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DUHEM AND KOYRÉ ON DOMINGO DE SOTO

ABSTRACT. Galileo's view of science is indebted to the teaching of the Jesuit professors at the Collegio Romano, but Galileo's concept of mathematical physics also corresponds to that of Giovan Battista Benedetti. Lacking documentary evidence that would connect Benedetti directly with the Jesuits, or the Jesuits with Benedetti, I infer a common source: the 'Spanish connection', that is, Domingo de Soto. I then give indications that the fourteenth-century work at Oxford and Paris on *calculationes* was transmitted via Spain and Portugal to Rome and other centers where Jesuits had colleges, and figured in the rise of mathematical physics at the beginning of the seventeenth century. A result of these researches is their vindication of Duhem, as contrasted with Koyré, on the origins of modern mechanics.

Pierre Duhem and Alexandre Koyré, both eminent French historians of science, held radically different views of the importance of Domingo de Soto for the evolution of modern science. For Duhem, Léonardo da Vinci was the linchpin in a development that stretched from the *Doctores Parisienses* to Soto, and Soto himself was the proximate source of Galileo's early writings and of the ideas contained in his later works (Duhem 1906–1913). Duhem based his analysis on two of Galileo's early manuscripts, now numbered 46 and 71 in the Galileo Collection in Florence, which had been transcribed and published by Antonio Favaro in the National Edition of 1890 with the titles *Juvenilia* and *De motu* respectively. For Koyré, on the other hand, Soto was merely an enigma, a Spanish scholastic isolated from the main flow of European thought (Koyré 1964). In his view neither Soto nor the Parisian doctors nor Léonardo figured importantly as sources of Galileo's science. Following Favaro's lead, Koyré preferred to see the whole of that tradition summarized in the writings of two of Galileo's Italian predecessors, Francesco Buonamici and Giovan Battista Benedetti (Koyré 1939, 1978). The first he discerned behind Galileo's MS 46 and the second behind his MS 71. The medieval and Renaissance development that had been traced in such detail by Duhem might be of antiquarian interest, but it was not at all necessary for Koyré's understanding of Galileo and the 'new science' he had brought into being.

Some years ago, at a conference in this Center, I focused on the

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debate between Duhem and Favaro as recorded in Favaro's 1916 review of Duhem's *Études sur Léonard de Vinci* and his 1918 resumption of that review in an essay entitled 'Galileo Galilei e i Doctores Parisienses' (Favaro 1916, 1918). My conclusion then was that, eminent though both were as scholars, neither had gone far enough in his researches; if they had, the dispute between them could have been dissolved in terms of what I was then developing as a 'qualified continuity thesis' (Wallace 1978, 1984b). In this essay I wish to enlarge on that theme by focusing not on Favaro but on Koyré, and by doing so in light of a third Galileo manuscript that was completely misjudged by Favaro, excluded by him from the National Edition, and as a consequence was unknown to both Duhem and Koyré. I refer to MS 27, the manuscript containing Galileo's treatises on Aristotle's *Posterior Analytics*, recently transcribed and edited by William F. Edwards and myself (Galilei 1988). This manuscript provides yet stronger support for Duhem's continuity thesis – but in a way that is somewhat surprising in that it allows one to integrate Koyré's findings into it and so include Benedetti as another possible link between Galileo and Soto. The intermediary that makes the linkage possible is the one that enabled me to dissolve the Duhem–Favaro controversy well over a decade ago, namely, the Jesuit tradition at the Collegio Romano. It is clear now that Galileo's MS 27 derives from lectures given at the Roman College, and, in light of that derivation, that MSS 46 and 71 derive similarly from the same source (Wallace 1986). What is more problematical is how to relate Benedetti to the Roman Jesuits. I shall therefore start with the Benedetti–Jesuit relationship and then work back from this to Domingo de Soto.[1]

BENEDETTI AND JESUIT SCIENCE

At first glance there would seem to be little that would connect the Collegio Romano, the Jesuit university founded by Ignatius Loyola in Rome in 1551, with Giovan Battista Benedetti, the patrician of Venice whose life spanned the years from 1530 to 1590. Benedetti's visits to Rome apparently were few. He is recorded as having lectured there on the science of Aristotle in the winter of 1559–1560, when the Collegio was but a fledgling institution, but to my knowledge had no contact with Jesuits at that time. From the Collegio side, in the years up to Benedetti's death there seems to have been little appreciation of his scientific work on the part of its philosophy professors, although he was known to the eminent mathematician on the faculty, Christopher Clavius. Such tenuous connections offer little basis for a documentary analysis of possible ties between Benedetti and the Roman Jesuits (Wallace 1987b, nn. 1–5).

In the absence of such evidence, I shall turn to a conceptual study and focus instead on the role of mathematics in the study of nature as an apt basis for comparison. In it I aim to show that by the time of Benedetti's death in 1590, the faculty at the Collegio Romano had come to a view of mathematical physics very similar to his. This would seem to be an important consideration, for it was such a conception of mathematical physics that channeled into MSS 27, 46, and 71 of the young Galileo and thence exerted an influence on the development of his science. Thus the *terminus ad quem* of my investigation is Galileo's writings on motion and mechanics around the year 1590. The *terminus a quo* is somewhat more problematical, and I will come to that later. For the moment I shall identify it simply as 'the Spanish connection', based on the facts that, on the one hand, Benedetti's father was a Spanish philosopher and physicist (or physician) and that many of his own professional contacts were with Spaniards, and, on the other, that the early Jesuit professors at the Collegio Romano were also Spanish and imported from the Iberian peninsula several ideas that proved seminal in the new mathematical physics (Wallace 1987b, nn. 6–9).

