The Fuzzy Syllogistic System*

Bora İ. Kumova and Hüseyin Çakır

Department of Computer Engineering
İzmir Institute of Technology
35430 İzmir, Turkey
{borakumova,huseyincakir}@iyte.edu.tr

Abstract. A categorical syllogism is a rule of inference, consisting of two premisses and one conclusion. Every premiss and conclusion consists of dual relationships between the objects M, P, S. Logicians usually use only true syllogisms for deductive reasoning. After predicate logic had superseded syllogisms in the 19^{th} century, interest on the syllogistic system vanished. We have analysed the syllogistic system, which consists of 256 syllogistic moods in total, algorithmically. We have discovered that the symmetric structure of syllogistic figure formation is inherited to the moods and their truth values, making the syllogistic system an inherently symmetric reasoning mechanism, consisting of 25 true, 100 unlikely, 6 uncertain, 100 likely and 25 false moods. In this contribution, we discuss the most significant statistical properties of the syllogistic system and define on top of that the fuzzy syllogistic system. The fuzzy syllogistic system allows for syllogistic approximate reasoning inductively learned M, P, S relationships.

Keywords: Syllogistic reasoning; fallacies; automated reasoning; approximate reasoning; human-machine interaction.

1 Introduction

Although syllogism were superseded by propositional logic [8] in the 19^{th} century, they are still matter of research. For instance philosophical studies have confirmed that syllogistic reasoning does model human reasoning with quantified object relationships [2]. For instance in psychology, studies have compared five experimental studies that used the full set of 256 syllogisms [5], [12] about different subjects. Two settings about choosing from a list of possible conclusions for given two premisses [6], [7], two settings about specifying possible conclusions for given premisses [9], and one setting about decide whether a given argument was valid or not [10]. It has been found that the results of these experiments were very similar and that differences in design appear to have had little effect on how human evaluate syllogisms [5]. These empirically obtained truth values for the 256 moods are mostly close to their mathematical truth ratios that we calculate with our algorithmic approach [11].

* This research was partially funded by the grant project 2009-İYTE-BAP-11.

G. Sidorov et al. (Eds.): MICAI 2010, Part II, LNAI 6438, pp. 418–427, 2010.

Although the truth values of all 256 moods have been analysed empirically, mostly only logically correct syllogisms were used for reasoning or modus ponens and modus tolens, which are generalisations of syllogisms [13]. Uncertain application environments, such as human-machine interaction, require adaptation capabilities and approximate reasoning [15] to be able to reason with various sorts of uncertainties. For instance, we know that human may reason purposefully fallacious, aiming at deception or trickery. Doing so, a speaker may intent to encourage a listener to agree or disagree with the speaker's opinions. For instance, an argument may appeal to patriotism, family or may exploit an intellectual weakness of the listener. We are motivated by the idea for constructing a fuzzy syllogistic system of possibilistic arguments for calculating the truth ratios of illogical arguments and approximately reason with them.

Firstly, the syllogistic system is discussed briefly, including its most significant statistical properties, followed by our main contribution, which is the fuzzy syllogistic system with its possible application for recognising fallacies and reasoning with them.

2 The Syllogistic System

A categorical syllogism can be defined as a logical argument that is composed of two logical propositions for deducing a logical conclusion, where the propositions and the conclusion each consist of a quantified relationship between two objects.

2.1 Syllogistic Propositions

A syllogistic proposition or synonymously categorical proposition specifies a quantified relationship between two objects. We shall denote such relationships with the operator Ψ. Four different types are distinguished $\Psi \in \{A, E, I, O\}$ (*Table 1*):

- A is universal affirmative: All S are P
- E is universal negative: All S are not P
- I is particular affirmative: Some S are P
- O is particular negative: Some S are not P

One can observe that the proposition I has three cases (a), (b), (c) and O has (a), (b), (c). The cases I (c) and O (c) are controversial in the literature. Some do not consider them as valid [3] and some do [14]. We have experimentally proven that including these cases, harmonically completes the symmetry of the statistical structures of the syllogistic system [11].

