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Enhancement of spatial thinking with Virtual Spaces 1.0

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ABSTRACT

Developing a software environment to enhance 3D geometric proficiency demands the consideration of theoretical views of the learning process. Simultaneously, this effort requires taking into account the range of tools that technology offers, as well as their limitations. In this paper, we report on the design of Virtual Spaces 1.0 software, a program that exercises the user's abilities to build spatial images and to manipulate them. This paper also reports on a study that aimed to assess whether those abilities affected achievements in the spatial thinking of 10th graders who worked with the software. Additionally, we investigated whether self-regulating questions can improve the effect of exercising with Virtual Spaces 1.0. The sample was 192 students, who were randomly assigned to four groups, two of which used Virtual Spaces 1.0 (Group 1 with virtual reality and self-regulating questions $N = 52$, Group 2 with virtual realty only $N = 52$) and the other two the non-Virtual Spaces 1.0 (Group 3 self-regulating questions only $N = 45$, Group 4 non-treatment group $N = 45$). The results suggest that spatial thinking was enhanced by exercising with Virtual Spaces 1.0 and asking self-regulating questions. In addition, it was found that the self-regulating questions make the use of virtual reality more efficient, and that the influence of self-regulating questions is especially manifested in tasks that make use of high order skills.

1. Introduction

In recent years, software environments have emerged that are designed to assist the teaching of geometry in schools, e.g., Cabri-Geometer (Laborde, 2000), and Geometer's SketchPad (Jackiw, 1995). These programs were successfully used in the teaching and learning of geometry because of an interactive style that allowed direct manipulation of geometric objects (Christou, Jones, Mousoulides, & Pittalis, 2006). However, at present, such use remains primarily restricted to the 2D drawing canvas on the computer screen (Christou et al., 2006, p. 168). Even in software environments designed to enhance the 3D effect, e.g., 3D-Lab (Hidaka, 1994) and 3DMath (Christou et al., 2006), students are still viewing flat shapes on the computer screen. It seems that this sort of 3D data presentation is not necessarily appropriate for dealing with the difficulties encountered in the teaching of 3-D geometry, where most students find it difficult to imagine simple rotations of objects (Pani, Chariker, Dawson, & Johnson, 2005).

In general, there are several reasons why students have difficulty understanding 3-D geometry: (1) The transition from drawing two-dimensional constructs to imagining and manipulating three-dimensional objects is neither natural nor easy (Guttiérez, 1996); (2) Students are unable to make accurate drawings of spatial objects; (3) Students lack a visual vocabulary pertaining to spatial geometry; (4) There is insufficient interaction with three-dimensional objects (O'Driscoll-Tole, 1998); and (5) There is a lack of attention paid to verbal processes involved in learning 3-D geometry (Battista, 1994). It is, therefore, reasonable to assume that the effect of interaction with two-dimensional objects in the classroom, such as drawing on the blackboard or in a notebook, clicking on a PC's screen, or watching three-dimensional objects, which the teacher brings to class, does not sufficiently enhance the student's ability to construct a spatial image and to manipulate it when trying to solve a problem in 3-D geometry (Garrity, 1998; Gurny, 2003).

The question arises as to what extent the technology of virtual reality (hereafter VR) can help students acquire proficiency in geometry that will help them cope with these difficulties. In this paper, we suggest that the advancement of spatial thinking, by using the software Virtual Spaces 1.0, may be an effective starting point.

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2. Related literature

2.1. Spatial thinking

According to Duval (1995, as cited in Jones, 1998, p. 32) proficiency in geometry can be advanced by three processes: visualisation processes (e.g., perception of spatial relations between two objects and perceptual constancy), construction processes (e.g., creation of spatial images and mental rotation), and reasoning processes (e.g., solving simple problems and exercises). These processes are basic elements of spatial thinking and the 3-D geometry curriculum in our schools (Yakimanskaya, 1991).

Moreover, spatial thinking is a vital constituent in Gibson's "ecological principle." According to this principle, human beings process spatial information indefinitely, consciously and unconsciously, from their immediate environment. This information acts as an instinctive trigger that activates imagination, curiosity and cognitive processes, e.g., spatial thinking (Elzer, Zellerfeld, & Beuthel, 1999). Some researchers assumed there is a "mental code" in human cognition that enables each human to process spatial information while interacting with his environment at each age (e.g., Cohen, 1985, p. 5; Olson, 1983, pp. 30–31). In the process of this interaction, spatial cognition as a component of human cognition develops.

Spatial cognition includes three elements (Liben as cited in Cohen, 1985, p. 6): (1) spatial products are the concrete results of the processing of spatial information (e.g., the spatial images of a cube); (2) spatial storage is information stored about space and relationships within space; and (3) spatial thinking is mechanisms and capabilities that help to realise cognitive events in which information processing takes place through interaction with the content of spatial storage and spatial products.

In light of the above, it may be assumed that spatial thinking is a mental process that realises the interaction between the mechanisms and the information contained in spatial cognition. In this process, one constructs spatial images and manipulates them to solve practical or theoretical problems. The question arises, "What is a spatial image?"

2.2. Spatial image

A spatial image, or "pictures in the mind" (Lavy & Shriki, 2006), are a central concept in teaching mathematics (e.g., Clausen-May, 2005, p. 3; Kynigos & Gavrilis, 2006; Lavy & Shriki, 2006). A spatial image is the end product of a mental process that uses various aspects of a concrete object (or objects) to create a picture of that object in our mind. This process depends heavily on the signs (visual givens) an individual perceives as he interacts with that object. From a semiotic point of view they determine the meaning he attaches to the image (Forrester, 2000, p. 4–5). The image created in our mind is similar to the object in reality. Although the transfer from the concrete object in reality to the symbol in our mind involves a process of abstraction, the construct created is structurally identical to the object perceived (Denis, 1991). An example of this is spatial place representation in human cognition (Liben, 1997, p. 62). Therefore, spatial images preserve the information of an object in a form accessible to cognitive processes, and to the realisation of simulations in which the individual can manipulate objects that are not in his/her immediate environment. Additionally, they function as cognitive activation patterns (Denis, 1991, pp. 8–9).

Among the theories that describe the creation of a spatial image, Bryant's (1992) theory maintains that representations created in our conscious are the combination of two kinds of input: sensory and verbal. These two kinds of input are processed by three mechanisms, which create a spatial representation in our conscious: the perception-sensory input mechanism, the verbal input mechanism, and a mechanism that places the representation in a coordinative system.

What is special in this theory is the emphasis placed on the role of these two kinds of input in the creation of the spatial image.

In the research literature (e.g., Denis 1991), we find mention of processes that are likely to advance the creation of spatial images, for example, the active study of a real object that is the model for a spatial image, transformation of the spatial image, and the process in which the individual is enabled to use spatial images to solve a problem in 3-D geometry. The question arises: what abilities are essential for advancing processes of this sort?

