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# XLVII. A contribution to electrodynamics

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## XLVII. *A Contribution to Electrodynamics.* *By* Bernhard Riemann*.

I BEG to communicate to the Royal Society an observation which brings into close connexion the theory of electricity and of magnetism with that of light and of radiant heat. I have found that the electrodynamic actions of galvanic currents may be explained by assuming that the action of one electrical mass on the rest is not instantaneous, but is propagated to them with a constant velocity which, within the limits of errors of observation, is equal to that of light. On this assumption, the differential equation for the propagation of the electrical force is the same as that for the propagation of light and of radiant heat.

Let S and S′ be two conductors traversed by constant voltaic currents but not moved towards each other; let $\epsilon$ be an electrical particle in the conductor S, which at the time $t$ is in the point $(x, y, z)$; $\epsilon'$ an electrical particle of S′, which at the time $t$ is in the point $(x', y', z')$. As regards the motion of the electrical particles, which in each particle of the conductor is opposite in the negative to what it is in the positive, I assume it at each moment to be so distributed that the sums

$$\Sigma\epsilon f(x, y, z), \quad \Sigma\epsilon' f(x', y', z')$$

extended over all the particles of the conductor may be neglected as compared with the same sums if they are distributed only over the positively electrical, or only over the negatively electrical particles, as long as the function $f$ and its differential quotients are constant.

This supposition can be fulfilled in various ways. Let us assume, for instance, that the conductors are crystalline in their smallest particles, so that the same relative distribution of the electricity is periodically repeated at definite distances which are infinitely small compared with the dimensions of the conductors; then, if $\beta$ be the length of one such period, those sums are infinitely small, like $c\beta^n$, if $f$ and their derivatives to the $(n-1)$th degree are continuous, and infinitely small like $e^{-\frac{c}{\beta}}$ if they are all continuous.

### *Experimental Law of Electrodynamic Actions.*

If the specific intensities expressed in mechanical measurement are $u, v, w$ at the time $t$ in the point $(x, y, z)$, parallel to the three

* Translated from Poggendorff's *Annalen*, No. 6, 1867. This paper was laid before the Royal Society of Sciences at Göttingen on the 10th of February 1858, by the author (whose premature death was such a loss to science), but appears, from a remark added to the title by the then Secretary, to have been subsequently withdrawn.

axes, and $u', v', w'$ in the point $(x', y', z')$, and if $r$ is the distance of the two points, $c$ the constant determined by Kohlrausch and Weber, experiment has shown that the potential of the forces exerted by S upon S$'$ is

$$-\frac{2}{cc}\iint\frac{uu'+vv'+ww'}{r}d\mathrm{S}\,d\mathrm{S}',$$

this integral being extended over the whole of the elements $d$S and $d$S$'$ of the conductors S and S$'$. If, for the specific intensities of the current, we substitute the products of the velocities into the specific gravities, and then for the products of these into the elements of volume the masses contained in them, this expression passes into

$$\Sigma\Sigma\frac{\epsilon\epsilon'}{cc}\frac{1}{r}\frac{dd'(r^2)}{dt\,dt},$$

if the alteration of $r^2$ during the time $dt$, which arises from the motion of $\epsilon$, be denoted by $d$, and that arising from the motion of $\epsilon'$ be denoted by $d'$.

This expression, by taking away

$$\frac{d\Sigma\Sigma\frac{\epsilon\epsilon'}{cc}\frac{1}{r}\frac{d'(r^2)}{dt}}{dt},$$

which disappears when summed with respect to $\epsilon$, passes into

$$-\Sigma\Sigma\frac{\epsilon\epsilon'}{cc}\frac{d\left(\frac{1}{r}\right)}{dt}\frac{d'(r^2)}{dt};$$

and this again, by the addition of

$$\frac{d'\Sigma\Sigma\frac{\epsilon\epsilon'}{cc}rr\frac{d\left(\frac{1}{r}\right)}{dt}}{dt},$$

which by summing with respect to $\epsilon'$ becomes null, is changed into

$$\Sigma\Sigma\epsilon\epsilon'\frac{rr}{cc}\frac{dd'\left(\frac{1}{r}\right)}{dt\,dt}.$$

