Measurement of Strong Galvanic Currents According to Absolute Measure

Wilhelm Weber

Editor's Note: An English translation of Wilhelm Weber's 1842 paper "Messung starker galvanischer Ströme nach absolutem Maasse", [Web42b].

Posted in October 2020 at www.ifi.unicamp.br/~assis

By Wilhelm Weber[1,2,3]

The instrument which is to be used for this measurement is set up in such a way that the current remains noticeably the same, it may or may not be passed through the instrument for the purpose of the measurement. This important point of measuring the current without decreasing it, when it is used, is achieved by making the resistance of the measuring device negligible compared to the remaining resistance of the circuit.

The instrument therefore consists, as Figure 1 shows, of a single thick copper ring,[4] which is set up in the plane of the magnetic meridian, and in its axis a small magnetic needle is located (the length of which is only about the fourth part of the ring diameter). The supply and removal of the current[5] is set up in such a way that only the current which goes through the ring can act on the needle, as can be easily understood from the illustration in Figures 1, 2, 3 and 4.[6]

[1] [Web42b].

[2] Translated by H. Härtel, haertel@astrophysik.uni-kiel.de and http://www.astrophysik.uni-kiel.de/~hhaertel/index_e.htm. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis. We thank R. W. Gray for relevant suggestions.

[3] The notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:]; the notes by W. E. Weber are represented by [Note by WEW:]; while the notes by A. K. T. Assis are represented by [Note by AKTA:].

[4] [Note by AKTA:] In the original: *aus einem einzigen starken Kupferrringe*.

[5] [Note by AKTA:] In the original: *Die Zuleitung und Ableitung des Stroms*.

[6] [Note by AKTA:] Further details of this instrument and its utilization can be found in [Web41] with English translation in [Web20].

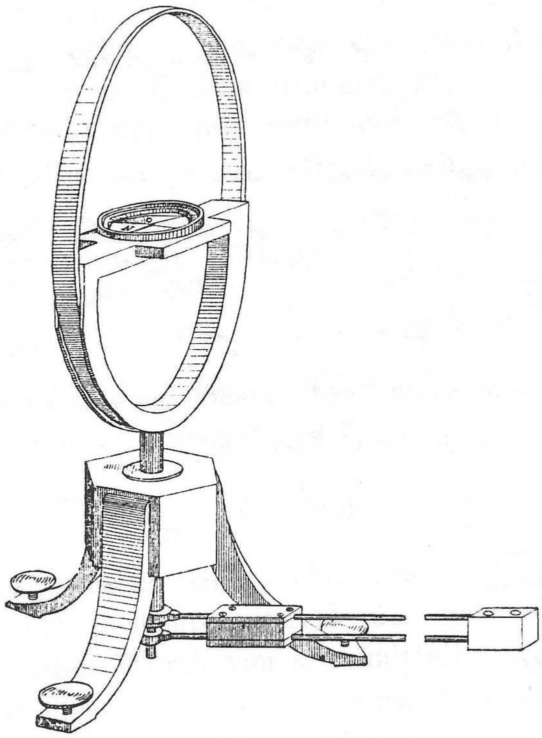


Fig. 1.

Fig. 2.

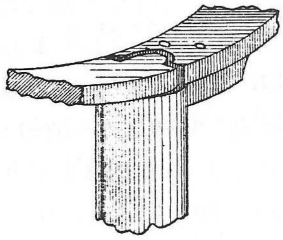


Fig. 3.

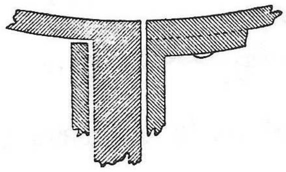
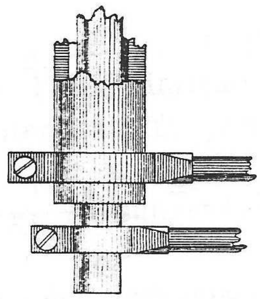


Fig. 4.



This setup of the instrument does not need any further explanation, since it essentially corresponds to the setup of a *tangent galvanometer*,[7] as it has already been used frequently. It should therefore only be shown in more detail how one can use it to determine the strength of a galvanic current according to *absolute measure*, which is easily accomplished if one assumes as known the Gaussian method of measuring magnetism according to absolute measure (see Ann., Vol. 28, pp. 241, 591).[8]

[7][Note by AKTA:] *Tangenten-Boussole* in the German original text.

