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A NOTE ON THE RELATIVISTIC COVARIANCE OF THE B-CYCLIC RELATIONS

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It is shown that the Evans-Vigier modified electrodynamics is compatible with relativity theory.

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Recently a new version of non-Maxwellian theories of electromagnetism was proposed [1,2] by Evans and Vigier. As a matter of fact, the Evans-Vigier $\mathbf{B}^{(3)}$ theory includes a spin variable in the classical theory and presents itself as a straightforward development of the ideas of Belinfante, Ohanian, and Kim [3-5]. In the present note, I restrict myself to only one particular question, namely that of the relativistic covariance of this theory. I would not like to speak here about numerous other generalizations of the Maxwell's theory, referring the reader to a recent review [6] instead. All these theories are either strongly (albeit not always reasonably) criticized or ignored. Only in the nineties several new versions appeared at once, which now ensure that the question is justly getting serious and careful attention. The $\mathbf{B}^{(3)}$ model is not an exception. A list of works criticizing this theory appeared in Ref. [8], where the author, E. Comay, has added critical comments of his own [7,8]. A serious objection to the Evans-Vigier theory which was presented by Comay is his belief that the "modified electrodynamics" is not a relativistic covariant theory. Questions like these posed by Comay may arise in future analyses of the $\mathbf{B}^{(3)}$ theory because, as correctly indicated, some notational misunderstandings do exist in the work of Evans and Vigier. For this reason, these questions merit detailed answers.

According to [9, Eq. (11.149)], the Lorentz transformation

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rules for electric and magnetic fields are

$$\mathbf{E}' = \gamma(\mathbf{E} + c\boldsymbol{\beta}\times\mathbf{B}) - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{E}), \tag{1a}$$

$$\mathbf{B}' = \gamma(\mathbf{B} - \boldsymbol{\beta}\times\mathbf{E}/c) - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{B}), \tag{1b}$$

where $\boldsymbol{\beta} = \mathbf{v}/c$, $\beta = |\boldsymbol{\beta}| = \tanh\phi$, $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\phi$, with ϕ being the parameter of the Lorentz boost. We shall further use the natural unit system with $c = \hbar = 1$. After introducing the spin matrices $(\mathbf{S}_i)_{jk} = -i\epsilon_{ijk}$ and deriving the relevant relations

$$(\mathbf{S}\cdot\boldsymbol{\beta})_{jk}\mathbf{a}_k \equiv i[\boldsymbol{\beta}\times\mathbf{a}]_j\,,$$

$$\boldsymbol{\beta}_j\boldsymbol{\beta}_k \equiv [\boldsymbol{\beta}^2 - (\mathbf{S}\cdot\boldsymbol{\beta})^2]_{jk}\,,$$

one can rewrite Eqs. (1a,1b) to read

$$\mathbf{E}'_i = \left(\gamma\mathbb{1} + \frac{\gamma^2}{\gamma+1}\left[(\mathbf{S}\cdot\boldsymbol{\beta})^2 - \boldsymbol{\beta}^2\right]\right)_{ij}\mathbf{E}_j - i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{B}_j\,, \tag{2a}$$

$$\mathbf{B}'_i = \left(\gamma\mathbb{1} + \frac{\gamma^2}{\gamma+1}\left[(\mathbf{S}\cdot\boldsymbol{\beta})^2 - \boldsymbol{\beta}^2\right]\right)_{ij}\mathbf{B}_j + i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{E}_j\,. \tag{2b}$$

First one should mention that these equations are valid for electromagnetic fields of various polarizations. Next, Eqs. (2a) and (2b) preserve properties of the vectors $\mathbf{B}$ (axial) and $\mathbf{E}$ (polar) with respect to space inversion operations. Furthermore, if we consider other field configurations, like $\phi_{L,R} = \mathbf{E}\pm i\mathbf{B}$ or $\mathbf{B}\mp i\mathbf{E}$, the Helmoltz bivectors, which may already not have definite properties with respect to the space inversion operation (viz., they transform like $\phi_R \leftrightarrow \pm\phi_L$), we obtain

$$(\mathbf{B}'\pm i\mathbf{E}')_i = \left(1 \pm \gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}(\mathbf{B}\pm i\mathbf{E})_j\,, \tag{3}$$

which obviously transform like the right and the left parts of the Weinberg's $2(2S+1)$-component field function [10].

