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CHARGE CONJUGATION AND THE $B^{(3)}$ FIELD: A REPLY TO COMAY

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The argument for non-existence of the $\boldsymbol{B}^{(3)}$ field proposed by E. Comay is based on adding π radians to the phase of a plane wave. This is trivially incorrect because $\boldsymbol{B}^{(3)}$ is a vacuum component of a C conserving Yang-Mills gauge field theory.

Key words: $\boldsymbol{B}^{(3)}$ field, charge conjugation symmetry.

1. INTRODUCTION

Recently, Comay [1] has claimed that the $\boldsymbol{B}^{(3)}$ field is unphysical because it violates charge conjugation symmetry. Comay uses a slight variation on the argument by Barron [2] and Buckingham [3], one which has been shown already to be trivially in correct [4–6]. In this reply it is shown that the vacuum definition of the $\boldsymbol{B}^{(3)}$ field in Yang-Mills gauge field theory trivially conserves the charge conjugation symmetry C, defined classically as a symmetry that reverses the sign of charge while leaving all space-time quantities unchanged. Therefore Comay's paper [1] is a repetition of the earlier paper by Barron [2], and the same arguments are used in reply.

2. CHARGE CONJUGATION SYMMETRY AND THE $B^{(3)}$ FIELD

In fairness to Comay the essence of his argument is summarized

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here: it is that $\boldsymbol{B}^{(3)}$ is disallowed simply by rotating two charges of opposite sign through π radians, adding π radians to the phase of an electromagnetic plane wave. It is as serted that this physical or dynamical rotation process and charge conjugation are equivalent [1] in classical electrodynamics (Comay's Fig. 1 and his Eqs. (11) and (12)). It is further asserted that such a rotation does not change the sign of $\boldsymbol{B}^{(3)}$, so $\boldsymbol{B}^{(3)}$ violates C, the charge conjugation operator. It is then further asserted that $\boldsymbol{B}^{(3)}$ is unphysical and incompatible with both the Maxwell and Proca equations. Voluminous recent argument to the contrary [4-12] is not cited and neither are replies to previous criticisms by Comay [13, 14].

The flaw in Comay's argument is clear and trivial: The charge conjugation operation in classical electrodynamics is an operation which leaves all dynamics unchanged. It is never equivalent to a dynamical process such as rotation through π radians. It changes the sign of the equal and opposite charges and changes the sign of the $\boldsymbol{B}^{(3)}$ field. The charge conjugation operator does not affect space or time by definition, and is never equivalent, therefore, to a dynamic or kinematic process such as rotation through π radians.

It is trivially clear, also, that the vacuum definition of $\boldsymbol{B}^{(3)}$ in Yang-Mills gauge field theory [15] conserves the C operator if the latter is defined as above. The definition in vacuo of $\boldsymbol{B}^{(3)}$ is

$$\boldsymbol{B}^{(3)*} := -i\left(\frac{e}{\hbar}\right)\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{1}$$

and is part of the definition of the field tensor in a Yang-Mills gauge field theory with internal group symmetry $O(3)$. Here e is the elementary charge (C negative) and $\hbar$ the Dirac constant (C positive), while $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ is a plane wave solution in vacuo of the d'Alembert equation (C negative). Applying C to each quantity in Eq.(1) leaves it unchanged but changes the sign of $\boldsymbol{B}^{(3)}$. Thus $\boldsymbol{B}^{(3)}$ is a C conserving field component, i.e., is C negative.

If we add π radians to the phase of a plane wave, it is trivially clear that the conjugate product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is unchanged, because the phases cancel on forming the product. The C operator has no effect on $\boldsymbol{A}^{(1)} \times vecA^{(2)}$ because both $\boldsymbol{A}^{(1)}$ and $\boldsymbol{A}^{(2)}$ are C negative. Therefore Comay's argument collapses and the rest of his paper is sequentially erroneous. Comay's other recent papers on $\boldsymbol{B}^{(3)}$ [13] are trivially erroneous, because $\boldsymbol{B}^{(3)}$ is part of a Yang-Mills gauge field theory that conserves all the discrete symmetries such as C and CPT of quantum field theory and which is Lorentz and gauge covariant [14].

3. DISCUSSION

It has become clear in recent months that $\boldsymbol{B}^{(3)}$ theory is a standard Yang-Mills theory with internal gauge symmetry $O(3)$ [4–12, 14,15]. It begins to correct quantum electrodynamics [12] only at the fifth order in the fine structure constant, roughly the tenth decimal place in the g factor of the electron and therefore for many practical purposes $O(3)$ electrodynamics is identical with $U(1)$ electrodynamics. The advantages of $O(3)$ electrodynamics include its ability to account from gauge theoretical fundamentals for the existence of the well known empirical observable $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ [4–12]. The $U(1)$ electrodynamics, or Maxwellian electrodynamics, is linear and does not use $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$. Many other advantages of $O(3)$ electrodynamics are developed in Refs. [4–12], not cited by Comay [1].

