetary Astronomy in the Renaissance: Giovanni Battista Amico's Homocentric Spheres," *Journal for the History of Astronomy*, **3** (1972): 36–48. Mario di Bono, "Copernicus, Amico, Fracastoro and Tusi's Device: Observations on the Use and Transmission of a Model," *Journal for the History of Astronomy*, **26** (1995): 133–154.

[19] This was not always the understanding of Aristotle's passage. In 1795, for example, Adam Smith understood the third sphere in the Eudoxian system to oscillate back and forth like the wheel pendulum in a clock (Adam Smith, *Essays on Philosophical Subjects*, Edited by W.P.D. Wightman and J.C. Bryce, (Oxford: Clarendon Press, 1980), p. 58).

account that accommodated the structure of Schiaparelli's reconstruction as well as the alternative described above because both of them had been considered by Eudoxus. The generality of this account reflects not a basic lack of interest in astronomical geometry, but a considered attempt to emphasize basic common features, which did not include the hippopede. But if that were the case, we must ask ourselves why Simplicius did give the hippopede such a prominent place in the Eudoxian scheme. A possible answer to this question lies in the assessment of Simplicius's reliability, which will be further examined here.

We have already seen that Simplicius's seemingly more specific account actually suffers from several ambiguities of which Aristotle's testimony is free. To these must be added Simplicius's problematic account of the Eudoxian lunar theory, which accounts for the motion of the moon by the combined rotations of three homocentric spheres. According to Simplicius, the first sphere rotates about the celestial pole every 24 hours, the second rotates about the ecliptic axis once every month, and the third sphere, which is inclined to the second, carries the moon and rotates once every 18.5 years. Unfortunately, this would make the moon stay to one side of the ecliptic for about nine years before it crosses to the other side where it remains for an equal length of time, while in fact the moon crosses the ecliptic twice a month. Martin in the 19thcentury, and Dicks in the 20th, have taken Simplicius at face value as representing the state of Eudoxus's knowledge of the moon's motion.[20] The error, however, is easily corrected by simply switching the rotation periods of the second and third spheres without changing the orientation of their respective poles. The majority of historians tend to consider Simplicius's account as an incorrect representation of the Eudoxian theory, and this is also the point of view adopted here.[21]

Turning now to Aristotle's account, we find it there stated that for the five planets as for the sun and the moon, the first two spheres are the same: the first spins in the plane of the celestial equator, the second spins in the plane of the ecliptic. Keeping in mind the observations of the previous paragraph, we note that while the second sphere rotates with the period of mean ecliptic revolution in the case of the five planets, it should not do so in the case of the moon if proper agreement with observations is desired. Some historians therefore suggested that Simplicius's error goes all the way back to Aristotle,[22] but as we shall presently see, this can be justified only by reading Simplicius into Aristotle - a rather dubious procedure in this case. It has already been pointed out that nowhere did Aristotle say anything about the speeds of these spheres, so that as far as it goes, his

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[20] D.R. Dicks, *Early Greek Astronomy to Aristotle*, (London: Thames and Hudson, 1970), pp. 180–181, 256 (note 338).

[21] It is unlikely that Eudoxus could precisely position the ecliptic - being the trace of the sun's path on the background of the fixed starts - and trace the moon's path relative to it. The moon, however, executes significant and easily observable latitudinal oscillations relative to the zodiacal belt within the course of a single lunation, resulting in an eclipse pattern that is inconsistent with Simplicius's depiction. No reference to the ecliptic is required for making this observation, and it does not seem very likely that Eudoxus was actually that ignorant of knowledge that was within easy reach in his day.

[22] Heath, *Aristarchus of Samos*, p. 197. Dicks (*Ibid.*), who took the error to be Eudoxus's, also read Aristotle's passage as identical to Simplicius's.

description is quite unproblematic: in all cases, the first sphere rotates about the celestial axis, and the second sphere rotates about the ecliptic pole. This remains true of the first and second lunar spheres regardless of the speeds we might wish to assign to the second and third spheres. Simplicius specified beyond Aristotle that in all cases the second sphere rotates with the mean ecliptic *period* of the member it pertains to, be it the sun, moon, or any of the planets. In the case of the moon this amounts to a gross incongruence with the observed appearances. But the only way to suggest that this was also Aristotle's view, is by reading Simplicius's specification of rotational speeds into Aristotle's text. This is a possible, but by no means the only plausible, reading of these texts. It is equally plausible to take these problematic passages to suggest that Simplicius's testimony is less trustworthy than Aristotle's.

Knorr recently observed that "the mere survival of a document need not compel us to believe what it says."[23] This cannot be ignored considering the existence of several problematic instances in Simplicius's commentary on Aristotle's description of the Eudoxian homocentric theory. In particular, it would not be an isolated misrepresentation on Simplicius's part to inappropriately emphasize the hippopede which Aristotle did not even see fit to mention. Furthermore, in this particular case specific considerations regarding the mathematical nature of the hippopede might have encouraged Simplicius to do so. The hippopede of Schiaparelli's arrangement possesses a great mathematical value because it can be generated by the intersection of two simple solids - a sphere and a cylinder - and its entire trace may therefore be reduced to the classical geometrical procedure of synthetic construction.[24] This may be the reason for the emphasis given to it by Simplicius. There seems to be no reason to doubt that Eudoxus constructed this curve, and used it as a rigorously analyzable example of oscillatory motion generated by the combination of uniform homocentric rotations. It is not at all clear, however, that in Eudoxus's overall astronomical work, the hippopede occupied a central position.

From the mathematical point of view, the curves generated by the alternative arrangement are not as attractive as the hippopede. They may, of course, be generated by the intersection of a sphere with other surfaces. However, unlike the cylinder that generates the hippopede, the new generating surfaces cannot be defined independent of the curve they create while intersecting a sphere. In other words, we cannot avoid treating the desired curves as generated by mechanical motion. In terms of rigorous Greek geometry, therefore, the alternative curves may have been considered problematic as mathematical entities, and progressively more so in the post-Euclidean Hellenistic tradition. Indeed, Simplicius seems to have rated mechanical construction as an inferior approach to ge-

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[23] Wilbur Richard Knorr, *The Ancient Tradition of Geometric Problems*, (New York: Dover Publications, Inc., 1993), p. 10. "Many scholars doubt Simplicius's authority and question Plato's putative role in the development of astronomical theory." B.R. Goldstein and A.C. Bowen, "A New View of Greek Astronomy", *ISIS*, **74** (1983): 330–340, p. 330.

[24] Following Schiaparelli, Heath showed how in addition to a cylindrical intersection, the hippopede is the intersection of a sphere with two cones joined at the apex, but this does not exhaust the mathematical beauty of this curve. John North ("The Hippopede," in A. von Gotstedter (ed.), *Ad Radioes*, (Stuttgart: Franz Steiner Verlag, 1995), pp. 143–154) recently showed that the intersection of a sphere with a parabolic surface also produces the hippopede of Schiaparelli's interpretation. This can easily be demonstrated in modern analytical terms as shown in the appendix.

ometrical problems.[25] As an astronomical entity, however, the alternative curve is both legitimate in Greek usage and highly useful. We possess some evidence to the effect that in the study of geometrical problems the Greeks were not exclusively dependent on rigorous geometrical analysis as exemplified in Schiaparelli's classical paper, but that they made considerable use of physical models where rigorous analysis failed them. In a well-known paragraph, Plutarch specifically mentioned Eudoxus and his Pythagorean mentor Archytas in this connection:

> For the art of mechanics, now so celebrated and admired, was first originated by Eudoxus and Archytas, who embellished geometry with its subtleties, and gave to problems incapable of proof by word and diagram, a support derived from mechanical illustrations that were patent to the senses.[26]

With elementary trigonometry, equally elementary use of the group of spherical rotations and a low-power home computer, it is possible to try different setups of homocentric spheres and to generate various hypothetical planetary paths. Eudoxus, and others that worked with his planetary scheme, had neither systematic trigonometric tables to execute the necessary calculations nor personal computers. However, the spherical arrangement described in Fig. 4 may very well have functioned as a crude analogue computer. It must be emphasized that there is no need to actually build nested homocentric spheres for this purpose. A set of concentric rings, consecutively attached to each other at diametrically opposed swivel points would suffice, and an object attached to the innermost ring can be made to demonstrate the basic motion of a planet according to any of the alternatives discussed thus far, including Schiaparelli's. A finer study of the traces generated by such motions requires no more than a single sphere and a compass for drawing a set of points on the sphere that may be joined to form continuous paths. It is time-consuming work, but not any more so than Kepler's manual calculations of planetary orbits. We have no evidence regarding the way Eudoxus and his immediate followers went about the practical business of studying the details of their newly invented cinematic theoretical astronomy. Consequently, we cannot reject out of hand the possibility that their theoretical astronomy was more an analogue graphical than a rigorous geometrical affair, whatever "rigorous" may have meant in those days. We do possess indications that portray Eudoxus as interested in the mapping of stars on solid spheres. Cicero, when recounting the sack of Syracuse, described some booty brought to Rome by Marcellus, that contained a spherical mechanization of planetary motion created by Archimedes. The mechanization, Cicero explained, was Archimedes's innovation, but plotting stars on solid spheres was already an old tradition; and Cicero explicitly mentioned Eudoxus of Cnidus in this respect. It was, Cicero reported "... a very early invention, the first one

[25] Knorr argued at length (*The Ancient Tradition of Geometric Problems*, [New York: Dover Publications, Inc., 1993]), that this gradual shift toward formalization and rigorization is responsible for an over-rigorized image of early Greek geometrical practices created by the doxographers of the early Christian era.

[26] *Plutarch's Lives*, "Marcellus," XIV. 5–6. Plutarch's story that Plato strongly disapproved of Eudoxus and Archytas for using such mechanical devices, and "... inveighed against them as corrupters and destroyers of the pure excellence of geometry" is open to doubts.

of that kind having been constructed by Thales of Miletus, and later marked by Eudoxus of Cnidus ... with the constellations and stars which are fixed in the sky ...."[27] Of far greater importance are the poem by Aratus that supposedly describes Eudoxus's stellar sphere and the detailed criticism of this poem by Hipparchus, who claimed that Aratus closely followed Eudoxus. It has been suggested that the sculpted depiction of the stellar sphere carried by Atlas was also modeled on Aratus's poem.[28] Now, if Eudoxus devoted time to mapping the fixed stars, the equator, the ecliptic, the tropics, and the Arctic circles on a sphere, it would not be out of character for him to try to plot at least the outstanding aspects of planetary motion, namely, the retrograde phases, on a sphere. Thus, in an astronomical tradition that makes use of both physical models and geometrical analysis, inability to achieve rigorous geometrical demonstration would not represent an insurmountable obstacle to further progress. Analogue graphical "computations" in the above sense, using the alternative loop as the generator of retrograde motion, would generate traces that do agree qualitatively with the observed paths traced by the planets in their motion. If true, this suggests that we view the Greek astronomer in the 4th century BC as a practical geometer interested more in the generation of astronomically useful forms than in their rigorous justification.

Of course, without conflicting with this view of Greek astronomy, the successful synthetic demonstration of one model of homocentric spheres, namely, the one that creates the hippopede of Schiaparelli's interpretation, could very well encourage further study and development of other such models. At the same time, it would be natural for a doxographer of Simplicius's taste and inclinations to give special emphasis to the mathematically justified model as representing a first triumph in the attempt to bring spherically symmetric astronomy and rigorous geometry into harmony. About nine hun-

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[27] Cicero, *De re publica*, I, XIV, 21–22. Dicks noted that the attribution of such a sphere to Thales does not add credence to Cicero's account. It may also be mentioned that Cicero reported on the Archimedean spheres in the course of recounting a conversation that may or may not have taken place in reality between Scipio Aemilianus and some friends. The object of the conversation was to present a typically Roman emphasis on the overriding importance of practical issues pertaining to public life, as opposed to Greek idealistic and abstract thought. When the older and highly respected Laelius joined the conversation, he asked Philus, who initiated it: "Do you really think then, Philus, that we have already acquired a perfect knowledge of those matters that relate to our own homes and to the State since we are now seeking to learn what is going on in the heavens?" and he continued to state his opinion that the usefulness of the abstract and mathematical sciences is that "... if for anything at all, they are valuable only to sharpen somewhat and, we may say stir up the faculties of the young, so that they find it easier to learn things of greater importance." After this, the conversation quickly came down to earth, but not before Laelius repeated the point attributed by Scipio to Socrates about the unusefulness of natural philosophy. Under these circumstances, it is not surprising to find modern scholars expressing doubts regarding the reliability of Cicero's report. On the other hand, why lie about the use of such spheres? Indeed, why bring them up in the first place? At the very least, the passage reflects the existence of such devices in Cicero's time, while their attribution to Thales in the above manner suggests some confusion regarding Thales's state of knowledge. But regarding Eudoxus, the poem of Aratus, its critique by Hipparchus, and the statements by Plutarch suggest that Cicero used a real historical fact to make a political point.

[28] J.B. Harley and David Woodward, *The History of Cartography*, 3 vols, (Chicago: The University of Chicago Press, 1987), vol. 1, pp. 140–143.

dred years separate Simplicius from the Aristotelian texts that he commented on, and planetary astronomy changed considerably in the interim. In Aristotle's time the Eudoxian homocentric spheres represented the state of the art in Greek planetary theory. As such this theory was as important as Eudoxus's contributions to mathematics. Aristotle's text, which accommodates both the hippopede and its alternative equally well, would fit nicely with this state of affairs if we actually allow it to include both interpretations. Indeed to Aristotle, who in the *Metaphysics* was demonstrably more interested in the mechanization than in the mathematization of astronomy, there would be no reason to give undue emphasis to one out of several known spherically symmetrical arrangements, merely on account of its mathematical neatness. By Simplicius's time, planetary theory was dominated by the Ptolemaic model, while Eudoxus's homocentric spheres were primarily an ancient museum piece. Eudoxus's contributions to geometry, on the other hand, were still as relevant as ever. If Simplicius's taste was more philosophical than astronomical, he may have been naturally inclined to single out the "mathematically correct" hippopede at the expense of several other alternatives whose days of astronomical usefulness have long gone by.[29]

e. Difficulties Associated with the Astronomical Function of the Hippopede

The reconstructions described in this discussion assume that the primary purpose of the hippopede in Simplicius's account is to generate the retrograde motion of a planet. To do that, the ecliptic component of the motion along the hippopede must exceed the rotational speed of the second sphere, which accounts for motion around the ecliptic. Now, the exterior planets Jupiter and Saturn retrogress many times during a single revolution around the heavens. This means that the third and fourth spheres rotate several times faster than the second sphere. In other words, the planet traverses the hippopede generated by the third and fourth spheres several times during a single revolution of the second sphere. Under such circumstances, motion even on a small hippopede becomes swift enough to produce retrograde motion, and, as Schiaparelli showed, hippopedes may be defined for these two planets to produce retrogressions of the correct arc lengths (see Fig. 14 and 15).

