

Notice of a Forthcoming Essay on the Principles of Electrodynamics

Carl Neumann

Abstract

English translation of Carl Neumann's 1869 paper "Notizen zu einer kürzlich erschienenen Schrift über die Principien der Elektrodynamik", [Neu69].

By Carl Neumann in Leipzig^{1,2,3}

My paper written for the Jubilee of the University of Bonn (*The Principles of Electrodynamics*, Tübingen, 1868),⁴ a preliminary report of which had already appeared in the *Nachrichten der Göttinger Societät der Wissenschaften* (June 1868), has been subjected by Clausius in the latest volume of *Poggendorff's Annalen* (Vol. 135, p. 606)⁵ to a judgment with which I cannot agree, and which prompts me to make these brief comments.

Looking back now on that paper, it can be considered as composed of two different parts, one of which is preceded by the other both in respect to the nature of its content and the strength of its reasoning. Accordingly, it seems to me appropriate first to address the more important and then

¹[Neu69].

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³The Notes by Carl Neumann are represented by [Note by CN:], the Notes by Laurence Hecht are represented by [Note by LH:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

⁴[Note by AKTA:] [Neu68].

⁵[Note by AKTA:] [Cla68] and [Cla69].

the subordinate part, such that no separation in the content of my paper be allowed to occur.

§1.

The First Part of the Cited Paper

As starting point of this part, two ideas are to be considered: One is the idea that for every electrical force there must exist a potential, which, however, depends not only upon the relative position of the electrical masses, but at the same time on their velocity. The other idea is that the well-known Hamiltonian Principle (ruling over the whole of Mechanics) is just as applicable to potentials of this kind, as to the common potential dependent only upon relative position. My research proceeds accordingly from a certain hypothetical formula for the potential of electrical masses:

$$w = \frac{mm_1}{r} \left[1 + \frac{1}{c^2} \left(\frac{dr}{dt} \right)^2 \right]. \quad (1)$$

Here m and m_1 signify the two masses, and r their distance apart at time t ; and by c is understood the constant contained in Weber's law.⁶

My research now shows how, by application of Hamilton's Principle, one is led directly from hypothesis (1) to the known laws of electrical repulsion and induction, and simultaneously also to a comprehensive electrodynamic form of the principle of *vis viva*.—The latter result lends itself directly to an investigation recently carried out by me, on the oscillating charge of a Franklin pane (cf. *Nachrichten der Göttinger Gesellschaft der Wissenschaften*, 13 January 1869), in which it is shown that in addition to Kirchhoff's differential equation for this phenomenon, one can achieve success in a completely different way already indicated by W. Thomson, namely by direct application of the principle of *vis viva*.—It should be noted at this occasion, that in judging the magnitude of a given mass, no distinction is made in my paper between the effect the mass produces and its inertial mass. However, such a distinction must be made when one is dealing with the masses of different matter (for example electrical and ponderable masses) simultaneously. If one

⁶[Note by CN:] This expression is identified right at the beginning of my paper (page 2) as the proper starting point for my observations; the constant $1/c^2$ is there called G . Regarding that, permit me to remark that due to a disturbing printing error in another place in my paper (page 24) this same expression appears as $1/cr$ instead of $1/cc$.

therefore considers the magnitude of the mass, as measured by its effect, as equal to m , as is usually the case with electrical masses, then the measured value of the inertial mass of the same is in no way equal to m , but is rather designated as fm , where f represents a constant factor whose value depends only on the nature of the matter under consideration. This constant factor, included in some places in my paper, has been neglected. The oversight is easily corrected; the results obtained remain completely intact.

Also in my paper, the mutual potential of two masses is brought under consideration using Formula (1) at the same time as the general formula

$$w = mm_1 \left[\varphi(r) - \frac{r}{c^2} \frac{d\varphi(r)}{dr} \left(\frac{dr}{dt} \right)^2 \right], \quad (1a)$$

where $\varphi(r)$ represents a certain function of r . Just as one arrives, on the basis of hypothesis (1), at the law of electrical repulsion and induction, one may also arrive, on the basis of hypothesis (1a), as my paper shows, at that law which I had supposed (in 1858) from my investigation of the magnetic rotation of the plane of polarization of light for the force obtaining between electricity and ether.⁷

Clausius has raised no concerns against the part of my paper discussed up to this point. His concerns are directed only against that portion which I have here designated as standing second in line. I will therefore address these latter in detail.

§2.

The Second Part of the Cited Paper

Clausius is concerned with the already identified hypothetical formula (1) assumed for the potential. He seeks to give this formula a further basis; he seeks to replace the formula with concepts.

I view the Newtonian potential (the product of the masses divided by the distance) of two bodies or masses, m and m_1 , as an impulse for motion,⁸ or (better expressed) an order or command,⁹ which is given and emitted by one body, and received and followed by the other. At the same time it is assumed that the order requires a certain time to get from the place of emission to

⁷[Note by AKTA:] [Neu58] and [Neu63].

