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Electrodynamics for Non-Relativistic Point Charges in Electrical Engineering

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Abstract: In electrical engineering, electric charges usually move at speeds substantially slower than the speed of light. Therefore, relativistic mechanics and the Lorentz transformation are rarely applied; instead, the much simpler Newtonian mechanics and the Galilean transformation are used. This approximation is hoped to yield useful results. In this article, the exact solution of Maxwell's equations for arbitrarily moving point charges is used to demonstrate that this approximation is not suitable for point charges, even if they move extremely slowly. Nevertheless, in electrical engineering, great practical demand exists for a functioning and consistent electrodynamic model for non-relativistic point charges. This article presents such a model and demonstrates that it ensures the universal constancy of the speed of light for all receiving antennas, while satisfying the Galilean principle of relativity regarding the point charges. The framework can describe electromagnetic waves as well as all static and quasistatic effects with excellent quality if they are generated by non-relativistic point charges. The framework is based on the exact solution of Maxwell's equations for arbitrarily moving point charges and a simplified Lorentz force law, which has been calibrated so that the force for slow velocities matches Ampère's original force law and thus fulfills the predictions of magnetostatics and quasistatics.

Index Terms: Electrodynamics, Magnetic forces, Simulation software, Energy conservation

1. Introduction

Theoretical physics has indicated that Maxwell's electrodynamics does not provide correct results for moving test charges without Lorentz transformation and relativistic mechanics, particularly when the test charges are moving very fast. In the daily engineering practice, however, charges rarely have high speeds. In most cases, speeds are below the speed of sound. Only the emitted electromagnetic waves show fast movement in these applications.

In engineering textbooks, for reasons of simplicity, such tasks in electrical engineering are often approached as if the special theory of relativity does not exist; i.e., the Maxwell equations are solved in the rest frame of the transmitter, and the calculated fields are then inserted in the Lorentz force without a Lorentz transformation. This method implicitly involves a Galilean transformation. The results of the calculations are hoped to be practically useful despite the approximation.

The systematic use of this approach can be seen in textbooks on electronics, energy technology, communications engineering, electromagnetic compatibility or measurement technology. Terms such as Lorentz transformation or relativity are absent in such textbooks. Textbooks intended to introduce engineers to electromagnetic field theory are an exception. However, even in such textbooks, the special theory of relativity is not always mentioned, and if so, then only at the very end [1]–[3].

The approach of electrical engineering to implicitly apply a Galilean transformation leads to several major disadvantages. Firstly, the Doppler effect is absent; this aspect is problematic in radar applications, for example, even though the objects to be detected by radar move at extremely non-relativistic speeds. Furthermore, both of Einstein's postulates are violated: the solution follows neither the Galilean nor the special principle of relativity, and the calculated electromagnetic wave moves for only the transmitter and for stationary test charges (resting probes, resting receiving antennas) at the speed of light. Another substantial disadvantage is that the results obtained in this manner violate the classical conservation laws. The violation of the conservation of momentum is particularly unpleasant, because completely correct calculations (apart from the approximation) predict the existence of so-called EmDrives.

The problem with the conservation of momentum becomes apparent when the exact solution of Maxwell's equations for arbitrarily moving point charges is generalized to arbitrarily moving test charges without Lorentz transformation. This article demonstrates that such an approximation is not useful and that not even the laws of magnetostatics are correctly reproduced. However, there is a solution, because the approximation can be fairly easily corrected by simplifying the Lorentz force law. This correction ultimately takes advantage of the circumstance that the model is already nearly relativistically correct, because of the application of retarded potentials.

The resulting non-relativistic electrodynamics has numerous advantageous properties with respect to the commonly applied basic laws of electrical engineering. For example, electrodynamics is reduced to two elementary functions that no longer contain any differential operators and can therefore be calculated directly. In addition, both equations have the same form in every inertial and non-inertial frame, and depend only on relative quantities, such as distance vectors, relative velocity and relative acceleration. Simultaneously, the radiated fields for each test charge are ensured to always move at the speed of light, regardless of their relative speed in relation to the source of the force. The magnetic effects and the quasistatics are also correctly included, and how the various manifestations of electromagnetic induction are produced becomes apparent. In addition, the conservation laws are met, and it is easy to demonstrate mathematically that an isolated system of point charges never changes its total momentum, even if the point charges are accelerated and emit electromagnetic waves.