Lohman (1988, as cited in Christou et al., 2006, p. 2) proposed three major components of spatial abilities: *spatial orientation* describes the ability to imagine how a stimulus array will appear from another perspective; *spatial relations* is defined by the speed in manipulating simple visual patterns and the ability to mentally rotate a spatial object quickly and correctly; and *spatial visualisation* is the ability to comprehend imaginary movements in a three-dimensional space or the ability to manipulate objects in the imagination. With regard to 3-D geometry, Guttiérez (1996) considered visualisation a kind of reasoning activity, which uses visual and spatial elements, either imagined or real, to solve problems or prove properties. This kind of reasoning involves four main elements: mental images, external representations, processes of visualisation, and visualisation abilities (Guttiérez, 1996, pp. 7–10). It follows that students of 3-D geometry should acquire and improve a certain set of visualisation abilities to advance proficiency in the subject (Duval, 1995 as cited by Jones, 1998).

In the research literature we came upon a number of the prerequisite abilities (e.g., Arnold, 1984; Clements, 1998; Del Grande, 1987; Guttiérez, 1996; Christou et al., 2006): (a) "perceptual constancy," the ability to recognise that some properties of an object are independent of size, colour, texture, or position, (b) "mental rotation," the ability to produce dynamic mental images and visualise a configuration in movement, (c) "perception of spatial relationships," the ability to relate several objects, figures, and/or mental images to each other, or simultaneously to oneself, (d) "visual discrimination," the ability to compare several objects or mental images and to identify similarities and differences among them, and (e) "visual memory," the ability to imagine things in a side-by-side order. Moreover, if this final ability is missing, visual imagination cannot organise visual memory images (Arnold, 1984).

In light of the above, it can be assumed that the connection between the prerequisite abilities, the creation and manipulation of spatial images, and spatial thinking appears as follows (Fig. 1).

To verify whether it is possible to enhance spatial thinking with VR, as defined by Fig. 1, a software environment, Virtual Spaces 1.0, was designed. Its design was informed by the theories of constructivism, semiotics, and component display theory.

2.3. Theory and design

Constructivism is a theory that maintains there is a real world that we experience and its meaning is determined by each individual who produces it (Duffy & Jonassen, 1992, p. 3). There are two approaches for designing teaching processes based on this theory (Perkins, 1992).

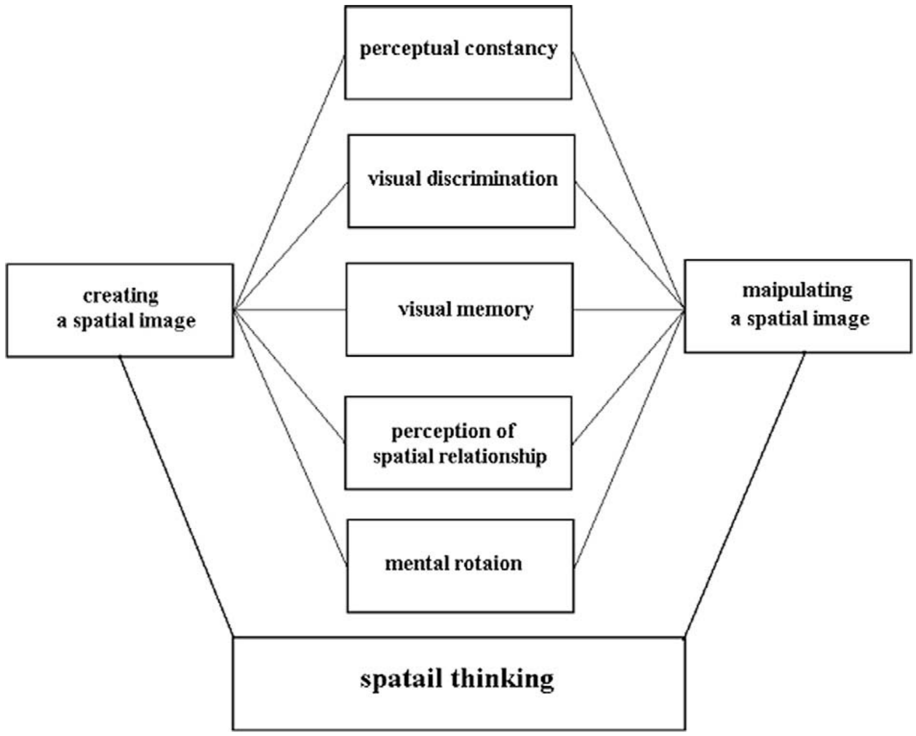

Fig. 1. Processes and variables affecting spatial thinking.

The first is WIG (Without Information Given). In this approach, the learner is given no information from the environment. He produces knowledge *solely* through his/her personal experimentation. The second is BIG (Beyond Information Given). According to this approach, the learner is presented with the central idea of a new topic, is given the basic knowledge necessary for different applications of the topic, and *only thereafter*, following a range of individual, thought-oriented activities, is it possible to advance the learner's retention, understanding and active use of knowledge skills (Perkins, 1992, p. 50).

Winn (1997) maintains that, in applying this theory to the field of learning with VR, one must take into account that simply turning students loose in a virtual learning environment, with the task of constructing understanding, is not likely to succeed, because learning experiences without appropriate feedback or regulation are liable to lead to misconceptions (Winn, 1997). Goo et al. (2006) provided evidence of this by showing that in VR learning, guidance seemed to affect visual attention and interest in searching for new objects/events, as opposed to unguided tasks.

Similarly, researchers (e.g., Azevedo, 2005) maintain that the learner in a software environment is *alone*, and that support for him at the point where he is having a hard time proceeding in the learning process is *critical* for the success of that process. Therefore a software environment can become a very powerful learning environment when *combined with* self-regulating questions (Azevedo, 2005).

Following this, we assumed that the adding of self-regulating questions and multiple forms of feedback would contribute to the improvement of the process of advancing spatial thinking, according to the BIG approach.

2.3.1. Self-regulating questions

The research literature points to a number of important aspects of the contribution of Self-Regulating Questions (SRQ, hereafter) to the learning process. First of all, self-regulating questions determine how students activate, alter, and sustain their learning (Zimmerman & Martinez-Pons, 1986), and it is a part of a self-directive process through which learners transform their mental abilities into task-related academic skills (Zimmerman, 2001, p. 1). Secondly, self-regulated students are active participants in their own learning process and take responsibility for, and initiate their own efforts to acquire skills and knowledge instead of depending on external sources. Thirdly, students who self-regulate learning were more aware of their mistakes and took steps to correct their deficiencies (Zimmerman, 2001, p. 1).