Now we can consider the question of the Lorentz transformations for transverse modes $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$, $\mathbf{E}^{(1)}$, $\mathbf{E}^{(2)}$ and hence draw a correct conclusion about the transformation of $\mathbf{B}^{(1)}\times\mathbf{B}^{(2)} = \mathbf{E}^{(1)}\times\mathbf{E}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*}$. In the first frame, transverse modes of the electromagnetic field have the following explicit forms [11], where is $\phi = \omega t - \mathbf{k}\cdot\mathbf{r}$:

$$\mathbf{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}(i\mathbf{i}+\mathbf{j})e^{i\phi}, \quad \mathbf{E}^{(1)} = -i\mathbf{B}^{(1)} = \frac{E^{(0)}}{\sqrt{2}}(\mathbf{i}-i\mathbf{j})e^{i\phi}, \tag{4a}$$

$$\mathbf{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}(-i\mathbf{i}+\mathbf{j})e^{-i\phi}, \quad \mathbf{E}^{(2)} = +i\mathbf{B}^{(2)} = \frac{E^{(0)}}{\sqrt{2}}(\mathbf{i}+i\mathbf{j})e^{-i\phi}. \tag{4b}$$

We have implied that, for free-space circularly-polarized radiation, $B^{(0)} = E^{(0)}$. Pure Lorentz transformations (without inversions) do not change the sign of the phase of the field functions, so we should consider separately the properties of the set $\mathbf{B}^{(1)}$, $\mathbf{E}^{(1)}$, which can be regarded as the negative-energy solutions in QFT (cf. the Dirac case [12]), and another set, $\mathbf{B}^{(2)}$, $\mathbf{E}^{(2)}$, the positive-energy solutions. The opposite interpretation is also possible and in fact was used by E. Comay ($\phi \to -\phi$). But these issues are not relevant in the present discussion. Thus, in the present framework, one can deduce from Eqs. (2) that

$$\mathbf{B}_i^{(1)\prime} = \left(1 + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}\mathbf{B}_j^{(1)} + i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{E}_j^{(1)}, \tag{5a}$$

$$\mathbf{B}_i^{(2)\prime} = \left(1 + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}\mathbf{B}_j^{(2)} + i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{E}_j^{(2)}, \tag{5b}$$

$$\mathbf{E}_i^{(1)\prime} = \left(1 + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}\mathbf{E}_j^{(1)} - i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{B}_j^{(1)}, \tag{5c}$$

$$\mathbf{E}_i^{(2)\prime} = \left(1 + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}\mathbf{E}_j^{(2)} - i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{B}_j^{(2)}. \tag{5d}$$

On using relations between transverse modes of electric and magnetic field (4), one can formally write[1]

$$\mathbf{B}_i^{(1)\prime} = \left(1 + \gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}\mathbf{B}_j^{(1)}, \tag{6a}$$

[1] One would wish to study properties of this physical system with respect to the space inversion operation. Since explicit forms of transverse modes of electric field in the first frame are proportional (with imaginary coefficients) to the transverse modes of the magnetic field (4), some fraction of $\mathbf{E}^{(k)}$ or $\mathbf{B}^{(k)}$ can be formally replaced by a vector of another parity (as we are doing in the consequence of calculations). Furthermore, one can take any combinations of Eq. (5a) and Eq. (5c) multiplied by an arbitrary phase factor, or

$$\mathbf{B}_i^{(2)\prime} = \left(1 - \gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij} \mathbf{B}_j^{(2)} , \qquad (6b)$$

$$\mathbf{E}_i^{(1)\prime} = \left(1 + \gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij} \mathbf{E}_j^{(1)} , \qquad (6c)$$

$$\mathbf{E}_i^{(2)\prime} = \left(1 - \gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij} \mathbf{E}_j^{(2)} . \qquad (6d)$$