*Deduction of this Law from the new Theory.*

According to the current assumption as to electrostatic action, the potential function U of arbitrarily distributed electrical masses, if $\rho$ denote their density at the point $(x, y, z)$, is defined by the condition

$$\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}+\frac{d^2U}{dz^2}-4\pi\rho=0$$

and by the condition that U is continuous and is constant at an infinite distance from acting masses. A particular integral of the equation

$$\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}+\frac{d^2U}{dz^2}=0,$$

which remains continuous everywhere outside the point $(x', y', z')$, is

$$\frac{f(t)}{r};$$

and this function forms the potential function produced from the point $(x', y', z')$, if at the time $t$ the mass $-f(t)$ is there.

Instead of this, I assume that the potential function U is defined by the condition

$$\frac{d^2U}{dt^2}-\alpha\alpha\left(\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}+\frac{d^2U}{dz^2}\right)+\alpha\alpha 4\pi\rho=0,$$

so that the potential function from the point $(x', y', z')$, if the mass $-f(t)$ is there at the time $t$, becomes

$$=\frac{f\left(t-\frac{r}{\alpha}\right)}{r}.$$

If the coordinates of the mass $\epsilon$ at the time $t$ are denoted by $x_t, y_t, z_t$, and those of the mass $\epsilon'$ at the time $t'$ by $x'_{t'}, y'_{t'}, z'_{t'}$, and putting for shortness' sake

$$\left((x_t-x'_{t'})^2+(y_t-y'_{t'})^2+(z_t-z'_{t'})^2\right)^{-\frac{1}{2}}=\frac{1}{r(t, t')}=F(t, t'),$$

on this assumption the potential of $\epsilon$ upon $\epsilon'$ at the time $t$ becomes

$$=-\epsilon\epsilon' F\left(t-\frac{r}{\alpha}, t\right).$$

The potential of the forces exercised by all the masses $\epsilon$ of the conductor S upon the masses $\epsilon'$ of the conductor S′ from the time 0 to the time $t$ becomes therefore

$$P=-\int_0^t \Sigma\Sigma\epsilon\epsilon' F\left(\tau-\frac{r}{\alpha}, \tau\right)d\tau,$$

the sums being extended over the entire masses of both conductors.

Since the motion for opposite electrical masses is opposite in each particle of the conductor, the function $F(t, t')$ by derivation with respect to $t$ acquires the property of changing its sign with $\epsilon$, and by derivation with respect to $t'$ the property of changing its sign with $\epsilon'$. Hence on the supposed distribution of the electricities, if derivation with respect to $t$ be designated by upper, and derivation with respect to $t'$ by lower accents, $\Sigma\Sigma\epsilon\epsilon' F^{(n)}_{n'}(\tau,\tau)$ distributed over all the electrical masses only becomes infinitely small as compared with the sum extended over the electrical masses of one kind when $n$ and $n'$ are both odd.

Let it now be assumed that during the time occupied in the transmission of the force from one conductor to the other the electrical masses pass over a very small space, and let us consider the action during a length of time compared with which the time of transmission vanishes. In the expression for P,

$$F\left(\tau-\frac{r}{\alpha}, \tau\right)$$

can be replaced by

$$F\left(\tau-\frac{r}{\alpha}, \tau\right)-F(\tau, \tau)=-\int_0^{\frac{r}{\alpha}} F'(\tau-\sigma, \tau)\, d\sigma,$$

since $\Sigma\,\Sigma\,\epsilon\,\epsilon' F(\tau, \tau)$ may be neglected.