The fundamental vacuum field equation of $\boldsymbol{B}^{(3)}$ theory is the Feynman-Jacobi identity of the $O(3)$ electrodynamics, a Yang-Mills gauge field theory [4–12]

$$D^{\mu} \tilde{G}_{\mu\nu} := \mathbf{0}, \tag{2}$$

where D^{μ} is a covariant derivative of $O(3)$ symmetry operating on the $O(3)$ symmetry field tensor [4–12]. The integral form of Eq.(2) is the equation that Comay should have used in his Ref. [13a] on the Stokes theorem and $\boldsymbol{B}^{(3)}$. Equations (1) and (2) are gauge and Lorentz covariant equations of Yang-Mills gauge field theory. From them it is shown straightforwardly that, in the vacuum,

$$\nabla \times \boldsymbol{B}^{(3)} = \mathbf{0}. \tag{3}$$

There is no "Faraday induction in vacuo" due to $\boldsymbol{B}^{(3)}$ [4–12], a five year old theoretical prediction which has been verified empirically with great thoroughness by Raja et al. [16].

In field matter interaction Eq. (1) becomes the inverse Faraday effect [4–12], and the vacuum constant $e/\hbar$ is replaced by a material molecular hyperpolarizability. Whenever $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is observed we observe $\boldsymbol{B}^{(3)}$' s effect on matter. The effect of $\boldsymbol{B}^{(3)}$ in vacuo occurs at the fifth order in the fine structure constant as above. The self-consistency of the $\boldsymbol{B}^{(3)}$ theory has been evaluated thoroughly in work uncited by Comay [4–12].

3.1. Quantum Field Theory

The issue of whether $O(3)$ electrodynamics is invariant under charge conjugation is easily examined. If we let $U = e^{i\phi}$ be a unitary matrix that describes a gauge transformation then for a sufficiently small phase shift in the wave function under this gauge transformation, we have

$$\psi(\boldsymbol{r},\, t) \;\rightarrow\; \psi'(\boldsymbol{r},\, t) \;=\; e^{i\phi}\psi(\boldsymbol{r},\, t) \;\simeq\; (1 \,+\, i\phi)\psi(\boldsymbol{r},\, t). \tag{4}$$

We now apply the charge conjugation operator $C : e \;\rightarrow\; -e$ on this gauge-transformed wave function:

$$C\psi'(\boldsymbol{r},\, t) \;=\; C(1 \,+\, i\phi)\psi(\boldsymbol{r},\, t),$$

$$= (1 \,+\, i\phi)C\psi(\boldsymbol{r},\, t) \,+\, i[C,\, \phi]\psi(\boldsymbol{r},\, t). \tag{5}$$

If this gauge transformation is not invariant under the charge conjugation operator then $[C,\, \phi] \neq 0$.

Now quantum field theory obeys the CPT symmetry $CPT\psi \;=\; \psi$. The parity and time operators are defined by $P : x \;\rightarrow\; -x$ and $T : t \;\rightarrow\; -t$. The CPT symmetry means that $CPT \;=\; 1$. With this it is easy to see that $C \;=\; T^{-1}P^{-1} \;=\; PT$. A substitution of this form of the charge conjugation operator where it acts on the wave function reveals that

$$\psi'(-\boldsymbol{r},\, -t) \;=\; (1 \,+\, i\phi)\psi(-\boldsymbol{r},\, -t) \,+\, i[C,\, \phi]\psi(\boldsymbol{r},\, t). \tag{6}$$

However, by definition the first term on the right-hand side is the gauge transformation of the wave function $\psi(-\boldsymbol{r},\, -t)$. This means that $[C,\, \phi] \;=\; 0$. This is a result that is generic to all gauge theories.

We now consider the case where the generator of this gauge transformation is given by the $O(3)$ gauge theory. We consider the operlap between the wave function $\psi(\boldsymbol{r},\, t)$ and $\psi(\boldsymbol{r},\, t) \,+\, \delta\psi(\boldsymbol{r},\, t)$,

$$\psi^*(\boldsymbol{r},\, t)(\psi(\boldsymbol{r},\, t) \,+\, \delta\psi(\boldsymbol{r},\, t)) \;=\; \psi^*(\boldsymbol{r},\, t)\psi(\boldsymbol{r},\, t)$$

$$+\, \psi^*(\boldsymbol{r},\, t)(1 \,+\, i\delta\phi)\psi(\boldsymbol{r},\, t). \tag{7}$$

We then see that $\delta\phi \;=\; \nabla\phi\cdot\delta\boldsymbol{r}$ when the variation or overlap is due to a phase shift that depends upon spatial position. If we consider this overlap parameterized around a loop $\mathcal{C}$ that encloses the area $\mathcal{A}$, it is apparent that we have the phase shift due to the gauge transformation

$$\phi \;=\; \oint_{\mathcal{C}=\partial\mathcal{A}} A_i^a dx^i. \tag{8}$$

Stokes' law then indicates that this is equal to

$$\phi \;=\; \int_{\mathcal{A}}(\partial_i A_j^a \,+\, \epsilon^{abc}A_i^b A_j)dx^{[i}dx^{j]}. \tag{9}$$