We still observe that $\mathbf{B}^{(2)}$ can be related to $\mathbf{B}^{(1)}$ by a unitary matrix:

$$\mathbf{B}^{(2)} = U\mathbf{B}^{(1)} = e^{-2i\phi} \begin{pmatrix} 0 & -i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{B}^{(1)} \quad . \qquad (7)$$

Since this unitary transformation results in a change of the basis of spin operators only, we deduce that the concepts of properties of some geometrical object with respect to Lorentz transformations and with respect to space-inversion transformations can be simultaneously well-defined concepts only after one defines corresponding "bispinors" of the $(j,0) \oplus (0,j)$ representations and keeps the same spin basis for both parts of the bispinor; see also [13] for the example in the $(1/2,0) \oplus (0,1/2)$ representation.

We still maintain: (a) The proportionality of $\mathbf{E}^{(k)\prime}$ to $\mathbf{B}^{(k)\prime}$ with imaginary coefficients are preserved, and (b) $\mathbf{B}^{(1)}$ and $\mathbf{E}^{(1)}$ in Eqs. (6a-6d) transform like $\mathbf{B} + i\mathbf{E}$ of the Cartesian basis, i.e., like the right part of the Weinberg field function, and $\mathbf{B}^{(2)}$ and $\mathbf{E}^{(2)}$ like $\mathbf{B} - i\mathbf{E}$, i.e., like the left part of the Weinberg field function. With the aid of the above rules, finding the transformed 3-vector $\mathbf{B}^{(3)\prime}$ is only an algebraic exercise. One has

$$\begin{aligned}\mathbf{B}^{(1)\prime} \times \mathbf{B}^{(2)\prime} &= \mathbf{E}^{(1)\prime} \times \mathbf{E}^{(2)\prime} \\ &= i\gamma (B^{(0)})^2 (1 - \boldsymbol{\beta}\cdot\hat{\mathbf{k}}) \left[\hat{\mathbf{k}} - \gamma\boldsymbol{\beta} + \frac{\gamma^2(\boldsymbol{\beta}\cdot\hat{\mathbf{k}})\boldsymbol{\beta}}{\gamma+1}\right] ,\end{aligned} \qquad (8)$$

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that of Eq. (5b) and Eq. (5d) multiplied by a phase factor. The parity properties of the field functions in the general case would be different on the left-hand side and on the right-hand side of resulting equations. Generally speaking, the notation (6) is used in this paper only for simplification of calculations. In fact, one can also proceed further with the forms (5).

where $\hat{\mathbf{k}}$ is the orth vector of the axis OZ. We know that the longitudinal mode in the Evans-Vigier theory is defined as $\mathbf{B}^{(3)} = \mathbf{B}^{(3)*} = B^{(0)}\mathbf{k}$. Thus, considering that $B^{(0)}$ transforms as the zero-component of the four-vector and $\mathbf{B}^{(3)}$ as space components of the four-vectorxi [9, Eq. (11.19)], i.e.,

$$B^{(0)\prime} = \gamma(B^{(0)} - \boldsymbol{\beta}\cdot\mathbf{B}^{(3)}), \tag{9a}$$

$$\mathbf{B}^{(3)\prime} = \mathbf{B}^{(3)} + \frac{\gamma-1}{\beta^2}(\boldsymbol{\beta}\cdot\mathbf{B}^{(3)})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}B^{(0)}, \tag{9b}$$

we find from (8) that the relation between transverse and longitudinal modes preserves its form:

$$\mathbf{B}^{(1)\prime} \times \mathbf{B}^{(2)\prime} = iB^{(0)\prime}\mathbf{B}^{(3)*\prime}. \tag{10}$$

A reader interested in these matters can work to prove the covariance of other cyclic relations [1,11]. Next, when the boost is made in the x direction, we obtain

$$\mathbf{B}^{(3)\prime} = \left(-\gamma\beta B^{(0)}, 0, B^{(0)}\right), \text{ in the coordinates of the old frame,} \tag{11a}$$

$$\mathbf{B}^{(3)\prime} = (0, 0, B^{(0)\prime}) \qquad \text{in the coordinates of the new frame.} \tag{11b}$$

It is seen that the transformations (9) are the ones for a light-like 4-vector of the Minkowski space, formed by $(B^{(0)}, \mathbf{B}^{(3)})$. They are similar (while not identical) to the transformation rules for the spin vector [9, Eq. (11.159)]. The difference with that consideration of a massive particle is caused by the impossibility to find a rest system for the photon, which is believed to move with the invariant speed c. Nevertheless, some relations between the concept of the Pauli-Lubanski vector of the antisymmetric tensor fields and the $\mathbf{B}^{(3)}$ concept have been derived elsewhere [14, 6,15].