The cases of Mars and Venus, by contrast, present severe difficulties because their rates of revolution around the sun are comparable to that of the earth. The case of Venus is the worst. The synodic period of Venus is 584 days. Being an interior planet, its ecliptic motion must be taken into account by a sphere that rotates once in 365 days. This means that its 3rd and 4th spheres rotate at 365/584, or 62.5% the speed of its ecliptic (2nd) sphere. In other words, Venus completes nearly two revolutions around the ecliptic for every retrograde episode in its composite motion. In order to give the planet sufficient speed around the hippopede so as to produce retrogressions, the size of the hippopede must

[29] That philosophical considerations favoring rigorous demonstrations over mechanical arrangements colored the observations of many late Hellenistic commentators - Simplicius included - on Greek geometry is argued in W.R. Knorr, *The Ancient Tradition of Geometric Problems*, (New York: Dover Publications, Inc., 1993), pp. 7, 364.

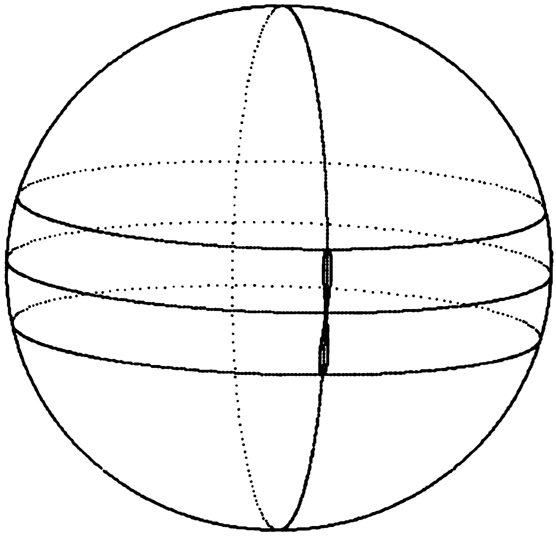

Figure 14

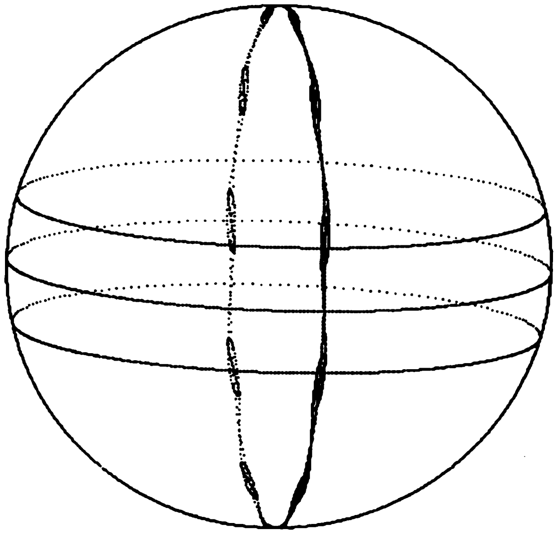

Figure 15

Figure 14 shows the hippopede created by the equal and opposite rotations of two spheres around axes that are inclined by 14° relative to one another. When a planet moving along this hippopede is simultaneously carried around the ecliptic at a rate that allows it to complete 11 hippopedes in the course of one lap around the ecliptic, it traces the course shown in Fig. 15. Jupiter's sidereal period is 11.86 years; its synodic period is 1.092 years, so it completes about 10.8 retrogressions during a single circuit of the ecliptic. Its deviation from the ecliptic in latitude is very small, and the arc length of its retrogressions is on the order of 15°. On the whole, then, the hippopede appears to provide a fairly good qualitative representation of its motion

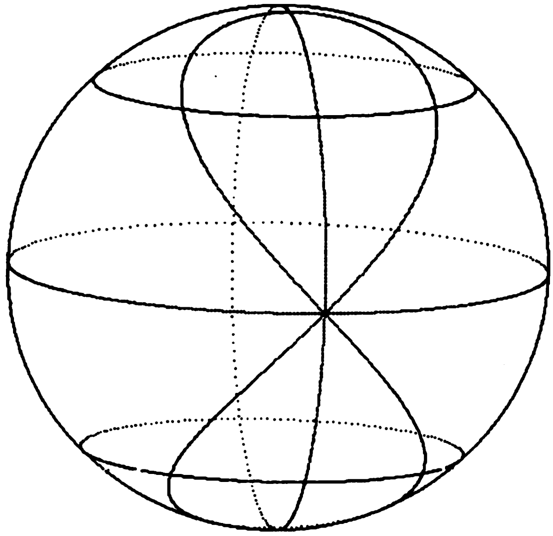

Figure 16

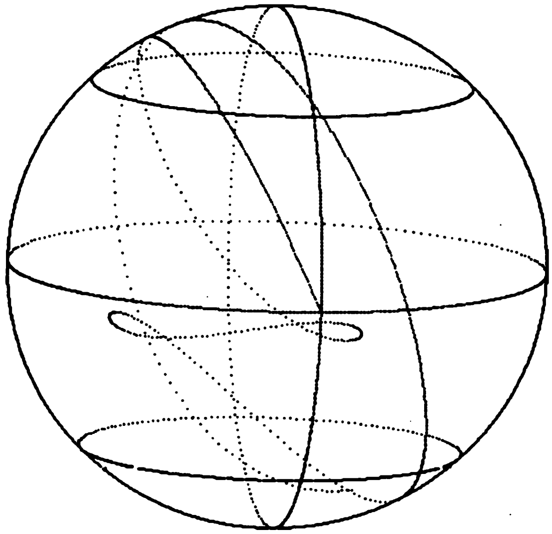

Figure 17

Figure 16 shows a hippopede created by an inclination of 87° between the axes of the third and fourth spheres. Figure 17 shows the planetary trace created when the planet completes 88% of the hippopede by the time its ecliptic sphere completes a whole revolution, which represents the ratio of Mars's mean ecliptic motion to its synodic period. Note how far the planet is made to stray from the ecliptic while it traces two retrograde loops of small arc length in a single synodic period be increased (note that the planet will complete 5/8 of its hippopedal motion regardless of the hippopede's size during one revolution of the 2^{nd} sphere). In particular, a large angle of inclination (at least 100°) between the axes of the third and fourth spheres must be specified. Unfortunately, this produces an undesirable increase in the maximum angular breadth of the hippopede in addition to the desired increase in its angular length. Fortunately, Venus performs its retrogressions when it is close to the sun, which makes direct observation difficult. Martian retrogressions, by contrast, occur when Mars is in opposition to the sun, and are therefore not similarly rescued from observation.

Mars circles the sun in 687 earth days and its mean synodic period is 780 days. That is to say, it must complete 88% of its hippopede by the time its ecliptic sphere completes a full revolution. The result of attempting to generate Martian retrogressions with a hippopede bears no resemblance at all to the observed ones. The hippopedal motion creates excessive deviations in latitude – dozens of degrees – from the ecliptic. Such deviations would place the planet well outside the zodiacal belt. Even the crudest of observations would have disclosed it, but nothing of the sort ever occurs (see Fig. 16 and 17), and Simplicius did note that Eudoxus was criticized for the motion in latitude generated by his theory.

This brings to mind the often-made statement that the hippopede's failure in the cases of Mars and Venus shows that it served only as a qualitative account of planetary motion and as no more than a demonstration of general feasibility.[30] There seems to be little reason to doubt that Eudoxus's homocentric astronomy served primarily as a general, qualitative account and not as a precise quantitative model. At the same time, we ought to exercise some caution and be a bit more clear about what we mean by "qualitative." Hargreave's discussion suggests one meaning for "qualitative" in this connection.[31] To be even moderately successful, any planetary theory must account for three readily observed phenomena: 1) All of the planets possess a mean eastward motion relative to the fixed stars; 2) the eastward motion is periodically interrupted by retrograde episodes during which a planet appears to move from east to west relative to the fixed stars; 3) in addition to this motion in longitude – that is, along the ecliptic – each planet shows some latitudinal deviations from the ecliptic in the course of its motion. Look again at the theoretical Martian path in Fig. 17 created by the hippopede in Fig. 16. It possesses a mean motion around the ecliptic, it shows distinct retrograde phases, and it certainly produces latitudinal deviations from the ecliptic. So strictly speaking, there is no qualitative failure in the above sense of the word in Eudoxus's theory of Mars. At the same time, there is no denying the gross incompatibility of the theoretical and observable paths of Mars in retrogression. It appears, then, that Eudoxus's "qualitative" failure in the case of Mars boils down to these large deviations of theory from observation.[32] To

[30] See, e.g. A. Aaboe, "Scientific Astronomy in Antiquity", *Philosophical Transaction os the Royal Society of London*, Series A, **276** (1974): 21–42, p. 40.

[31] D. Hargreave, "Reconstructing the Planetary Motions of the Eudoxean System," *Scripta Mathematica*, **28** (1967): 335.

[32] Heath stated that Eudoxus's Martian theory fails to produce retrogressions because inclinations larger than 90° are required for the two inner spheres, and that would make them rotate in the same direction. The problematic nature of this observation has been discussed in the previous sections, but regardless of this discussion, Heath's observation (which is common in the secondary

reject the theoretical trace as qualitatively unacceptable because of these deviations is to compare the shape of the theoretical trace to the observed traces with a definite criterion for qualitative lack of congruence. Unfortunately, this poses the difficult problem of deciding how large such deviations must be in order to be called a qualitative failure.

Neugebauer appears to have used a somewhat different sense of the word "qualitative" in this respect, which both sharpens the issue and avoids the hopeless problem of setting quantitative criteria for qualitative differences.[33] Quantitative methods, he observed, appear to have become central to Greek astronomy only in the time of Hipparchus, whose work was significantly enriched by exposure to Babylonian arithmetical astronomy. This suggests a sense of the word "qualitative" as opposed to "quantitative" that does not necessarily equate "qualitative" with "imprecise." The concepts of geometrical congruence and similarity are quite precise without being quantitative, and one may be qualitatively sensitive to very subtle differences between geometric forms. Subtle as they may be, however, such differences must remain qualitative as long as no method of quantifying the compared forms is available. This at once reveals the immense importance of Hipparchus's new quantitative astronomy, without rendering the old qualitative astronomy insensitive to small differences of geometrical form.

The distinction, then, is between quantitative and qualitative modeling, but "qualitative" in this sense does not mean "grossly inaccurate." The hippopede certainly produces retrograde motion, but for all intents and purposes, it is still a qualitative failure. Fig. 19 shows the retrograde trace generated by a hippopede that creates three retrogressions per ecliptic revolution, which is about the ratio Eudoxus used for Mars according to Simplicius. Being 200% off the mark, this ratio is questionable, considering that current values for the synodic and ecliptic periods of all the other planets are well within 10% of the Eudoxian values, as the table below shows. Both Ideler and E.F. Apelt, who anticipated much of Schiaparelli's reconstruction, attributed the aberrant Martian synodic period to a scribe's error.[34] Schiaparelli was more cautious, and quite reasonably preferred to suspend judgement on the issue in the absence of other evidence.[35] As Heath

literature) is simply false: as Figs. 16 and 17 above show, an inclination of 87° clearly produces two small retrograde phases in the path of Mars. One may choose to follow Simplicius's lead, and nail Eudoxus's failure to the widely exaggerated deviations from the ecliptic, which exceed the observed ones by about 25°. However, consider that Mars's retrograde arc can be up to 19.5° long, while the retrogression produced by the Eudoxian theory as depicted in Fig. 17 is only about 2°–3° long. Eudoxus's theory of Mars is an order of magnitude off both in longitude and latitude. The large deviations in latitude are perhaps the easiest to single out, but the traditional emphasis on latitudinal deviation in this context should be taken as a matter of convenience more than an identification of the fundamental problem.

[33] O. Neugebauer, *The Exact Sciences in Antiquity*, 2nd edition, (New York: Harper & Brothers, 1957), pp. 156–162.

[34] Ideler, "Uber Eudoxus (Zweite Abtheilung)," *Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin*, (1830): 49–88, p. 78. E.F. Apelt, "Die Sphärentheorie des Eudoxus und Aristoteles," Abhandlungen der Fries'schen Schule, Zweites Heft (Leipzig: Verlag von Wilh. Engelmann, 1849): 27–49, p. 42.

[35] G.V. Schiaparelli, "Le Sfere Omocentriche di Eudosso, di Callippo e di Aristotele," in *Scritti Sulla Storia della Astronomia Antica*, 3 vols., (Bologna: Nicola Zanchelli, Editore), vol. 2, p. 71.

Planet	Synodic periods			ecliptic periods		
	Eudoxus	modern	%error	Eudoxus	modern	%error
Saturn	390 days	378 days	3	30 years	29 years 166 days	2
Jupiter	390 days	399 days	2	12 years	11 years 315 days	1
Mars	260 days	780 days	200	2 years	1 year 322 days	6
Venus	110 days	116 days	5	1 year	1 year	0
Mercury	570 days	584 days	2	1 y ear	1 year	0

already noted, a 3:1 ratio of synodic to ecliptic period would make Mars undergo two superfluous retrograde phases out of opposition in addition to the one in opposition, and that is undoubtedly the most significant discrepancy associated with this ratio.[36] However, even ignoring this discrepancy and restricting the comparison only to the theoretical and observed forms of the retrograde phase itself, the inadequacy of the hippopede immediately stands out. Basically, this is because a figure of 8 is not qualitatively similar to a simple stretched loop. In most cases, a retrogressing Mars describes a simple elongated and slightly skewed loop on the background of the fixed stars. The entire loop is located to one side of the ecliptic. By contrast, retrograde motion generated by Schiaparelli's hippopede is always exactly symmetrical with respect to the ecliptic, which is crossed by the theoretical planet precisely in the middle of the retrograde track. Furthermore, the retrograde path itself usually looks like a double loop that bears no resemblance to the observed motion.[37] It takes no more than a score of random observations of Mars during the 5 months it takes to trace its retrograde loop to see that a hippopede could never produce an acceptable semblance of its trace, even if the hippopedal path has the correct arc length (compare Figs. 18 and 19).

The only time that a hippopede could create a very rough semblance of a planet's actual retrograde path is when the middle of the retrograde phase occurs near the intersection of the planet's path with the ecliptic, in which case the retrogression ceases to look like a loop, and assumes the form of a simple zig-zag motion. On the average, Mars traces such paths in about 3 out of 10 retrogressions. A closer comparison of such a path with the one created by a hippopede (see Figs. 20 and 21) again shows that because of the hippopede's crossing self-intersection and orientation relative to the ecliptic component of the motion, it makes the planet curve into and out of the retrograde track in a manner that cannot be reconciled with the observed trace.