This article consists of two parts. The first part is theoretical and shows step by step in a brief and economical manner the calculation of the fields produced by arbitrarily moving point charges. The fields are then generalized to arbitrarily moving test charges through a Galilean transformation. As expected, the solution is useless if it is inserted into the Lorentz force. However, the second part of this article demonstrates a simple and obvious means of correction[1]. Subsequently, several examples are provided to show how the resulting framework can be used. Moreover, explanations are provided regarding how certain aspects of the model are to be interpreted correctly and the conditions under which the presented framework cannot be used.

2. General solution to Maxwell's equations for point charges

In this section, Maxwell's equations for point charges are solved. We start with the Liénard-Wiechert potentials, which are the exact solution of the Maxwell equations for the electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ generated by a moving point charge q_s with trajectory $\boldsymbol{r}_s(t)$ in the rest frame of a test charge q_d located at time t and location $\boldsymbol{r}$.

The Liénard-Wiechert potentials are derived and have been well-described in physics textbooks (e.g., [4]). In summary, the Liénard-Wiechert potentials state that the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ can be calculated by using the formulas

$$\boldsymbol{E} = -\nabla\,\Phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{1}$$

and

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{2}$$

if the potentials

$$\Phi = \frac{q_s\,c}{4\,\pi\,\varepsilon_0\,\left(c^2\,(t-\tau) - \dot{\boldsymbol{r}}_s(\tau)\cdot(\boldsymbol{r}-\boldsymbol{r}_s(\tau))\right)} \tag{3}$$

and

$$\boldsymbol{A} = \frac{1}{c^2}\,\dot{\boldsymbol{r}}_s(\tau)\,\Phi \tag{4}$$

are known. The parameter τ is a certain moment in the past defined by

$$\tau = t - \frac{1}{c}\,\|\boldsymbol{r}-\boldsymbol{r}_s(\tau)\|\,. \tag{5}$$

[1] All mathematical proofs in this article have been validated in Mathematica, and the corresponding script is available from the author on request.

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The electromagnetic force $\boldsymbol{F}$ in Maxwell's electrodynamics on a test charge q_d can be found by means of the Lorentz force

$$\boldsymbol{F} = q_d\,\boldsymbol{E} + q_d\,\boldsymbol{u} \times \boldsymbol{B} \tag{6}$$

where $\boldsymbol{u}$ is the velocity of the test charge relative to an observer [5].

To evaluate the right side of equation (6), we must calculate the derivatives in equations (1) and (2). For this reason, we calculate the derivatives of the potentials Φ and $\boldsymbol{A}$. The difficulty in doing so is that τ is an unknown function of $\boldsymbol{r}$ and t. By calculating these derivatives, we obtain

$$\nabla\,\Phi = \frac{q_s\,c\,h_2(\tau)\,\nabla\,\tau + q_s\,c\,\dot{\boldsymbol{r}}_s(\tau)}{4\,\pi\,\varepsilon_0\,h_1(\tau)^2}, \tag{7}$$

$$\frac{\partial}{\partial t}\,\boldsymbol{A} = -\frac{q_s\,\boldsymbol{h}_3(\tau)\,\frac{\partial\tau}{\partial t} + q_s\,c^2\,\dot{\boldsymbol{r}}_s(\tau)}{4\,\pi\,\varepsilon_0\,c\,h_1(\tau)^2} \tag{8}$$

and

$$\nabla \times \boldsymbol{A} = \frac{q_s\,\boldsymbol{h}_3(\tau) \times \nabla\tau}{4\,\pi\,\varepsilon_0\,c\,h_1(\tau)^2}. \tag{9}$$

Here, h_1, h_2 and $\boldsymbol{h}_3$ are auxiliary variables that were introduced so that the equations above could be presented more concisely. They are defined as

$$h_1(\tau) := (\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \dot{\boldsymbol{r}}_s(\tau) - c^2\,(t - \tau), \tag{10}$$

$$h_2(\tau) := c^2 - \dot{\boldsymbol{r}}_s(\tau) \cdot \dot{\boldsymbol{r}}_s(\tau) + (\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \ddot{\boldsymbol{r}}_s(\tau) \tag{11}$$

and

$$\boldsymbol{h}_3(\tau) := h_1(\tau)\,\ddot{\boldsymbol{r}}_s(\tau) - h_2(\tau)\,\dot{\boldsymbol{r}}_s(\tau). \tag{12}$$

