# Measurement of Maxwell's displacement current

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### Abstract

The measurement of Maxwell's displacement current has remained a veritable challenge. Here we present a novel approach that allows a detailed verification in a lecture hall-type experiment.

Keywords: electromagnetism, Maxwell's equations, displacement current

(Some figures may appear in colour only in the online journal)

## Introduction

In a series of papers between 1865 and 1875, Maxwell addressed the inconsistencies of Ampère's circuital law when applied to electric circuits with capacitors. He introduced the concept of the dielectric displacement $\mathbf{D}$ and consequently also its temporal derivative $\mathbf{j} = \partial_t \mathbf{D}$, the dielectric displacement current. This allowed connecting electric current and magnetic field $\mathbf{B}$ also for insulators, even vacuum, in electric circuits on the basis of a modified version of Ampère's law, the Maxwell–Ampère law. For vacuum and to a very good approximation also for air, it reads:

$$\oint_{\partial A} \mathbf{B} \cdot \mathrm{d}\ell = \iint_A \mu_0 \, \varepsilon_0 \, \frac{\partial}{\partial t} \mathbf{E} \cdot \mathrm{d}\mathbf{A}. \tag{1}$$

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[1] G Scheler passed away on 9 February 2014, shortly before the completion of the manuscript.

Together with Faraday's law of induction

$$\oint_{\partial A} \mathbf{E} \cdot \mathrm{d}\ell = - \iint_A \frac{\partial}{\partial t}\mathbf{B} \cdot \mathrm{d}\mathbf{A} \qquad (2)$$

and the two material equations div $\mathbf{D} = \rho$ and div $\mathbf{B} = 0$, this equation establishes a set of coupled equations that became the basis of the entire field of electrodynamics and are known as Maxwell's equations. A profound consequence of Maxwell's equations has been the prediction of electromagnetic waves with a wave velocity equalling the speed of light. This suggested that Maxwell's equations not only put electric and magnetic phenomena on an equal footing, but also unify the oldest discipline of physics, optics, with the, at Maxwell's time, newest ones. Not much later, physicists would realize that the theory of relativity is an inevitable consequence of these equations.

The profound consequences of the displacement current, originally introduced for purely theoretical reasons, called for experimental verification. On Hermann v. Helmholtz' suggestion, the Prussian Academy of Science even launched a competition, endowed with a prize, for an experimental proof or disproof of the displacement current. Helmholtz then encouraged his student Heinrich Hertz to work on the problem who indeed won the prize by famously discovering radio waves [1]. Hertz dismissed any suggestions of possible applications of his experiment and saw its relevance exclusively in proving Maxwell right. The displacement current is indeed fundamental to the combination of Ampère's and Faraday's laws into a wave equation and Hertz' discovery thus a compelling although indirect proof of its existence.

Notwithstanding the pivotal significance of the displacement current in physics, there appears to be hardly any direct, let alone quantitative measurement of the displacement current under lecture-room conditions to the present day. A student's lab or even lecture-room experiment demonstrating the displacement current is therefore highly desirable. Ideally, it is detected in the same context in which it had been predicted in the first place, namely a straight wire that is interrupted by a capacitor and the associated magnetic fields. As any discussion of the displacement current is necessarily preceded by a discussion of Ampère's law, a natural approach appears to be to measure the magnetic field generated by the displacement current. This, in fact, has been achieved in the experiment of Bartlett and Corle. In their experiment, a SQUID magnetometer was used as a probe and the capacitor was shielded by a superconductor [2]. Unfortunately, such a setup is hardly suitable for the class room: its subsequent discussion in the Nature magazine under the title 'Measuring the unmeasurable' has characterized the experimental complications as 'horrendous' [3]. The technical complexity of that experiment certainly prevents its use for teaching the displacement current to undergraduate students. A much simpler demonstration was presented by Carver and Rajhel [4]. However, as also admitted by the authors, the evidence is not entirely conclusive. An interesting historical fact is that Röntgen measured the displacement current in a glass plate in 1888 [5], an achievement he considered more important than his discovery of x-rays. Colleagues agreed: physicists like Lorentz and Poincaré would refer to the displacement current as 'Röntgen current' while Sommerfeld thought that the experiment was worth a Nobel prize [6].