⁸[Note by LH:] *Bewegungsantrieb* in the original.

⁹[Note by LH:] *Befehl* in the original.

the place of receipt, in other words, that it requires a certain time to traverse the space between the two bodies.

The distance of the two bodies from one another at the time t_0 may be designated as r_0 . In the instant t_0 , a certain order is issued by one body, and that with reference to the instantaneous relationship, that is the instantaneous distance r_0 ; the command rings out accordingly: mm_1/r_0 . Given and emitted at time t_0 , the order traverses the space between the two bodies without suffering any change en route. Because the passage of that space requires a certain time, the order is therefore not received and obeyed by the obedient body at time t_0 , but at a later time t , a time when the distance between the two bodies is no longer r_0 , but another magnitude r .

The command (the value of the potential mm_1/r_0) can accordingly be designated on the one hand as the emissive potential corresponding to the time t_0 , and on the other hand it can also be designated as the receptive potential corresponding to the time t . In the time t_0 the order is given; in the time t it goes into force.¹⁰

¹⁰[Note by CN:] This is stated in fuller detail, for example on pages 6 and 7 of my paper:

“If we consider only two points m and m_1 , there are, proceeding from the concept of a progressive propagation of the potential for each instant of time t , we must distinguish between two different potentials, the emissive and the receptive.”

“The emissive potential is that which is sent out at the time t by each of the two points, and which thus only reaches the other point a little later...”

“The receptive potential on the other hand is that which is received in time t , by each of the two points in time t , and which thus has already been sent out a little earlier by the other point. The receptive potential belonging to a given time is accordingly identical with the emissive potential of an earlier time...”

On the basis of the just cited definition for emissive and receptive potential, the last theorem could also be stated as:

“The value of the potential received by a point in a given time is identical with that which is emitted at an earlier time by the other point.”

Because, in the cited pages of my paper, the potential is understood to be completely determined by the value expressed in the formulas.

It is clear from this, however, how little is said in my writing about a direct analogy between the propagation of the potential and that of light. It would be completely absurd to say that the value of the light (either in quantity or intensity) received by a point at a given instant would be identical to that emitted at an earlier time by the other point.

The interval $t - t_0$ is that which the command requires to traverse the space between the two bodies. In my paper, the concept of this traversal is based on what seems to me the **simplest** representation, namely, to assume that the command proceeds forward, at the constant velocity c , along the radius vector which originates at the body giving the order and ends at the obeying body. The velocity designated as c is thus a relative motion, because the radius vector along which the order proceeds is itself in motion, carried away along the same path as the moving body.¹¹

The order mm_1/r_0 received by the obeying body in time t , must now obviously traverse that length of radius vector which is present at the instant of its reception; thus it must traverse the length of radius vector r which exists at time t . The time required is r/c ; therefore $t - t_0 = r/c$.

Let the receptive potential mm_1/r_0 at time t be designated by ω , and also let $t_0 = t - \Delta t$, $r_0 = r - \Delta r$, then it results that:

$$\omega = \frac{mm_1}{r_0} = \frac{mm_1}{r - \Delta r} ; \quad (2)$$

and at the same time for the interval Δt corresponding to length Δr , we get the formula:

$$\Delta t = t - t_0 = \frac{r}{c} . \quad (3)$$

However, by further manipulation, and neglecting the third power of $1/c$, Formulas (2) and (3) give for the value of the receptive potential ω :¹²

$$\omega = w + \frac{d\mathbf{w}}{dt} ,$$

Overall, as one sees, my suppositions about the potential bear such an extraordinarily glaring difference to the laws of light, that it could scarcely have occurred to me to instead propose a similarity.

¹¹[Note by CN:] This is the first time I have made use of these words to explain this concept. Earlier when writing my paper, I had clothed the concept differently, more pictorially. I thought then of the body giving the order as surrounded by an infinitely extended atmosphere, which was to a certain extent rigidly bound to the body and took part in all its motions; thus I thought of the order from the emitting body as proceeding in this pure ideal atmosphere with constant velocity and without suffering any change in its original constitution. I thus made use of the word “*propagation*”, which might better have been replaced by “*transmission*”.

¹²[Note by AKTA:] In the original paper Neumann utilized in the next equation the letter \mathfrak{w} . Here we are replacing it by \mathbf{w} in order to facilitate the edition of the equations in LaTeX.

where w represents the expression (1), while on the other hand, \mathbf{w} is a rational combination of $\log r$ and dr/dt ; and further, by application of Hamilton's Principle, the just named value is equivalent to the simpler value: $\omega = w$;— to which there are no objections, and in fact no one is in doubt.

Consequently it is shown that the given concepts really lead to the hypothetically assumed potential formula (1). Whether this substitution of an odd formula by means of a no less odd concept implies progress, is very difficult to judge at present. In writing my paper I also did not attach the same weight to this second line of thought. This explains the striking brevity with which I treated it, taking up only 3 pages of my 38-page paper. So it happens that this part of the underlying concept in my paper is not explained in detail, but implied only briefly and in passing.¹³ It is left to the reader to some extent, first to extract these concepts from the given formulae; and that, I gladly concede, were no easy task and certainly a thankless one. For, as one sees, these ideas, at least in their present form, are very varied in comparison with those usually employed in the explanation of physical processes.