In addition, the SRQ is an important tool for focusing the learner's attention on those aspects of knowledge relevant to the process of solving a problem or an exercise (e.g., Schoenfeld, 1987). The effective acquirement of knowledge of this kind of process depends on the learner's use of signs and symbols (e.g., symbol, letters, numbers, icons, formulas), which the learning environment provides. Signs, symbols and their interpretation are important factors of meaningful knowledge construction, in general (Osberg, 1997 as cited in Yeh & Nason, 2004a). According to semiotic theory, the interaction between representations (*signs*) of an object (the *object* itself) and the *interpretant* (meaning of those signs) are the central elements of meaningful knowledge one constructs from his environment (Otte, 2001, p. 32). The signs mediate (Cunningham, 1992) and function as "thinking helpers" in the interaction between the object and its interpretant (Radford, 2001).

On these premises, Yeh and Nason (2004a) conceptualised 3-D geometry within a semiotic framework consisting of an external material world (objects), representing all geometric components (e.g., shapes of triangle and cube, angle, and height). Internal spatial ability (interpretant) is the human potential and abilities requisite for the perception of various aspects of external geometric objects, e.g., perceptual constancy, visual discrimination, and perception of spatial relationships. The enhancement of these abilities relies on the mediation of sign systems – communication (signs). Similarly, Salomon's (1988) theory states that the individual internalises a system of signs and symbols, while the interaction between computer and student is being carried out.

This allows us to hypothesise that a design that exploits the unique VR visual effects will turn a learning environment into a unique sign environment. This, in turn, would make the interaction among the interpretant–object–communication in the process of spatial image creation more efficient and effective than other methods that do not use VR.

2.3.2. Feedback in Virtual Spaces 1.0

Merrill's (1983) component display theory specifies that an effective instructional unit would consist of an objective followed by some combination of rules, examples, recall, practice, and *feedback* appropriate to the subject matter and learning task. Moreover, feedback involves providing learners with information about their responses. Informing learners how well they are performing a task will act as an incentive for investing greater effort in the task, and learners will adopt a more effort-intensive strategy than they would without feedback (Vollmeyer & Rheinberg, 2005, p. 591).

In sum, the design of Virtual Spaces1.0 features was informed by the theories and practices detailed above.

2.4. Virtual Spaces 1.0 – main features

Virtual Spaces 1.0, which was designed specifically for this study, is a software environment based on the technology of immersive virtual reality, which merges elements of teaching with PCs (the use of textual feedback) with the advantages of virtual reality (the manipulation of objects that are not on a flat screen). This version of Virtual Spaces 1.0 is designed as an exercise environment that includes 52 exercises, and not as a constructing tool for studying geometrical objects.

The objects in the virtual worlds were constructed using the WorldUP5 patch 7 program. With the help of the 5DT stereoscopic (40° fov, 800×600 resolution) HMD, students could actually see three-dimensional objects and manipulate them by using a three-dimensional mouse (Logicad3D).

Generally speaking, the software environment's main goal is to advance spatial thinking according to the BIG approach, by practicing the five capabilities that appear in Fig. 1.

A basic assumption in designing Virtual Spaces 1.0 was that the unique VR's instructional potential would contribute to the realisation of the software's main goal. This potential expresses itself in a number of ways. First, a VR software environment diminishes the need for verbal codes in the learning process. Everyone uses language codes for realising the interaction of transferring information. VR technology, however, has the ability of simulating those codes and diminishing their strength by adding a small number of visual codes (Lanier & Biocca, 1992, p. 159).

Second, the student's intuitive interaction with the virtual world, which is a copy of the real world, creates a tie between motor activity and the creation of the objects the learner sees in the virtual world. This sense of creation helps in assimilating the content related to the objects in cognition (Regain & Shebilsky, 1992, p. 13; Wilson, 1997).

Moreover, immersive VR gives the learner intuitive freedom of action to interact in an unlimited manner with the objects in his or her environment. Essentially, it is the software environment best suited for advancing the learning process according to constructivist theory (Winn, 1997).

Virtual Spaces 1.0 has three different, virtual worlds. Each one has its own central features:

- The world of concepts – In this world, the learner navigates the streets of a city, and arrives at two parks. In each one, the learner carries out assignments, with the goal of gaining information about special concepts related to 3-D geometry. For example, "a line perpendicular to a plane" – by setting up a mast.
- The world of construction and manipulation - In this world the learner has to construct a spatial image and manipulate it in order to solve an exercise (Figs. 2 and 3, Appendix A).
- While doing the exercise, the learner has to use three self-regulating questions:
  a. What is the method for solving the given exercise?
  b. What are the signs that brought me to choose a particular solution?
  c. What are the signs that led me NOT to choose a particular solution?
- The world of constructing in motion - This world contains a game (Appendix B) in which the learner constructs an object according to a given model. There are five levels in the game: the given model is static, the model moves slowly, the model moves at a higher speed, the model disappears after 12 s, and the model disappears after 6 s.
- The goal of the game is to practice the capabilities vital to the building and manipulation of a spatial image. In the more advanced levels, the emphasis is on visual memory.
- The exercises for advancing perception of spatial relationships, mental rotation, perceptual constancy, visual discrimination, and visual memory are similar to the items of Gardner's (1982) TVPS test, which evaluates capabilities of this type.
- Forms of feedback: In Virtual Spaces 1.0 multiple forms of feedback were presented: Feedback-1 (after the first incorrect response) directs attention to the context and relevant information of the correct response: e.g., "Sorry...wrong choice. Move object to the right". Feedback-2 (after the second incorrect response) directs attention to relevant facts, which help to distinguish between a correct answer and an incorrect one: e.g., "Look for an object without this line." Feedback-3 directs attention to the right answer without any additional information: e.g., "The right answer is No. 2."

The next section will discuss the ways in which Virtual Spaces 1.0 differs from other software environments with similar orientation.

2.5. Related work

Recently, a number of training studies have shown the usefulness of virtual reality for training spatial ability, e.g., in navigation and way finding (Durlach et al., 2000) and rehabilitation (Rizzo et al., 1998). A survey of research performed since 1996 that has focused on training spatial abilities using pre-test/post-test design can be found in Kaufmann (2004, pp. 23–26). This survey clearly shows that spatial skills – as measured by standardized tests – can be substantially improved by means of training. However, little to no work has been done towards systematically developing virtual reality applications for practical education purposes in this field. Furthermore, no virtual reality application for use in high school or higher education has been developed with the *main* purpose of improving spatial skills (Kaufmann, Steinbügl, Dünser, & Glück, 2005, p. 1).

In general, the main difference between Virtual Spaces 1.0 and software environments that focus on spatial geometry e.g., 3DMath (Christou et al., 2006), or programs that advance the understanding of geometric concepts, e.g., 3D-lab (Hidaka, 1994), HyperGami (McLurge, Lee, Shalavier, & Jacobson, 1997), is that the objects in those environments appear on a 2D flat screen, while in Virtual Spaces 1.0 the objects are truly three-dimensional. Virtual Spaces 1.0 makes direct interaction possible with 3D objects that do not appear on a flat screen. It is as if the learner is working with a three-dimensional object taken from his or her day-to-day reality (Wilson, 1997, p. 182). The enhancement of the three-dimensional visual effect focuses the learner's attention on the very signs that may help him or her to extract meaning from the information given, thereby helping the learner to grapple with the difficulties in studying 3-D geometry, as mentioned above.