Irrespective of the challenges, the first and foremost requirement for a lecture room experiment is that it is not misleading. In several textbooks simple schemes allegedly being able to detect the displacement current by measuring the associated magnetic field are described: a coil is placed in a capacitor which is connected to an ac power supply. The voltage measured at the pins of the coil is then said to be induced by the magnetic field produced by the displacement current. What is detected, however, is nothing but the coupling

of the electric field of the capacitor with the unavoidable capacitance of the induction coil. An experimental proof for the detection of the displacement current should verify not only the (i) correct magnitude of the magnetic field, but also (ii) its geometry and (iii) dynamics. In addition, the setup should be affordable and, ideally, the essential components should be visible.

## Elementary theoretical analysis

Assuming the displacement current exists, one should be able to measure the corresponding magnetic field via the induced voltage in an inductance. As already explained, the respective setup for an experiment is quite straightforward, at least in principle: an inductance (i.e. a coil) is placed between a parallel plate capacitor and the induced voltage is measured. Calculating the induced voltage $V_{ind}$ is equally straightforward. According to Faraday's law, the induced voltage is given by

$$V_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \iint_A \mathbf{B} \cdot d\mathbf{A}, \tag{3}$$

where $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$ is the magnetic flux through the inductance.

   In order to calculate the magnetic field $\mathbf{B}(\mathbf{r})$ at the inductance's position $\mathbf{r}$, we model the displacement current as a uniform current in a regular conductor, see figure 1. This, of course, is a classic textbook problem in the context of Ampère's law, see also figure 1. Its solution is a linearly increasing magnetic field strength

$$B(r) = \frac{\mu_0 \, I \, r}{2\pi \, R_0^2}, \tag{4}$$

inside the conductor and an $1/r$-dependence outside. $I = I_0 \sin \omega t$ is the time-dependent current through the conductor of radius $R_0$ modelling the displacement current in the capacitor. $r = 0$ corresponds to the centre of the conductor (viz. plate capacitor). Substituting equation (4) into (3), we arrive at

$$V_{ind} = -V_0 \, \varepsilon_0 \mu_0 \frac{nAr\omega^2}{2d} \cos \omega t, \tag{5}$$

where $n$ is the winding count and $A$ the cross sectional area of the inductance.

   At this point, a reference to a recent discussion appears to be in place. There is, of course, no dispute whether this result is correct. However, there have been arguments concerning the interpretation. It has already been pointed out by Planck that $\partial \mathbf{D}/\partial t$ will not produce a magnetic field under quasi-static conditions, i.e. when the changing electric field is conservative [7]. Indeed, it can be shown that the Biot–Savart law delivers the same result as above if all conduction currents, in particular the radial currents in the capacitor plates, are taken into account [8]. The displacement current does not contribute which can be seen from the fact that its curl must be zero because the curl of the electric field in the capacitor is zero [2]. These observations gave rise to some discussions [9–11]. Our view is that semantic and logical arguments are superimposed in the discussion. At the end of the day, however, it is clear that (i) the derivation of the magnetic field in the capacitor as outlined above is much simpler than a derivation based on the Biot–Savart law and (ii) Ampère's law has to be augmented by the displacement current. Nevertheless and despite of the difficulties of the discussion, it appears to be unavoidable to mention the problem in undergraduate courses when the displacement current is presented using the standard example of a charging plate capacitor.
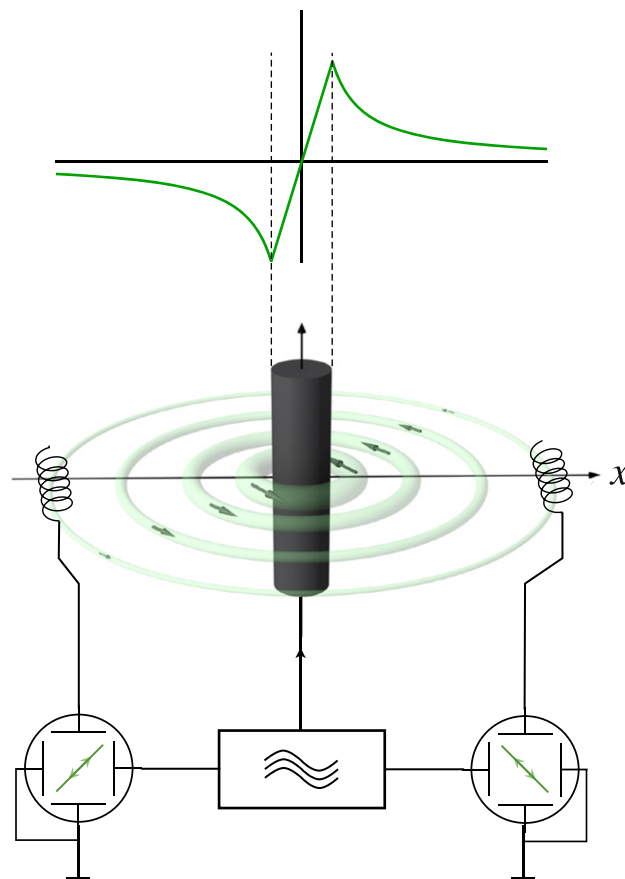
**Figure 1.** Magnetic field inside and outside an electric conductor. The phase of the magnetic field at symmetrically opposing positions differs by 180°. This can be exploited using a phase-sensitive detection scheme.