These qualitative incompatibilities between the observed and theoretical paths remain blatant even when the observed path is traced only relative to the local group of stars that form its immediate background without reference to the ecliptic, which may not have been mapped out with sufficient accuracy for critical comparison with a

[36] Heath, *Aristarchus of Samos*, p. 210.

[37] See O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Part III, (Berlin: Springer-Verlag, 1975), pp. 1255–1256 for illustrations of retrograde paths for Jupiter and Saturn. Mars's retrograde loops are considerably wider and more easily observable.

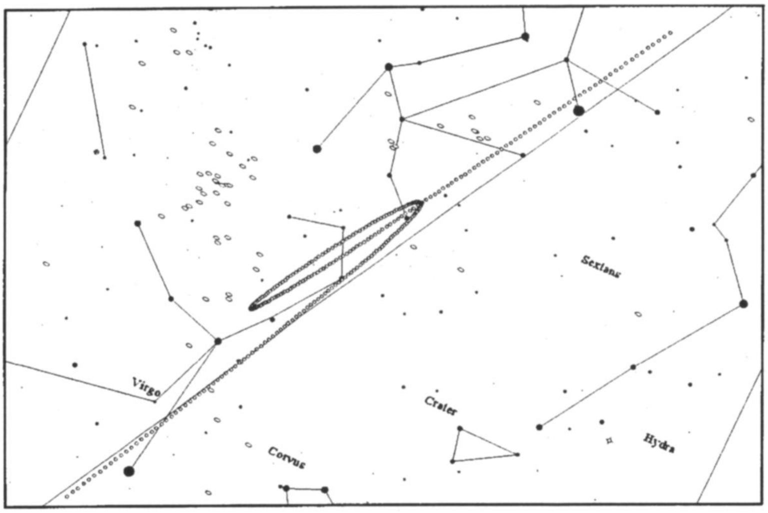

Fig. 18. Mar's between Leo and Virgo, 10/96-8/97; arc length ~19°

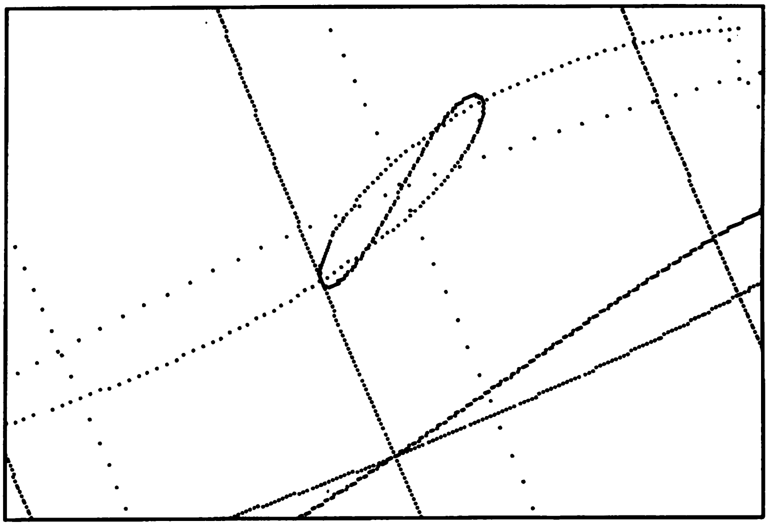

Fig. 19. Section of the stellar sphere (solid front, dotted back) showing parts of a hippopedal path around the ecliptic, with a retrograde loop (arc length ~20°) created by a 35° hippopede with a period 3 times shorter than the period of ecliptic revolution

planetary path in Eudoxus's time. In other words, it is difficult to see how the discrepancies between the observed and theoretical forms can be swept under the rug by any sensible use of the term "qualitative agreement" in this context. In principle, and regardless of the particular difficulties introduced by the ratios of synodic to ecliptic periods for Mars, Venus, and Mercury, Schiaparelli's version of the hippopede simply cannot reproduce the basic form of planetary retrogressions. Indeed, Schiaparelli himself emphasized precisely this qualitative incongruence of form in his general discussion of

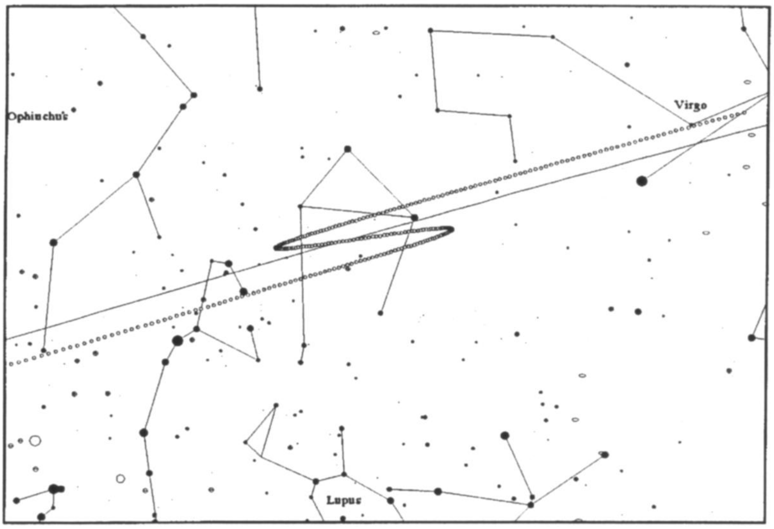

Fig. 20. Mars in Libra, 12/83-10/84

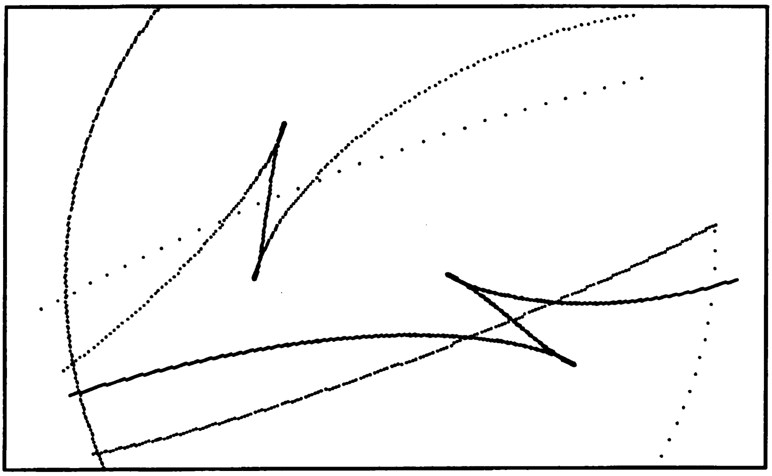

Fig. 21. Section of the stellar sphere (solid front, dotted back), traversed by a 40° hippopede making two retrogressions per ecliptic revolution

the Eudoxian theory,[38] but he was not generally followed in this by later historians of science. Heath, to give but one example, devoted 3 pages to quantitative comparisons of observed and predicted arc lengths and breadths, without once noting the formal incon-

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[38] G.V. Schiaparelli, "Le Sfere Omocentriche di Eudosso, di Callippo e di Aristotele," in *Scritti Sulla Storia della Astronomia Antica*, 3 vols., (Bologna: Nicola Zanchelli, Editore), vol. 2, pp. 75–76.

gruence between the theoretical and observed curves.[39] Now, Eudoxus was arguably the greatest geometer of his time, and much of his work involved the study of geometrical forms - not merely straight-edged ones, but curved forms for the creation of which he constructed fairly elaborate instruments. To claim that he never engaged in systematic quantitative observations of planetary motion is one thing; it is quite another to claim that as a geometrician of his stature Eudoxus was insensitive to the incongruence between the trace generated by a moving hippopede and a rudimentary observation of Mars's actual path. This does not add up to a sufficient reason for rejecting Schiaparelli's interpretation, but the significance of his interpretation must be clear - it implies that Eudoxus never bothered to take more than a casual look at Mars in retrogression.

In the past decade, Bowen and Goldstein produced a series of studies on the nature of early Greek astronomical observations.[40] For the purposes of the present analysis, the relevant conclusion from Bowen and Goldstein's studies is that it would be unlikely for Eudoxus or Callippus to map and date the full course of Mars relative to the ecliptic with the precision required for a critical comparison with the theoretical alternatives described in this paper. That is to say, any discrepancy in a pointwise comparison of the theoretical and observed position of the planet at a given date relative to some global mapping scheme would fall well within the range of observational inaccuracy in Eudoxus's time. However, a different picture emerges if it is admitted that rather than the precise reproduction of the planet's location in real time, the desired object was to reproduce the basic form of the planet's retrograde path. To begin with, this harmonizes with the assumption that the Eudoxian (as well as the Callippic) scheme was a qualitative rather than a quantitative planetary theory. As interpreted here, the Eudoxian and Callippic theories were designed to address the departures of planetary motions from uniform rotation. In this framework, retrograde motion is clearly the primary problematic phenomenon, not the prediction or retrodiction of a planet's coordinates at a given date. Significantly enough, observational procedures that fall considerably short of Bowen and Goldstein's standards for being considered as precise, dated observations are more than sufficiently precise to reveal the inadequacy of the hippopede as a realistic retrograde generator. In other words, observational practices in violation of Bowen and Goldstein's conclusions need not be attributed to Greek astronomers of the 4th century BC in order for them to have shown that the hippopede could not pass the test of observation. To see this, consider that Mars's retrograde loops are not tiny forms that require refined instruments to be observed. They vary in arc length roughly from 11° to 19°, and their breadth ranges up to 2.5°. Using three or four prominent stars in the constellation against which Mars retrogresses, the form of its path can be easily traced by a small number of observations unsystematically spread over the course of five months without ever needing to measure angles in excess of 10°. No particular standard of angular measurement is required for this, nor does the planet's position need to be fixed relative to any global

[39] Heath, *Aristarchus of Samos*, pp. 208–210.

[40] Of particular importance to this study are "Hipparchus' Treatment of Early Greek Astronomy: The Case of Eudoxus and the Length of Daytime," *Proceedings of the American Philosophical Society*, **135** (1991): 233–254, and "The Introduction of Dated Observations and Precise Measurement in Greek Astronomy," *Archive for History of Exact Sciences*, **43** (1991): 94–132.

coordinate system or a timed table of stellar risings and settings. Because of the small angles involved, the difficulties associated with the sectioning of a circle disappear, and the problem reduces to one of measuring linear separations on a practically flat patch of sky. In fact, a T-bar 1 meter long, with a 15 cm long cross beam on which 16 equidistant segments are marked provides a primitive dioptra capable of measuring small angles by increments of roughly 0.53°. Time measurements of any kind are not required. In other words, significantly less sophisticated observational techniques of a sort that is not ruled out by the conclusions of Bowen and Goldstein suffice for clearly demonstrating the inadequacy of Schiaparelli's arrangement as a basis for accounting for the appearances. We shall see that a similar comparison would also indicate some deviations from observed results on the part of the alternative reconstruction of Eudoxus's theory. However, as a step toward a qualitative theory the alternative certainly appears to point in the right direction, and, we shall also see, it could provide Callippus with a starting point to a far better theory than the one ascribed to him by Schiaparelli.

All of this does not necessarily mean that the hippopede was completely unimportant in Eudoxus's astronomy, or that Simplicius's text is totally misleading, or that Schiaparelli's reconstruction should be rejected. It does not necessarily follow that Eudoxus and his contemporaries carried out certain observations just because such observations were within their capabilities. It does suggest, however, that if Schiaparelli's hippopede was central to the Eudoxian astronomical system, then Eudoxus's interest in elementary observational astronomy was at best marginal. It appears more likely in this case, that for Eudoxus the problem of accounting for planetary motion was no more than a springboard to a purely mathematical problem of analyzing superimposed spherical rotations. We have seen that regardless of its astronomical usefulness the hippopede represents a triumph of geometrical analysis. Furthermore, Riddell has shown that Schiaparelli's arrangement with the third sphere rotating twice as fast as the fourth may be used as a mechanical contrivance for solving the problem of doubling the cube.[41] This may be used to portray a Eudoxus who transported a primarily mathematical device into an astronomical context as a quick afterthought that he had no particular interest to follow up with observational comparisons. Unfortunately, we do not really know what Eudoxus's attitude to observational astronomy was. Strabo reported that Eudoxus had observatories in his home island of Cnidus and in Heliopolis, Egypt, which still existed in the time of Augustus when Strabo wrote about them.[42] If the poem of Aratus is based on

[41] R.C. Riddell, "Eudoxan Mathematics and the Eudoxan Spheres," *Archive for History of Exact Sciences*," **20** (1979): 1–19, esp. pp. 14–17.

[42] *The Geography of Strabo*, 8 vols., Tr. H.L. Jones, (London: William Heinemann, 1908), vol. VIII, pp. 83–85 (C 806–C 807), "... for a kind of watchtower is to be seen in front of Heliupolis, as also in front of Cnidus, with reference to which Eudoxus would note down his observations of certain movements of the heavenly bodies." Strabo's story that Eudoxus visited Egypt with Plato for thirteen years, no less (alternative readings of three years and sixteen months have been suggested), seems rather questionable. But Strabo's indication that Eudoxus occupied himself with astronomical observations of some sort need not be discarded along with the less credible parts of the story. A. Berry, *A Short History of Astronomy*, (New York: Dover Publications, 1961, reissue of the original 1898 text), p. 29, took Strabo's testimony at face value, and thought of Eudoxus as an active observational astronomer. Paul Tannery, *Recherches Sur L'Histoire De L'Astronomie*

observations carried out by Eudoxus, then he did have more than a passing interest in active astronomical observations.