The main difference between Virtual Spaces 1.0 and other software environments that *employ* VR lies principally in the forms of feedback and in employing the SRQ. An example is Merickel's (1991) "Creative Technology Project." This project was designed to enhance certain cognitive abilities (e.g., imagery, spatial relations, creativity, spatial displacement and transformation, and spatial-related problem solving) by using 2D and 3D objects. According to Merickel (1991), training in visualisation and mental manipulation can enhance spatially related problem-solving skills, both in two and three-dimensions (p. 9).

The improvements in achievement reported in his research do not demonstrate the contribution of the unique advantages of VR because of the use of two-dimensional objects in the process of learning. Additionally, the students solved problems, but the kind of feedback they received when they failed at a task was not reported.

In Osberg's (1997) "Puzzle-World," the students built 3D puzzle pieces using VR technology. After finishing their task they built a puzzle made up of the pieces they had built in the first stage of the program. The research question was, "Can thinking intensively about spatial concepts enhance spatial cognition?" Even though the findings give a positive answer to that question, the use of workbooks during the experiment raises some problems: "Though these data support the hypothesis that intensive training in three-dimensional thinking can help a child gain skills necessary for spatial cognition, it does not say which particular component of the week-long program did the most good. Was it working in Swivel environment? Was it the workbook approach? Was it the experiential nature of virtual reality?"(Osberg's, 1997).

The Virtual Spaces 1.0 courseware has no learning material aside from the virtual world. In this way, the students' achievements are primarily a function of their interaction with the software environment content.

In Yeh and Nason's (2004b) VR Math, the central idea is to empower the intuitive sense of three-dimensional objects in space. To realise this idea, the authors put at the learner's disposal an interface with an attractive and wonderfully varied range of options for building objects and manipulating them in many different ways. In addition, the learner had use of a programming language similar to Logo, which allows for the investigation of the objects' qualities. The assumption behind the designing of the interactions was that constructing and investigating objects would lead to the learner's involvement in the process of procedural thinking and the Tutee mode of computer use.

However, VRMath is not designed to enhance capabilities defined in advance, as is done with Virtual Space 1.0 (e.g., visual memory). Also, VRMath does not feature the process of self-regulating questions.

In light of the review of the literature and the rationale of the design, the goal of the current research was to find an answer to two questions:

a. Is it possible to advance spatial thinking by employing Virtual Spaces 1.0?
b. To what extent does the addition of SRQ to the exercises in Virtual Spaces 1.0 contribute to the advancement of spatial thinking?

To answers these questions, we constructed an experimental treatment design in which there were four groups:

- Group 1 practiced with Virtual Spaces 1.0 and SRQ.
- Group 2 practiced with Virtual Spaces 1.0 but without SRQ.
- Group 3 used booklets but with SRQ.
- Group 4 used booklets without SRQ.

In light of the research goals, the review of the research literature, and the research system, the following hypotheses were formulated:

1. Students who learn with Virtual Spaces 1.0 will receive higher grades on tests of spatial thinking than those who learn without Virtual Spaces 1.0.
2. Students who practice with Virtual Spaces 1.0 and SRQ will receive higher grades on tests of spatial thinking than those who practice without SRQ.

3. Method

3.1. Participants

Participants in this study were 194 tenth-grade students in six comprehensive high schools (52% boys and 48% girls, average age 15:2). In each school, all the students in the tenth grade were invited to participate in a "VR project" and 213 students enlisted. Based on the results of the demographic questionnaire, only 194 were accepted because they used a PC at home on a daily basis (88% use a PC for games and 12% for other uses). Nineteen students either did not have a PC at home (three students) or reported that working with a PC makes them feel uneasy or anxious (16 students).

3.2. Design and procedure

3.2.1. Participants

The students' participation was voluntary, and the anonymity of students' responses and their confidentiality as participants were explained before distributing the instruments. The students were randomly assigned to one of four treatments groups:

Group 1 ($N = 52$): practiced with Virtual Spaces 1.0 and SRQ.
Group 2 ($N = 52$): practiced with Virtual Spaces 1.0 but without SRQ.
Group 3 ($N = 45$): practiced from a booklet with the same exercises as Group 1 and with SRQ.
Group 4 ($N = 45$): non-treatment group practiced from a booklet the same exercises as Group 1 but without SRQ.

3.2.2. Design

To evaluate two experimental interventions (VR and SRQ) not only separately, but also in combination and against a control group, the current study utilised a pre-test–post-test 2×2 design.

3.2.3. Procedure

The research took place in five comprehensive schools over a period of five months. Each school chose a day of the week, and on that day, we summoned students to the computer lab there they did exercises with Virtual Spaces 1.0 (Groups 1 and 2) or worked on the same exercises in booklets (Group 3 or Group 4) in vacant classrooms. The hours schools allotted were 12:00–16:00. During these hours, each student in Groups 1 and 2 worked with Virtual Spaces 1.0 for 15 min a day for 3 weeks.

In the *first week* all the students were tested on MRT and APTS-E. In the *next 2 weeks* the researcher met with the participants of Groups 1 and 2, and a short presentation of the new technology (the stereoscopic 5DT HMD and the three-dimensional mouse) was given. Each student put on the HMD and was shown how to fasten the plastic bands to fit his or her head and then practiced for 5–12 min, navigating in the "world of concepts" and the usage of the manipulating tools (Fig. 4, Appendix C). During the *next 15 weeks*, students in Groups 1 and 2 finished doing exercises with Virtual Spaces 1.0 at the same time that students in Groups 3 and four finished their exercises in their booklets. They studied with a teacher who explained the instructions for each exercise and asked the students to solve the exercises. After two lessons they finished all the exercises. In the *19th week*, all the students were tested again on MRT and APTS-E. In the *20th week*, the researcher met with students and thanked them for their efforts.