## Theoretical treatment including fringe fields

Another subtlety are the effects of the unavoidable fringe fields. Any quantitative or semi-quantitative experiment will unavoidably reveal their impact on the experiment, The electric field **E** of a plate capacitor can approximately be deduced from the field of a semi-infinite plane opposing an infinite plane, if the radius $R$ of the plate capacitor is much larger than the plate separation $d$.[2] This problem has already been analyzed by Maxwell and Kirchhoff in order to obtain precise formulas for the capacity of plate capacitors [12]. At a distance $r$ from the centre of the capacitor, the electric field in the symmetry plane is given by

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[2] The validity of Maxwell's (and many other authors' similar) analysis decreases as $d/R$ increases. It is a matter of the specific problem at hand at which point the approximation becomes insufficient. The practical applicability of Maxwell's approach even for rather large $d/R$ can be seen by the success of the Rogowski profile in high-voltage technology.

$$\mathbf{E}(r) = \frac{\mathbf{E}_0}{1 + \exp\left[\hat{r} - W(\exp\hat{r})\right]}, \tag{6}$$

where $\mathbf{E}_0 = V/d$ is the field strength in an infinite plate capacitor, $A = d/2\pi$, and $\hat{r} = (r - R - A)/A$. $W(x)$ is the Lambert $W$-function, i.e. the inverse function of $z = W(z)\exp[W(z)]$. Closed-form expressions for the electric field also exist outside the symmetry plane.

Using the Ampère–Maxwell law (1), it is straightforward to compute the magnetic field in a more realistic way as compared to the elementary analysis above. However, to our knowledge, the evaluation of the integral

$$\mathbf{B}(r) = +\frac{1}{2\pi r}\frac{\omega^2}{c^2}\int_0^r \mathbf{E}(\rho)\,\rho\,\mathrm{d}\rho \tag{7}$$

cannot be written in a closed-form expression; it rather has to be evaluated numerically. In figure 4 the result is plotted on top of the respective experimental data that will be discussed below.

## Experimental approach

The layout of the experiment to be discussed is based on the following estimates: the plate capacitor has a radius of $R_0 = 5$ cm and a plate separation of $d = 2$ cm, resulting in a capacitance of 3.5 pF. When a voltage with an amplitude $V_0 = 2$ V and a frequency $f = \omega/2\pi = 1$ MHz is applied to this capacitor, the amplitude of the current will be $I_0 = V_0 \cdot \omega C \approx 45\,\mu$A. According to equation (4), the magnetic field at the edge of the capacitor will be $B \approx 1.7 \times 10^{-10}$ T. Using a winding count of $n = 50$ and an area of $A = 1$ cm$^2$ for the inductance, one arrives at $V_{\text{ind}} \approx 5\,\mu$V at the edges of the capacitor.