In the next step, we substitute equations (7), (8) and (9) into equations (1) and (2) to obtain

$$\boldsymbol{E} = \frac{q_s\,\boldsymbol{h}_3(\tau)\,\frac{\partial\tau}{\partial t} - q_s\,c^2\,h_2(\tau)\,\nabla\,\tau}{4\,\pi\,\varepsilon_0\,c\,h_1(\tau)^2} \tag{13}$$

and

$$\boldsymbol{B} = \frac{q_s\,\boldsymbol{h}_3(\tau) \times \nabla\tau}{4\,\pi\,\varepsilon_0\,c\,h_1(\tau)^2}. \tag{14}$$

Although the function τ is usually an unknown function of $\boldsymbol{r}$ and t, the derivatives $\nabla\tau$ and $\partial\tau/\partial t$ can be calculated in equations (13) and (14). To do so, we first apply the differential operators to both sides of equation (5). This process yields

$$\nabla\,\tau = \frac{-(\boldsymbol{r} - \boldsymbol{r}_s(\tau)) + (\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \dot{\boldsymbol{r}}_s(\tau)\,\nabla\,\tau}{c\,\|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\|} \tag{15}$$

and

$$\frac{\partial\tau}{\partial t} = 1 + \frac{(\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \dot{\boldsymbol{r}}_s(\tau)}{c\,\|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\|}\,\frac{\partial\tau}{\partial t}. \tag{16}$$

We now have two linear equations in terms of $\nabla\tau$ and $\partial\tau/\partial t$. These equations are easy to solve. Using the equation $\|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\| = c\,(t - \tau)$ and definition (10), we obtain the following equations

$$\nabla\,\tau = \frac{\boldsymbol{r} - \boldsymbol{r}_s(\tau)}{h_1(\tau)} \tag{17}$$

and

$$\frac{\partial\tau}{\partial t} = -\frac{c^2\,(t - \tau)}{h_1(\tau)}. \tag{18}$$

These two equations can be substituted into equations (13) and (14), thus yielding

$$\boldsymbol{E} = -\frac{q_s\,c\,\left(\boldsymbol{h}_3(\tau)\,(t - \tau) + h_2(\tau)\,(\boldsymbol{r} - \boldsymbol{r}_s(\tau))\right)}{4\,\pi\,\varepsilon_0\,h_1(\tau)^3} \tag{19}$$

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and

$$\boldsymbol{B} = \frac{q_s\, \boldsymbol{h}_3(\tau) \times (\boldsymbol{r} - \boldsymbol{r}_s(\tau))}{4\,\pi\,\varepsilon_0\, c\, h_1(\tau)^3}. \tag{20}$$

Equations (19) and (20) are the formal solutions of Maxwell's equations for a resting test charge at location $\boldsymbol{r}$ at time t. Assuming that the test charge at rest is located exactly at the origin of the coordinate system, the substitution

$$\boldsymbol{r} \to \boldsymbol{0} \tag{21}$$

in the equations (19), (20), (10), (11) and (5) can be applied.

Now let us assume that we are situated in a frame of reference in which the origin of the coordinates of the previously used frame (and thus also the test charge q_d) is moving with trajectory $\boldsymbol{r}_d(t)$. If the relative speed $\|\dot{\boldsymbol{r}}_d(\tau) - \dot{\boldsymbol{r}}_s(\tau)\|$ between q_d and q_s is much smaller than the speed of light in vacuum, c, it should be possible to apply non-relativistic mechanics and Galilean relativity. As described in the Introduction, use of this approximation is standard in electrical engineering because a Lorentz transformation would be too complicated for small velocities. We will now carry out such a transformation and subsequently investigate the consequences of this approximation.