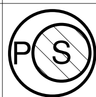
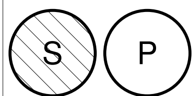
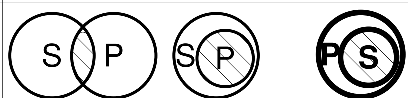
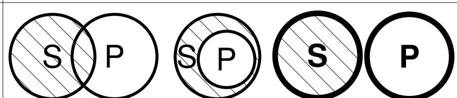
2.2 Syllogistic Figures

A syllogism consists of the three propositions major premise, minor premise and conclusion. The first proposition consist of a quantified relationship between the objects M and P, the second proposition of S and M, the conclusion of S and P (Table 2). Note the symmetrical combinations of the objects.

Since the proposition operator Ψ may have 4 values, 64 syllogistic moods are possible for every figure and 256 moods for all 4 figures in total. For instance, AAA-1 constitutes the mood MAP, SAM - SAP in figure 1.

We shall denote a propositional statement with Φ_1, in order to distinguish between possibly equal propositional operators of the three statements of a particular mood, where i∈{1, 2, 3}.

Table 1. Syllogistic Propositions Consist of Quantified Object Relationships

Operator ψ	Proposition $\Phi(\psi)$	Set-Theoretic Representation of Logical Cases*
A	All S are P	 $\lvert\Phi(A)\rvert=2$
E	All S are not P	 $\lvert\Phi(E)\rvert=2$
I	Some S are P	 (a)=3 (b)=2 **(c)=**2 (a)+(b)+(c)= $\lvert\Phi(I)\rvert=7$
O	Some S are not P	 (a)=3 (b)=2 **(c)=**2 (a)+(b)+(c)= $\lvert\Phi(O)\rvert=7$

* Number of sub-sets of a case (a), (b), (c) and total number of sub-sets of a proposition $|\Phi(\psi)|$.

2.3 Statistics About the Syllogistic System

The algorithm that we have introduced earlier [11] enables revealing various interesting statistics about the structural properties of the syllogistic system [4]. The most significant once are as follows.

First we calculate the truth values for every mood in form of a truth ration between its true and false cases, so that the truth ratio becomes a real number, normalised within [0.0, 1.0]. Thereafter we sort all moods in ascending order of their truth ratio (*Fig 1*). Note the symmetric distribution of the moods according their truth values. 25 moods have a ratio of 0 (false) and 25 have ratio 1 (true), where each is 25/256 = % 10.24 of all moods. 100 moods have a ratio between 0 and 0.5 and 100 have between 05 and 1, where each is 100/256 = % 0.390625. 6 moods have a ratio of exactly 0.5, which is % 0.0234375 of all moods.

For any three set, like M, P, S, in total 41 distinct intersections can be drawn. The 256 moods have in total 2624 truth cases, which map those 41 intersections multiple times. These mapping structures are also inherently symmetric. A complete discussion of all statistical details is presented in [4].

Table 2. Syllogistic Figures

Figure Name	I	II	III	IV
Major Premise Minor Premise	MψP SψM	PψM SψM	MψP MψS	PψM MψS
Conclusion	SψP	SψP	SψP	SψP

3 Fuzzy Syllogistic Reasoning

Based on the symmetrical properties of the syllogistic system, we now define the fuzzy syllogistic system and a sample application for recognising fallacies.

3.1 Fuzzy Syllogistic System

From the structural properties of the syllogistic system [4], we elaborate now a fuzzified syllogistic system.

One can see (*Fig 1*) that every syllogistic case is now associated with one truth ration. We utilise the symmetric distribution of the truth ratios, for defining the membership function FuzzySyllogisticMood(x) = {CertainlyNot; Unlikely; Uncertain; Likely; Certainly} with a possibility distribution that is similarly symmetric (*Fig 1*). the possibility distribution of FuzzySyllogisticMood that was presented earlier, has been adapted to the values of the moods, such that moods with equal values have now equal linguistic values. The linguistic variable was adopted from a meta membership function for a possibilistic distribution of the concept likelihood [16]. The complete list with the names of all 256 moods is appended (*Table A1*).