3.3. Instrumentation

The dependent variable of the current study was achievement in spatial thinking. Achievement measurement was carried out using two tests:

1. *Spatial-Visual Reasoning test of the Aptitude Profile Series - Educational (APTS-E) from* Morgan, Stephanou, and Simpson (2000). The APTS-E is a battery of normed cognitive ability tests, which contains four tests: the Verbal Reasoning Test, the Quantitative Reasoning Test, the Abstract Reasoning Test, and the Spatial-Visual Reasoning test. Each test can be used alone or in conjunction with the other tests (Morgan et al., 2000, p. ix). They are suitable for middle secondary school students who can follow the instructions and older students. The Spatial-Visual Reasoning test contains three parts: Part 1, visualisation/deduction in two dimensions (20 items in 12 min); Part 2, visualisation/ deduction in three-dimensions (10 items in 8 min); and Part 3, visualisation/deduction of object arrangement when observer view is changed (18 items in 10 min). In the current research, the Spatial-Visual Reasoning test was used to assess spatial thinking capacity and visualisation abilities. A perfect score for the APTS-E is 48. This test has a Cronbach's alpha reliability of 0.90.
2. *Vandenberg & Kuse Mental Rotation Test (MRT)*. The MRT is a cognitive test used to estimate test-takers' three-dimensional thinking capacity and spatial ability through identification of rotated objects. The test contains 20 items. Each item consists of a criterion figure, two correct alternatives, and two incorrect ones or "distractors." Correct alternatives are always identical in structure to the criterion but are shown in a rotated position (Vandenberg & Kuse, 1978, p. 599). The recommended method of scoring is to give two credits for two correct choices, none if one choice is correct and the other is incorrect, and none if both are incorrect. If only one design is chosen and it is correct, one point is given. A perfect score for the MRT is 40. The time limit is 6 min (3 min for each part). It is appropriate for ages 14 and older. Reliability was measured using a Kuder–Richardson reliability coefficient = 0.88 (Wilson et al., 1975). The test–retest reliability coefficient was 0.83 after an interval of more than one year (Vandenberg & Kuse, 1978).

3.4. Data analysis

In this study, the independent variables include achievements on the APTS-E test and MRT tests. The dependent variable was spatial thinking in nonverbal dimension measured by the MRT test and verbal dimension measured by the APST-E test. Data were analysed with the Statistical Package for the Social Sciences (SPSS) for Windows version (release 9.0). One-directional analysis of variance (ANOVA) was performed on the pre-test to determine if there were differences between the groups. Afterwards, multi-directional analysis of variance (MANOVA) was performed with repeated measurements over time. A post hoc analysis (LSD test) was performed to examine the specific differences in achievement between the experimental groups.

4. Results

The recent study was designed to test two hypotheses that were put forward in light of the theories and assumptions presented above. The first hypothesis was that the achievements of the students who practiced with Virtual Spaces 1.0 and SRQ would be higher than that of students who practiced without this combination.

To test the first hypothesis, one-directional analysis of variance (ANOVA) was performed on the pre-test grades. The analysis showed that there were no differences between the four groups on the MRT test ($F(3, 190) = .79$, $p = .50$) or on the APTS-E test ($F(3, 190) = 2.14$, $p = .10$). Then, a multi-directional variation analysis (MANOVA) with repeated measurements was performed to test whether practicing with Virtual Spaces 1.0 made a difference. The results clearly show a significant difference between the pre-test and post-test grades on both of the MRT tests ($F(1, 190) = 107.80$, $p < .001$). Similarly, there was a significant improvement in grades ($F(3, 190) = 12.87$, $p < .001$). Furthermore, post hoc analysis was performed to examine specific differences in achievement between the experimental groups. An LSD test revealed that the Group 1 grades (Virtual Spaces 1.0 and SRQ) were significantly higher than those of Group 3, who practiced with SRQ only ($M = 20.788$ vs. $M = 14.422$) ($p < .001$). Additionally, the Group 1 grades were significantly higher than those of Group 4 the control group ($M = 20.788$ vs. $M = 11.257$) ($p < .001$). The Group 2 (Virtual Spaces 1.0 only) grades were also significantly higher than those of the control group ($M = 17.81$ vs. $M = 11.26$) ($p < .05$). No significant difference was found between Groups 1 and 2. Table 1 illustrates the differences between the groups on the MRT test.

In the MANOVA analysis with repeated measurements of the APTS-E grades, the results show, as hypothesised, a significant difference between pre-test and post-test grades ($F(1, 190) = 58.55$, $p < .001$). It was also found that the improvement in grades was significant ($F(3, 190) = 3.25$, $p < .05$). Furthermore, post hoc analysis was performed to examine the specific differences in achievement between the experimental groups. The LSD test revealed that the achievements of Group 1, who practiced with Virtual Spaces 1.0 and SRQ, were significantly higher than the achievements of Group 3, who practiced with SRQ only ($M = 33.86$ vs. $M = 28.58$) ($p < .001$). Additionally, Group 1 achieved significantly higher grades than the control group ($M = 33.86$ vs. $M = 24.96$) ($p < .001$). Also the Group 2 grades were significantly higher than those of the control group ($M = 30.49$ vs. $M = 24.96$) ($p < .05$). No significant difference was found between Groups 1 and 2. Table 2 illustrates the differences between the groups on the APTS-E test.

These findings show that the achievements of Group 1, who practiced with Virtual Spaces 1.0 and SRQ, were higher than those of Group 2, who practiced with Virtual Spaces 1.0 but without SRQ (see Table 2), but the difference was not significant. Since the APTS-E test is more complex than the MRT test, we considered the possibility that the findings could indicate that SRQ contributes more than VR for more complex tasks, while for simpler tasks the SRQ is less effective.

To perform an examination of this possibility, we compared the achievements on the third subtest of the APTS-E (Part 3: items 31–45) of students who learned with both Virtual Spaces 1.0 and SRQ, with those of students who learned with Virtual Spaces 1.0 alone. The subtest included exercises with complex tasks, like organising imaginary objects in a way that requires a process with a number of acts of imaginary creation and their manipulation.

For the purpose of comparison, an analysis of variance with repeated measurements was performed. The repeated measurements were the result of the grades on the subtest before and after studying the course material, in which the independent variable was the group. When the analysis of variance was performed, a significant difference was found between the repeated measurements: $F(1, 102) = 105.77$, $p < 0.001$. Similarly, significant interaction was found between the repeated measurements and the group: $F(1, 102) = 19.1$, $p < 0.001$.

Table 3 illustrates the differences between the groups on that test. Additionally, Table 3 points out the impact of SRQ on high order skill tasks.

The comparison produced findings that clearly indicate the influence of SRQ is primarily recognisable in the more complex tasks in spatial thinking.

In summary, the findings indicate that practice with Virtual Spaces 1.0 is more effective for advancing spatial thinking than the conventional method. The use of VR becomes more efficient with SRQ in carrying out complex tasks.

Table 1
MRT grades: means and standard deviations by method and time.

	Pre		Post	
	M	*SD*	*M*	*SD*
Group 1 VR & SRQ	12.17	6.80	20.79	9.08
Group 2 VR without SRQ	12.00	7.27	17.81	8.00
Group 3 without VR with SRQ	10.47	8.39	14.42	7.87
Group 4 non treatment group	10.53	6.19	11.27	6.57

Table 2
APTS-E grades: means and standard deviations by method and time.