I would like to indicate the reasons why Comay achieved the opposite, incorrect result:

(1) Obviously, one is *not* allowed to identify B_z with $\mathbf{B}^{(3)}$ (as the authors of previous papers did; see, e.g., formula (6) in Ref. [7]), since the first is a component of an antisymmetric tensor and the second is a 3-vector quantity, a part of a 4-vector. They are different geometric objects. Of course, the Poynting vector must be perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, the Cartesian 3-vectors, whose components define the antisymmetric tensor field. $\mathbf{B}^{(3)}$ is a vector of different nature, which, in its turn, forms an "isotopic" vector with $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$

in a circular *complex* basis[2] and whose physical effect is similar to that of the Cartesian **B**, namely, magnetization.

(2) One must not forget that $B^{(0)}$ is not a *scalar quantity*; it is a *zero component* of a 4-vector. Thus Comay's "appropriate units" would also transform from the first to the second frame.[3]

(3) As we have found, the axial 3-vector $\mathbf{B}^{(3)}$ is always aligned with the OZ axis in all frames like the Poynting vector (polar) is, provided that the ordinary electric and magnetic fields lie in the XY plane in these frames. If this is not the case, one can always achieve that by rotation, using the unitary matrix. So, while the questions raised by Comay in the paper [7] may be useful for deeper understanding of the Evans-Vigier theory and relativity theory, his conclusion is *unreasonable*. Briefly referring to the paper [8a], let me note that, in my opinion, the $\mathbf{B}^{(3)}$ field is a property of a single photon and, when considering the many-photon problem with various types of polarizations in a superposition, the question whether the circulation of this vector would be different from zero (?) must be approached more carefully. Furthermore, the applicability of the dynamical equations to this vector, which Comay refers to, is not obvious to me.

Our conclusion follows in a straightforward manner: The $\mathbf{B}^{(3)}$ Evans-Vigier modified electrodynamics is a relativistic covariant theory if it is mathematically interpreted correctly. Indeed, this construct may be the simplest and most natural classical representation of particle spin. The B-cyclic relations manifest relations between Lorentz group generators that account for the angular momentum [1,2,14,11]. Moreover, as was written by E. Comay himself, "the modified electrodynamics relates its longitudinal magnetic field $\mathbf{B}^{(3)}$ to the expectation value of the quantum mechanical intrinsic angular momentum operator" (but then he doubts it on the basis of obscure arguments!?). Therefore, recent criticism by L. D. Barron, A. D. Buckingham, E. Comay and others of the Evans-Vigier $\mathbf{B}^{(3)}$ theory appears to signify that these authors doubt the existence of the helicity variable for a photon (of additional discrete phase-free variable according to Wigner) and hence all related developments

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[2] Let me remind that the Cartesian basis is a pure *real* basis. On introducing *complex* vectors, we in fact enlarge the space; the number of independent components may increase, and the bases in general are not equivalent mathematically.

[3] Surprisingly, Comay noted this fact himself in his Sec. 4 but ignored it in his Sec. 3.

