

Weber's Electrodynamics

Fundamental Theories of Physics

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Weber's Electrodynamics

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To the memory of my grandfather,
Eng. David Koch,
for all that I owe to him.

Table of Contents

Preface	xi
Chapter 1 / Vector Analysis	1
1.1. Definitions and Notation	1
1.2. Vector Algebra	2
1.3. Gradient	5
1.4. Divergence, Curl and Laplacian	7
1.5. Integral Calculus	9
1.6. The Dirac Delta Function	12
1.7. Cylindrical and Spherical Coordinates	13
Chapter 2 / Review of Classical Electromagnetism	17
2.1. Introduction	17
2.2. Equations of Motion	20
2.3. Electric and Magnetic Forces	26
2.4. Lorentz's Force Law	36
2.5. Maxwell's Equations	38
2.6. Derivation of Gauss's Law	45

Chapter 3 / Weber's Electrodynamics	47
3.1. Wilhelm Weber and His Electromagnetic Researches	47
3.2. Weber's Force	56
3.3. Weber's Potential Energy	61
3.4. Conservation of Linear Momentum, of Angular Momentum and of Energy	63
3.5. Lagrangian and Hamiltonian Formulations of Weber's Electrodynamics	68
3.6. Maxwell and the Electrodynamics of Weber	73
Chapter 4 / Forces of Ampère and Grassmann Between Current Elements	78
4.1. Ampère's Force Between Current Elements	78
4.2. Derivation of Ampère's Force from Weber's Force	84
4.3. Grassmann's Force and Biot-Savart's Law	89
4.4. Derivation of Grassmann's Force from Lorentz's Force	91
4.5. Ampère Versus Grassmann	92
4.6. Force Between Circuits from the Coefficient of Mutual Inductance	98
4.7. Derivation of the Magnetic Circuital Law and of the Law of Nonexistence of Magnetic Monopoles	103
4.8. Modern Experiments Related to the Controversy Ampère Versus Grassmann	108
Chapter 5 / Faraday's Law of Induction	118
5.1. Faraday's Law	118
5.2. Franz Neumann	122
5.3. Derivation of Faraday's Law from Weber's Force	126
5.4. Derivation of Faraday's Law from Weber's Electrodynamics Without Utilizing Fechner's Hypothesis	134
Chapter 6 / Forces of Weber and of Lorentz	142
6.1. Introduction	142
6.2. Retarded and Liénard-Wiechert's Potentials	143
6.3. Derivation of Grassmann's Force from the Liénard-Schwarzschild's Force	148

6.4. Comparison Between Weber's Force and Liénard-Schwarzschild's Force	150
6.5. Two Charges in Uniform Rectilinear Motion	153
6.6. Electric Field Due to a Stationary, Neutral and Constant Current	161
6.7. Weber's Law and Mass Variation	169
6.8. Darwin's Lagrangian	177
Chapter 7 / Important Topics Related to Weber's Law	180
7.1. Two Body Problem According to Weber's Law	180
7.2. Motion of a Charge Orthogonal to the Plates of a Capacitor	189
7.3. Charged Spherical Shell	193
7.4. Centrifugal Electrical Force	199
7.5. Weber's Law Applied to Gravitation	203
7.6. Mach's Principle	208
7.7. The Mach-Weber Model	215
Chapter 8 / General Discussion	223
8.1. Weber's Electrodynamics and Maxwell's Equations	223
8.2. Action at a Distance Versus Contact Action	227
8.3. Weber's Electrodynamics in Terms of Fields and Retarded Time	232
8.4. Weber's Law and Plasma Physics, Quantum Mechanics, Nuclear Physics, Etc.	234
8.5. Limitations of Weber's Electrodynamics	236
Appendix A / The Origins and Meanings of the Magnetic Force	
$\vec{F} = q\vec{v} \times \vec{B}$	238
Appendix B / Alternative Formulations of Electrodynamics	242
References	246
Index	270

Preface

“Great progress has been made in electrical science, chiefly in Germany, by cultivators of the theory of action at a distance. The valuable electrical measurements of W. Weber are interpreted by him according to this theory, and the electromagnetic speculation which was originated by Gauss, and carried on by Weber, Riemann, F. and C. Neumann, Lorenz, etc., is founded on the theory of action at a distance, but depending either directly on the relative velocity of the particles, or on the gradual propagation of something, whether potential or force, from the one particle to the other. The great success which these eminent men have attained in the application of mathematics to electrical phenomena, gives, as is natural, additional weight to their theoretical speculations, so that those who, as students of electricity, turn to them as the greatest authorities in mathematical electricity, would probably imbibe, along with their mathematical methods, their physical hypothesis.