The straightforward setup would be to move around the inductance inside the plate capacitor in order to probe the position-dependent magnetic field. In fact, in our first attempts we tried exactly this. However, a real selenoid has a finite capacitance. Due to its size, it can be modelled as a small plate capacitor with a plate separation of a few millimeters. Accordingly, there will be a voltage at this parasitic capacitance of the order of a fraction of $V_0$, i.e. orders of magnitude more than $V_{\text{ind}}$. Another indication that this signal has nothing to do with $V_{\text{ind}}$ is that it is independent of the position inside the plate capacitor.

We have realized an unambiguous measurement of the induced voltage by (i) placing the induction coil away from the capacitor and (ii) by using a phase-sensitive (heterodyne) detection scheme. The phase-sensitive detection of $V_{\text{ind}}$ not only provides the required sensitivity and noise suppression, but is also able to determine the geometry and dynamics of the **B**-field.

This is illustrated in figure 1 for the already discussed analogous example of the magnetic field of a straight wire carrying alternating current. The magnetic field **B** is perpendicular to the radial coordinate $r$, i.e. if **B** is measured along the $x$-axis, **B** will be parallel to the $y$-axis. Evidently, the sign of **B** will reverse as the sign of $x$ is reversed and **B** vanishes for $x = 0$. In other words, the fields at opposing positions in the conductor differ in phase by 180° and so will the respective voltages induced in coils placed there.

The simplest approach of a phase-sensitive measurement would be to plot the time- (and position-)dependent **B**-field versus the time-dependent voltage applied to the capacitor. In principle, this could be realized by an oscilloscope in *XY*-mode. According to equations (1) and (3), the probe on the right-hand side of the wire will produce a straight line with negative
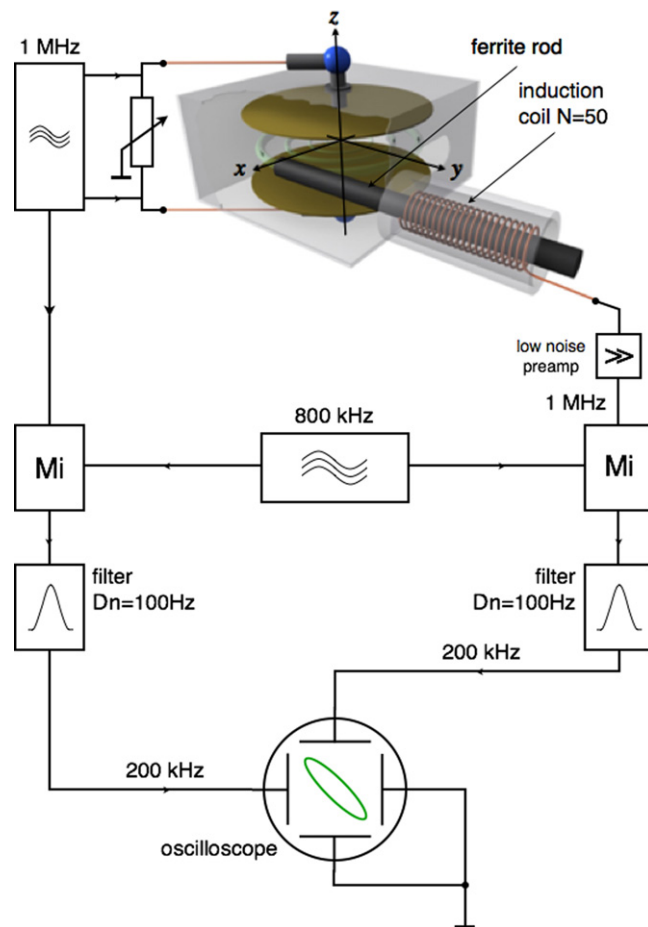
**Figure 2.** Heterodyne detection of the displacement current: the displacement current is generated by applying 1 MHz voltage from a frequency generator to the plates symmetrically to ground. The magnetic field due to the displacement current is conducted to a shielded induction coil by a ferrite rod. After pre-amplification, the induced voltage is mixed with a 800 kHz frequency of a local oscillator and, after suitable filtering, detected at the $Y$-input of an oscilloscope or a level meter (not shown). Analogously, also the frequency generator's signal is mixed with the frequency of the local oscillator, filtered and detected at the $X$-input of the oscilloscope.

slope on the oscilloscope's screen and a positive slope for $x < 0$, as indicated in figure 1. For the symmetry axis ($x = 0$), the slope will be zero.