Starting, then, in somewhat ahistorical fashion with the *terminus ad quem*, let me characterize briefly the concept of science that emerges clearly in Galileo's early treatises on motion in MS 71 and that continued to dominate his later writings down to the *Two New Sciences* of 1638. This was very much a mathematical physics that proposed itself as a *scientia* and presented its reasonings in the form of *demonstrationes*; its model was ostensibly that of Archimedes, but the ideal was already Aristotle's as formulated in his *Posterior Analytics*. Working out the implications of his new *scientia* (in effect a *scientia mixta* or *scientia media*, subalternating physics to mathematics), Galileo was sharply critical of the causal analyses found in Aristotle's *Physics* and *De caelo*, while at the same time he was intent on searching out, in an Aristotelian mode, the *verae causae* of natural phenomena. Local motion (*motus localis*) was his major concern; to explain this he invoked the principal concepts used by Aristotle – nature and violence, time, place and space, force and resistance, causality – although he rejected others associated with the medium through which the moving object passed, e.g., Aristotle's teaching on the void and his solution to the projectile problem. In their place Galileo substituted the scholastic concept of *impetus*, which he used to explain both violent and natural motion. His most important methodological innovation was his clever use of *suppositiones* when framing his demonstrations, making them amenable to the use of limit concepts and to applications in experimental situations where a mathematical ideal could be closely approximated in the physical world (Wallace 1987b, nn. 10–14).

Much of my recent research has been directed at showing how this view of science is indebted to the teaching of Jesuit professors at the Collegio Romano, whose lecture notes on logic and natural philosophy were the proximate source of Galileo's MSS 27 and 46 and prepared for the *De motu antiquiora* of MS 71. But those who are acquainted with the works of Benedetti will surely have noticed how closely Galileo's concept of mathematical physics just sketched by me corresponds to that of Benedetti. Such correspondence suggests points of comparison between Benedetti and the professors of the Collegio. To develop it, we must look in detail at Benedetti's main theses and then see how these compare with related teachings among the Jesuits whose lecture notes I have studied (Wallace 1987b, nn. 15–16).

BENEDETTI'S MATHEMATICAL PHYSICS

For convenience let me divide my consideration of Benedetti's physics into two parts, the first concentrating on its logical and methodological foundations, the second on its treatment of problems relating to local motion. With regard to the first, there can be no doubt that, from the outset of his career, Benedetti wished to reinforce his arguments as much as possible with 'mathematical demonstrations' (Benedetti 1553); in his last and most important work he identifies his basic disagreement with Aristotle as based "on the unshakable foundation of mathematical philosophy, on which I always take my stand" (Benedetti 1585, p. 196). This presupposes, of course, a difference between physics and mathematics, of which Benedetti was well aware: "balances or levers are not pure mathematical lines", he writes, "but are physical, and as such exist in material bodies" (144).[2] Again, "since balances are material and are sustained . . . not by a mathematical point but by a line or a physical surface having a material existence, some resistance arises to the motion of the arms" (153). Yet he wished to use mathematical principles, such as that "a sphere touches a plane at only one point" (155), to establish physical conclusions. The only way he could do this, he recognized, was through the use of appropriate suppositions and thought experiments. It is thus important to recognize how frequently the terms *suppositio* and *imaginemur* (and their variants) recur in Benedetti's writings. Well known are his disagreements with Tartaglia and Jordanus Nemorarius in his solution of mechanical problems; invariably these are occasioned by the divergent *suppositiones* on which the respective solutions are based. For example, Benedetti frequently reproves Tartaglia for supposing that the "lines of inclination" going from the ends of a balance to a distant center of gravity are parallel (150). Yet on some occasions he makes the same supposition himself, noting that the line of inclination is *fere perpendicularis* to the beam of the balance or that, if the angle it makes is not a right angle, the deviation is negligible. But, when discussing the imagined case of a balance situated close to the earth's center, he rightly insists that the approximation cannot be made and that the simplifying supposition cannot be employed in a rigorous proof (143).

Such suppositions are important in the treatment of problems in statics, but they are crucial for the development of a science of dynamics. Benedetti was intent on discovering the *verae causae* – an expression that occurs repeatedly in his writings – of various kinds of motion in the universe, both natural and forced. An important contribution was his study of horizontal motions on the earth's surface; here he was convinced that the only truly natural motion is circular, for this alone can be perpetual (184). But, he reasons, there is "no noteworthy difference" between a perfect sphere and a plane surface of small extent. For this reason one will encounter no difficulty in moving a sphere along a horizontal surface; indeed, it can be moved by "a force no matter how small" (156). In another context he qualified an argument to specify that it holds only "when all impediment is removed" (154). Such insights, plus Benedetti's frequent allusion to the natural tendency of a body when released from a sling to move in a straight line, shows how close he came to the principle of inertia later formulated by Sir Isaac Newton.