As we have mentioned earlier, the algorithmically calculated truth ratios of the 256 moods (*Fig 1*) mostly comply with those empirically obtained truth ratios in psychological studies [5]. Hence the suggested possibilistic interpretation should reflect an approximately correct model of the syllogistic system.

3.2 Fuzzy Syllogistic Reasoning

Our objective is to design a new model for automated reasoning, which uses the fuzzy syllogistic system as reasoning mechanisms. For this purpose, we specify following methodology:

- Inductively accumulate sample instances of relationships between the objects M, P, S and classify them into the above mentioned 41 distinct sub-sets.
- Calculate the truth values of the 256 moods for these M, P, S relationships.
- Based on the cardinalities of the 41 sub-sets, calculate possible fallacies.
- Fuzzy syllogistic reason with the mood that has the highest truth value.

Fallacies may be identified manually, by a human, who is deciding on the proper semantics of the M, P, S relationships. However, in this methodology, we identify fallacies fully automated, based on the cardinalities of the sample 41 sub-sets.

3.3 Fallacies in Categorical Syllogisms

In logic, a fallacy is a misconception resulting from incorrect reasoning in argumentation. 7 fallacies are known in the literature for categorical syllogisms:

- Equivocation fallacy or fallacy of necessity: Unwarranted necessity is placed in the conclusion, by ignoring other possible solutions.
- Fallacy of undistributed middle: Middle term not distributed in at least one premiss.
- Illicit major/minor: Major/minor term undistributed in major/minor premiss, respectively, but distributed in the conclusion.
- Fallacy of exclusive premisses: Both premisses negative.
- Affirmative conclusion from negative premiss: Positive conclusion, but at least one negative premiss.
- E xistential fallacy: Both premisses universal, but particular conclusion.

These fallacies comply exactly with the 7 rules for eliminating invalid moods, which were discovered already by Aristotle [1].

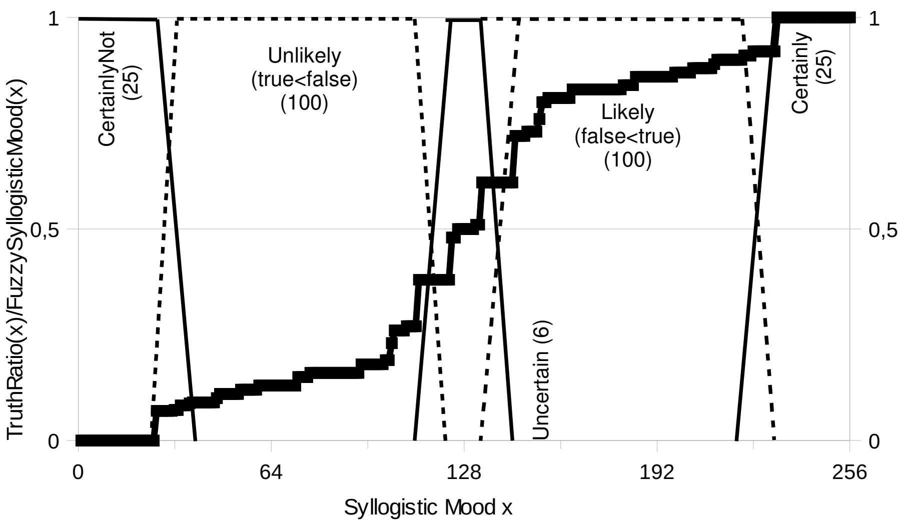


Fig. 1. 256 syllogistic moods sorted in ascending order of their TruthRatio(x) true/false, if number of truth cases of a mood is true<false and 1-false/true ratio, if false<true. Definition of the possibility distribution FuzzySyllogisticMood(x) with the linguistic variables CertainlyNot, Unlikely, Uncertain, Likely, Certainly and their cardinalities 25, 100, 6, 100, 25, respectively