in physics since its discovery.[4] Undoubtedly such a viewpoint could lead to deep contradictions with experimental results (the spin-spin interaction, the inverse Faraday effect, the optical Cotton-Mouton effect, the Tam and Happer experiment (1977), etc.). Whether this criticism has sufficient reasons? On an equal footing, claims of "it is *unknowable*" and/or "*not fundamental*" seem to me to be based on the *unknowable* logic. As opposed to the authors of critical papers, with introduction of helicity in a classical manner [3,4.1,2,14,11], I believe, nobody wishes to doubt all theoretical results of QED and other gauge models. As a matter of fact, the existence of the spin variable and of different polarization states are accounted for in calculations of QED matrix elements. The proposed new development of Maxwell's theory does not signify the necessity of rejecting the results which have been obtained in areas where the old models work well. Moreover, it was recently shown [6] that both transverse and longitudinal *classical* modes of electromagnetism are naturally incorporated in the Weinberg formalism [10]. Thus, the aim of my work (and, I believe, also that of M. Evans and others who try to generalize Maxwell's formalism) is to systematize results on the basis of the Poincaré group symmetries, to simplify the theory, to unify interactions and, perhaps, to predict yet unobserved phenomena. One should follow the known advice of A. Einstein and W. Pauli to build a reliable theory on the basis of first principles, that is, on the basis of relativistic covariance (irreducible representations of the Poincaré group), and of causality. Some progress in this direction has already been made by authors starting from different viewpoints of each other [6,16,17].

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[4] Surprisingly, opposite claims (of the *pure* "longitudinal nature" of the *massless* antisymmetric tensor fields) by several authors are still further unexplained statements. This was pointed out as early as 1939 by F. Belinfante in his comment on the paper by Durandin and Erschow [*Phys. Z. Sowjet.* **12** (1937) 466]: "Three directions of polarization are possible for a Proca quantum with given momentum and charge." While the question for neutral particles (self/anti-self charge conjugate states) should be regarded properly in both the Majorana and the Dirac constructs, even in this case one can see at first sight that those claims of the pure "longitudinal nature" contradict a classical limit and the Weinberg theorem $B - A = \lambda$ [10b]. By the way, I do not understand the reasons for naming this field after the paper by M. Kalb and P. Ramond (1974). As a matter of fact, the antisymmetric tensor fields (and their "longitudinality") have earlier been investigated by many authors; in the first place by E. Durandin and A. Erschow (1937), F. Belinfante (1939), V. Ogievetskiĭ and I. Polubarinov (1966), F. Chang and F. Gürsey (1969), Y. Takahashi and R. Palmer (1970), and by K. Hayashi (1973).

I realize that further discussions of the Evans-Vigier model will be desirable. First of all, the question arises whether this theory implies a photon mass? And does this theory account for it (mass appears to manifest itself here in a somewhat different form)? If "yes" what is the massless limit of this theory? What are the relations between the $E(2)$ and $O(3)$ groups and the group of gauge transformations of the 4-potential electrodynamics [5] and of other gauge models? Can a massless field be particulate? Finally, what is mass itself? It is also necessary to somehow link this construct with ideas presented by L. Horwitz, M. Sachs, A. Staruszkiewicz, D. Ahluwalia, and myself. This should be the aim of forthcoming papers.

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Note Added. The main addition to the final version of Prof. Comay's Letter is the claim that the object $(B^{(0)}, \mathbf{B}^{(3)})$ does *not* form the (pseudo) 4-vector. I lightly touched on the question of the parity properties in the above Note. But, after writing it I have become aware of several more papers from his pen, which are aimed at the destruction of the Evans-Vigier modified electrodynamics. Moreover, questions concerning the properties of spin-1 massive/massless fields with respect to the discrete symmetry operations deserve much attention. Thus I think it would be useful to discuss matters of parity covariance of the $\mathbf{B}$ cyclic relations and other claims of Comay in more detail in a separate paper. Here I want to note only that, in the final version of his Letter [7] Comay has made conventions which require rigorous proof. As a matter of fact, he assumes that the complex-valued $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ are axial vectors, in the sense that they transform under space inversion operations as $\mathbf{B}^{(1)} \to \mathbf{B}^{(1)\prime}$ and $\mathbf{B}^{(2)} \to \mathbf{B}^{(2)\prime}$ (in his opinion!). This point (if one takes into account the Lorentz transformation rules of $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$) in fact shades the rules $\phi_R \leftrightarrow \phi_L$ with respect to the space inversion operation [13] in the $2(2j+1)$-component theories.

Finally, I want to note that my paper is *not* a Reply to Comay. In fact, it criticizes Evans' work as well.

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