Moving on to his study of problems relating to local motion, we can treat these under three headings, namely, those relating to motion in general, those relating to falling motion, and those relating to the movement of projectiles. With regard to the first, Benedetti was Aristotelian in his conviction that nature is an inner source of motion in a body; even forced or violent motion he saw as caused by an *impetus* or *virtus movens* impressed within a body. But unlike Aristotle he seems not to have invoked a sharp dichotomy between natural and violent motion, or between curvilinear and rectilinear motion, regarding the latter two as mathematically comparable (194). He does not discuss explicitly the possibility of a *motus medius*, i.e., one intermediate between the natural and the violent, but for him horizontal motion for limited distances would answer to that description. And in the case of reflex motion, he invokes the principle that a circle touches a line at only one point to argue that no intermediate rest (*quies medius*) interrupts the upward and downward motion of a body, making its motion truly continuous (184).

Benedetti is most known, of course, for his study of falling motion, especially for his argument *contra Aristotelem et omnes philosophos* that velocity of fall is dependent not on weight but on specific gravity, and therefore is conditioned by the medium in which the body falls and the resistance it encounters (Maccagni 1983). He proposed that velocity of fall increases with distance of travel because impetus builds up naturally in the falling body, and that all bodies would fall with the same speed *in vacuo*, where buoyancy and resistive effects can be neglected. Gravity and levity became for him relative concepts, so that air has no weight in air, nor water in water. And he analyzes the case of a body falling through the center of the earth to argue that it would oscillate about the center, on the analogy of the motion of the bob of a pendulum of exceedingly long length (Benedetti 1985, pp. 174–85, 368–69).

Equally ingenious is Benedetti's study of projectile motion, which is dominated by his skillful use of the concept of impetus, already referred to. This he regarded as a force impressed on a body from without but that moves it from within, decreasing gradually and continually with the body's motion (160). Most motions involving trajectories of bodies he saw resulting from a composition of motions, partly natural and partly forced (161), and in this is seen as having noticeably advanced beyond the teaching of Tartaglia.

COUNTERPARTS IN JESUIT TEACHINGS

Such was the contribution of Benedetti to the foundations of mathematical physics by the time of his death in 1590. The question I now would raise is this: how similar were the teachings of Benedetti as I have just outlined them to those at the Collegio Romano during the years, say, from 1560 to 1590? An answer is difficult because of the paucity of records that have survived from this period. At the beginning, Francisco Toletus taught the physics course in Rome during the academic year 1560–61, and his printed text gives indication that his ideas were fairly similar to Benedetti's. But only a year or two later, Benedictus Pererius took over the course in 1562–63, and, as his textbook indicates, set it on a path almost diametrically opposite to his predecessor's. Decidedly antimathematical and Averroist in his approach, Pererius combatted most of the Benedetti's theses concerning motion, not naming him explicitly or even being aware of his teachings, but simply rejecting out of hand the principles on which they were based (Giacobbi 1977).

This mentality apparently persisted at the Collegio for some fifteen years, and then gradually changed owing to two factors: the influence of Clavius, who fought strenuously to give mathematics a respectable place in the curriculum, not merely in its own right but also as an adjunct to natural philosophy; and the advent of a new physics professor, Antonius Menu, who was much interested in 'calculatory' techniques and imported them where possible into his lectures. A series of professors who followed Menu – Paulus Vallius, Muzius Vitelleschus, and Ludovicus Rugerius – developed their teachings on motion and the heavens along lines more acceptable to Clavius, and thus closer to Benedetti. Finally one of Clavius's special students, Giuseppe Biancani, synthesized all this work by systematically elaborating a mathematical physics capable of dealing with the problems of natural philosophy (Giacobbi 1976).

I shall elaborate more fully on this development later in the essay. Suffice it here to call attention to Vallius's commentary on the *Posterior Analytics*, particularly to his treatise *De praecognitionibus*, which was appropriated by Galileo in his MS 27 (Galilei 1988); this, taken in conjunction with Clavius's preface to his *Elements* and Biancani's later emendations, shows how *suppositiones* can be employed to uncover causes and supply *demonstrationes* in these difficult subject matters. Menu and Vallius recovered the concept of *impetus* and showed how it, and other notions in the scholastic tradition, could improve Aristotelian teachings as these were being advanced by the peripatetics of their day (Wallace 1981c). Vitelleschus and Rugerius built on these foundations. Vitelleschus is particularly important for his awareness of Benedetti's analysis of falling motion, though he knew it only through a work by Jean Taisnier that plagiarized Benedetti's *Demonstratio* of 1554 (Maccagni 1967). The reference occurs in Vitelleschus's lectures on the *De caelo* of Aristotle (given in 1590), where he questions Aristotle's laws of motion as stated in Books 4 and 7 of the *Physics*, and directs his students to the treatises of Bradwardine and Taisnier on the ratios of motions. In the same manuscript Vitelleschus echoes Benedetti's sentiment against the authority of Aristotle, stating that it is safer to abandon some of his teachings than it is to interpret them, for the authority of a philosopher should be used to confirm the truth, not abandon it, seeing that truth is the philosopher's friend. Rugerius then took up Vitelleschus's teachings on the ratios of motion, noting that Aristotle's rules for comparing motions labor under severe difficulties. For a fuller discussion of how they might be revised he then refers his students to the commentaries of Toletus and Soto, among others, in their commentaries on the *Physics* (Wallace 1987b, nn. 53–57).