3.4 Recognising Fallacies: Procedure

Our objective is to use the whole set of 256 syllogistic moods as one system of possibilistic argument for recognising fallacies and reasoning with them. For that purpose, we specify the following steps:

1. Calculate all truth cases and the truth ratio of a given mood.
2. Try to recognise fallacies with following rules, for identifying
   (a) possible false instances: reduction of A to I
   (b) possible true instances: reduction of E to O
   (c) further possible true instances: generalisation of I to A
   (d) further possible false instances: generalisation of O to E
   (e) complementing false instances: complementation of I to O
   (f) complementing true instances: complementation of O to I
3. Try to map the initial mood x to any mood y with a truth ratio closer to 1: TruthRatio(x) < TruthRatio(y)
4. Approximately reason with the truth ratios.

Rules (a)-(f) are generalisations of the above discussed reduction and conversion techniques.

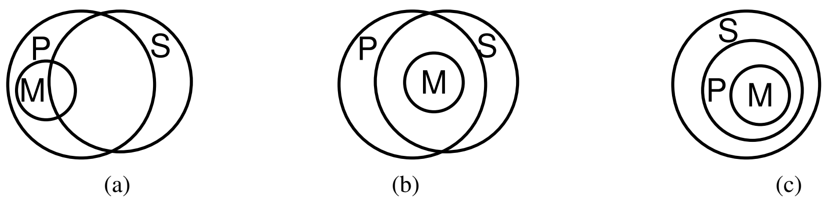


Fig. 2. False syllogistic cases of the mood AIA-1

3.5 Recognising Fallacies: Sample Application

We will now discuss these steps experimentally on the following example (*Fig 4*).

Firstly, we calculate the 3 true (*Fig 3*) and 3 false (*Fig 2*) cases of mood AIA-1 and its truth ratio of 0.5.

Secondly, we identify following fallacies:

- rule (a): Not all stories in The Child's Magic Horn (TCMH) are sad $\neg\Phi_1$(A). The truth is that only some stories in TCMH are sad Φ_1 (I).
- rule (a): Not all stories I cry at are stories in TCMH, because I will possibly cry at some other stories as well $\neg\Phi_3$(A). The truth is that only some of all the stories I cry at are stories in TCMH Φ_3(I).

Based on the identified fallacies and reductions Φ_1(A) to Φ_1(I) and Φ_3(A) to Φ_3(I), we can easily calculate the mood III-1 to be "more true" for the given sample propositions. In dead, mood III-1 has with 4 false/19 true cases 1-0.21=0.79, a better truth ratio.

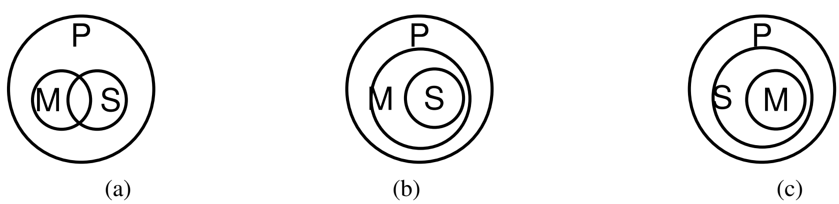


Fig. 3. True syllogistic cases of the mood AIA-1

P:	Sad	A: all M are P
M:	Stories in The Child's Magic Horn	I: some S are M
S:	Tales I cry at	
		A: all S are P

Φ_1(A): All "Stories in The Child's Magic Horn" are "Sad"
Φ_2(I): Some "Tales I cry at" are "Stories in The Child's Magic Horn"
Φ_3(A): All "Tales I cry at" are "Sad"