	Pre		Post	
	M	*SD*	*M*	*SD*
Group 1 VR & SRQ	26.15	5.75	33.87	7.45
Group 2 VR without SRQ	25.28	3.75	30.40	11.50
Group 3 without VR with SRQ	22.67	8.12	28.58	7.88
Group 4 non treatment group	24.76	8.02	24.96	9.88

Table 3
APTS-E part 3 grades: means and standard deviations by method and time.

	Pre		Post	
	M	*SD*	*M*	*SD*
Group 1 VR & SRQ	4.92	1.04	7.12	1.11
Group 2 VR without SRQ	4.15	1.91	5.04	1.17

4.1. Research limitations

Every study has its advantages and limitations. We assume that awareness of this study's limitations will provide a better perspective for evaluating its contribution. In the current research, despite the collaboration of the schools, the schools' scheduling requirements did not allow us to conduct the research at fixed times. The times we were allotted were between 12:00 and 16:00. Some of the students that arrived after 15:00 were tired by that time of day and perhaps did not have the opportunity to show their full abilities.

5. Discussion and implications

The main purpose of the current research was to determine if exercising abilities that advance the building and manipulation of a spatial image with virtual software environments can enhance spatial thinking.

Before we discuss the results of the present research, we must point out that the discussion of the findings relating to this question is somewhat limited. This is because, to the best of our knowledge, there has been almost no research performed that dealt with this question in the same manner as our study.

In any case, the research findings allow us, with the greatest of caution, to conclude that a design that takes into account the theoretical implications of constructivism, semiotics, and Merrill's theory (1983) in the area of teaching with VR can contribute significantly to the enhancement of spatial thinking. This bears out the study's first hypothesis that practicing with Virtual Spaces 1.0 enhances this kind of thinking.

Moreover, the findings support the constructivist BIG approach: providing basic knowledge, examples, and explanations, combined with individual inquiry while doing exercises, does advance thinking skills. The findings agree with research that emphasises the power of the visual effect of virtual reality in simplifying the understanding of abstract concepts (e.g., Dede, Salzman, Loftin, & Ash, 1997; McLellan, 1996) and the resulting acceleration in the processes of internalisation and acquisition of spatial knowledge (e.g., Osberg, 1997; Merickel, 1991).

The unique nature of this study's findings expresses itself in a number of areas. First, they indicate a significant improvement of achievement in the nonverbal aspect of spatial thinking due to the special visual effects of virtual realty learning environments (Regain & Shebilsky, 1992; Wilson, 1997). Additionally, the improvement in the verbal aspect of spatial thinking can be explained by the activating of verbal processes in the creation and manipulation of spatial images, which occurred in the asking of self-regulating questions. The improvement in both aspects strengthens the Bryant's theory (1992) that the level of spatial information processing depends on verbal input and nonverbal, sensory input. Second, they strengthen Yakimanskaya's (1991) claim that enhancing the processes of creating and manipulating a spatial image is a basic condition for advancing spatial thinking. Third, our findings agree with the conclusions of a number of researchers who state that the addition of a meta-cognitive process to computer-assisted learning in general, and specifically in VR, would improve the quality of learning (e.g., Winn, 1997; Azevedo, 2005).

However, when we tested the efficacy of practice with both VR and SRQ, as opposed to without SRQ, an unexpected result occurred. The students who practiced with the combined program of VR and SRQ were more successful, but not significantly so, thereby nullifying the second hypothesis, which was that students who learn with the combined program of VR and SRQ would obtain greater achievements than those who learned without SRQ.

There may be a number of reasons for this result. First, the use of self directing questions may encounter a motivation problem. One must motivate the student to ask themselves the right questions at the right moment (e.g., Pressley, Van Etten, Yokoi, Freeburn, & Meter, 1998). However, since learning with VR causes great excitement among students (e.g., Bricken & Byrne, 1993), as was observed in the present study, it is reasonable to assume that this thinking is irrelevant to our findings. Second, since the student has to ask the questions he asks himself out loud during the solution of the exercise, it's possible that the questions being asked may distract the student, rather than help him or her focus their attention (Allen & Armour-Thomas, 1993). Analysis of the findings on the difficult items, as we shall see below, shows that this claim is not corroborated by the findings in this study.

Third, a number of researchers (e.g., Kramarski, Lieberman, & Mevarech, 2001; Koriat & Goldsmith, 1998) maintain that the efficacy of SRQ expresses itself in tasks that the students define as complex. This claim is corroborated by an additional analysis of the findings that was done only on the difficult items of the APTS-E. The analysis shows that on the complex items, the achievements of students who practiced with both VR and SRQ were significantly higher than the achievements of those who used VR alone. Therefore, the findings allow us, with the greatest of caution, to conclude that SRQ leads to a more efficient use of VR, especially in conjunction with complex tasks.

5.1. Conclusions and further research

In this paper we presented a virtual software environment designed to enhance spatial thinking by exercising certain abilities that influence the process of creating a spatial image and manipulating it. Additionally, we used multi form feedback and self-regulating questions to assist learners and focus their attention and awareness on that process. The main research question concerned whether this software could advance spatial thinking. Results reveal that the creation of a spatial image and manipulating it is influenced by visual nonverbal effects of the software – achievements on MRT test, and verbal information processing e.g., textual multi form feedback, SRQ-achievements on APTS-E test. Therefore we may conclude that combing verbal processes in virtual reality learning environments does not diminish the effect of visual signs in spatial information unique to this environment. Furthermore, they do not distract the learners' attention or lessen the motivation in doing the exercises. On the contrary, their significant contribution is shown clearly when solving more complicated problems (e.g., difficult items of the APTS-E test). Additionally, the fact that the achievements resulted from exercising with VR and SRQ were not significantly higher than those who exercised with VR only implies that further research is needed to focus on finding the best method of using verbal information in virtual environments.

The software environment, Virtual Spaces 1.0, did not include building and exploring 3D objects relevant to curriculum materials of 10th graders. Our future research will investigate the new version, Virtual Spaces 2.0 that includes object building and exploration, and

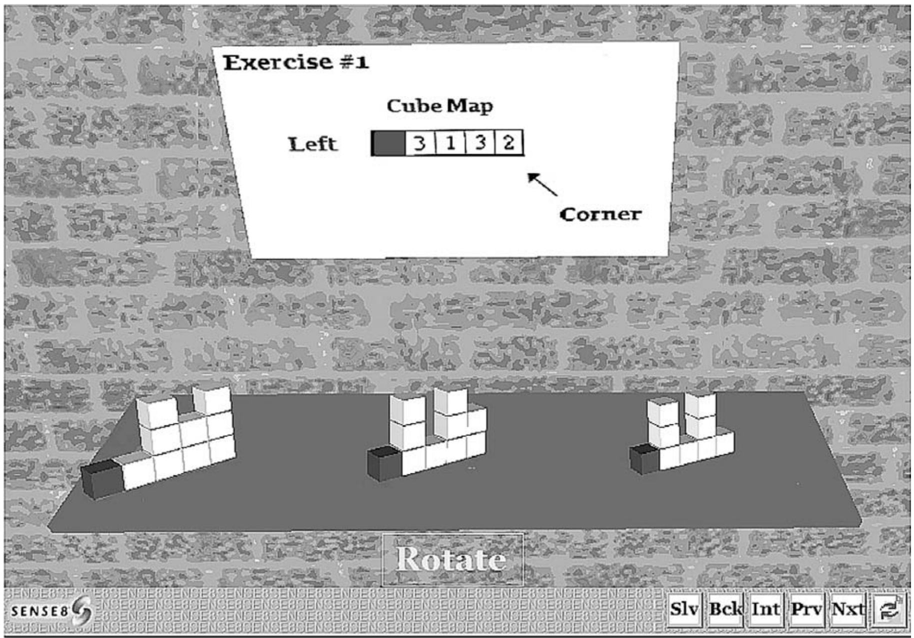

Fig. 2. Cube-map.

can investigate whether combing self-regulating questions with object building and exploring will improve students' geometric proficiency.