[8][Note by AKTA:] Weber is quoting here Gauss's work on the intensity of the Earth's magnetic force reduced to absolute measure, [Gau33]. It was announced at the Königlichen Societät der Wissenschaften zu Göttingen in December 1832, [Gau32]. The original paper in Latin was published only in 1841, although a preprint appeared already in 1833 in small edition, [Gau41a] and [Rei19]. Several translations have been published. There are two German versions, one by J. C. Poggendorff in 1833 and another one in 1894 translated by A. Kiel with notes by E. Dorn; a French version by Arago in 1834; two Russian versions, one by A. N. Drašusov of 1836 and another one by A. N. Krylov in 1952; an Italian version by P. Frisiani in 1837; an English extract was published in 1935, while a complete English translation by S. P. Johnson and edited by L. Hecht appeared in 1995; and a Portuguese version by A. K. T. Assis in 2003: [Gau33], [Gau34], [Gau36], [Gau37], [Gau94], [Gau35],

Just like the *moment of a magnet*, the *moment of a closed galvanic circuit* can also be measured in *absolute units* (if the Earth's magnetism is known) from the deflection of a magnetic needle from the magnetic meridian caused by the current. Essentially the same rules must be observed here as there in order to obtain a safe and precise result.

If the *moment of a magnet* is to be measured, the deflection of a needle is observed at two different distances from the magnet. It is assumed that the magnet is always positioned in the horizontal plane of the needle and perpendicular to the magnetic meridian, and that its extended axis meets the centre of the needle. The deflection should be observed 4 times at every distance, where the magnet should be positioned now to the East, now to the West of the needle and its North pole should turn now to the East, now to the West. Let the mean values of the deflection found for the distances R and R' be v and v'. Now put:

$$\tan v = \frac{L}{R^3} + \frac{L'}{R^5} ,$$

$$\tan v' = \frac{L}{R'^3} + \frac{L'}{R'^5} .$$

This can be accepted if R and R' are so large compared to the length of the magnet and the needle, that the terms of the series which contain the 7th or higher order of R and R' can be neglected. By eliminating L' one obtains from this:

$$L = \frac{R^5 \tan v - R'^5 \tan v'}{R^2 - R'^2} ,$$

where v, v', R and R' are known by measurements. The theory has proven that the following relation exists between the calculated value of L and the desired moment M of the magnet:

$$L = \frac{2M}{T} \qquad \text{or} \qquad M = \frac{1}{2} LT ,$$

where T denotes the horizontal intensity of the Earth's magnetism in absolute measure.

This method, presumed to be known, of measuring the *moment of a bar magnet* according to absolute measure, could be applied directly to the measurement of the *moment of a closed galvanic circuit*, if this complete

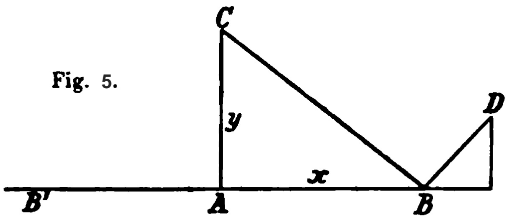
[Gau52], [Gau75], [Gau95] and [Ass03].

circuit did not occupy a larger space than the magnet, and thereby would cause at the same distance an equally large deflection of the needle. But since these two conditions cannot be met at the same time, the following modification of the method can be accepted when it is applied to galvanic circuits.

The galvanic current is passed through a large and thick copper ring positioned in the plane of the magnetic meridian. The supply of the current to the ring is done through a long and thick copper rod, the discharge of the current is through a copper tube that surrounds the rod without touching it. The magnetic needle is set up in such a way that it is at equal distance from all parts of the ring; the centre of the needle lies in the axis of the ring either at its centre or near to it, so that the current flows almost all the way around the needle.

Let the point A in Figure 5 be the centre of the ring, AB its axis, $AC = y$ its radius; the intensity of the current is called g.

Fig. 5.



Let a North magnetic element μ be located on the axis at the distance $AB = x$ from the centre. If the current g passes through the ring element $yd\varphi$ at point C (from back to front in the Figure), μ will be moved from B to D and this motion will be perpendicular to that plane through B and through the ring element at C. The magnitude of this moving force is directly proportional to the product $g\mu yd\varphi$ and inversely proportional to the square $(x^2 + y^2)$ of the distance CB,[9] or it can be expressed through

$$\frac{fg\mu yd\varphi}{x^2 + y^2} ,$$

where f denotes a constant factor. If one decomposes this force BD along the direction of the ring axis by multiplying that value by $y/\sqrt{x^2 + y^2}$, which gives $fg\mu y^2 d\varphi/(x^2 + y^2)^{3/2}$, the resultant of the force is obtained with which all elements $yd\varphi$ of the circular current try to move the element μ in the direction of the axis, [namely:]

[9] [Note by WEW:] The sine of the angle that CB makes with the direction of the ring element in C should be added as a factor, which in our case is equal to 1 because that angle is a right one.

$$= \frac{2\pi f g \mu y^2}{(x^2+y^2)^{3/2}} \; .$$

The forces perpendicular to the direction of the axis cancel each other.

