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NON-ABELIAN ELECTRODYNAMICS AND THE VACUUM $\boldsymbol{B}^{(3)}$

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By using an O(3) gauge group, a non-Abelian theory of vacuum electrodynamics is developed in which the newly discovered longitudinal vacuum fields $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ appear self-consistently with the usual plane waves $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$, $\boldsymbol{E}^{(1)}$, and $\boldsymbol{E}^{(2)}$ in the circular basis (1), (2), (3), a complex representation of space. Using the charge quantization condition $\hbar\kappa = eA^{(0)}$ the vacuum Maxwell equations are given in the non-Abelian representation.

Key words: $\boldsymbol{B}^{(3)}$ field, O(3) gauge.

1. INTRODUCTION

It is well known that three-dimensional space can be described in the complex circular basis [1–3]:

$$\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)*} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \qquad \boldsymbol{e}^{(3)} = \boldsymbol{k}, \tag{1}$$

where $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$, and $\boldsymbol{e}^{(3)}$ are unit vectors related to the usual Cartesian unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$. The circular basis (1), (2), and (3) is the geometrical reason why there exists in free space the newly discovered [4–12] longitudinal vacuum field $\boldsymbol{B}^{(3)}$. The latter is related to the usual plane waves $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ through a cyclically symmetric, non-Abelian, Lie algebra [6]:

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$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \qquad \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} = iB^{(0)}\boldsymbol{B}^{(1)*}, \qquad \boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} = iB^{(0)}\boldsymbol{B}^{(2)*}. \tag{2}$$

It is immediately clear that if $\boldsymbol{B}^{(3)}$ were zero, then so would be $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, resulting in the disappearance of electromagnetism. Here $B^{(0)}$ is the scalar magnitude of the magnetic flux density of the electromagnetic beam propagating in the axis (3) in the vacuum. It is possible, furthermore, to define precisely [6,7] the experimental conditions under which $\boldsymbol{B}^{(3)}$ can be isolated through its magnetization of an electron plasma [13,14]. The reason is that magnetization, $\boldsymbol{M}^{(3)}$, due to $\boldsymbol{B}^{(3)}$ is to first order in $B^{(0)}$ and therefore to order $I_0^{1/2}$, where I_0 is the power density of the circularly polarized beam (W m^{-2}). An $I_0^{1/2}$ profile is not obtainable from the plane waves $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ because magnetization at first order due to these waves averages to zero. Therefore the experimental observation of an $I_0^{1/2}$ profile signals conclusively and unequivocally the existence of $\boldsymbol{B}^{(3)}$ in the vacuum. These inferences arise [6] from the relativistic Hamilton-Jacobi equation of one electron (e) in the classical electromagnetic field represented by the four potential A_μ [15,16]. It has been shown, furthermore [6,12], that $\boldsymbol{B}^{(3)}$ emerges directly from the Dirac equation of the relativistic quantum theory of e in A_μ. These rigorous calculations [6] demonstrate that if $\boldsymbol{B}^{(3)}$ were zero, the well known intrinsic electron spin would not emerge from the Dirac equation and therefore could not exist. The interaction Hamiltonian between intrinsic electron spin and field is governed *entirely, and at first order in* $\boldsymbol{B}^{(3)}$, by the product of $\boldsymbol{B}^{(3)}$ with the electronic spin angular momentum itself. Similarly, the induced, classical, orbital, electronic angular momentum in the electromagnetic field is governed [6] entirely by $\boldsymbol{B}^{(3)}$ and $B^{(0)}\boldsymbol{B}^{(3)}$, respectively, at first and second order in $B^{(0)}$ (or at order $I_0^{1/2}$ and I_0). The spinning trajectory (intrinsic and orbital) of one electron in the field is therefore due to $\boldsymbol{B}^{(3)}$ which is related to $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ by Eqs. (2). Conversely, $\boldsymbol{B}^{(3)}$ is observable through this type of electronic trajectory, i.e., through the magnetization set up in an electron plasma by a pulse of electromagnetic radiation. The precise conditions under which the first order effect of $\boldsymbol{B}^{(3)}$ is observable are defined elsewhere [6,12], using microwave pulses of about 10 to 100 MW power at 30 GHz directed into a pyrex tube [13] producing and magnetizing an electron plasma from helium gas. It is overwhelmingly probable that the $I_0^{1/2}$ profile will be observed because it is the direct result [6,12] of the relativistic Hamilton-Jacobi equation itself.