There is thus obtained

$$P=\int_0^t d\tau \Sigma\Sigma\epsilon\epsilon' \int_0^{\frac{r}{\alpha}} F'(\tau-\sigma, \tau)\, d\sigma\,;$$

or if the order of the integrations be inverted and $\tau+\sigma$ put for $\tau$,

$$P=\Sigma\Sigma\epsilon\epsilon' \int_0^{\frac{r}{\alpha}} d\sigma \int_{-\sigma}^{t-\sigma} d\tau F'(\tau, \tau+\sigma).$$

If the limits of the inner integral be changed to 0 and $t$, at the upper limit the expression

$$H(t)=\Sigma\Sigma\epsilon\epsilon' \int_0^{\frac{r}{\alpha}} d\sigma \int_{-\sigma}^{0} d\tau F'(t+\tau,\ t+\tau+\sigma)$$

will be thereby added, and at the lower limit the value of this expression for $t=0$ will be taken away. We have thus

$$P=\int_0^t d\tau \Sigma\Sigma\epsilon\epsilon' \int_0^{\frac{r}{\alpha}} d\sigma F'(\tau,\ \tau+\sigma)-H(t)+H(0).$$

In this expression $F'(\tau, \tau+\sigma)$ can be replaced by $F'(\tau,\tau+\sigma)-F'(\tau,\tau)$, since

$$\Sigma\Sigma\epsilon\epsilon'\frac{r}{\alpha}F'(\tau,\ \tau)$$

may be neglected. An expression is thereby obtained as a factor of $\epsilon\epsilon'$, which changes its sign both with $\epsilon$ and $\epsilon'$; so that in the additions the members do not cancel one another, and infinitely small fractions of the individual members may be disregarded. Hence, substituting $\sigma\dfrac{dd'\left(\frac{1}{r}\right)}{d\tau\, d\tau}$ for $F'(\tau,\ \tau+\sigma)-F'(\tau,\tau)$, and integrating with respect to $\sigma$, we obtain

$$P=\int_0^t \Sigma\Sigma\epsilon\epsilon'\frac{rr}{2\alpha\alpha}\frac{dd'\left(\frac{1}{r}\right)}{d\tau\, d\tau}d\tau-H(t)+H(0),$$

to a fraction which may be neglected.

It is easily seen that $H(t)$ and $H(0)$ may be neglected; for

$$F'(t+\tau,\ t+\tau+\sigma)=\frac{d\left(\frac{1}{r}\right)}{dt}+\frac{d^2\left(\frac{1}{r}\right)}{dt^2}\tau+\frac{dd'\left(\frac{1}{r}\right)}{dt\, dt}(\tau+\sigma)+\dots,$$

and therefore

$$H(t)=\Sigma\Sigma\epsilon\epsilon'\left(\frac{rr}{2\alpha\alpha}\frac{d\left(\frac{1}{r}\right)}{dt}-\frac{r^3}{6\alpha^3}\frac{d^2\left(\frac{1}{r}\right)}{dt^2}+\frac{r^3}{6\alpha^3}\frac{dd'\left(\frac{1}{r}\right)}{dt\, dt}+\right).$$

But here only the first member of the factor of $\epsilon\epsilon'$ is of the same order with the factor in the first constituent of P; and this, on account of the summation with respect to $\epsilon'$, yields only a fraction of it which may be neglected.

The value of P obtained from our theory agrees with the experimental one,

$$P=\int_0^t \Sigma\Sigma\epsilon\epsilon'\frac{rr}{cc}\frac{dd'\left(\frac{1}{r}\right)}{d\tau\, d\tau}d\tau,$$

if we assume $\alpha\alpha=\frac{1}{2}cc$.

According to Weber and Kohlrausch's determination,

$$c=439450\,.\,10^6\ \frac{\text{millimetre}}{\text{second}},$$

which gives $\alpha=192965$ miles in a second, while for the velocity of light Busch has calculated the number 193172 miles from Bradley's observation of aberration, and Fizeau has obtained the number 192757 by direct measurement.