We know that Eudoxus devoted a treatise to the theory of homocentric spheres, but neither it, nor any other astronomical work by Eudoxus and Callippus has survived. Later sources, particularly Ptolemy, do not refer to any observations by Eudoxus and his contemporaries for the purpose of ascertaining the form of planetary retrograde loops. But then, the sort of observations that Eudoxus was likely to carry out for this purpose would have been far too crude and imprecise for Ptolemy's more refined purposes, so he had little motivation for citing such old knowledge, even if it did exist. In the absence of any extant references to observations carried out by Eudoxus for the purpose of ascertaining the general shape of retrograde loops, we would normally conclude that he did not, in fact, engage in such observations. However, the general loss of nearly all the astronomical works from his period requires that we qualify such statements as follows: It seems more likely that Eudoxus did not embark on an observational program designed to elucidate the form of planetary retrogressions, but technically they were certainly within his reach, and for lack of sufficient evidence we cannot reject the possibility completely. Considering this, it seems unjustifiable to close our historical view to the possibility that observational planetary astronomy was of more than marginal interest to Eudoxus, and that consequently the hippopede as reconstructed by Schiaparelli could not play a central role in his planetary theory.

f. Astronomical Advantages of the Alternative Interpretation

From the astronomical point of view, the alternative arrangement possesses several decided advantages over Schiaparelli's, and goes a considerable way toward alleviating the difficulties outlined in the previous section. This is particularly true of the general alternative interpretation of Aristotle's text, but even the restricted alternative based on Simplicius's text creates more realistic depictions of planetary retrogressions than the hippopede of Schiaparelli's reconstruction. In the general case, the availability of two independent inclinations instead of one makes it possible to create simple long and narrow loops lying along the ecliptic as opposed to the self-crossing figure of eight that characterizes the hippopede of Schiaparelli's interpretation. The addition of phase angles to the alternative arrangement skews the resulting loops relative to the ecliptic (see Figs. 8 and 9). When coupled to a rotating ecliptic sphere, this alternative arrangement gives rise to simple elongated and slightly skewed retrograde loops resembling the observed ones (see Fig. 10). We have seen in the previous section that the hippopede fails to provide an adequate semblance of Martian retrogressions even if the grossly erroneous synodic period of 260 days is taken. This is no longer the case in the framework of the alternative arrangement. The loop it generates still diverges from the real one in

Ancienne, (Paris: Gauthier-Villars & Fils, 1893, Reprinted by Arno Press, New York, 1976), pp. 44–45, who was equally aware of Strabo's testimony, remained unconvinced, and considered that Eudoxus's many interests suggest that he did not possess the patient character required for lengthy systematic observations.

details such as the particular curvatures of its upper and lower sections, the point of self-intersection, and its general position relative to the ecliptic (see Figs. 22 and 23). But it clearly possesses the correct qualitative form and all of the above discrepancies require far more precise observations and path analysis to perceive than those that characterize the comparison with the hippopedal traces.[43] Having said all of that, it is not very likely that Eudoxus actually used 260 days for the mean synodic period of Mars, when the real figure is 780 days. When this number is used, the alternative interpretation, either in its restricted or general form, clearly diverges from observed Martian retrogressions. It is possible, for example, to enlarge the inclinations and to produce retrograde motion without unacceptable departures from the ecliptic. However, Figs. 24 and 25 below show that as the inclinations increase, they accentuate the differences between the geometry of a plane and the geometry of a spherical surface. In excess of a certain size, the alternative loop looks like a rubber band that overlaps itself twice instead of the merely larger ellipse that a deferent and an epicycle would create on a plane surface. A proper choice of phase angle will produce a looped retrogression, but the basic discrepancy of form clearly reveals itself by excessive motions in latitude that prevent the loop from assuming the long flat shape of typical planetary retrogressions. Hence, while the alternative arrangement satisfies the general requirement of qualitative representation of planetary retrograde motion, it still falls short of reproducing the particulars of Mars.

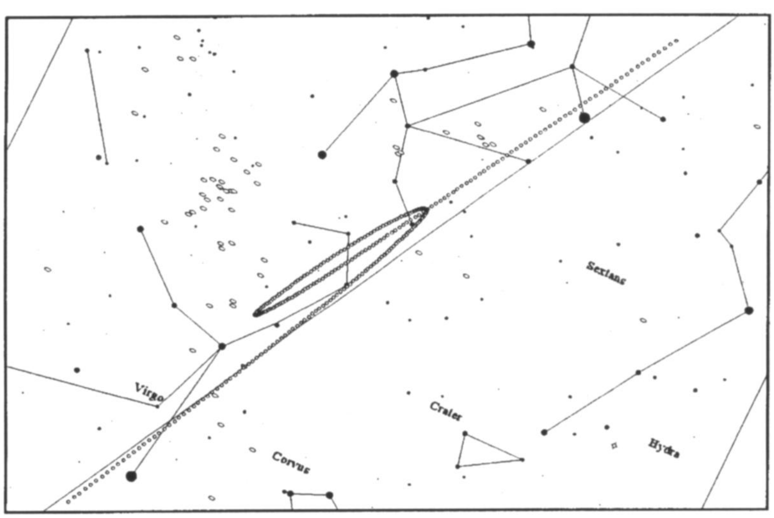

Fig. 22. Mars between Leo and Virgo, 10/96 - 8/97; arc length ~19°

[43] By varying the parameters (e.g. planet on latitude 17.5° measured from the pole of sphere 4, inclination of 18.5° between spheres 3 and 4, and a phase angle of 4°), the alternative arrangement can yield a far better qualitative semblance of the zig-zag path (shown in Figure 20 above) than Schiaparelli's hippopede. The zig-zag path can also be produced by the restricted form of the alternative arrangement (e.g. 18° for the inclination of spheres 3 and 4 and for the latitude of the planet on sphere 4, and a phase angle of 5°).

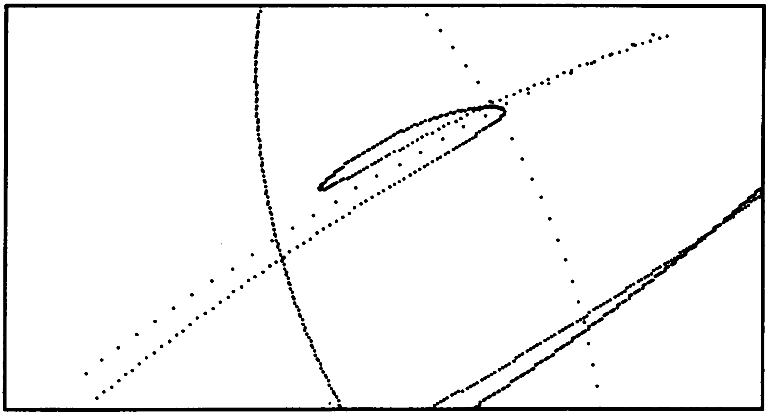

Fig. 23. The retrograde path generated by the alternative arrangement, with the planet carried on latitude 17° (measured from the pole) of sphere 4, an inclination of 19° between the axes of spheres 3 and 4, and a phase angle of 2°. The ratio of ecliptic to synodic period is 730/260

From the point of view of historical reconstruction, this is precisely the sort of effect we need, because historical reconstructions of Eudoxus's work that *do* allow him to account for the motions of the inner planets would be hard put to explain the references to Callippus, who found it necessary to amend Eudoxus's theory for Mars, Venus, and Mercury. Thus, contrary to the unavoidable implication of Schiaparelli's interpretation, we now have a Eudoxus who did capture the basic qualitative form of planetary retrograde

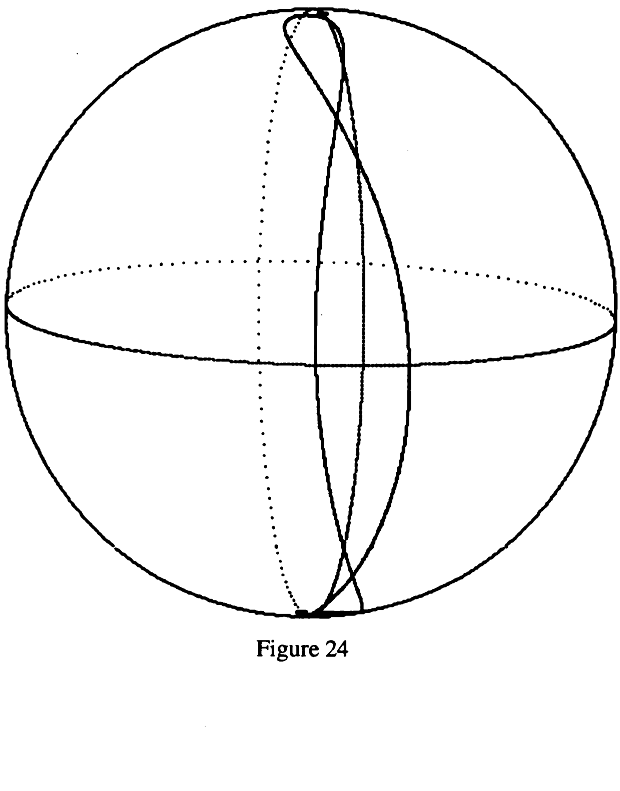

Figure 24

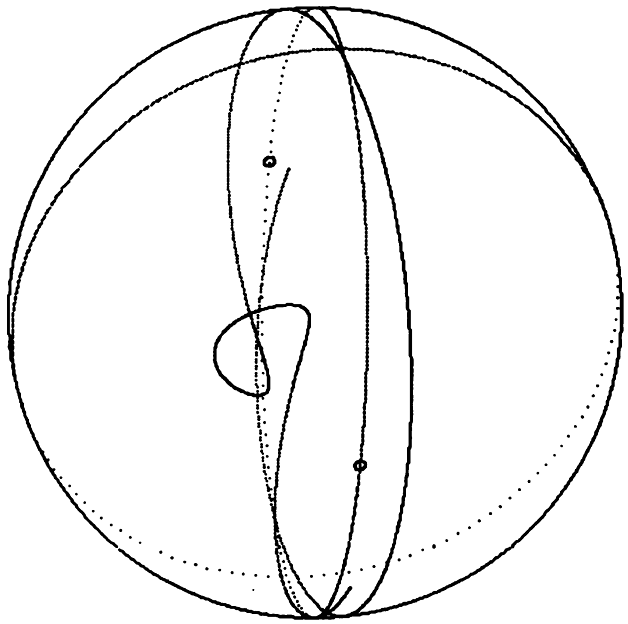

Figure 25

Figure 24 shows the curve that results from the combined motion of the alternative arrangement with the planet on latitude 40° as measured from the pole of the fourth sphere, while the axes of the third and fourth spheres are inclined by 49° to each other and a phase angle of 4° is used. Figure 25 shows the trace that a planet leaves on the stellar sphere if it completes 88% of the curve in Fig. 24 by the time that its ecliptic sphere completes one full revolution in accordance with Mars's ratio of sidereal to synodic period

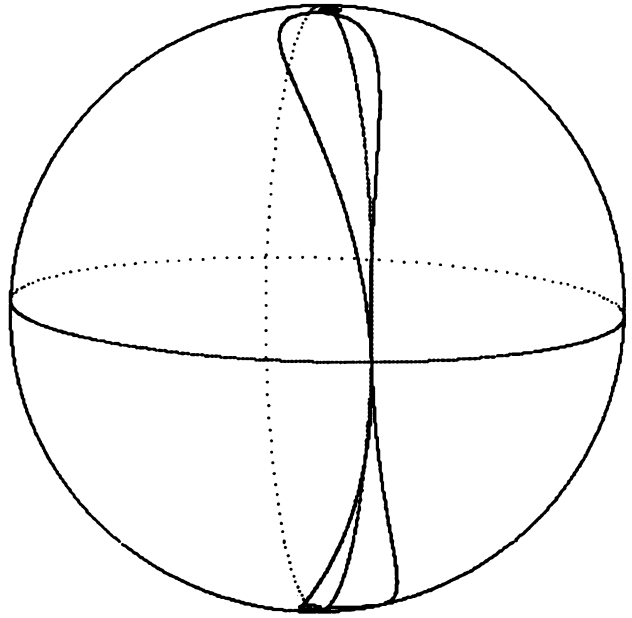

Figure 26

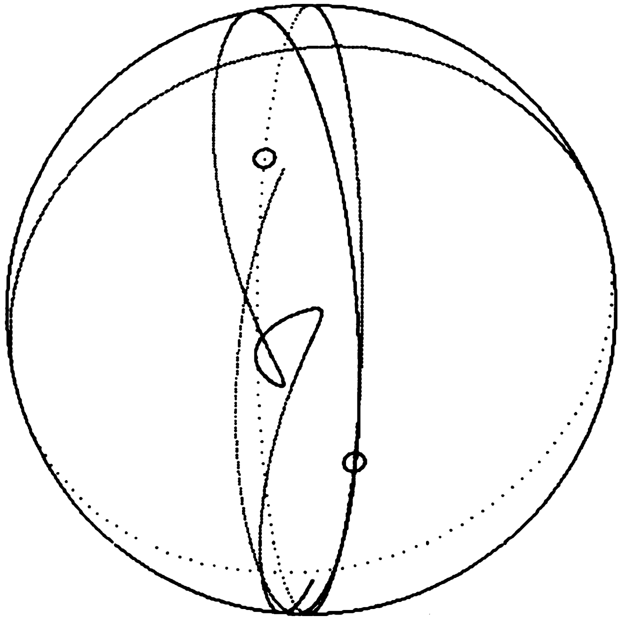

Figure 27

Figure 26 shows the curve generated by the alternative arrangement, restricted to two equal inclinations, 44° each, with a phase angle of 4°. Figure 27 shows the resulting planetary path around the ecliptic, when the Martian ratio of .88 for the sidereal/synodic periods is chosen

motion, but left it for Callippus to produce a satisfactory account for the actual motion of the three inner planets.

g. What Callippus May Have Done

We know practically nothing about the details of Callippus's contributions, save the general statement that he added two spheres each to the motion of the sun and moon, and one sphere to each of Mars, Venus, and Mercury.[44] For Mars, Schiaparelli showed how an additional sphere can create a curve more complicated than the hippopede with a construction that is a clever variation on his basic arrangement.[45] The first two spheres, as before, account for the diurnal rotation and for motion in the ecliptic plane. Again

[44] See Heath, *Aristarchus of Samos*, pp. 212–213.

[45] G.V. Schiaparelli, "Le Sfere Omocentriche di Eudosso, di Callippo e di Aristotele," in *Scritti Sulla Storia della Astronomia Antica*, 3 vols., (Bologna: Nicola Zanchelli, Editore), vol. 2, pp. 79–81.

as before, the third sphere's poles are attached to the second sphere at the ecliptic. The fourth sphere is inclined by 90° to the third, *i.e.*, the pole of the fourth sphere rests on the equator of the third. The fifth sphere's pole is inclined at 45° relative to the pole of the fourth sphere, in the direction of the pole of the third sphere. In other words, the starting position has the pole of the fifth sphere halfway between the pole of the third and fourth spheres. The planet rests on the equator of the fifth sphere. The fourth sphere rotates twice as fast as the third, in the opposite sense of the third sphere's rotation. The fifth rotates at the same speed and direction as the third. (Note that by fixing the pole of the fifth sphere directly under the pole of the fourth, the fifth sphere is made to rotate at the speed of the third, but in the opposite direction. In other words, this would create a hippopede). The resulting curve lies symmetrically along the ecliptic. It is just over 90° long, and contains two triple points near its ends as Fig. 28 below shows. The advantage of this curve is twofold. To begin with, it is narrow, so that the planet's deviation in latitude does not exceed the observed deviation. Then, since the planet is made to spend a long time near the ends of the curve because of the extra loops it must perform there, its motion along the interior part of the curve between the two triple points is very quick. Therefore, this relatively small curve can generate retrograde motion even when the ecliptic period of the planet is shorter than the synodic period (which is the period of time it takes to complete a circuit of the retrograde generating curve). Retrograde motion may be obtained with this new curve for Mars, that has a ratio of 0.88 between its ecliptic and synodic periods (see Figs. 28 and 29).