In order to minimize capacitive coupling of the electric field to the induction coil, the coil is placed *outside* the capacitor. In addition, both, capacitor and coil, are separately shielded, i.e. enclosed in Faraday cages, see figure 2. The magnetic field is conducted to the coil by a ferrite rod[3]. The rod protrudes through a slit in the capacitor's shielding into the centre of the capacitor and can be moved by a translation stage such that the rod remains parallel to the magnetic field lines at the tip of the rod.

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[3] Keramische Werke Hermsdorf, Manifer 340, diameter 10 mm, length 150 mm, $\mu_i = 100$, $f_{max} = 8$ MHz.
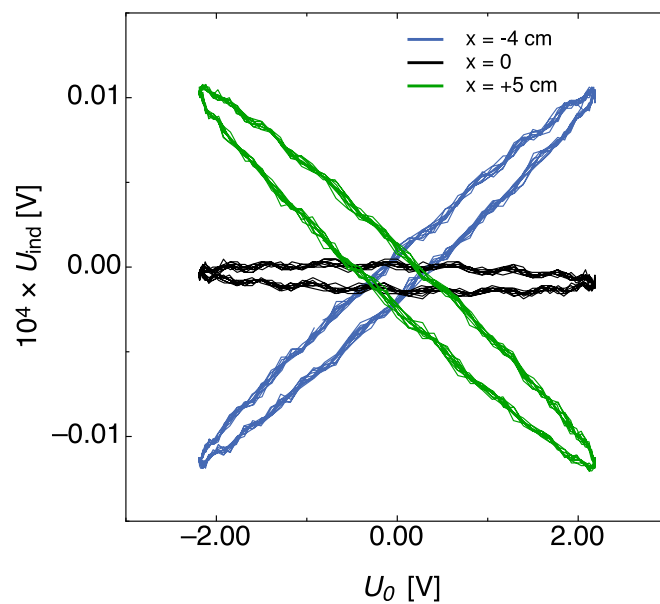
**Figure 3.** Phase-sensitive measurement of the induced voltage. The three oscillograms correspond to probe positions at approximately opposing positions in the plate capacitor ($x = -4$ cm, blue and $x = +5$ cm green) relative to its centre ($x = 0$ cm, black). The fundamental symmetry properties of the magnetic field corresponding to the displacement current become very obvious: the field at opposing positions symmetric to the centre of the capacitor has opposite sign. Consequently, the field strength on the symmetry axis is zero.

Another challenge of measuring the displacement current's magnetic field originates from the small magnitude of the induced voltage—micro-volts as explained above. Noise from the environment has to be filtered efficiently. Therefore, a heterodyne amplifier has been used, for a slightly simplified scheme see figure 2. The voltage delivered by the coil is first fed into a low-noise preamplifier and then mixed with a 800 kHz local oscillator. The output oscillating at 200 kHz is cleaned in a narrow-band filter ($\Delta\nu = 100$ Hz) before it is connected to the $Y$-input of the oscilloscope. Similarly, also the plate capacitor's voltage is mixed with the same local oscillator, filtered, and connected to the $X$-input of the oscilloscope[4]. It should be noted, however, that the heterodyne detection is not effective against capacitive coupling because the respective signal has the same frequency as the real signal.

## Calibration

The use of a ferrite rod for coupling the magnetic field from the capacitor to the induction coil requires to calibrate the coupling coefficient. To this end, small Helmholtz coils (10 turns in each coil, coil radius 15 mm) each are slid onto the tip of the ferrite while the other end remains in the induction coil. A current of 16.5 mA at a frequency of 1.63 MHz is applied to the Helmholtz coils. This generates a magnetic field of 25 $\mu$T corresponding to 1.5 $\mu$T mA$^{-1}$.