Now in the case of the third Lie algebraic index we have that $A_i^3 = 0$, and the phase due to the gauge transformation is

$$\phi = \frac{1}{2} \int_{\mathcal{A}} \boldsymbol{A}^1 \times \boldsymbol{A}^2 \cdot d\mathcal{A}. \tag{10}$$

Now consider the commutator $[C, \phi]$. The action of the charge conjugation operator on the gauge connections $\boldsymbol{A}^1$ and $\boldsymbol{A}^2$ is C : $\boldsymbol{A}(\boldsymbol{r}, t) \rightarrow \boldsymbol{A}(-\boldsymbol{r}, -t)$. Yet, since these gauge potentials correspond to the quantized f ield of the electromagnetic interaction, we associated this field with a vector boson. Boson wave functions are seen to be symmetric on the interchange of coordinates in the wave function. Further, this gauge field is time reversal invariant. Further, we have that each gauge potential commutes with the charge conjugation operator. This means that

$$C\boldsymbol{A}^1(\boldsymbol{r}, t) \times \boldsymbol{A}^2(\boldsymbol{r}, t) = \boldsymbol{A}^1(-\boldsymbol{r}, -t) \times \boldsymbol{A}^2(\boldsymbol{r}, t)CC$$

$$= \boldsymbol{A}^1(\boldsymbol{r}, t) \times C\boldsymbol{A}^2(\boldsymbol{r}, t)C = \boldsymbol{A}^1(\boldsymbol{r}, t) \times \boldsymbol{A}^2(\boldsymbol{r}, t)C. \tag{11}$$

Since the commutator of the charge conjugate operator with the phase of the gauge transformation is essentially the commutator $[C, \boldsymbol{A}^1 \times \boldsymbol{A}^2]$ it is apparent that this commutator vanishes in $O(3)$ electrodynamics. It is then apparent that $O(3)$ electrodynamics is invariant under charge conjugation.

4. SUMMARY

Dr. Comay's article is essentially a series of statements and unproven assertions which are made in total disregard of recent developments, developments that show beyond all reasonable doubt that his arguments are repeatedly and trivially erroneous. If Comay were correct, Yang-Mills theory would violate C and CPT, Lorentz covariance, and gauge covariance. The Yang Mills theory is in fact the most succesful gauge theory of the twentieth century and was originally intended to generalize electrodynamics. The $\boldsymbol{B}^{(3)}$ theory was therefore developed in this spirit.

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point described in this reply. The quoted paper by Comay appeared in print entirely without my knowledge or that of any member of this discussion group, and to date, has not been cited in the literature.

REFERENCES

1. E. Comay, *Physica A* **242**, 522 (1997).
2. L. D. Barron, *Physica B* **190**, 307 (1993).
3. A. D. Buckingham, *Science* **266**, 665 (1994).
4. M. W. Evans, *Physica B* **190**, 310 (1993).
5. M. W. Evans, *Found. Phys. Lett.* **8**, 563 (1995).
6. M. W. Evans, *Found. Phys. Lett.* **8**, 187 (1995).
7. M. W. Evans and S. Kielich, eds., *Modern Nonlinear Optics* (Wiley, New York, 1997, paperback).
8. M. W. Evans, *The Photon's Magnetic Field* (World Scientific, Singapore, 1992).
9. A. A. Hasanein and M. W. Evans, *The Photomagneton in Quantum Field Theory* (World Sceintific, Singapore, 1994).
10. M. W. Evans, J.-P. Vigier, S. Roy, S. Jeffers, G. Hunter, and M. Meszaros (authors and editors), *The Enigmatic Photon*, Vols. 1–5 (Kluwer, Dordrecht, 1994 to 1999).
11. M. W. Evans, L. B. Crowell, and J.-P. Vigier, *Classical and Quantum Electrodynamics and the* $B^{(3)}$ *Field* (World Scientific, Singapore, 1999), in preparation.
12. M. W. Evans and L. B. Crowell, *Found. Phys. Lett.*, in press and submitted, 1998/1999.
13. E. Comay, *Chem. Phys. Lett.* **261**, 601 (1996);*Found. Phys. Lett.* **10**, 245 (1997); *Physica B* **222**, 150 (1996).
14. M. W. Evans and S. Jeffers, *Found. Phys. Lett.* **9**, 687 (1996). M. W. Evans, *Found. Phys. Lett.* **10** 255, 403 (1997); Vol. 4 of Ref. 10, pp. 391 and 399. See also a comments by V. V. Dvoeglazov, *Found. Phys. Lett.* **10**, 383 (1997).
15. M. W. Evans, Volume 4 of Ref. 10, pp. 413 ff. M. W. Evans and L. B. Crowell, *Found. Phys. Lett.*, submitted, 1998.
16. M. Y. A. Raja, W. N. Sisk, M. Youssaf, and D. Allen, *Appl. Phys Lett.* **67**, 2123 (1995). M. Y. A. Raja, W. N. Sisk, and D. Allen, *Appl. Phys. B* **64**, 79 (1997). See also M. W. Evans, *Found. Phys. Lett.* **10**, 487 (1997) and *Apeiron* **4**, 80 (1997).