If one compares this force with that which an infinitely small *magnet*, whose axis coincides with the direction AB and whose moment is M, at the distance $CB = \sqrt{x^2+y^2}$ from the element μ located in B, would exert, [namely:]

$$= \frac{2M\mu}{(x^2+y^2)^{3/2}} \; ,$$

(see Gauss in the *Resultaten des magnetischen Vereins für das Jahr 1840*, p. 26 and the following, and the article: "Bemerkungen über die Wirkungen eines Magnetes in die Ferne" (Remarks on the actions of a magnet at a distance),[10,11] so you can see that both expressions become identical for

$$M = \pi f g y^2 \; .$$

If, in analogy with the magnetic moment, $\pi f g y^2$ is called the *moment of the galvanic circular current*, and denoting it with G, then G can be determined by deflection experiments according to absolute measure, just like M. If one denotes by u the observed deflection[12] of a magnetic needle in A, with u' the observed mean deflection of the same in B and B' (where $B'A = BA$), and sets $y = R$, $\sqrt{x^2+y^2} = R'$, it follows in a similar way:

$$\tan u = \frac{L}{R^3} + \frac{L'}{R^5} \; ,$$

$$\tan u' = \frac{L}{R'^3} + \frac{L'}{R'^5} \; .$$

It follows by elimination of L':

$$L = \frac{R^5 \tan u - R'^5 \tan u'}{R^2 - R'^2} = \frac{2G}{T} \; ,$$

[10] [Note by HW:] Gauss' *Werke*, Vol. V, p. 427 and Wilhelm Weber's *Werke*, Vol. II, p. 242.

[11] [Note by AKTA:] [Gau41b, pp. 26 and the following of the *Resultate* and pp. 427 and the following of Gauss' *Werke*] and [Web42a].

[12] [Note by WEW:] It is assumed here that each deflection observation is repeated after the current in the ring has been reversed.

or

$$G = \frac{1}{2} \frac{R^5 \tan u - R'^5 \tan u'}{R^2 - R'^2} \cdot T = \pi f g R^2 \ .$$

From this one finally finds the searched *intensity* g of the current when one determines the unit on which its determination is based. If one takes that current intensity as the unit,[13] *whereby the current, when it circulates in the plane the unit of an area, exerts the same action at a distance as the unit of free magnetism*, then the indeterminate factor f is determined, because then at the same time the intensity $g = 1$, the moment $G = 1$ and the area $\pi R^2 = 1$, from which the value of f results:

$$f = 1 \ ;$$

it follows

$$g = \frac{LT}{2\pi R^2} \ ,$$

where L can be calculated from the measured quantities u, u', R, R'.

This determination of the absolute intensity of the galvanic current becomes even easier if the length of the needle can be neglected compared to the diameter of the circle, because one can then restrict oneself to the first term in the series expansion for $\tan u$:

$$\tan u = \frac{L}{R^3} \qquad \text{or} \qquad L = R^3 \tan u \ .$$

One then only needs to measure the deflection u when the needle is in the center of the circle and then get

$$g = \frac{1}{2\pi} R T \tan u \ .$$

This approximation formula can still be regarded as sufficient even for measurements of higher quality, if the length of the needle does not exceed the fourth or fifth part of the diameter, as one can convince oneself if one carries out the observations completely, as stated above, and then compares the results of the approximation formula with the result of the more precise calculation.

The accuracy of the result finally depends on the accuracy with which the deflection u is measured. If the error du is made in this measurement,

[13] [Note by AKTA:] That is, $g = 1$.

an error is caused in the current intensity calculated from it, which is in parts of the total intensity $= 2du/\sin 2u$. This error has a minimum for $u = 45°$. From this follows the rule for the construction of the instrument that the copper ring is of the most advantageous size when the current to be measured produces a deflection of 45°, which is the case only with strong currents.

References

[Ass03] A. K. T. Assis. Tradução de uma obra de Gauss. *Revista Brasileira de Ensino de Física*, 25:226–249, 2003. Paper in Portuguese: "Translation of a work by Gauss".

[Gau32] C. F. Gauss. Anzeige der "Intensitas Vis Magneticae Terrestris ad Mensuram Absolutam Revocata". *Göttingische gelehrte Anzeigen*, 205-207:2041–2058, 1832. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 293-304 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867). Corrected version in: Astronomische Nachrichten 10, 1833, Nr. 238, col. 349-360.

[Gau33] C. F. Gauss. Die Intensität der erdmagnetischen Kraft, zurückgeführt auf absolutes Maass. *Annalen der Physik und Chemie*, 28:241–273 and 591–615, 1833. Translated by J. C. Poggendorff.