Fig. 4. Sample syllogistic inference with the mood AIA of the syllogistic figure 1 (AIA-1)

Using mood III-1, we can now try to recognise further fallacies, by applying all combinations for complementing true or false possibilities, which yields the moods:

- OII-1: 1-7/17 = 0.59
- OOI-1: 1-7/17 = 0.59
- IIO-1: 1-6/17 = 0.65
- IOO-1: 1-5/17 = 0.71
- IOI-1: 1-5/17 = 0.71
- III-1: 1-4/19 = 0.79
- OOO-1: 1-3/21 = 0.86
- OIO-1: 1-3/21 = 0.86

Thirdly, these are 8 further candidates for replacing the initial mood AIA-1: 3/3 = 0.5. We may now chose OOO-1 or OIO-1, since both have equal truth ratio of 0.86.

In the last step, we may use the truth rations of the moods for fuzzy syllogistic reasoning as a model for approximate reasoning with quantified propositions.

3.6 Discussion

In the initial example (*Fig 4*), one can suspect possible fallacies in the positive generalisations Φ_1(A) and Φ_3(A), by intuitively assuming possible false instances in them.

The moods OOO-1 and OIO-1 have higher truth ratios than the initial mood 0.5 < 0.86. Consider now the case OII-1 = 0.86

- Φ_1(O): Some "Stories in TCMH" are not "Sad"
- Φ_2(O): Some "Tales I cry at" are not "Stories in TCMH"
- Φ_3(O): Some "Tales I cry at" are not "Sad"

and OOI-1 = 0.86

- Φ_1(O): Some "Stories in TCMH" are not "Sad"
- Φ_2(I): Some "Tales I cry at" are "Stories in TCMH"
- Φ_3(O): Some "Tales I cry at" are not "Sad"

Although humans usually get confused from multiple existentially quantified propositions, we mostly assume intuitively that they are usually correct cases for reasoning, ie that they should have truth ratio close to 1.0. Nevertheless, these moods are mathematically not fully correct, as their truth ratios are considerably below 1.0. Now consider the case OOII-1 = 0.59

- Φ_1(O): Some "Stories in TCMH" are not "Sad"
- Φ_2(I): Some "Tales I cry at" are "Stories in TCMH"
- Φ_3(I): Some "Tales I cry at" are "Sad"

Usually, anyone will assume that this mood, like the both previous moods, is a correct case for reasoning. Although, their truth ratios differ with 0.86 – 0.59 = 0.27 considerably within the value range [0.0, 1.0]. This experimentally proves, what was known since medieval time, that humans tend to assume that reasoning with existential quantifiers are mostly confusion, but possibly correct. Possibly, because humans fail to combine multiple such fuzzy propositions logically correct. We can explain this phenomenon with the possible sub-sets of the propositions $|\Phi(A)| = 2$, $|\Phi(E)| = 2$, $|\Phi(I)| = 7$ and $|\Phi(O)| = 7$ (*Table 1*). Any figure including solely A or E propositions will have 6 sub-sets in total. Any figure including solely I or O propositions will have 21 sub-sets in total. Deciding about the correctness of a particular example requires approving or disapproving the truth of every single sub-set. Thus, propositions that consist of multiple existential quantifications are “too fuzzy” for humans to be decided logically correctly. However, as soon as at least several true sub-sets exist, humans tend to assume that the whole syllogism should be correct.

Finally, consider the mood AII-1 = 1.0

- Φ_1(A): All "Stories in TCMH" are "Sad"
- Φ_2(I): Some "Tales I cry at" are "Stories in TCMH"
- Φ_3(I): Some "Tales I cry at" are "Sad"

Assuming that Φ_1(A) is really true, ie M is a real sub-set of P, then we get a tautology. However, if we always strictly apply the above rules for recognising fallacies, then we should be able to identify almost always possible true or false instances within a given proposition. Hence, tautologies should rather be rare cases in real life.