In the writings of Jesuit professors from Menu to Rugerius, therefore, one can find illuminating discussions of the internal causes of motion, of the possibility of motion in a void, and of an intermediate or neutral motion (neither natural nor violent) that can endure perpetually. One can find too a rejection of the *quies medius* in reflex motion; a rejection of Aristotle's dynamical laws of motion; a sophisticated discussion of gravity, including the distinction between extensive and intensive gravity, similar to Benedetti's notion of specific gravity; a rejection of the notion that air has weight in air based on Archimedian principles; and detailed analyses of the factors that cause bodies to accelerate as they fall. Not all these teachings are the same as Benedetti's, but one gains the impression that, had the Venetian mathematician visited the Collegio in the years following the publication of his last work, he would have found a compatible atmosphere in which to advance his researches (Wallace 1987b, nn. 58–59).

THE SPANISH CONNECTION

This, then, brings me back to *terminus ad quem* with which I began this discussion. Most of the ideas I have just sketched are to be found, in various ways, in the lectures of Jesuits in Rome around 1590, in Galileo's MSS 27, 46, and 71, likewise composed around 1590, and in Benedetti's publications, probably known to Galileo through Jacopo Mazzoni, with whom he studied in 1590. I suspect that it was a fusion of ideas gleaned from Benedetti and the Jesuits that lay behind the various drafts on motion in Galileo's MS 71. Yet I have found no documentary evidence that would connect Benedetti directly with the Jesuits, or the Jesuits with Benedetti, in the development of these concepts. Was there a common source from which they could have derived? I suspect that there might have been, and I would like to speculate about this as the *terminus a quo* of my investigation – the 'Spanish connection' to which I have alluded above.

A plausible candidate for the origin of a mode of thought that would allow mathematics to enter into an experimental study of motion is none other than the Spanish Dominican who first proposed that the motion of falling bodies is uniformly accelerated with respect to time – *uniformiter difformis* is the expression he used – and who was seen by Duhem on this account to be a scholastic precursor of Galileo (Duhem 1906–1913). I refer, obviously, to Domingo de Soto. Soto was known to the Jesuits; indeed, Toletus had studied under him at Salamanca before joining the faculty of the Collegio, and Rugerius, as we have seen, called attention to his superior treatment (along with Toletus's) of Aristotle's dynamic laws of motion. Now, in his commentary and his questions on the *Physics*, Soto assimilated his doctrine on impetus to his teaching on *gravitas* and taught that a falling body accelerates continuously because of the impetus being built up in it during its travel – ideas very similar to those found in Benedetti. These notions are not fully developed in an incomplete edition of Soto's *Physics*, published at Salamanca around 1545, but they are present in the edition of 1551 as well as in the more widely diffused second edition of 1555, both also printed in Salamanca. Between 1545 and 1550, moreover, Soto was present at the Council of Trent, which took place just north of Venice. As the most illustrious theologian in the Dominican Order, he was surely known to Abbot Gabriel de Guzman and the two Spanish Dominicans Benedetti praises so lavishly in his *Resolutio* of 1553 and his two versions of the *Demonstratio* of 1554 and 1555, directed, as we saw, "against Aristotle and all philosophers". While in northern Italy, it is also possible that Soto became acquainted with experimental work being done there on laws of fall, which would have buttressed his own rejection of Aristotle's teaching on this subject. And finally, though I have found no mention of Soto in Benedetti's *Speculationes* of 1585, it may be no mere coincidence that Soto's *Physics*, both commentary and *quaestiones*, was reprinted in Venice in 1582 with an introduction that gives fulsome praise to his ability as a natural philosopher (Wallace 1987b, nn. 63–68). All bits of coincidental evidence, but all pointing to Soto as a link that could ultimately tie together Benedetti, Galileo, and the Jesuits of the Collegio Romano.

SOTO'S SECOND ENIGMA

Earlier I remarked that Soto was an enigma for Koyré, but I did not elaborate on Soto's enigmatic status. Actually two enigmas can be associated with Domingo de Soto. The first is how he came to know that the motion of heavy bodies in free fall is *uniformiter difformis* with respect to time, and the second is how this knowledge might have been transmitted to Galileo. The first enigma was what puzzled Koyré and served as the subject of an essay I published years ago with the title 'The Enigma of Domingo de Soto: *Uniformiter Difformis* and Falling Bodies in Late Medieval Physics' (Wallace 1968). The second enigma, to my knowledge, was not explicitly addressed by Koyré, although it posed the problematic on which much of his *Études galiléennes* was based. Let us address this second enigma now, for, if we can cast light on that, we may additionally be able to fill a lacuna in Duhem's thesis about Soto and his importance for Galileo's science. We can do so through a study of books and manuscripts written by Jesuits in Italy and Portugal in the century following Soto's publication of his *uniformiter difformis* doctrine.

Galileo mentions Soto twice in MS 46, in a *Tractatus de elementis* that occupies the last part of the manuscript. We now know that this *Tractatus*, as well as other treatises written by Galileo at Pisa around 1589–1591, were based on lectures given by the Jesuits mentioned above (Wallace 1977). Some of these lectures were published, but the majority survive only in manuscript. They were based on scholastic and Renaissance authors, whom they cite extensively, and are otherwise prosaic teaching notes. What makes them somewhat distinctive is the attention they pay to nominalist teachings deriving from the *Calculatores* of Oxford University and the *Doctores Parisienses*.