Acknowledgement

I would like to thank Prof. A. Cohen for his guiding comments.

Appendix A. Sample exercises

Legend: In this exercise the student is requested to select the (only) 3D model from the ones on the table according to the "map" with numbers, The "map" consists of squares, marked with digits (except for the left, dark one). Each square corresponds to a vertical box, in which the number of cubes is indicated by the number inside the square. Several control buttons permit the student to select a solution, or move back and forth between exercises. Example Fig. 2.

Legend: In this exercise, a 3D figure is presented on a table. The student has to choose, from among the four figures in the left-hand side, the correct image, representing the exact view from the location the arrow is indicating. Although the student is allowed to rotate the shape (along with the table), the amount of rotation is limited, to avoid solving the problem trivially. For example, in the figure above, since the arrow is located at the left of the table, the bottom figure represents the correct view. Example Fig. 3.

Appendix B. Construction in motion game

The initial configuration of "constructing in motion" is shown in Fig. 7.

Phase one: Before the game commences, a shape-manipulation phase takes place, in which the student is given an opportunity to study the model (located in MDL frame), which he or she will later be required to construct. This is accomplished by clicking either on the left side of the object (the blue cube) or on the right side (the yellow cube). Each "click" rotates the object in 30° (blue to the right, yellow to the left[1]). Fig. 8 and Fig. 9 present the tools ("buttons") one uses to build the object in MDL frame (Fig. 7).

Phase two: the student constructs the model shown in MDL (although in more advanced levels, it either rotates, or disappears after a fixed amount of time). The model is constructed by adding a cube clicking on the arrows at the bottom as shown in Fig. 8.

Appendix C

Legend: Example Fig. 4
An Answer – presents an answer for an exercise
?: Help- by clicking "help", instructions or a strategy are displayed. For example: Fig. 5
Arrows: moves the point of view either left, right, down or up
Zm: Zoom in – enlarges an object, for example in Fig. 6

[1] For interpretation of color in Fig. 7, the reader is referred to the web version of this article.

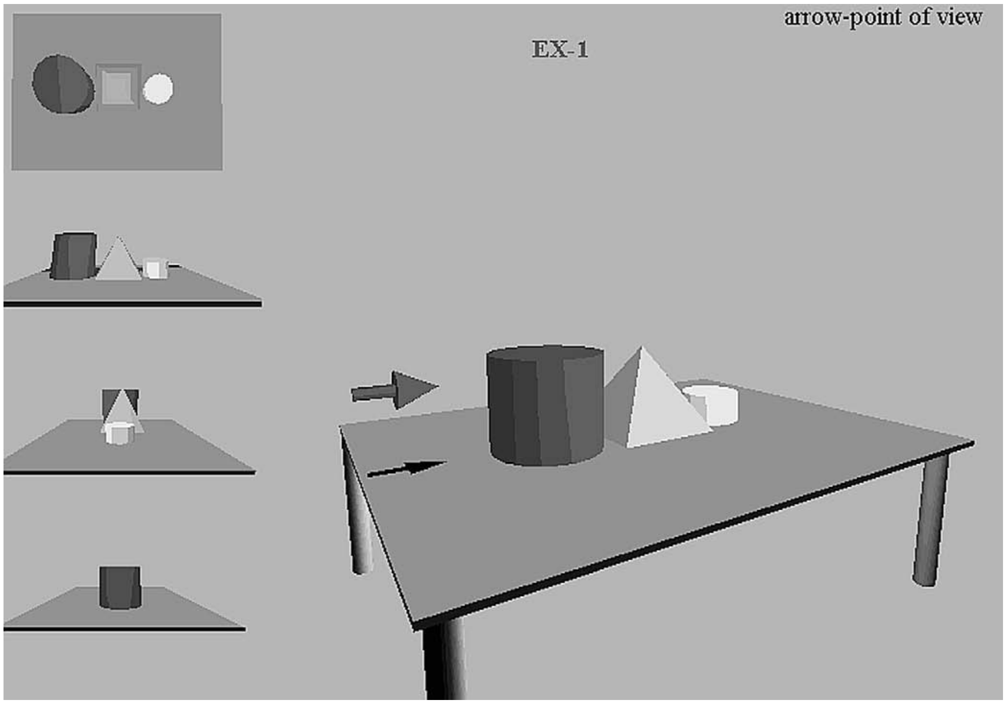

Fig. 3. Point of view.

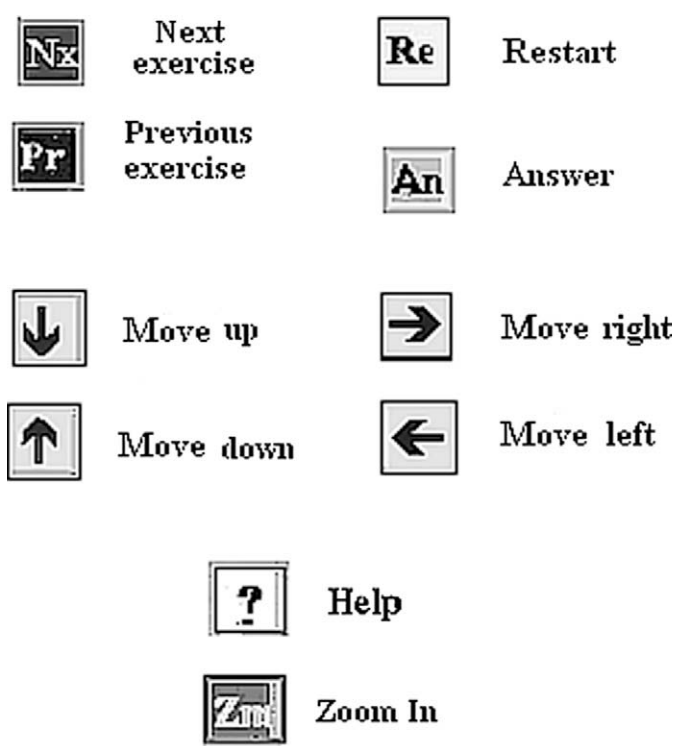

Fig. 4. Control panel.