Still it seems remarkable, to me¹⁴ at least, that the same concepts also lead to the law of the mutual interaction of electricity and ether. Thus if one replaces the function $1/r$ in the Newtonian potential by any function $\varphi(r)$, then, based on those ideas, instead of formulas (2) and (3), there result

$$\omega = mm_1\varphi(r_0) = mm_1\varphi(r - \Delta r) , \quad (2a)$$

$$\Delta t = t - t_0 = \frac{r}{c} . \quad (3a)$$

¹³[Note by CN:] I thought it permissible earlier, as I considered that paper only provisional, to be followed by a more extensive publication on the subject, and such was also my intention.

The disproportionately large attention paid to some of its parts is not in keeping with the provisional character of the paper; since in some parts cited by others in the greatest detail, only the end results were given. This is also why I did not publish my paper more widely, and intentionally withheld it from the book sellers.

¹⁴[Note by CN:] I cannot insist that the argument made here be recognized as generally applicable. For the law of the mutual interaction of electricity and the ether, on which my case here rests, could be put in doubt by the experimental investigations of Verdet on the dispersion arising from the rotation of the plane of polarization light by a magnet (*Ann. d. chim.* (3) Vol. 69, p. 415). As for me, I believe for obvious reasons that the provisional results of the dispersion observations should be accorded no significant weight as to the correctness or incorrectness of that law.

However, on further treatment this result leads immediately to the potential formula (1a), consequently to the law of the mutual interaction of electricity and ether.

Overall, I would like to view this second part of my paper as by no means completely superfluous. Rather I am inclined to consider it as a preparatory work, allowing a deeper insight into any obstacles, which, if not eliminated, can at least be analyzed and illuminated. Clausius' objections are directed against this second part of my work. In the following paragraphs I take the opportunity to respond to them.

§3.

Potential and Light

My suppositions concerning potential show a very great difference with the laws of light. So, for example, the following differences between emission, transmission and reception come into view: The light emitted by a luminous body is independent of the illuminated body, while the potential emitted in any instant by an attracting body is in the strictest sense dependent on the instantaneous position of the attracted body, (they are the same, namely $= mm_1/r$, or $= mm_1\varphi(r)$, where r signifies the instantaneous distance). Further: The light emitted by the luminous body in a given instant diminishes in intensity the further away it is from the body; while the emitted potential travels without any change in its original value up to the attracted body. Finally: the light received (i.e. absorbed) by the illuminated body is in general a fraction of the outgoing light; while the potential received by the attracted body is identical (i.e. equal) to the arriving potential.

The laws by which the potential of a body is transmitted to another are thus (according to my suppositions) so extraordinarily different from the corresponding laws of light, that one can scarcely speak of an analogy. At least there would be only one circumstance in which a kind of analogy could be asserted. This consists in the fact that light, like potential, propagates with a very large constant velocity; and even this analogy is not perfect, for the constant velocity for light and potential possess different values, and it applies in the case of light to an absolute motion, while for potential it is a relative motion.

Coincidentally, the just mentioned similarity in a certain passage in the introduction to my paper (page 3) has been pointed out. There it is noted that Riemann had assumed that the "*Potential—similar to light—propagates*

through space with a certain constant velocity”; at the same time it is added that this assumption is also the basis of my research.¹⁵

That this comparison with light was completely accidental and unintended should be clear not only from the text of my paper, but from the fact that one finds such a comparison in only **one** place in my whole paper, and it is even clearer when it is recognized that such a comparison with light is **not found anywhere** in my communication to the Göttingen Society. (The word “*light*” is not contained anywhere in that communication.)

In any case I must also add that the referenced passage from the introduction to my paper, which gave only a casual historical note, namely that Riemann should be designated as the originator of the idea of a progressive motion of the potential, possesses less exactness. While writing my paper, it seemed petty to wish to emphasize in the introduction that my idea on this progressive motion differs fundamentally from that of Riemann (so far as the latter is understood by me). Thus I omitted mention of this difference; which could more easily lead to misunderstanding as the disputes in question are treated extremely briefly in the text of my paper.

Thus, it is easily understandable that Clausius, in judging my paper, proceeded from the opinion that I had supposed that the potential propagates from one body to another in a way similar to that of light; while in reality my suppositions concerning the progressive movement of the potential exhibit the greatest difference with the laws of light.—I can only welcome the fact that I have been made aware of the danger of such a misunderstanding, and begun to eliminate it.

Leipzig, January 19, 1869.

¹⁵[Note by AKTA:] Carl Neumann is referring to Riemann’s posthumous 1867 work, [Rie67b], [Rie67a] and [Rie77].

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