4 Conclusion

Our algorithmic approach for calculating the truth ratios of syllogisms has enabled us to reveal all structural properties of the complete syllogistic system. On top of the syllogistic system we have proposed a fuzzy syllogistic system that consists of possibilistic arguments, which we have used in a sample application for recognising fallacies and fuzzy syllogistic reasoning with them.

We believe that this approach may prove a practical approach for reasoning with inductively learned knowledge, where P, M, S object relationships can be learned inductively and the "most true" mood can be calculated automatically for those relationships. That shall be our future work, alon with examples including recognising intentional or unintentional fallacies, with the objective to facilitate automated human-machine interaction.

References

[1] Aristotle.: The Works of Aristotle, vol. 1. Oxford University Press, Oxford (1937)
[2] Geurts, B.: Reasoning with quantifiers; Department of Philosophy; University of Nijmegen (2002)
[3] Brennan, J.G.: A Handbook of Logic. Brennan Press (2007)
[4] Çakır, H., Kumova, B.İ.: Structural Analysis of the Syllogistic System. In: International Conference on Fuzzy Computation (ICF 2010), Valencia/Spain (2010)
[5] Chater, N., Oaksford, M.: The probability heuristics model of syllogistic reasoning. Cognitive Psychology 38, 191–258 (1999)
[6] Dickstein, L.S.: The effect of figure on syllogistic reasoning. Memory and Cognition 6, 76–83 (1978)
[7] Dickstein, L.S.: Conversion and possibility in syllogistic reasoning. Bulletin of the Psychonomic Society 18, 229–232 (1981)
[8] Frege, L.G.F.: Begriffsschrift, eine der Arithmetischen Nachgebildete Formalsprache des Reinen Denkens. Verlag von Louis Nebert (1879)
[9] Johnson-Laird, P.N., Steedman, M.: "The psychology of syllogisms. Cognitive Psychology 10, 64–99 (1978)
[10] Johnson-Laird, P.N., Bara, B.G.: Syllogistic inference. Cognition 16, 1–61 (1984)
[11] Kumova, B.İ., Çakır, H.: Algorithmic Decision of Syllogisms. In: García-Pedrajas, N., Herrera, F., Fyfe, C., Benítez, J.M., Ali, M. (eds.) IEA/AIE 2010. LNCS, vol. 6097, pp. 28–38. Springer, Heidelberg (2010)
[12] Oaksford, M., Chater, N.: The probabilistic approach to human reasoning. Trends in Cognitive Sciences 5, 349–357 (2001)
[13] Russell, S., Norvig, P.: Artificial Intelligence - A Modern Approach. Prentice-Hall, Englewood Cliffs (2009)
[14] Wille, R.: Contextual Logic and Aristotle's Syllogistic. Springer, Heidelberg (2005)
[15] Zadeh, L.A.: Fuzzy Logic and Approximate Reasoning. Syntheses 30, 407–428 (1975)
[16] Zadeh, L.A., Bellman, R.E.: Local and fuzzy logics. In: Dunn, J.M., Epstein, G. (eds.) Modern Uses of Multiple-Valued Logic, Reidel, Dordrecht (1977)

Appendix A: Truth Degree of Syllogistic Moods

The table (*Table A1*) shows the x = [1, 256] moods in 5 categories with TruthRatio(x) normalised in [0.0, 1.0]. False and true moods are sorted according their number of false and true cases, respectively. Unlikely and Likely moods are sorted in ascending order of their truth ratio. The table also shows the possibility distribution of the membership function FuzzySyllogisticMood(x) $\in$ {CertainlyNot, Unlikely, Uncertain, Likely, Certainly}.