As Dreyer noted, the new curve crosses the ecliptic eight times – unlike the hippopede, which crosses the ecliptic four times.[46] In the course of a single synodic period, therefore, the planet would oscillate across the ecliptic several times more than it actually does. The observational rejection of such a deviation, however, requires careful plotting of the planet's course around the entire ecliptic and comparison of this plot with the sun's motion. This involves systematic observations that are considerably more demanding than the few random observations relative to a single zodiacal constellation that suffice to portray the form of the retrograde phase itself. Unfortunately, even such a limited observation of just the retrograde portion of the planet's course is incongruous with the theoretical path created by Schiaparelli's arrangement, because planets never curve in and out of their retrograde phase as the model suggests. Furthermore, Mars in particular traces distinct loops, which Schiaparelli's arrangement cannot reproduce. In the case of Callippus, this requires us to stretch the meaning of "qualitative" agreement with the phenomena even beyond the level of discomfort associated with the case of Eudoxus. Recall that Callippus allegedly amended Eudoxus's solar theory because it yielded seasons of equal length. Callippus added two spheres to the sun's motion, presumably to create a hippopede (or an alternative retrograde generating loop) much too small to produce retrograde motion, but sufficient to slow the sun down in part of its course and speed it up in the remaining part so as to produce four seasons of unequal length. It does not seem reasonable that the same Callippus, who supposedly worried about disagreements of one or two days out of 90 in the case of the sun, would allow

[46] J.L.E. Dreyer, *A History of Astronomy from Thales to Kepler*, (New York: Dover Publications, Inc., 1953 reprint of the 2nd edition, 1906), pp. 104–106.

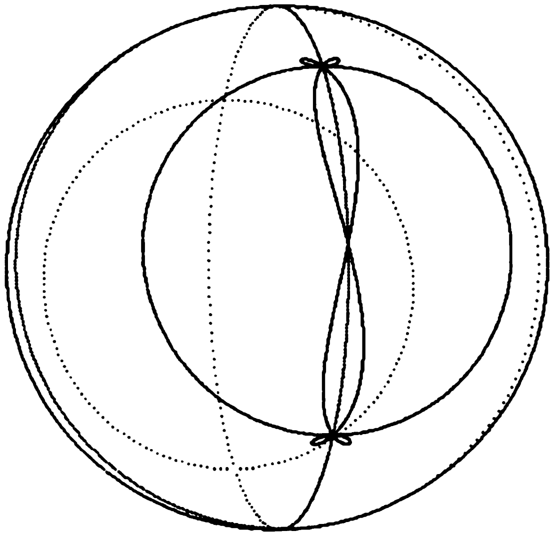

Figure 28

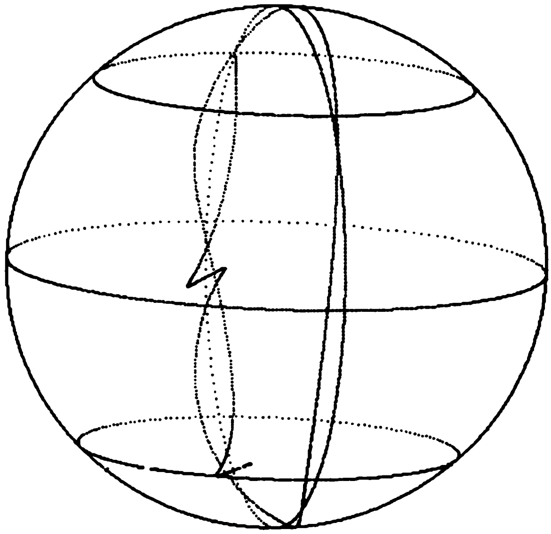

Figure 29

Figure 28 shows the trace created by the arrangement used by Schiaparelli to reconstruct Callippus's variation on the Eudoxian theme. Figure 29 shows the path for Mars generated by this arrangement. Such traces, as explained above, occur only when the planet crosses the ecliptic during the retrograde phase. Note how the model makes the planet curve into and out of the retrogression in a manner that is never encountered in reality significantly more blatant discrepancies between the theoretical and observed traces of planetary retrograde motion.

It should be noted that the arrangement Schiaparelli used to reconstruct Callippus's amendment is useful only within a narrow range: it produces a retrogression generator in the form of a relatively long (approximately 90° of arc), narrow closed curve along the ecliptic if the angle between the fourth and fifth spheres is 45°.[47] However, for inclinations of 20 or 70 degrees, the arrangement creates quite unacceptable forms as Figs. 30 and 31 show. In other words, while the 90°, −45°, 90°arrangement above could provide barely acceptable retrograde motion for Mars, its one variable parameter (being the angle between the poles of the fourth and fifth spheres, set here to 45°) cannot be varied to accommodate the cases of Venus or Mercury.

Echoing Schiaparelli, Heath referred to this arrangement as the simplest of several possible alternatives which he did not proceed to describe. Indeed, other variations using a fourth sphere that rotates twice as fast as the third, and a fifth sphere that rotates at the same speed and the same direction as the third are made possible by abandoning the restriction of the angle of inclination between the third and fourth spheres to 90°. For example, instead of the 90°, −45°, 90°arrangement that Schiaparelli showed, use 27°, 53°, and 90°. This yields a hippopede that cannot be reduced to intersecting spheres and cylinders. It is very narrow, but spans an arc of 170° along the ecliptic. This would create a differently shaped retrogression for Mars (see Figs. 32 and 33). This form is acceptable for an ecliptic-crossing retrogression. As already noted, however, such retrogressions are relatively rare.

In contrast with these variations on Schiaparelli's reconstruction, the alternative scheme creates none of the above difficulties when taken as Callippus's point of departure. The addition of a fifth sphere to the alternative interpretation is analogous to the development of the next term in a series expansion, and its effect is intuitively explicable. The additional sphere, placed beneath the fourth sphere of the original arrangement, makes the elongated, ellipse-like curve carry an additional epicycle to which the planet is attached. On the ecliptic, the long axis of the original curve is added to the (angular) radius of the epicycle, while at 90° from the ecliptic the short axis of the curve is subtracted from the epicycle's radius. This creates a longer and narrower loop as shown below. With a small phase angle as described in section *c* above, the new curve may be slightly rotated relative to the ecliptic. By specifying that 88% of this loop be traversed while its center of symmetry completes one revolution around the ecliptic, we can produce retrograde motion in accordance with the basic characteristics of Mars's motion (see Figs. 34 and 35). The graphs at the end of the appendix show that every form of Martian retrogression can be quite nicely reproduced with such a four-sphere arrangement. The position of the retrograde loops relative to the ecliptic, and the speed of the

[47] Schiaparelli noted that several alternative 3-sphere structures can create retrograde generating curves in keeping with Callippus's needs, but did not proceed to describe them. Instead, he described only one arrangement and characterized it as the "... simplest way which preserves the natural limits of the [motion in] latitude..." G. Schiaparelli, *Scritti Sulla Storia della Astronomia Antica*, Vol. 2, (Bologna: Nicola Zanichelli, Editore), p. 79. Both Dreyer and Heath simply reproduced the figures from Schiaparelli's original paper.

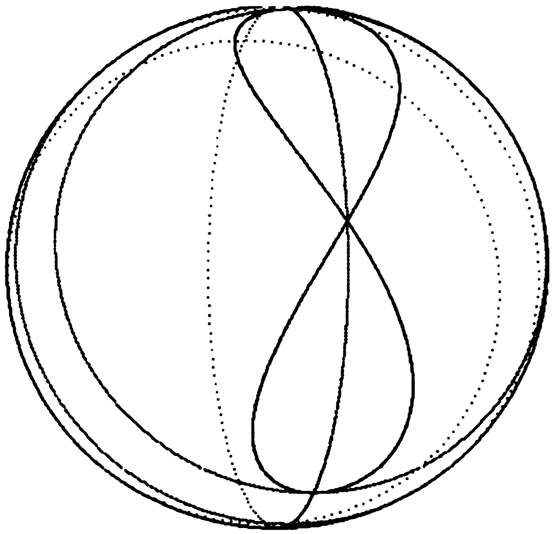

Figure 30

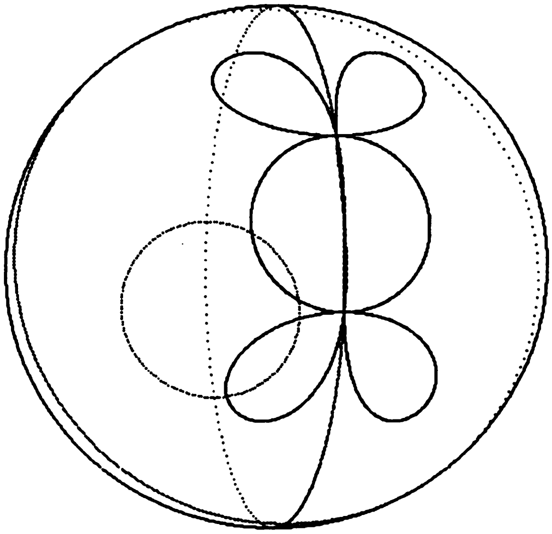

Figure 31

Figure 30 shows the curve created by inclining the fifth sphere by 20° relative to the fourth in Schiaparelli's arrangement. The similarity between this curve and Schiaparelli's reconstruction of Eudoxus's hippopede is deceptive: unlike the hippopede, this figure of 8 is created by intersecting a sphere with a pipe that has an oval cross-section, not a circular one. Figure 31 shows the curve resulting from inclinations of 90°, −70°, and 90°. Both generate retrograde traces that are unacceptable for reproducing planetary motions

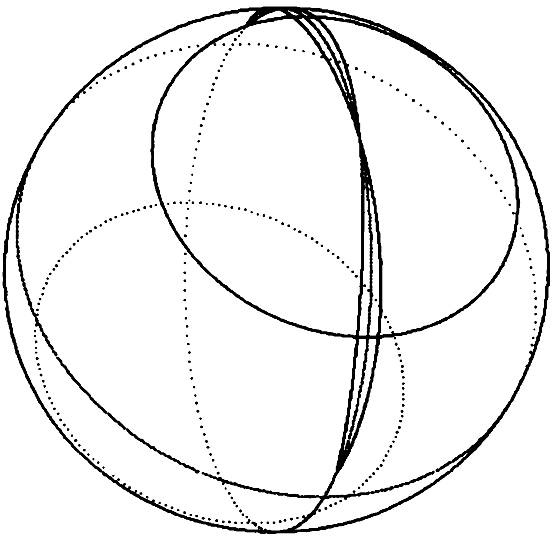

Figure 32

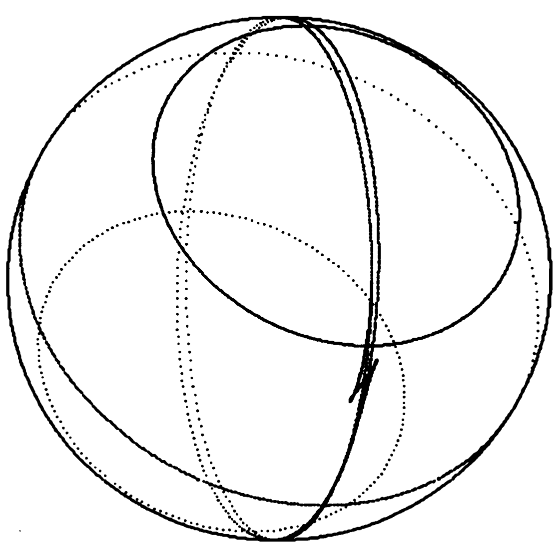

Figure 33

Figure 32 shows the modified figure of 8 generated by using angles of 27°, 53°, and 90° in Schiaparelli's scheme instead of the 90°, −45°, 90° used by Dreyer and Heath to illustrate the arrangement. Figure 33 shows the resulting planetary motion using Mars's ratio of synodic to ecliptic to synodic period (0.88). Retrograde motion is produced, but it still fails to reproduce the basic looped form that characterizes typical Marian retrogressions

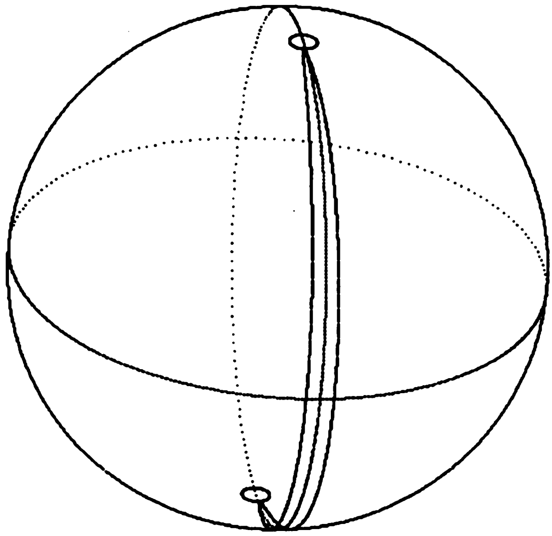

Figure 34

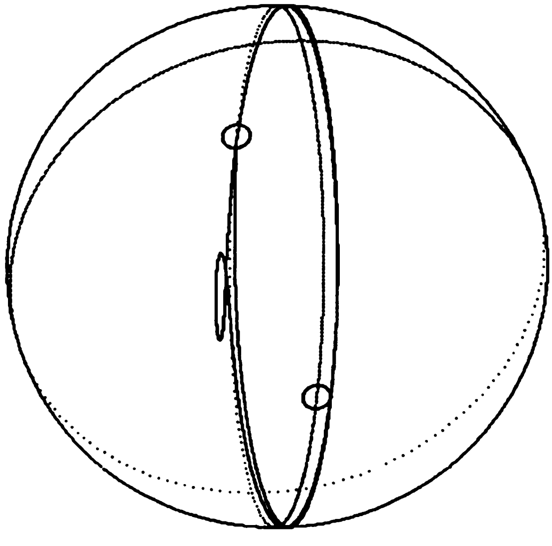

Figure 35

Figure 34 shows the curve created by three spheres. The poles of the first are fixed to the ecliptic (depicted by the dotted great circle that bisects the two small latitude circles). The poles of the second are tilted by 17° relative to the poles of the first; the poles of the third are tilted by 45° with respect to those of the second, and the planet is placed on latitude 25° of the third sphere measured relative to its pole. By starting the planet at an angle of −.4° on its latitude relative to the ecliptic, the whole resulting curve is slightly turned with respect to the ecliptic. Figure 35 shows the trace that results when the planet is moved like Mars around the ecliptic, completing 88% of its motion around the curve of Fig. 34 by the time it completes one revolution around the ecliptic. The angle of view in Fig .35 is rotated to show a view of the retrograde phase as seen by an observer at the center of the stellar sphere. This should be compared with the observed retrogression of Mars in Leo from June '94 to April '95 planet in this theoretical course will not quite match the motion of Mars. The observation of such discrepancies, however, would require the sort of careful timed observations that are not evident in Greek astronomy prior to the third century BC at the earliest. Barring such errors, the general shape of the theoretical loop may quite reasonably be described as having excellent qualitative agreement with the observed trace of Mars's retrograde motion. Venus is slightly more difficult to satisfy, because its synodic period is only 5/8 of its sidereal period, but here also, close likeness to its actual path may be obtained by selecting three larger inclinations for the three inner spheres to create a longer retrograde generating loop.[48]