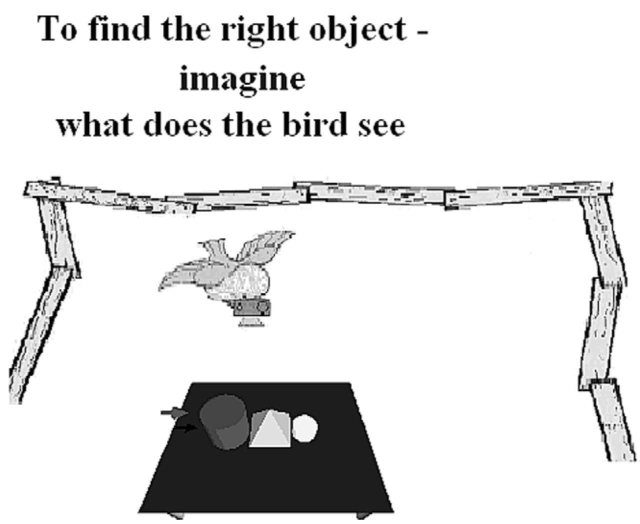

Fig. 5. Directing attention to a strategy.

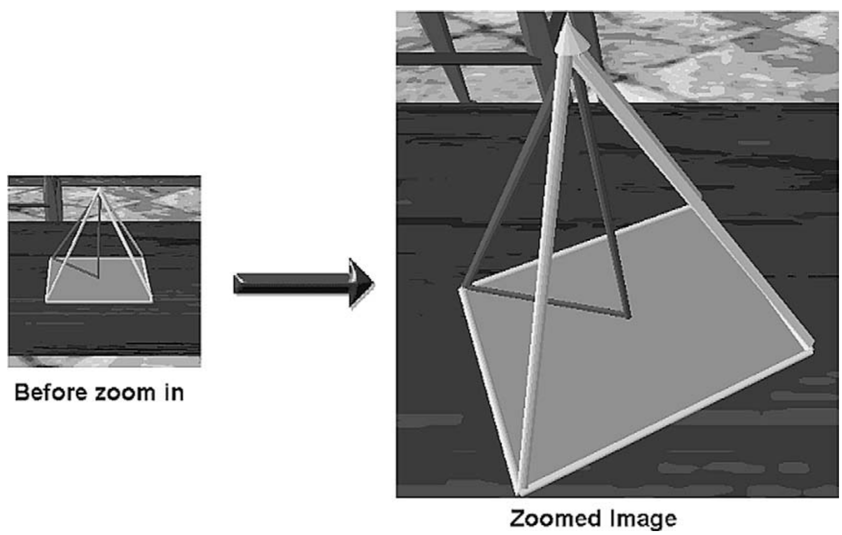

Fig. 6. Zoom in.

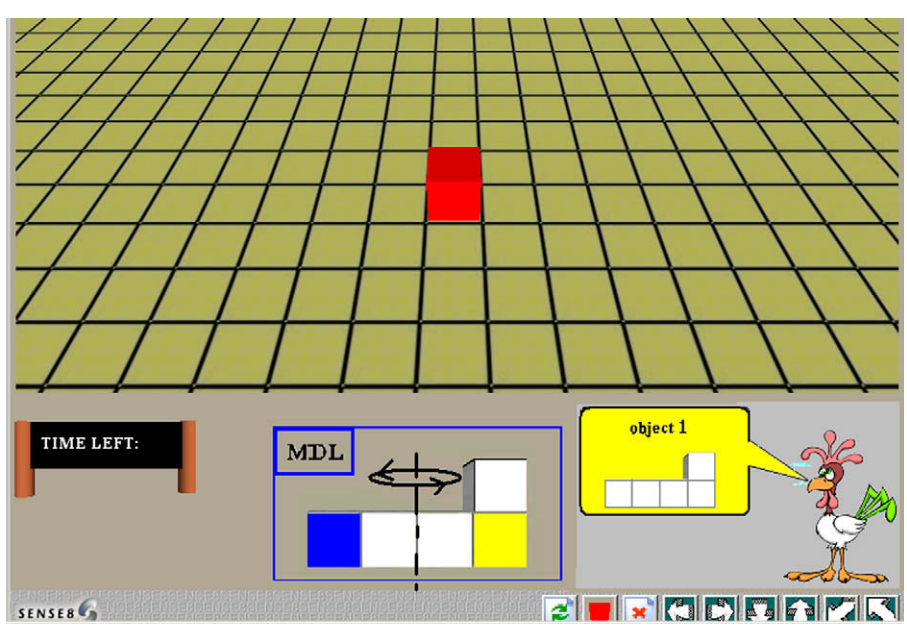

Fig. 7. Construction in motion phase one.

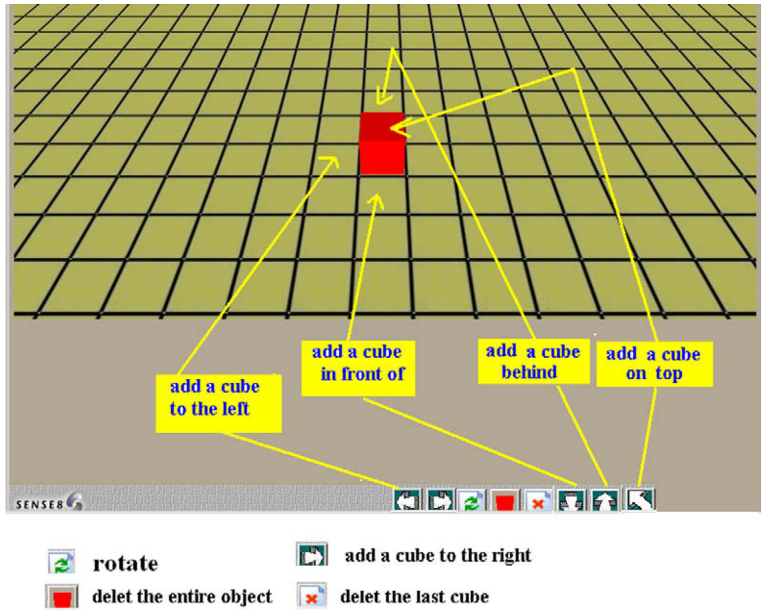

Fig. 8. Phase two: constructing an object.

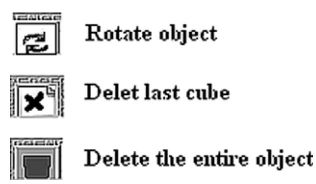

Fig. 9. Legend.

Restart: undo all rotations done in an exercise
Pr/Nx: Previous/Next exercise-navigates back and forth between exercises

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