Table A1. Possibility Distribution FuzzySyllogisticMood(x) over the Syllogistic Moods in Increasing Order of Truth Ratio of the Mood x

Linguistic Value	Sum	Mood x
CertainlyNot; false; ratio=0	25	EIA-1, EIA-2, EIA-3, EIA-4, AIE-1, AIE-3, IAE-3, OAA-3, IAE-4, AOA-2, AAE-3, EAA-3, EAA-4, AAE-1, AAO-1, EAA-1, EAI-1, AEA-2, AEI-2, EAA-2, EAI-2, AAA-4, AAE-4, AEA-4, AEI-4
Unlikely; rather false; 0<ratio<0.5	100	EIE-1, IEE-1, EIE-2, IEE-2, EIE-3, IEE-3, EIE-4, IEE-4, AOE-2, OAA-2, OAE-2, AOA-1, IAA-1, OAE-1, OEE-1, IAA-2, EOE-3, OEE-3, AOE-4, EOE-4, OOE-3, AEA-1, AEE-1, AAA-3, AEA-3, AEE-3, EAE-3, EAE-4, EOE-1, EOE-2, OEA-2, OEE-2, OEA-4, OEE-4, OIE-1, OOE-1, OOA-4, OOE-4, IOA-3, IOE-3, OIE-3, IOA-4, IOE-4, IEA-1, IEA-2, IEA-3, IEA-4, IIA-1, IIA-2, IIA-3, IIA-4, IAE-1, OAA-1, OEA-1, AIE-2, IAE-2, OEA-3, AIE-4, AAA-2, AAE-2, EAA-1, EEE-1, EEA-2, EEE-2, EEA -3, EEE-3, EEA-4, EEE-4, IOA-1, IOE-1, IOA-2, IOE-2, OIA-2, OIE-2, OIA-4, OIE-4, OOA-2, OOE-2, OOA-3, IIE-1, IIE-2, IIE-3, IIE-4, AOE-3, IAA-3, OAE-3, IAA-4, OOA-1, OIA-1, OIA-3, AOE-1, AIA-2, EOA-3, AIA-4, AOA-4, EOA-4, OAA-4, OAE-4, EOA-1, EOA-2
Uncertain; undecided; ratio=0.5	6	AIA-1, AIO-1, AIA-3, AIO-3, AOA-3, AOO-3
Likely; rather true; 0.5<ratio<1.0	100	EOO-1, EOO-2, OIO-1, OOO-1, OIO-3, AIO-2, EOO-3, AIO-4, AOI-1, AOO-4, EOO-4, OAI-4, OAO-4, IAO-3, IAO-4, OAI-3, AOI-3, III-1, III-2, III-3, III-4, OOO-3, OOI-2, OOO-2, IOI-1, IOO-1, OII-2, OIO-2, IOI-2, IOO-2, OII-4, OIO-4, IAI-1, OAO-1, OEO-1, AII-2, OEO-3, IAI-2, AII-4, AAI-2, AAO-2, EEI-2, EEO-2, EEI-3, EEO-3, EEI-4, EEO-4, EEI-1, EEO-1, IIO-1, IIO-2, IIO-3, IIO-4, IEO-1, IEO-2, IEO-3, IEO-4, OII-1, OOI-1, IOI-3, IOO-3, OII-3, IOI-4, IOO-4, OOI-4, OOO-4, EOI-1, EOI-2, OEI-4, OEI-2, OEO-2, OEO-4, AEI-1, AEO-1, AAO-3, AEI-3, AEO-3, EAI-3, EAI-4, OOI-3, AOO-1, IAO-1, OAI-1, OEI-1, IAO-2, EOI-3, OEI-3, AOI-4, EOI-4, AOI-2, OAI-2, OAO-2, IEI-1, EII-1, EII-2, IEI-2, EII-3, IEI-3, EII-4, IEI-4
Certainly; true; ratio=1.0	25	EIO-1, EIO-2, EIO-3, EIO-4, AII-1, AII-3, IAI-3, OAO-3, IAI-4, AOO-2, AAI-3, EAO-3, EAO-4, AAA-1, AAI-1, EAE-1, EAO-1, AEE-2, AEO-2, EAE-2, EAO-2, AAI-4, AAO-4, AEE-4, AEO-4