Figures 36–53 below demonstrate the ability of the alternative Callippic arrangement to reproduce the geometrical forms of individual Martian retrogressions. No attempt has been made to arrive at a precalculated best fit, which would be quite outside the most optimistic assessment of analytical abilities in classical Greece. The particular parameters for each retrogression represent one out of a range of possible choices that would satisfy a qualitative, visually gauged similarity of form. In all of the examples, the ecliptic sphere rotates in the same direction (west to east relative to the stellar sphere), with a period of 0.88 of the synodic period, which is the time it takes for the three inner spheres to complete a single revolution (recall that relative to an observer on the ecliptic sphere, all three inner spheres appear to spin equally fast; the third and fifth spheres rotate in the same direction, the fourth spins opposite to them). The main retrogression features the various skew directions of the loops and their locations on both sides of the ecliptic, are all produced by small variations of the phase and three inclination angles, keeping constant their rotational directions and speeds of the spheres about their respective poles. All of the theoretical paths are traced on the back side of the stellar sphere, to present a view "over the shoulder" of an earthbound observer at the common center of the spheres. They show only a small area of the sky centered on the retrograde loop. In most cases, in addition to the retrograde loop, the figures contain a non-retrograde trace from Mars's previous passage through this area. These are irrelevant to the comparison, and may be ignored. In all cases, the ecliptic is marked by a series of dots more widely spaced than those that plot the retrograde trace. The reproductions of Martian motion based on modern calculations are plotted approximately in the same scale as the theoretical curves produced by the alternative Callippic arrangement. To facilitate comparison, the dots that mark the ecliptic in the Callippic figures have been plotted every 2° of arc. The angular size of each theoretical loop may be guaged against this measure, and compared to the sizes noted below each reproduction of Mars's actual motion. Note that while the forms

[48] Use, for example, angles of 23°, 60°, and 32° for the inclination between spheres 3 and 4, 4 and 5, and the planet's latitude relative to the pole of the. (innermost) sphere 5 respectively, and a phase angle of 2°. Superimpose the resulting retrograde generating loop on a mean ecliptic motion whose period is 5/8 of the synodic period (being the time to go around the retrograde generator). This will result in a typically skewed retrograde loop roughly 10° long and 2° wide. Note again that comparison of the theoretical path with direct observation is practically impossible because of Venus's proximity to the sun.

of individual Martian retrogressions can be approximated quite well by the alternative Callippic arrangement of four concentric spheres, the location of the retrograde trace relative to the ecliptic can be up to 3° off. (For further details on the construction of the theoretical paths, see appendix, §*b*).

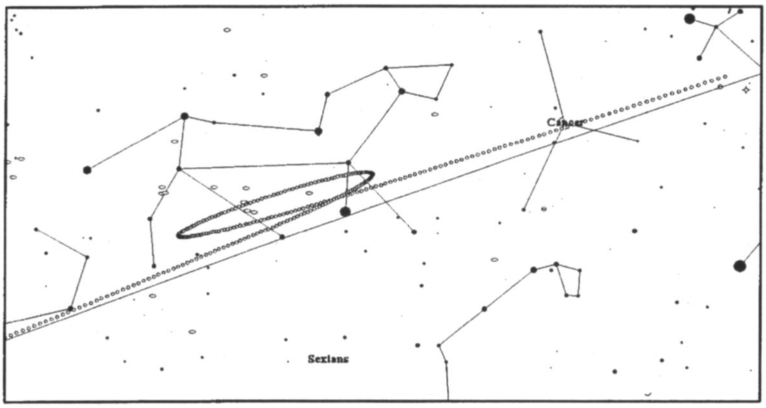

Fig. 36. Mars in Leo, 9/79-7/80; loop size: 19° X 2°6′

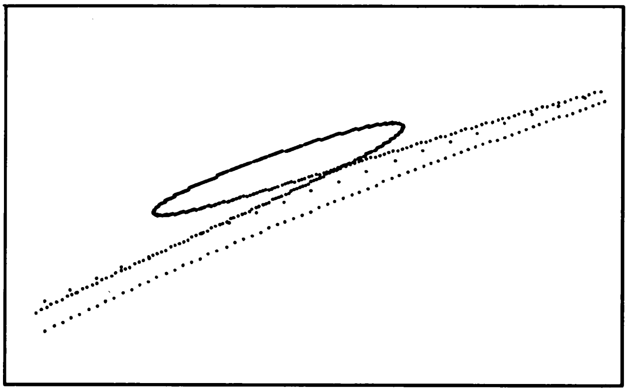

Fig. 37. Phase angle: 0.3°, 1st inclination: 16.75°, 2nd: 44.5°, 3rd: 24.75°

To the extent that reproducing the form of any given planetary retrogression was the problem to which Eudoxus and Callippus directed their efforts, it can be said that if Callippus developed the theory along the lines of the alternative reconstruction, then he successfully finished what Eudoxus had begun. All of this should not be taken to mean that Callippus solved the problem of planetary retrograde motion. Indeed, the alternative arrangement can reproduce a good likeness of any Martian retrograde loop. However, once fixed with a given set of specific parameters, the arrangement will keep

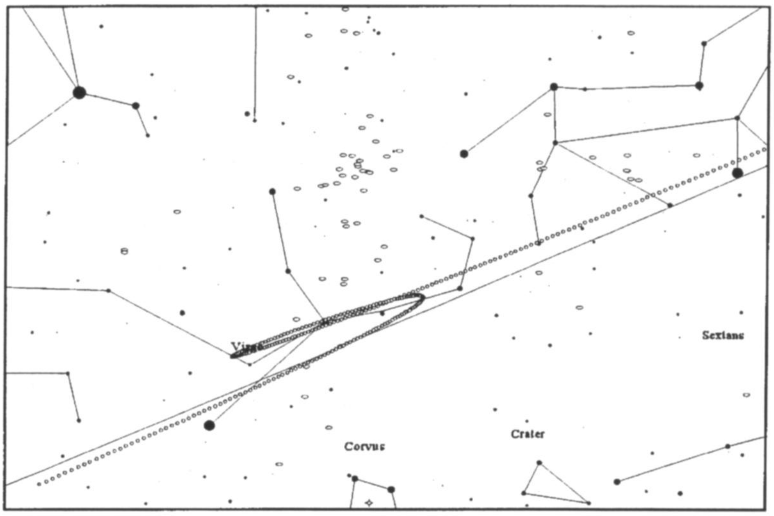

Fig. 38. Mars in Virgo, 10/81-8/82; loop dimensions: 18°32′ X 0°33′

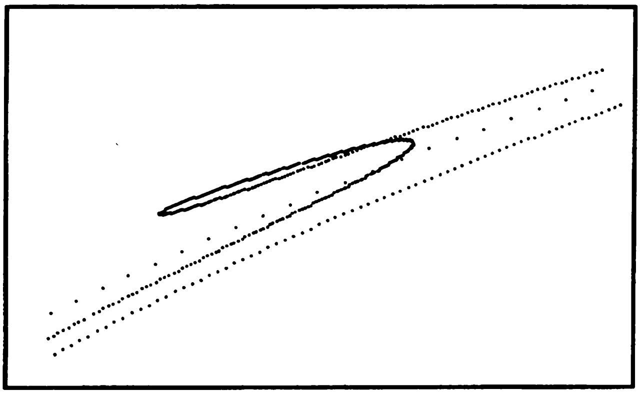

Fig. 39. Phase: 1.2°, 1st inclination: 17°, 2nd: 44°, 3rd: 24.75°

reproducing the same retrograde loop over and over. The scheme cannot automatically reproduce the variation in the form of the retrogressions from one to the next. The inclinations and phase angle must be reassigned for each new variation, and even the rotational directions of the three inner spheres have to be reversed in order to create the full range of Martian retrogressions (the speeds remain constant and always reflect the mean synodic period of the planet). Absence of variability remains a general failure of any Eudoxian or Callippic theory. In other words, Callippus successfully finished what Eudoxus had begun only to the extent of retrospectively accounting for the geometrical form of any given Martian retrogression, but not to the extent of predicting the variations in form from one synodic period to the next. This, however, cannot be ascertained by random observations. One might carefully observe Mars in retrogression once, then choose appropriate parameters for the spheres to reproduce the motion. Slightly more

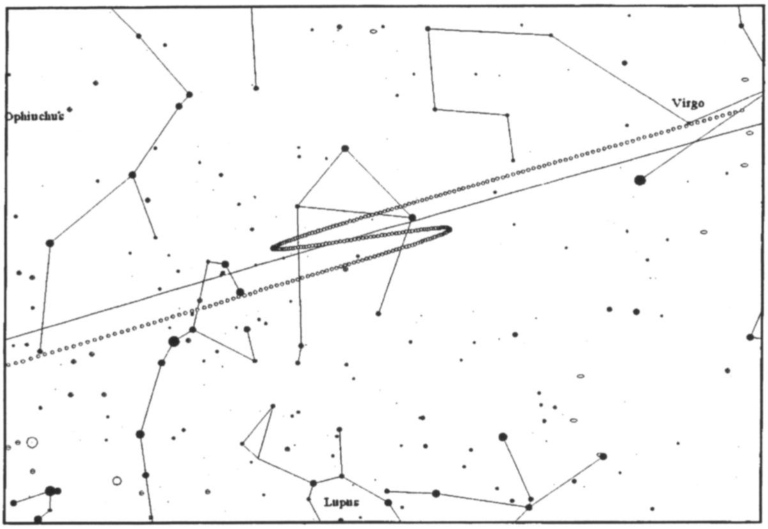

Fig. 40. Mars in Libra, 12/83-10/84; zigzag dimensions: 16°45′ X 3°56′

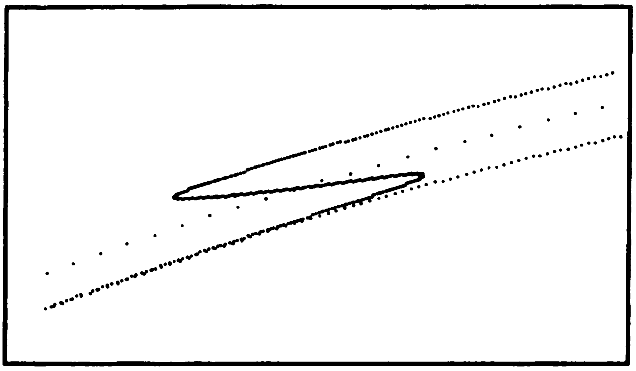

Fig. 41. Phase: 2°, 1st inclination: 17.25°, 2nd: 42.5°, 3rd: 25.25°

than two years later, another Martian retrogression might be observed, with the likely result of revealing the inadequacy of the previous choice of parameters. At this point, one might simply choose a new set of parameters, only to be frustrated again by the next retrograde phase. It takes systematic observations of Mars in retrogression for 15 years at the very least, and probably closer to twice as long, before any sense of the cyclical nature of these variations can begin to be perceived with any degree of confidence. There might not have been sufficient time for such data to be collected in Callippus's lifetime. However, the very existence of cinematic theories such as Eudoxus and Callippus created could provide a powerful incentive for systematic observations of planetary motion. That the earliest known records of such observations in Greece date back to the beginning of the 3rd century BC is again in accordance with the fact that Callippus flourished in the second half of the 4th century BC.

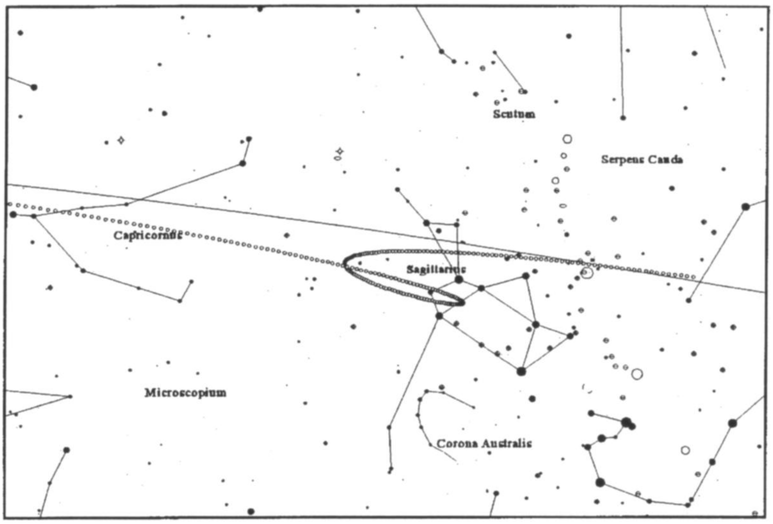

Fig. 42. Mars in Sagittarius, 3/86-11/86; loop dimensions: 11°47′ X 1°11′

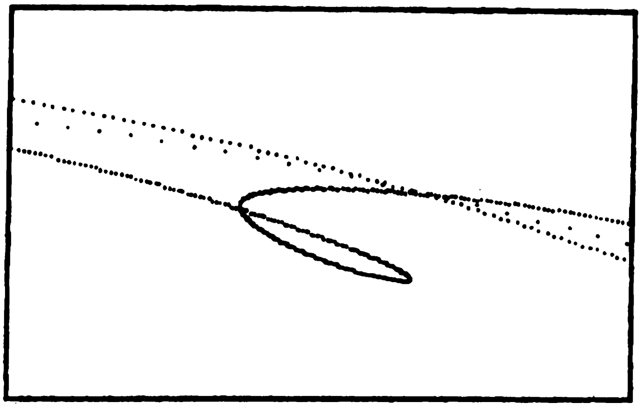

Fig. 43. Phase: 1.4°, 1st inclination: 15.5°, 2nd: 38°, 3rd: 27°.