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[4] In practice, we use a slightly modified Präzitronic GF61 frequency generator together with a MV61 voltage level meter.
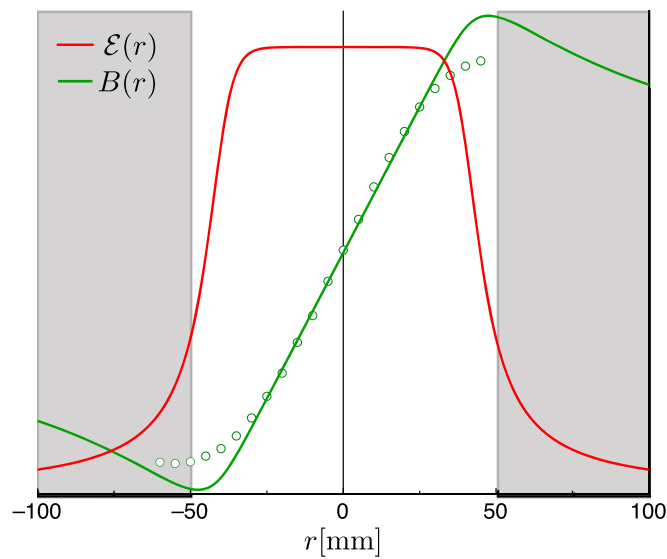
**Figure 4.** Electric (red) and magnetic (green) field strength in a finite plate capacitor according to the elementary theory already known to Maxwell. The data points correspond to the *y*-components of the Lissajous curves some of which are displayed in figure 3. The inner and outer regions of the capacitor are indicated by the white anf grey background.

The phase-sensitive voltmeter then displays +10 dBm or 2.5 V. Accordingly, a voltage of 100 mV is measured for a magnetic field of 1 $\mu$T.

According to the elementary analysis above, the maximum field strength in the plate capacitor is expected to be 4 orders of magnitude lower, i.e. a voltage of the order of 10 $\mu$V corresponding to $\approx -100$ dBm is expected to be detected. Within the experimental uncertainties, which we estimate to be on the order of 30%, the experiment agrees well with this prediction. The setup can also be used to calibrate the phase relation of the voltage applied to the plate capacitor / Helmholtz coils and the induced voltage. In this way, also the dynamics of the displacement current can be determined.

## Measurements

Figure 3 displays phase-sensitive measurements of the displacement current for three positions of the ferrite rod in the capacitor. The Lissajous curve corresponds to the parametric plot of the voltage $V_0$ applied to the plate capacitor and the induced voltage $V_{\text{ind}}$, both mixed down to 200 kHz as shown in figure 2. A slight phase shift of $V_{\text{ind}}$ versus $V_0$ is noted. This is due to the fact that the driving frequency is not very far from the resonance frequency of the induction coil. The corresponding phase error can be compensated by a phase shifter. For visual convenience, however, we usually keep some residual phase shift.

The *y*-amplitudes of the Lissajous curves for a series of different positions of the ferrite rod in the plate capacitor are displayed in figure 3. A high-frequency voltmeter has been used for increased accuracy and reading convenience. The respective data points are plotted in

figure 4 on top of the theoretical curves discussed above. Considering the uncertainties of the calibration, we have used the slope $\partial B/\partial x$ as a fit parameter. A very reasonable agreement is observed. The deviations close to the edges of the plates are mainly caused by averaging effects due to the finite thickness of the ferrite rod. Also not included are effects from the finite thickness of the plates and distortions of the magnetic field by the very presence of the ferrite rod.

## Conclusion

A simple experimental setup for a direct detection of Maxwell's displacement current via its magnetic field has been presented. The decisive experimental artifices are the use of a ferrite rod that allows placing the induction coil outside the plate capacitor thus greatly reducing shielding issues. In addition, heterodyne amplification of the induced voltage is employed in order to suppress noise and to prove the correct geometry of the magnetic field and potentially also its time-dependence. The setup is relatively simple and robust. Its conceptual simplicity along with accessibility of all relevant part make it suitable for lecture hall demonstration experiments.

## In memoriam

Professor Gerhard Scheler (1930–2014) was a pioneer in NMR spectroscopy. He was a leading member and finally the head of the probably most recognized NMR group behind the iron curtain. His NMR probes gained a particularly high international reputation, before and after German reunification. He continued his research and development efforts long beyond retirement and frequently in cooperation with the Bruker company. His superb knowledge in high-frequency electronics and keen interest in the history of physics were pivotal for the present work.

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