The development of these lecture notes took place in Rome at the Collegio Romano over a period of some thirty years. There the introduction of calculatory thought is traceable to Toletus, himself a Spaniard, who taught the course in natural philosophy in 1560 and imported ideas he had learned from Soto at Salamanca. Some of this material was taken up by two other Spanish Jesuits, Pererius, mentioned above, who taught natural philosophy at the Collegio between 1558 and 1566, and Francisco Suarez, who taught theology there between 1580 and 1585. Fortunately these authors published their materials, which have been analyzed in some detail by Christopher Lewis (Lewis 1980). Lewis picked out for examination the use by all three of calculatory language in the following areas of natural philosophy: (1) when discussing problems of action and reaction; (2) when treating the intension of forms in alteration and identifying distributions of qualities as uniform, uniformly difform, etc.; and (3) when analyzing the ratios of motions following the tradition of Thomas Bradwardine.

Of the three Jesuits, Toletus undoubtedly made fullest use of the *Calculatores* in these areas, even referring to "the calculator Suisset" (i.e., Swineshead) by name in his treatments of reaction and alteration. He also had the clearest understanding of calculatory terminology, although he frequently departed from positions held at Oxford and favored instead those developed at Paris. In treating expressions such as *uniformiter difformis*, moreover, Toletus made the interesting comment that "these [terms] should be very carefully considered in order to understand many matters that are met with in physics". Suarez likewise took up uniform difformity in some detail when analyzing the action of natural agents in his *Disputationes metaphysicae* of 1597, although he rejected the view (apparently subscribed to by Toletus) that velocity could be viewed as an intensity of motion, which would be expected of one subscribing to Mertonian developments in kinematics. Pererius, predictably, showed the least acquaintance with, or interest in, the calculatory tradition, although he was acquainted with some of its terminology. In discussing the dynamical laws given by Aristotle in the seventh book of the *Physics*, for example, Pererius accepted and defended them without even a nod in the direction of Bradwardine, thus showing little sympathy for the mathematical physics developed two centuries earlier at Merton College, Oxford (Lewis 1980).

As already noted, partially because of his antimathematical bias Pererius was replaced after 1566 and succeeded by a series of other professors. Lecture notes survive from only two who taught between then and 1585, viz., Hieronymus de Gregorio and Antonius Menu, but the second of these, Menu, enjoyed the longer tenure and seems to have had the greater influence. Menu revived the approach of Toletus and had a notable effect on the Jesuits mentioned above who taught natural philosophy at the Collegio between 1585 and 1592, namely, Vallius, Vitelleschus, and Rugerius, all of whose lecture notes survive in whole or in part. Although some details are lacking, these four professors supplied the materials on which Galileo's early notebooks on the *De caelo* and *De generatione* and the early versions of his *De motu* were based, and so serve to explain Galileo's knowledge of the calculatory tradition (Wallace 1987a, n. 14).

Menu is of particular importance for standing at the head of this fifteen-year tradition, which used Mertonian terminology but usually applied it in ways more consistent with teachings in vogue at Paris in the fourteenth century and so associated with the *Doctores Parisienses*. Indeed, Menu cites these doctors when treating the question whether the world could have existed from eternity and when discussing the ratios of the elements. He was also favorable to their adoption of impetus, or *virtus impressa*, as necessary to explain the motion of projectiles. Particularly striking are his arguments in favor of the proposition that "the motion of a simple or compound body through a void will be successive, for granted that it would encounter no extrinsic resistance, there would still be intrinsic resistance" to overcome. These are clearly those of the *Parisienses*, adopting the calculatory stance of the Mertonians but applying it to physical problems in the tradition of Jean Buridan, Albert of Saxony, and others who worked in fourteenth-century Paris (Wallace 1987a, nn. 15–16).

The lecture notes of Vallius, Vitelleschus, and Rugerius do not employ these particular arguments, but they nonetheless touch on all the matters pertaining to the mathematical or calculatory tradition that are to be found in Galileo's early writings. The latter's notes in MS 46 are written in the form of a questionary based on Aristotle's *De caelo* and *De generatione*. The questions wherein analytical languages in the Mertonian and Parisian traditions occur most frequently are in the treatises *De alteratione* and *De elementis*, where inquiries are made into the intension and remission of forms, the parts and degrees of qualities, and the number and quantity of the elements. There seems little doubt that all of these materials are derived from lectures given at the Collegio some time prior to 1591. The precise author is difficult to identify, however, since correspondences can be found between Galileo's notes and passages in Rugerius, Vitelleschus, Vallius, and Menu, and indeed all the way back to Pererius and Toletus. At the present stage of research Vallius seems to be the most likely candidate, for, although his surviving lecture notes are incomplete, the portions that survive show closest agreement with Galileo's text. There is excellent evidence, moreover, to connect Galileo's MS 27, the one containing questions on Aristotle's *Posterior Analytics*, with Vallius's lectures on logic, which were completed in the summer of 1588 and manifest a good knowledge of nominalist positions on science and demonstration (Wallace 1987a, nn. 19–21).