Paul Tannery found it difficult to believe that with their cinematic theories explicitly formulated, none of Eudoxus's immediate followers was moved to observe the motions of the planets. He conjectured that no record of their observations survived because they were perceived as lecture experiments designed to demonstrate a theory rather than as research experiments designed for the critical evaluation of a theory.[49] Regardless of

[49] "On pourrait les comparer à des expériences de physique d'amphithéâtre, non pas à des recherches de laboratoire." Paul Tannery, *Recherches Sur L'Histoire De L'Astronomie Ancienne*, (Paris: Gauthier-Villars & Fils, 1893, Reprinted by Arno Press, New York, 1976), p. 45. Heath viewed things differently: "Whether Callippus actually arranged his additional spheres in the way suggested by Schiaparelli or not, the improvements which he made were doubtless of the nature

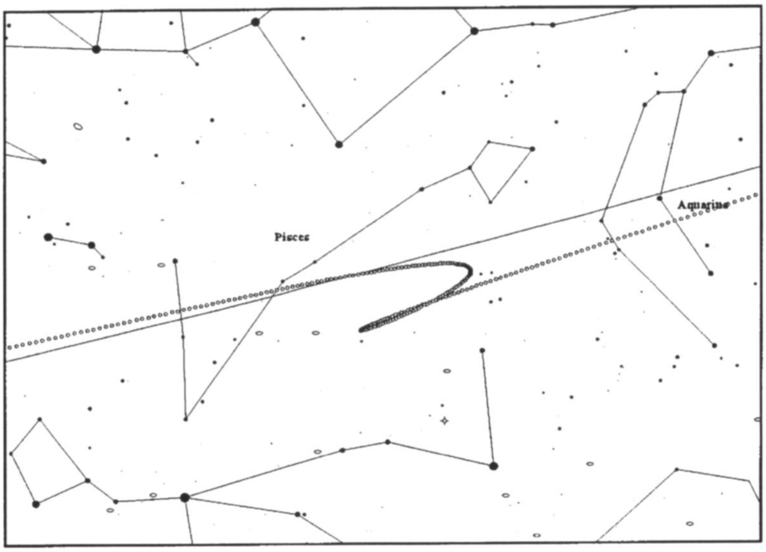

Fig. 44. Mars in Pisces, 5/88-2/89; loop dimensions: 11°47′ X 0°19′

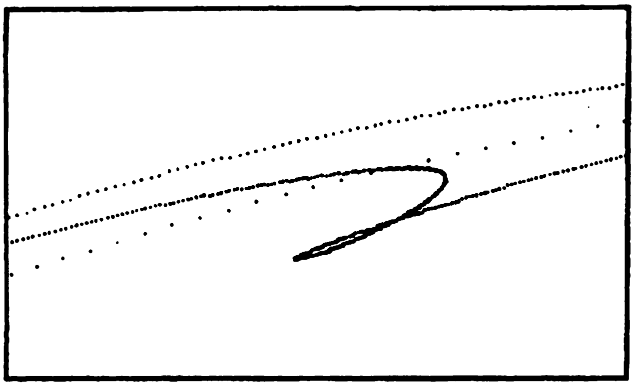

Fig. 45. Phase: $-1.9°$, 1st inclination: 16°, 2nd: 38.5°, 3rd: 26°.

why certain records did or did not survive, there is nothing to prevent what might have started as a demonstration experiment from turning into a critical comparison of theory with observation. Furthermore, Tannery's conjecture notwithstanding, nothing that we know forbids the possibility that Eudoxus, Callippus, or any of their contemporaries, purposefully turned their attention to planetary retrograde motion for the purpose of assessing the value of their most recent theories on the subject. All of the alternative homocentric reconstructions of Eudoxus and Callippus's work that have been discussed

indicated above; and his motive was that of better 'saving the phenomena', his comparison of the theory of Eudoxus with the results of actual observation having revealed differences sufficiently pronounced to necessitate a remodeling of the theory." (Heath, *Aristarchus of Samos*, p. 216).

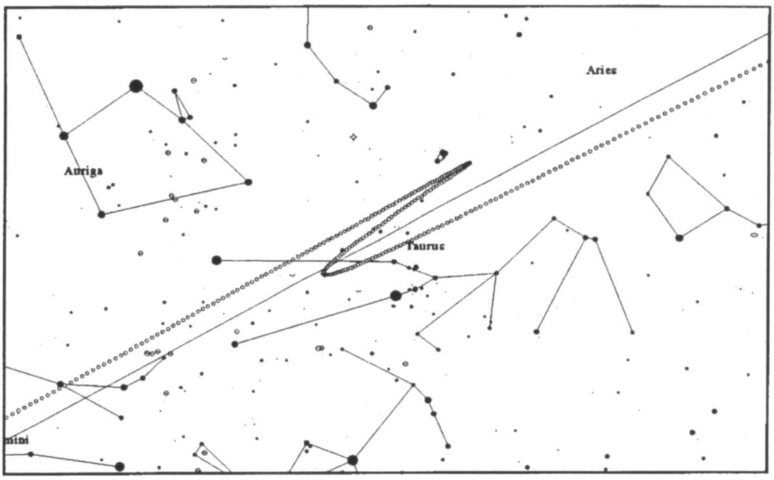

Fig. 46. Mars in Taurus, 6/90-5/91; zigzag dimensions: 16°52′ X 3°48′

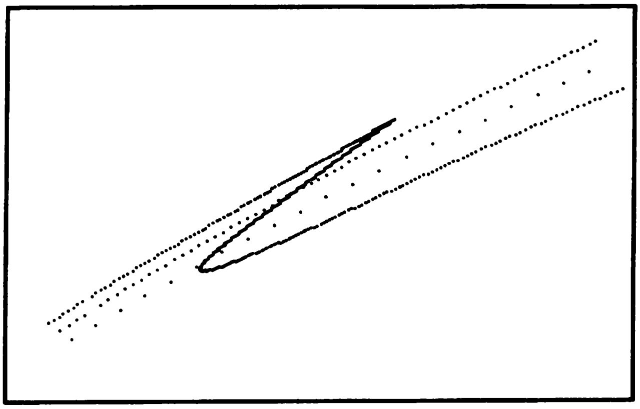

Fig. 47. Phase: −1.7°, 1st inclination: 17°, 2nd: 43°, 3°: 24.75°

in this paper (Schiaparelli's included) are consistent with these possibilities. However, they differ in the degree of harmony between theory and observation that they entail. Unfortunately, without additional information it does not seem possible to decide how Eudoxus and his followers regarded the relationship between astronomical theory and astronomical observations.

h. Conclusions and Conjectures

In general, expositions of the Eudoxian system in the current secondary literature reflect Heath's assessment that Schiaparelli's interpretation "... will no doubt be accepted by all future historians (in the absence of the discovery of fresh original documents) as

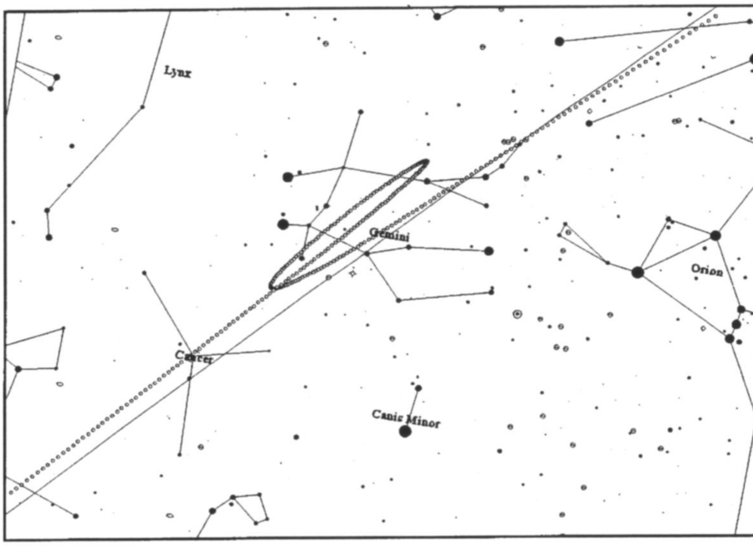

Fig. 48. Mars in Gemini, 8/92-6/93; loop dimensions: 18°48′ X 1°27′

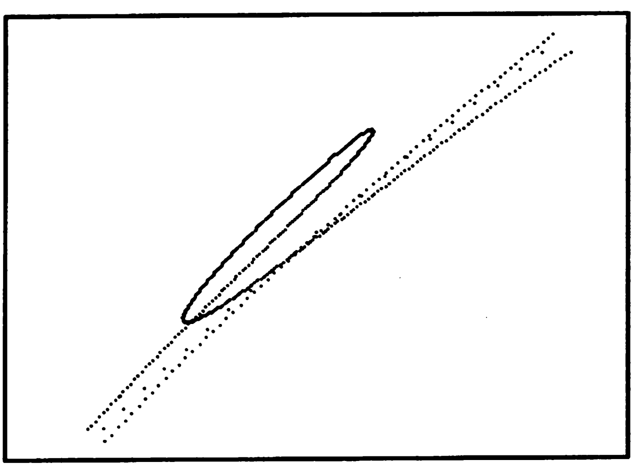

Fig. 49. Phase: −0.8°, 1st inclination: 17°, 2nd: 44.5°, 3°: 24.75°

the authoritative and final exposition of the system."[50] This exclusive identification of the Eudoxian homocentric scheme with Schiaparelli's reconstruction seems unusually specific and ambiguity-free compared to our uncertain knowledge of other aspects of

[50] Heath, *Aristarchus of Samos*, p. 194. An exception to this rule is D.R. Dicks's measured observation that "Schiaparelli's reconstruction (which is, of course, hypothetical, since the geometrical details are nowhere given in the ancient sources) has been generally accepted, and the accounts of Dreyer and Heath are both closely based on it." (*Early Greek Astronomy to Aristotle*, [London: Thames and Hudson, 1970], p. 177).

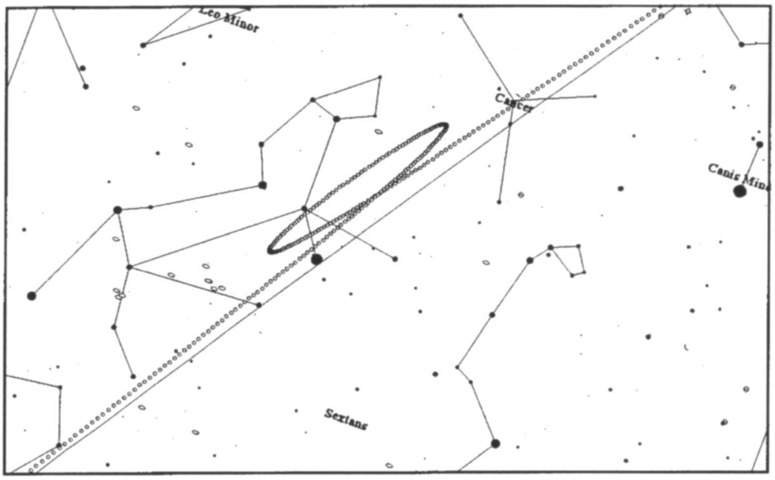

Fig. 50. Mars in Leo, 9/94-8/95; loop dimensions: 19°22′ X 2°31′

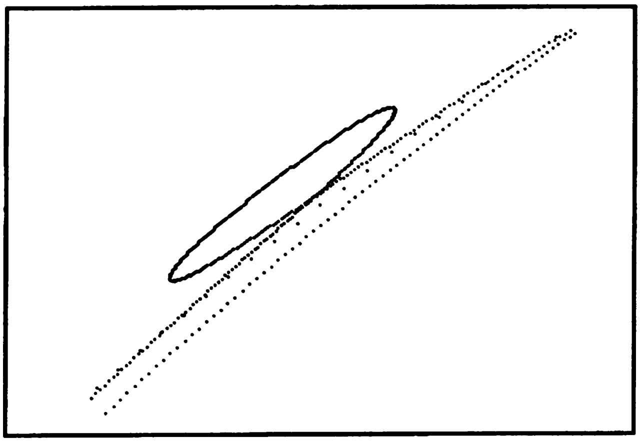

Fig. 51. Phase 0.1°, 1st inclination: 17°, 2nd: 45°, 3rd: 25°

early Greek astronomical thought. By contrast, this study suggests that unless further evidence can be brought to bear on the subject, our knowledge of Eudoxian astronomy is not as unusual as might be gathered from Schiaparelli's reconstruction. Aristotle's text is equally interpretable either according to Schiaparelli or according to the general version of the alternative view described here. Accommodation of both Simplicius's and Aristotle's texts does call for a further restriction on the alternative interpretation of Aristotle's text, but it does not require a full reversion to Schiaparelli's interpretation. Furthermore, there is no a-priori requirement that all aspects of both texts need to be satisfied. In fact, Schiaparelli himself found it necessary to correct Simplicius on some points of Eudoxus's system. At present we have no hard evidence, direct or indirect, that enables us to make a sound choice between the various alternatives. Such a choice would rely too heavily on an unduly restricted interpretation of the single word "hippopede"

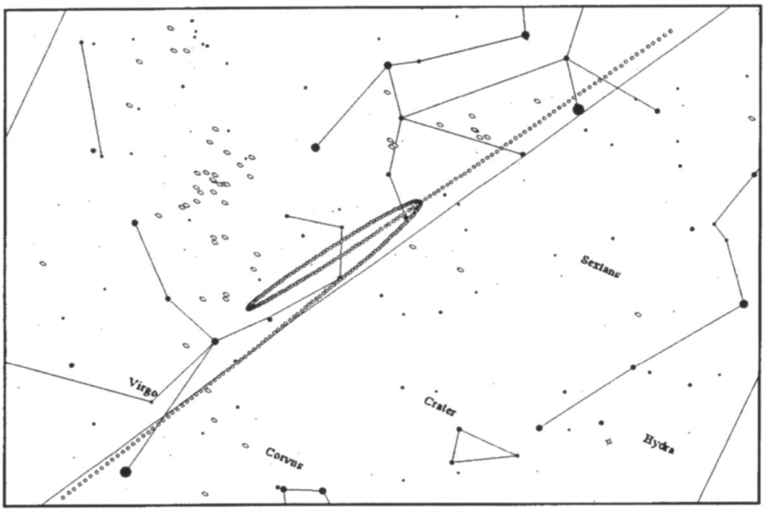

Fig. 52. Mars in Virgo, 10/96-8/97; loop dimensions: 19°14′ X 1°11′

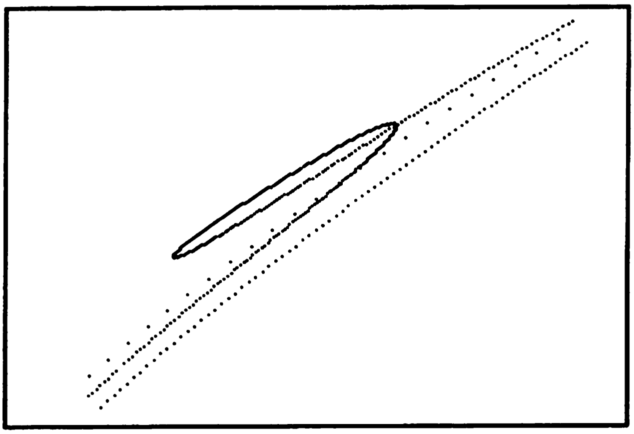

Fig. 53. Phase: 0.9°, 1st inclination: 17.25°, 2nd: 44.75°, 3rd: 25°

written 900 years after the fact, in a second-hand account by a writer who proved himself less than reliable in the very same context. Of course, this does not give license to pick and choose from Simplicius's text at will without historical justification, but this applies with equal force to Schiaparelli's reconstruction, as well as the new alternatives that have been developed here. In the final analysis, given our present state of knowledge, we may only advance the alternative interpretations of Aristotle's testimony as possibilities to be considered alongside Schiaparelli's reconstruction, not as substitutes for it. With the paucity of information that characterizes the history of Greek astronomy in the 4th century BC, it is practically impossible not to have several possible interpretive orientations, and it would appear that suspension of judgement is the better part of valor.