In this Letter, we infer from the non-Abelian algebra (2) and the concomitant existence of $\boldsymbol{B}^{(3)}$ in the vacuum that the gauge group of vacuum electromagnetism is O(3), and not the conventional, planar, O(2) (or U(1)). In Sec. 2 the rigorous geometrical theory of gauges [17] is used to write down the vacuum Maxwell equations in a gauge group O(3). These are developed in Sec. 3 with a charge quantization condition [6],

$$\hbar\kappa = eA^{(0)}, \tag{3}$$

which is derived independently from the relativistic Hamilton-Jacobi equation of e in A_μ.

2. THE O(3) GAUGE GROUP

The representation of three dimensional space is cyclically symmetric and non-Abelian, and in consequence so is the Lie algebra (2). A view of vacuum electrodynamics based on an Abelian gauge group, O(2), cannot accommodate the physical existence of $\boldsymbol{B}^{(3)}$, which is orthogonal to the plane of definition of the group O(2). If we are to accept the arguments of the introduction, the gauge group of vacuum electromagnetism must become non-Abelian. The natural group to choose is the rotation group O(3) of three dimensional space [6], a group which governs the fundamental relations (1) between unit vectors, and the fundamental relations (2) between three *physical* fields. We refer to this henceforth as the O(3) gauge group. Having realized this, it becomes a simple matter to adapt the literature [17] on the rigorous geometrical theory of gauges to write down the vacuum Maxwell equations in this gauge group. This literature is, however, expressed in the language of particle physics, in which the ordinary (1), (2), (3) space is referred to as *isospin space*. In what follows, the O(3) space is the ordinary three dimensional configuration space in the circular basis defined in Eq. (1).

The inhomogeneous Maxwell equations in an O(2) gauge group are the familiar

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu} = 0, \tag{4}$$

where $F_{\mu\nu}$ is the ordinary electromagnetic field four-tensor in the vacuum. In Minkowski notation, as used, for example, by Jackson [16]: $x_\mu = (X, Y, Z, ict)$, as usual. In vector notation, Eq. (4) becomes, in S.I. units,

$$\nabla \cdot \boldsymbol{E} = 0, \qquad \nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}. \tag{5}$$

In the O(3) gauge group, on the other hand, the equivalent of Eq. (4) is [6], in the circular basis of Eq. (1),

$$D_\nu \boldsymbol{G}_{\mu\nu} = \boldsymbol{0}, \qquad D_\nu = \frac{\partial}{\partial x_\mu} + \frac{e}{\hbar}A_\mu, \tag{6}$$

where e is the electronic charge and $\hbar$ the Dirac constant. Here $\boldsymbol{G}_{\mu\nu}$ is a four tensor in Minkowski space-time and a vector in the configuration space (1), (2), (3). This type of generalization was first introduced by Yang and Mills [17] and has been widely employed in contemporary field-particle theory [17]. The space in which $\boldsymbol{G}_{\mu\nu}$ is a vector is the isospin space. Usually this is an abstract O(3) space [17], but here it becomes the three-dimensional O(3) configuration space itself, expressed in the circular basis (1), (2), and (3). Therefore Eq. (6) has three circular components,

$$D_\nu G_{\mu\nu}^{(1)} = 0, \qquad D_\nu G_{\mu\nu}^{(2)} = 0, \qquad D_\nu G_{\mu\nu}^{(3)} = 0, \tag{7}$$

and we see that there are three field equations in the O(3) gauge group, one for each polarization (1), (2), and (3). Polarizations (1) and (2) are transverse [4–12] and polarization (3) is longitudinal. The geometrical reason for the existence of these three equations is simply that space has three dimensions. Similarly, the homogeneous Maxwell equations in the gauge group O(3) become

$$D_\rho \boldsymbol{G}_{\mu\nu} + D_\mu \boldsymbol{G}_{\nu\rho} + D_\nu \boldsymbol{G}_{\rho\mu} = \boldsymbol{0}. \tag{8}$$

In the following section, we simplify these equations using the charge quantization condition [6], which is derived independently from the classical Hamilton-Jacobi equation of e in A_μ.