My study of all these materials thus encourages me to go considerably beyond Christopher Lewis in identifying likely sources of Galileo's knowledge of the calculatory tradition. Suffice it to mention that citations from Walter Burley and William Heytesbury, as well as Bradwardine and Swineshead, are to be found in these Jesuit lectures. And not only do such citations occur in discussions involving intension and remission, latitudes of qualities, and maxima and minima, but they also occur in discussions of local motion and of Aristotle's dynamical laws involving ratios between forces, resistances, and velocities of motion. Vitelleschus, for example, cites experimental evidence against the Aristotelian formulations in Book 7 of the *Physics* and refers his students to Bradwardine's *De proportione motuum* for an alternative view. Rugerius likewise discerns difficulties with Aristotle's rules and, as already noted, sends his students to Toletus and Soto for more satisfactory treatments of the ways in which velocity varies at the beginning, middle, and end of motion (Wallace 1987a, nn. 22–24).

Of the natural philosophers who taught physics at the Collegio Romano after Rugerius down to 1626, I have thus far located *reportationes* of lectures by four other Jesuit professors: Robert Jones, an Englishman, who taught in 1592–93, Stefano Del Bufalo, who taught in 1596–97 and again in 1598–99; Andreas Eudaemon-Ioannis, who taught in the intervening year 1597–98, while Del Bufalo had the course in metaphysics; and Fabiano Ambrosio Spinola, who taught in 1625–26. Of these, the treatments of the first and the last, Jones and Spinola, show less concern with the calculatory tradition. Del Bufalo, on the other hand, has a rather full discussion of alteration, degrees of qualities, intension and remission of forms, and action and reaction – in the last of which he mentions the teaching of the *Calculator* and contrasts it with those of Pomponazzi, Buccaferreus, Flaminio Nobili, Franciscus Neritonensis, and Zabarella. In his discussions of *gravitas* and *levitas*, moreover, he mentions the *Parisienses* and compares their teachings with those of Geronimo Borro and Buonamici – and this in 1597, the year in which Buonamici's *De motu* had just appeared. It is noteworthy that all of Del Bufalo's notes located thus far are in the National Library at Lisbon, where they had been taken from the Jesuit college at Evora, having been sent there from Rome by October of 1603, as recorded in one of the codices containing them (Wallace 1987a, nn. 25–28).

The other professor who deserves mention for his calculatory interests is Eudaemon, who, as already mentioned, had the course in natural philosophy in 1597–98. In addition to his lectures on the *Physics*, *De caelo*, and *De generatione*, he left a *tractatus* in two books on action and passion and a *quaestio* on the motion of projectiles, both of which are written in the calculatory manner. As I have pointed out in my *Galileo and His Sources*, Eudaemon is of some importance for the fact that he discussed "the ship's mast" experiment with Galileo at Padua, and, along with Biancani, also teaching there, could have influenced Galileo's use of calculatory terms in his *De motu accelerato* fragment and later writings (Wallace 1984a; 1987a, nn. 29–30).

The first book of Eudaemon's work on natural agency, entitled simply *Tractatus primus*, is prefaced by five definitions and nine suppositions; it then develops twenty-one propositions, with proofs and corollaries, all relating to the ways in which qualities come to be mathematically distributed as a result of such agency, with occasional geometrical diagrams in the margins illustrating the text. Noteworthy among the definitions are the third and the fifth, the third stating that "something is said to be distributed uniformly difformly when it diminishes in the same ratio as the distance increases," and the fifth explaining how quantitative attributes can be ascribed to a quality that is uniformly difformly distributed. Following the definitions Eudaemon begins his suppositions, which he prefaces with the note:

> Because the matters with which we shall be concerned are physical, it is necessary to take some propositions from our physical disputations that can be presupposed as principles in this treatise. Things that are commonly conceded in physical science or are sufficiently proved and explained may be seen in our disputations on *De generatione* and on the *Physics*. And since this treatise is principally mathematical, it will not be necessary to note and prove propositions that come from mathematics.

This notation, and the nine suppositions that follow it, are important for the fact that they show Eudaemon adopting the stance of a mathematician and attempting to develop a mathematical physics of natural agency, even though he was a philosopher entrusted with the main sequence of courses at the Collegio. Also noteworthy is Eudaemon's first supposition, which reads as follows:

> We presuppose that every natural agent acts *uniformiter difformiter* on a quantified subject when applied to it. Physicists commonly concede this, at least with respect to some parts of the sphere of activity, because we see that when close the agent acts more vehemently and when farther way less so; therefore, the closer the greater, the farther the lesser; therefore, as the distance increases the action decreases; therefore the action is uniformly difform.

Noteworthy here and throughout the treatise is the preoccupation with the expression *uniformiter difformis* as applicable to natural agency (Wallace 1987a, nn. 31–33).

The second book of this same treatise is titled *De iis quae in actione et passione physica contingunt* and it begins, like the first, with definitions, ten in number, then notes a single supposition, and concludes with proofs of thirty-one propositions, some of which contain substantial numbers of corollaries. The reason for this development is not transparently clear at first reading, but it all becomes intelligible when we get to the *Quaestio de motu proiectorum* that follows immediately after the second book. The entire treatise on natural agency had occupied fifty-one closely written folios; that on the motion of projectiles coming after it takes up seventy-two more. Divided into three articles, it inquires first whether the projector moves the projectile immediately at a distance, then whether the *vis movens* is within the projectile itself, and finally whether the *virtus movens* is located in the medium, and if so, how (Wallace 1987a, n. 34). Somewhat surprisingly, considering the fact that his predecessors at the Collegio had all adopted the impetus explanation of projectile motion, Eudaemon ends up by rejecting an impetus in the projectile and by putting the *virtus movens* in the air. I have not yet analyzed his arguments in detail, but I suspect that his reason for doing so is to subsume projectile motion under natural agency so as to show that it slows down uniformly difformly. This, we may recall, was Soto's position, for he held that falling motion is accelerated and projectile motion decelerated in the same quantitative way, namely, *uniformiter difformiter.*