These physical hypotheses, however, are entirely alien from the way of looking at things which I adopt, and one object which I have in view is that some of those who wish to study electricity may, by reading this treatise, come to see that there is another way of treating the subject, which is no less fitted to explain the phenomena, and which, though in some parts it may appear less definite, corresponds, as I think, more faithfully with our actual knowledge, both in what it affirms and in what it leaves undecided.

In a philosophical point of view, moreover, it is exceedingly important that two methods should be compared, both of which have succeeded in explaining the principal electromagnetic phenomena, and both of which have attempted to explain the propagation of light as an electromagnetic phenomenon and have actually calculated its velocity, while at the same time the fundamental conceptions of what actually takes place, as well as most of the secondary conceptions of the quantities concerned, are radically different.”

These are the words of James Clerk Maxwell, in the Preface of his major book, *A Treatise on Electricity and Magnetism*. As we can see from these words, Maxwell perceived a conceptual difference between his conceptions, derived in great measure from those of Faraday; and the conceptions of Gauss, Weber, etc. Maxwell knew that both formulations succeeded in explaining the main phenomena of electromagnetism, and he emphasized the great importance in the comparison of the two methods.

And the goal of this book is exactly to follow this general idea. Our basic intention is to present in a fairly complete way Weber's Electrodynamics. As Maxwell said and showed more than once, Weber's theory is compatible with what we call Maxwell's equations (namely, laws of Gauss, Ampère and Faraday), although it is completely different from Maxwell's conceptions in philosophical matters. In this book we show how Maxwell's equations can be derived from Weber's force and the limitations of this compatibility.

In Maxwell's time the electrodynamic researches in the Continent were centered on the action at a distance laws of Coulomb, Ampère, Weber, Neumann, etc. In these theories only the charges, current carrying circuits and magnets, as well as their distances, velocities and accelerations are important. The ether or the field concept are not necessary. Maxwell had different conceptions, based essentially on the ether, and was trying to show that this new model could also explain the known facts of electromagnetism, as we can see from the middle paragraph quoted above. Nowadays we have the opposite situation. We only talk of fields, local action, finite velocity of propagation of the interactions, etc. The aim of this book is summarized in Maxwell's middle paragraph, but now reversing the methods or physical hypotheses.

Maxwell's admiration of Weber's work can also be seen by observing that he dedicated the last chapter of his most important book (the *Treatise*) to present Weber's Electrodynamics and to show its compatibility with the main known facts of electromagnetism.

This book is intended for students and scientists in the areas of physics, engineering, mathematics, history and philosophy of science. This work is intended to be complete in the sense that no previous knowledge of Weber's law is required to follow the text. A first Chapter on Vector Analysis including the main mathematical tools utilized in the text is included for completeness.

The subject of this work is within classical physics. For this reason we did not deal here with quantum mechanics nor with Einstein's theories of relativity. These topics are beyond the scope of this book.

At the end of the book a large bibliography has been included to allow interested readers further studies. It is not intended to be complete but only to indicate some of the subjects being researched nowadays along these lines and to mention authors working in this field. These recent references can be utilized as topics of research by graduate students. In the text each reference is indicated by the author's name and year of publication. For instance: (Edwards, Kenyon and Lemon, 1976).

This book can be utilized in a one or two semesters course. We have taught courses on Weber's Electrodynamics at undergraduate and graduate levels, and this book grew out of these experiences. We wrote a *Course of Weber's Electrodynamics*, with exercises, which has been utilized in these courses (Assis, 1992 a). The reception of the students to this material has been very encouraging and they always mention it has been helpful in their formation in science.

Whenever possible we present historical information relevant to the topic which is being treated. The reason is to give the historical context of some discoveries and to make a critical analysis of some topics. The sources of the major part of this information are the original papers, and the excellent books of Whittaker (*A History of the Theories of Aether and Electricity*), O'Rahilly (*Electromagnetic Theory - A Critical Examination of Fundamentals*), and Mach (*The Principles of Physical Optics*).

In this book we utilize the International System of Units. When we define any physical concept we utilize "≡" as a symbol of definition.

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June, 1994