[Gau34] C. F. Gauss. Mesure absolue de l'Intensité du Magnétisme terrestre. *Annales de Chimie et de Physique*, 57:5–69, 1834.

[Gau36] C. F. Gauss, 1836. Ob ismĕrenii zemnago magnitizma. (Soč[inenie] Karl[a] Frid[richa] Gaussa). Per[evël] A. Drašusov. Učenyja zapiski Imperatorskago universiteta čast 11, 1836, Nr. 7 (Januar), str. 3-22; Nr. 8 (Februar), str. 246-271; Nr. 9 (März), str. 341-381.

[Gau37] C. F. Gauss. Misura assoluta dell' intensità della forza magnetica terrestre. *Effemeridi astronomiche di Milano*, primo supplemento:3–132, 1837. Tradotta e commentata da P. Frisiani.

[Gau41a] C. F. Gauss. Intensitas vis magneticae terrestris ad mensuram absolutam revocata. *Commentationes Societatis Regiae Scientiarum Goettingensis Recentiores*, 8:3–44, 1841. Delivered before the Society in 15 December 1832. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 79-118 (Königliche Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau41b] C. F. Gauss. Vorschriften zur Berechnung der magnetischen Wirkung, welche ein Magnetstab in der Ferne ausübt. In *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, volume V, chapter II, pages 26–34. Weidmannschen Buchhandlung, Leipzig, 1841. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 427-435 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau94] C. F. Gauss. Die Intensität der erdmagnetischen Kraft auf absolutes Maass zurückgeführt. In E. Dorn, editor, *Ostwald's Klassiker der exakten Wissenschaften, Vol. 53*. Wilhelm Engelmann Verlag, Leipzig, 1894. Translation by A. Kiel, notes by E. Dorn. Reprinted in: *Carl Friedrich Gauss — Werke — Supplement* — Band 3: Varia: 15 Abhandlungen in deutscher Übersetzung. Mit einer Einleitung, einer Bibliographie und Registern herausgegeben von Karin Reich. Hrsg. von der Akademie der Wissenschaften zu Göttingen, Universitätsverlag Göttingen 2019, S. 533-594.

[Gau35] C. F. Gauss. The absolute measure of magnetic force. In W. F. Magie, editor, *A Source Book in Physics*, pages 519–524, New York, 1935. McGraw-Hill.

[Gau52] C. F. Gauss, 1952. Intensivnost semnoj magnitnoj sily, privedennaja k absoljutnoj mere. In: B. M. Janovskij (Hrsg): Karl Fridrich Gauss. Izbrannye trudy po zemnomu magnetizmu. Perevod A. N. Krylova. Moskau 1952, str. 23-75.

[Gau75] C. F. Gauss. Letter from Gauss to Schumacher, April 29, 1845. In *Carl Friedrisch Gauss, Werke, Ergänzungsreihe, Vol. V, Briefwechsel C. F. Gauss — H. C. Schumacher, Part 2*, pages 436–440. Georg Olms Verlag, Hildesheim, 1975.

[Gau95] C. F. Gauss. The intensity of the earth's magnetic force reduced to absolute measurement. Translated from the German by Susan P. Johnson, edited by L. Hecht. Available at http://21sci-tech.com/translation.html, 1995.

[Rei19] K. Reich, 2019. Die Intensität der erdmagnetischen Kraft: Nr. 15, pp. xxxix-xlii. In *Carl Friedrich Gauss — Werke — Supplement* — Band 3: Varia: 15 Abhandlungen in deutscher Übersetzung. Mit einer Einleitung, einer Bibliographie und Registern herausgegeben von Karin Reich. Hrsg. von der Akademie der Wissenschaften zu Göttingen, Universitätsverlag Göttingen 2019, S. 533-594.

[Web41] W. Weber. Messung starker galvanischer Ströme bei geringem Widerstande nach absolutem Maasse. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, volume V, chapter VIII, pages 83–90. Weidmannschen Buchhandlung, Leipzig, 1841. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 6-12.

[Web42a] W. Weber. Bemerkungen über die Wirkungen eines Magnetes in die Ferne. *Annalen der Physik und Chemie*, 55:33–42, 1842. Reprinted in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 242-249.

[Web42b] W. Weber. Messung starker galvanischer Ströme nach absolutem Maasse. *Annalen der Physik und Chemie*, 55:27–32, 1842. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 19-23.

[Web20] W. Weber, 2020. Measurement of strong galvanic currents with low resistance according to absolute measure. Posted in July 2020 at www.ifi.unicamp.br/~assis. Translated by H. Härtel. Edited by A. K. T. Assis.