If such was Eudaemon's thesis in this manuscript, undated but probably written in 1599, his discussions with Galileo at Padua around 1604 take on special significance. At that time Galileo was looking for a principle on which he could construct his new science of motion, as we know from his letter to Paolo Sarpi. His telling Eudaemon that he had experimented with a stone dropped from the mast of a ship first at rest and then in motion shows that both were still interested in the problem of impetus. Eudaemon could therefore have been a source that directed Galileo's attention around 1604 to calculatory treatments of uniform acceleration and deceleration, later to be reflected in the *De motu accelerato* fragment on which the *Two New Sciences* would be based (Wallace 1987a, nn. 35–37).

Let us look back, then, at the situation at the Collegio Romano from the time of Pererius, say 1566, when he taught, or 1576, when his textbook was published, to Eudaemon in 1599. In all of that time there were many references to the *Calculatores* and *Parisienses* and how they impacted on theses in natural philosophy. Not one, however, is to be found in a printed text – all occur only in manuscript sources. It is not surprising, therefore, that influences deriving from this tradition have thus far been overlooked by scholars and so have not been seen as a significant factor in the growth of mathematical physics among the Roman Jesuits toward the end of the sixteenth century.

THE JESUIT TRADITION IN PORTUGAL

To move now to the Iberian peninsula, a situation similar to that at the Roman College existed in the Jesuit colleges there, particularly in those at Evora and Coimbra. The Coimbran *Cursus philosophicus* was a five-volume course, first published at Coimbra between 1592 and 1605 and reprinted often thereafter. My researches have shown that natural philosophy in Portugal became less technical and mathematical from the end of the sixteenth century onward, and this possibly explains why there is no conspicuous use of calculatory terminology in the famous *Cursus*. A goodly number of manuscripts from Evora and Coimbra dating from the 1570s and 1580s are still extant, however, and these show the same patterns deriving from the *Calculatores* and the *Parisienses* as do the lecture notes from the Collegio Romano.

Lectures on the *Physics* and *De caelo* for the years 1570, 1582, 1587, and 1588 by professors named Juan Gomez de Braga, Luis de Cerqueira, Antonio del Castelbranco, and Manuel a Lima respectively are all extant. Some of these Jesuits taught at Evora, others at Coimbra, but the substance of their notes is all the same; in some cases the wording is repeated almost exactly, suggesting a transmission of notes from one professor to another. In addition, notes from a Trinitarian, Marcus de Moura, who taught at Lisbon in 1588 are available, and his lectures are substantially the same as those given by Manuel a Lima at Evora in the same year. The same could be said of an anonymous set of lectures on the *Physics*, *De caelo*, and *Meteorology* given at Coimbra in 1580 (Wallace 1987a, nn. 40–42).

The anonymous lectures of 1580 are a good place to start, for their author gives a key to the source of most of the materials the others contain. The fifth chapter of his commentary on the seventh book of the *Physics* begins with two questions: (1) whether the velocity of local motion is to be ascertained from the quantity of space it traverses as from an effect, and (2) whence the velocity of motion is to be judged as from a cause. Following his replies to these queries the author writes: "These last two questions are treated more fully by Domingo de Soto and can be studied there. For this reason, and especially because of limitations of time, we will pass over them quickly." And his reply to the first question indicates the extent of his dependence on Soto, which I give in the slightly clearer formulation of Cerqueira, who repeated this material at Coimbra two years later, in 1582:

> Sometimes the mobile is moved so difformly with respect to time that, taken [any] part of time in which it moves, the velocity it has at the middle instant will exceed the velocity it had or will have at one terminal instant of that time by the same amount as it is exceeded by the velocity it had or will have at another [terminal instant]. Such a motion is said to be *uniformiter difformis* with respect to time, and it is found in heavy and light bodies when they move naturally, since the more they depart from their starting point the greater is the velocity with which they move.

This, of course, is the teaching developed by Soto around 1550, which is reiterated in most of these lecture notes preserved in Portugal throughout the 1570s and 1580s. It is further explained and extended to projectile motion by Cerqueira, and by Manuel a Lima again in 1588, in the following terms:

> It is customary to ask at this place why it is that things that are moved naturally in rectilinear motion are moved more swiftly at the end than at the beginning of their motion, whereas those that are moved violently are moved more swiftly at the beginning . . . . The reason for this is that, just as the force that exists in the hand of the thrower when

conjoined with the stone . . . impresses on the stone a certain impulse that moves it when separated from the hand of the thrower, so also gravity and levity, impelling the heavy and light thing to its natural place, impresses by such motion a certain impulse through whose agency the motion of the heavy and light thing is made swifter. And this impetus gets more intense as the heavy and light objects come closer to their natural places, which is to be understood in terms of the relation of each to the *terminus a quo*. For if one and the same stone were now to descend from the middle of a tower and later from its top, it would descend much more swiftly at the end of the later motion than at the end of the earlier. For the longer the space that is traversed the greater is the impetus impressed by levity and gravity throughout the motion, since it is continually intensified until the thing arrives at its natural place. And since this impetus effects in the heavy or light thing a motion similar to that which arises in it from gravity or levity, Aristotle referred to it as an increase of gravity and levity; others, however, speak of it as accidental gravity and levity, since it is lost as soon as the motion stops.