3. THE CHARGE QUANTIZATION CONDITION

The O(3) theory of Sec. 3 shows [6] that

$$G_{\mu\nu}^{(3)*} = \partial_\mu A_\nu^{(3)*} - \partial_\nu A_\mu^{(3)*} - i\frac{e}{\hbar}\left[A_\mu^{(1)}, A_\nu^{(2)}\right] \tag{9}$$

which reduces to

$$\boldsymbol{B}^{(3)} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{10}$$

However, by re-expressing Eqs. (2) in terms of vector potentials, it is easily shown [6] that

$$\boldsymbol{B}^{(3)} = -i\frac{\kappa}{A^{(0)}}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{11}$$

and comparison of Eqs. (10) and (11) gives the charge quantization condition

$$\hbar\kappa = eA^{(0)}. \tag{12}$$

The physical meaning of Eq. (12) has been demonstrated [18] by considering one electron in the classical electromagnetic field and extending these considerations using the Planck quantization condition, the familiar $En = \hbar\omega$, which defines the photon as the quantum of energy of electromagnetic radiation. These considerations have shown [6,12], that in the relativistic limit

$$\omega < \frac{e}{m}B^{(0)}, \tag{13}$$

the charge quantization condition emerges directly from the Hamilton-Jacobi equation of e in A_μ. Equation (12) means that the momentum of an electron of mass m accelerated infinitesimally close to the speed of light is both $eA^{(0)}$ and $\hbar\kappa$. Radiation from such an electron becomes indistinguishable, classically, from the electromagnetic field and concomitant photon, as first shown by Jackson [16]. This author showed that under such conditions there cannot be a longitudinal electric field, but he did not consider the possibility of $\boldsymbol{B}^{(3)}$. We shall show that our $\boldsymbol{B}^{(3)}$ is indeed connected to an imaginary and unphysical $i\boldsymbol{E}^{(3)}$ through Eq. (7).

The condition (12) is that between the scalar parts of the four-vector relation

$$\frac{\partial}{\partial x_\mu} = \frac{e}{\hbar} A_\mu, \tag{14}$$

and it follows straightforwardly [6] that the O(3) Maxwell equations become

$$\frac{\partial F_{\mu\nu}^{(1)}}{\partial x_\nu} = 0, \qquad \frac{\partial F_{\mu\nu}^{(2)}}{\partial x_\nu} = 0, \qquad \frac{\partial F_{\mu\nu}^{(3)}}{\partial x_\nu} = 0. \tag{15}$$

The first two are the ordinary plane wave equations, while the third links the physical and real $\boldsymbol{B}^{(3)}$ to the unphysical and imaginary $i\boldsymbol{E}^{(3)}$, showing that there is no Faraday induction due to $\boldsymbol{B}^{(3)}$ in the absence of matter [6].

4. DISCUSSION

By considering an appropriately three dimensional gauge group, it has been shown that the Maxwell equations in three space self consistently yield the presence of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ through the charge quantization condition, which can be understood in terms of the equivalence of a photon and an electron accelerated infinitesimally near to the speed of light by an incoming radiation field. Experimental evidence for these assertions is available in principle by using pulses of intense microwave radiation to magnetize an electron plasma. Using this technique, the $I_0^{1/2}$ profile of $\boldsymbol{M}^{(3)}$ reveals the presence in the vacuum of the physical field $\boldsymbol{B}^{(3)}$. The concomitant unphysical field $i\boldsymbol{E}^{(3)}$ is related to $\boldsymbol{B}^{(3)}$ through Eq. (15), which has the same structure as the ordinary Faraday induction law, but since $i\boldsymbol{E}^{(3)}$ is always pure imaginary, no Faraday induction due to a hypothetically changing $\boldsymbol{B}^{(3)}$ occurs in the absence of matter, e.g. electrons, whereupon induction occurs through the induced magnetic field $\mu_0\boldsymbol{M}^{(3)}$, where μ_0 is the vacuum permeability.

These methods self consistently show the presence of the novel and centrally important field $\boldsymbol{B}^{(3)}$ in vacuum electromagnetic radiation.

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