We may have to be content with the understanding that spherically symmetrical astronomy could have been considerably more flexible in ancient Greece than Simplicius and Schiaparelli suggest. As such it may have encouraged a great deal of speculative study within its framework and in its terms. Indeed, in its general Aristotelian version, this spherical astronomy is capable of supplying the basic concepts with which later thinkers could break its homocentric constraints and lead to the epicycloidal astronomy of Ptolemy. The basic scheme illustrated in Figs. 4, 5, and 8 clearly indicates the deferent-epicycle arrangements that Apollonius began to study and that came to characterize Ptolemaic astronomy. Passage to the models of Ptolemy requires extrication of these superimposed rotations from the spherical surface, and embedding them in planes that are parallel, or nearly parallel to the plane of the ecliptic. This suffices to create the basic epicycloid evident in astronomy from Ptolemy to Copernicus. The alternative interpretation therefore suggests the possibility of a pre-existing geometrical motivation for this model and hence a natural conceptual affiliation between Eudoxus and Apollonius. The difficulties encountered by Eudoxus's system would thus not entail a complete overthrow of his geometrical tools, which with a relatively simple modification may have been found quite useful outside the general astronomical scheme that he wove around them.

Finally, both Schiaparelli's reconstruction and the alternative outlined in this paper are in keeping with the presently accepted limitations on the state of observational astronomy prior to the work of Timocharis in the early 3rd century BC, but they suggest different possible reconstructions of the evolution of Greek astronomy following Eudoxus. Under Schiaparelli's interpretation, the Eudoxian and Callippic models could not pass the test of rudimentary observations well within the capacity of Greek astronomers at the time. That such models were advanced just the same seems to indicate a relatively low level of interest in observational planetary astronomy, at least on the part of Eudoxus and Callippus. Schiaparelli's interpretation, then, depicts a wide gulf between theory and reality, and suggests a rather idealistic development of theory that was only marginally guided or corrected by observation. It inevitably leads to assessments such as the following from Pannekoek:

> The great Greek scientists were not observers, not astronomers, but keen thinkers and mathematicians. Eudoxus' theory of the homocentric spheres is memorable not as a lasting acquisition of astronomy but as a monument of mathematical ingenuity.[51]

The alternative Eudoxian model leads to a Callippic system that has the theoretical capability of reproducing a good likeness of any planetary retrograde loop. However, even if this was the scheme used by Eudoxus and Callippus, it is still possible that both they and their contemporaries remained unaware of its full potential. Indeed, we do not even know whether Eudoxus or Callippus attempted to extract specific planetary retrograde loops from their possible arrangements of homocentric spheres, let alone compare them with actual observations. Consequently, the superior inherent capability

[51] A. Pannekoek, *A History of Astronomy*, (London: George Allen & Unwin, LTD., 1961; New York: Dover, 1989), p. 111.

of the alternative model does not necessarily imply a finer observational knowledge of the loops traced by retrogressing planets, and Pannekoek's judgement could hold equally true under the alternative interpretation as under Schiaparelli's. However, unlike Schiaparelli's reconstruction the alternative one also makes possible – though not necessary – a more harmonious depiction of the coexistence of theory with observation. The observational practices required for such a coexistence are significantly less sophisticated than the precise dated observations of the 3rd century BC, but still sufficient for critically studying the primary astronomical phenomena that the theory was presumably designed to address, namely, planetary retrograde motion. Besides Pannekoek's view, then, the alternative interpretation also makes possible a more harmonious and gradual evolution of Greek theoretical and observational planetary astronomy from Eudoxus's and Callippus's studies of phenomena localized in small areas of the sky, to Ptolemy's global mapping relative to a standardized celestial grid. While our current state of knowledge makes it more probable that Eudoxus and Callippus did not engage in the sort of observations implied by this view, we do not possess sufficient evidence to reject it out of hand.

Appendix

a. Obtaining an analytical expression for the hippopede.

Throughout the discussion, the observer's coordinate system is defined with the z-axis pointing northward on the page, the y-axis pointing east, and the x-axis pointing up from the face of the page. Positive rotations around any axis are defined by the right hand rule. We first generate the rotation operators (this is just for completeness of discussion here, they may by found in almost any text on classical mechanics). Rotate by angle α round the z-axis. This yields:

$$z' = z; \quad x' = x\cos\alpha - y\sin\alpha; \quad y' = x\sin\alpha + y\cos\alpha$$

This may be represented by the matrix operation:

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where the 3×3 matrix represents the operator that rotates any point round the z-axis. It is easy to show by similar considerations that the operators for rotation around the y- and x-axes are respectively:

$$\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

Using these, we create the hippopede in three steps:

1) Turn a planet resting at (1,0,0) by α round the z-axis:

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix}$$

2) Turn the result by γ around the x-axis to create the inclination between the poles of the outer and inner spheres. This will complete the function of the inner sphere:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \cos\gamma\sin\alpha \\ \sin\gamma\sin\alpha \end{bmatrix}$$

3) We now have to turn the outer sphere, namely the result of the first two operations, by an angle $-\alpha$ around the z-axis:

$$\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \cos\gamma\sin\alpha \\ \sin\gamma\sin\alpha \end{bmatrix} = \begin{bmatrix} \cos^2\alpha + \cos\gamma\sin^2\alpha \\ (\cos\gamma - 1)\sin\alpha\cos\alpha \\ \sin\gamma\sin\alpha \end{bmatrix} \tag{1}$$

The right side of Eq. (1) will generate the hippopede when α is allowed to range from 0° to 360°. The size of the hippopede is determined by γ. One can see that the relationship between the x and z coordinates is:

$$x = 1 + \frac{(\cos\gamma - 1)}{\sin^2\gamma} z^2,$$

which defines a parabola in the $x-z$ plane, with the x coordinate as the axis of symmetry. If a parabolic sheet is created by extruding this parabola along the y-axis, then the hippopede is formed by the intersection of this sheet and a sphere.

b. Derivation of the trace left on a sphere by the alternative scheme

Consider a planet fixed on latitude δ (measured from the pole) of a sphere. Fix this sphere inside another sphere, whose own pole is inclined to the pole of the planet carrier by an angle γ. Spin the two spheres in opposite directions at equal speeds and observe the planet's motion. It is equivalent to the combined motion of an epicycle of angular size 2δ on a deferent of angular size 2γ lying on a spherical surface, and turning in opposite senses at equal speeds. The basic procedure is to tilt the vector (1, 0, 0) by δ around the y-axis, then turn it 2α round the x-axis, then tilt the whole thing a further γ round the y-axis, and finally turn by -α around the x-axis. The outcome results in the γ-tilted center of 2α rotation being rotated around the x-axis by $-\alpha$, while the net rotation around this tilted pole is reduced to α. So we follow four distinct steps:

1) Rotate the vector $(1, 0, 0)$ by δ round the y-axis, to create the rotating radius vector that will create the epicycle:

$$\begin{bmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\delta \\ 0 \\ -\sin\delta \end{bmatrix}$$

2) Turn this by 2α round the x-axis to start the epicyclic component:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix} \begin{bmatrix} \cos\delta \\ 0 \\ -\sin\delta \end{bmatrix} = \begin{bmatrix} \cos\delta \\ \sin 2\alpha \sin\delta \\ -\cos 2\alpha \sin\delta \end{bmatrix}$$

(Note that rotating twice by α gives the same result, as it must if the operators are to represent rotations properly).

3) Tilt by γ round the y-axis to place the epicycle on the circumference of a deferent:

$$\begin{bmatrix} \cos\gamma & 0 & \sin\gamma \\ 0 & 1 & 0 \\ -\sin\gamma & 0 & \cos\gamma \end{bmatrix} \begin{bmatrix} \cos\delta \\ \sin 2\alpha \sin\delta \\ -\cos 2\alpha \sin\delta \end{bmatrix} = \begin{bmatrix} \cos\gamma\cos\delta - \sin\gamma\cos 2\alpha \sin\delta \\ \sin 2\alpha \sin\delta \\ -\sin\gamma\cos\delta - \cos\gamma\cos 2\alpha\sin\delta \end{bmatrix}$$

4) Turn by $-\alpha$ around the x-axis to move the center of the epicycle along the deferent, and reduce the epicycle's rotation to α:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\gamma\cos\delta - \sin\gamma\cos 2\alpha \sin\delta \\ \sin 2\alpha \sin\delta \\ -\sin\gamma\cos\delta - \cos\gamma\cos 2\alpha\sin\delta \end{bmatrix} =$$

$$\begin{bmatrix} \cos\gamma\cos\delta - \sin\gamma\cos 2\alpha\sin\delta \\ \cos\alpha\sin 2\alpha\sin\delta - \sin\alpha\sin\gamma\cos\delta - \sin\alpha\cos\gamma\cos 2\alpha\sin\delta \\ -\sin\alpha\sin 2\alpha\sin\delta - \cos\alpha\sin\gamma\cos\delta - \cos\alpha\cos\gamma\cos 2\alpha\sin\delta \end{bmatrix}$$

This yields the observer's coordinates for the resulting motion.

More generally, the position of a point on a given latitude θ measured relative to the x-axis of a sphere that rotates α degrees about the x-axis may be specified by two matrix operators as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

To superimpose another spherical rotation on the above, apply another pair of matrix operators to the vector (x, y, z) with a new angle of inclination and a new rotation. This may be repeated any number of times, with different inclinations and different rotations. In the case of Callippus, we need three such operator pairs. The three angles of inclination must be chosen so as to create a retrograde-generating curve of the appropriate length and width, while the rotational angles must be α for the third sphere, -2α for the fourth, and 2α for the fifth. Throughout the operation, the coordinate system is fixed, and only the vectors are rotated. The coordinate system is chosen so that the x-axis points directly outward from the screen, the y-axis points horizontally to the right of the screen, and the z-axis points vertically up. The order, symbolically, is: $[\alpha+\phi]_x[\theta]_y[-2\alpha]_x[\delta]_y[2\alpha]_x[\gamma]_y$ $\mathbf{e_x}$. Here $\mathbf{e_x}$ is a unit vector in the x direction, α is the running variable, ϕ the phase angle (the angle between the long axis of the retrograde generator and the ecliptic), γ is the latitude (measured from the pole, not the equator) of the planet on the innermost (5^{th} overall) sphere, δ is the inclination of the 5^{th} sphere relative to the 4^{th}, θ is the inclination of the 4^{th} relative to the 3^{rd}, and the poles of the third sphere, as always, are fixed to the ecliptic, being the equator of the 2nd sphere. The subscripts outside the brackets signify the axis of rotation, so that $[\gamma]_y\, \mathbf{e_x}$ signifies a counter-clockwise rotation of (1,0,0) by γ around the y-axis. To complete the picture, one more rotation around the y-axis is required to create the mean ecliptic motion of the planet, say $[\beta]_y$. Both β and α are running variables, such that $\Delta\alpha/\Delta\beta$= (synodic angular speed)/(mean ecliptic angular speed) = (mean ecliptic period)/(synodic period) for the planet under consideration (about 0.88 in the case of Mars).

For instance, the curve in Fig. 34 was created by placing the planet on latitude 25° of the innermost sphere, which rotates about its axis at a speed of 2α. The innermost sphere's axis is inclined by 45° to the axis of the sphere above it, which rotates at a speed of -2α, and which is itself inclined by 17° relative to the axis of the next higher sphere which rotates at a speed of α. This last axis is attached to the ecliptic, and carried around it at the mean ecliptic period of the planet. To see how this creates the resulting curve, consider first that the axis of the innermost sphere is moving on the alternative Eudoxian curve created by placing a planet on latitude 45° of the Eudoxian 4th sphere whose axis is inclined by 17° relative to the Eudoxian 3rd sphere. This creates a curve with a 62° semi-major axis, and a 28°semi-minor. In the Callippean scheme, the planet is rotating on latitude 25° measured from a pole that follows the Eudoxian path. This results in a curve with an 87° semi-major axis, and a 3° semi-minor axis, which is reproduced in Fig. 34. In this figure the retrograde loop is situated to the left of the ecliptic. Planets sometimes retrogress to the left, and sometimes to the right of the ecliptic. Reflecting the retrograde loop in Fig. 35 to the right of the ecliptic requires reversing the direction of motion along the retrogression generator in Fig. 34. To obtain this effect, use inclinations of 20°, 42°, and 25°. This places the planet on latitude 25° of a spinning sphere whose pole travels along a Eudoxian generator with a same semi-major axis of 62° as before, and a semi-minor axis of 22°, as opposed to 28° before. Because the planet's latitude on the innermost sphere is now 3° larger than the difference between the 1st and 2nd inclinations instead of 3° smaller, the left and right sides of the resulting retrogression generator are switched, which has the effect of reversing the direction of motion. As Figs. 6 and 7 show, the reversed generator is not exactly congruent with the generator in Fig. 34, despite having the same semi-major and semi-minor axes (87° and 3° respectively). However, since the angular size differences between the inclinations that create the two versions are small, the divergence of form is negligible in application to the creation of Martian retrograde loops at the relevant level of accuracy.[52]

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