I give this as one example relating to the ratios of motions; similar materials relating to action and reaction, wherein the opinions of Heytesbury and the *Calculator* are discussed, could also be mentioned. But perhaps this is sufficient for present purposes to show evidences of a Jesuit mathematical tradition on the Iberian peninsula in the latter part of the sixteenth century (Wallace 1987a, nn. 43–46).

To return briefly to Italy, I would add that Biancani, who had studied under Clavius at Rome in the 1590s, wrote detailed defenses and justifications of mathematics and mathematical physics as sciences in the Aristotelian sense, wherein he shows considerable competence as both a philosopher and a mathematician. These he explicitly directed against Pererius and the authors of the Coimbran *Cursus philosophicus*. Biancani taught principally at Parma, where Giambattista Riccioli was in turn his student. And Riccioli is of some importance for his verification of Galileo's experiments on falling bodies. In his *Almagestum novum* of 1651 Riccioli recounts that he had first started experimental work in this field with two other Jesuits in 1629, and then with yet another in 1634. At that time he obtained permission, he says, to read Galileo, whom he first thought to be in error but later found to be correct. Of his early work Riccioli writes that

at that time I had not yet come to the better and more evident experiments manifesting not only an inequality in the motion of heavy bodies but the true increment of their velocity, increasing *uniformiter difformiter* toward the end of the motion.

What is interesting here is Riccioli's use of Parisian terminology deriving from Soto when describing the results to which he had finally come. This seems to me a fairly good indication that such terminology had been part of his training too, and persisted in his mind to the middle of the seventeenth century, i.e., to 1651 (Wallace 1987a, pp. 58–62).

CONCLUDING REMARKS

From all these indications it would seem that the fourteenth-century work at Oxford and Paris on *calculationes*, transmitted via Spain and Portugal to Rome and other centers where Jesuits had colleges, figured in the rise of mathematical physics at the beginning of the seventeenth century. The circumstances of this transmission may help to clear up two problems that have bothered historians of science. The first of these is the disparity between Galileo's use of calculatory language and that of the Mertonians, which has recently been analyzed by Edith Sylla (Sylla 1986). Such disparity is readily understandable when one considers that Galileo acquired that language at several removes from its initial formulators. The second is the lack of a consistent attitude on the part of the Jesuits toward the use of mathematics in the study of nature. This becomes intelligible in terms of the tensions that developed within the Order between the mathematicians and the philosophers, and the censorship that was invoked to present a 'united front' to the outside world. Vallius had difficulty with censors within his own Order when he attempted to have his Collegio Romano lectures on logic and natural philosophy published in the early 1600s, and we know that Biancani ran into the same difficulty with censorship when he wrote in 1615 and 1620 in support of Galileo (Wallace 1984a, pp. 141–48). Invariably the theologians and the leadership within the Order sided with conservative confreres among their philosophers rather than with progressive confreres among their mathematicians whenever Church teaching was involved. As a result, the period between about 1560 and 1650 presents a somewhat ambivalent picture of the Jesuits' role in the development of mathematical physics. But the manuscript record, official positions aside, shows that solid progress was being made during those decades, wherein foundations were laid for later important contributions to the sciences from within the Society of Jesus.

A yet more important result of these researches for this conference is their vindication of Duhem, as contrasted with Koyré, in the work of these French historians on the origins of modern mechanics. Koyré's fortunes have declined in recent years with the discoveries by Stillman Drake and others of the extensive experimental program on which

Galileo had embarked between 1604 and 1610. This has sounded the deathknell for Koyré's appraisal of Galileo as a Platonist or rationalist who had no need of experiment to found his 'new science'. My own work tarnishes Koyré's image a bit more, for it shows that his neglect of medieval and scholastic sources vitiated much of the reasoning behind his *Études galiléennes*, the part relating to Benedetti alone excepted. But if Koyré has been devalued, as it were, the same cannot be said of Duhem. Duhem's emphasis on Soto, it turns out, was well founded. One would no longer wish to maintain that Soto was the proximate source of Galileo's science. But whether one traces Soto's influence through the Jesuits in Italy or in Spain and Portugal, or by a parallel route through Benedetti, there seems little doubt that Soto played a pivotal role in promoting a mathematical analysis of local motion.[3]

NOTES

[1] The further development of this essay is a conflation of two previously published studies, the first focusing on the Benedetti–Jesuit relationship (Wallace 1987b) and the second on influences on the Jesuits that derived from Domingo de Soto (Wallace 1987a). Neither of these studies, on the one hand, is readily available; both, on the other, are heavily documented with references to source materials and to Latin texts. Since readers of this journal are not primarily interested in history, I have pruned much of the documentation from my presentation here. However, to make available a detailed justification of my thesis for those who might be interested, I have included parenthetical references to the footnotes of the two studies in the body of the text.

[2] The numbers here and following continue the page enumeration of the previous citation in the text.

[3] Preparation of this essay was supported in part by a grant from the National Endowment for the Humanities, an independent agency of the U.S. government.

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