In the rest frame of q_d, only the source charge q_s appears to be moving with the trajectory $\boldsymbol{r}_s(t)$. To change to the new frame, we can perform the replacements

$$\boldsymbol{r}_s(\tau) \to \boldsymbol{r}_s(\tau) - \boldsymbol{r}_d(\tau) := -\boldsymbol{r}, \tag{22}$$

$$\dot{\boldsymbol{r}}_s(\tau) \to \dot{\boldsymbol{r}}_s(\tau) - \dot{\boldsymbol{r}}_d(\tau) := -\boldsymbol{v} \tag{23}$$

and

$$\ddot{\boldsymbol{r}}_s(\tau) \to \ddot{\boldsymbol{r}}_s(\tau) - \ddot{\boldsymbol{r}}_d(\tau) := -\boldsymbol{a}. \tag{24}$$

Note that after the transformation, trajectories must be used that are valid in the new frame. The transformation therefore does not change the numerical values of the fields acting on the charge q_d.

Because of these replacements and equation $c\,(t - \tau) = r$, we obtain

$$\boldsymbol{E} = \frac{q_s}{4\,\pi\,\varepsilon_0} \left(\frac{(\boldsymbol{r}\,c + r\,\boldsymbol{v})\left(c^2 - v^2 - \boldsymbol{r}\cdot\boldsymbol{a}\right)}{(r\,c + \boldsymbol{r}\cdot\boldsymbol{v})^3} + \frac{r\,\boldsymbol{a}}{(r\,c + \boldsymbol{r}\cdot\boldsymbol{v})^2} \right) \tag{25}$$

and

$$\boldsymbol{B} = \frac{q_s}{4\,\pi\,\varepsilon_0\, c} \left(\frac{(\boldsymbol{r}\times\boldsymbol{v})\left(c^2 - v^2 - \boldsymbol{r}\cdot\boldsymbol{a}\right)}{(r\,c + \boldsymbol{r}\cdot\boldsymbol{v})^3} + \frac{\boldsymbol{r}\times\boldsymbol{a}}{(r\,c + \boldsymbol{r}\cdot\boldsymbol{v})^2} \right). \tag{26}$$

Incidentally, as can be seen directly, the equation

$$\boldsymbol{B} = \frac{\boldsymbol{r}}{r} \times \frac{\boldsymbol{E}}{c} \tag{27}$$

applies. This equation shows that, in Maxwell's electrodynamics, the force $\boldsymbol{F}$ that a point charge q_s with trajectory $\boldsymbol{r}_s(t)$ exerts on another point charge q_d with trajectory $\boldsymbol{r}_d(t)$ at time t can be expressed solely in terms of the electric field $\boldsymbol{E}$.

Of note, formulas (25) and (26) correspond to formulas (4-4.34) and (4-5.2) by O. Jefimenko [6], although Jefimenko's method for calculating the solutions is more laborious[2]. Equation (27) can also be found as formula (3-2.13) in Jefimenko's work.

The final solution for the force experienced by a moving test charge can be obtained by inserting equation (27) into the Lorentz force (6) as follows:

$$\boldsymbol{F} = q_d \left(\boldsymbol{E} + \frac{\boldsymbol{u}}{c} \times \left(\frac{\boldsymbol{r}}{r} \times \boldsymbol{E} \right) \right). \tag{28}$$

Force $\boldsymbol{F}$ is therefore a function of four parameters: $\boldsymbol{u}$, $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$.

[2]Because Jefimenko does not generalize his equations to arbitrarily moving test charges, the signs of $\boldsymbol{v}$ and $\boldsymbol{a}$ should be considered when the equations are being compared.

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The velocity $\boldsymbol{u}$ is defined as the velocity $\dot{\boldsymbol{r}}_d(t)$ of the charge q_d at the current time t. The next three parameters are the retarded distance vector

$$\boldsymbol{r} := \boldsymbol{r}_d(\tau) - \boldsymbol{r}_s(\tau), \tag{29}$$

the retarded relative velocity

$$\boldsymbol{v} := \dot{\boldsymbol{r}}_d(\tau) - \dot{\boldsymbol{r}}_s(\tau) \tag{30}$$

and the retarded relative acceleration

$$\boldsymbol{a} := \ddot{\boldsymbol{r}}_d(\tau) - \ddot{\boldsymbol{r}}_s(\tau). \tag{31}$$

Interestingly, these parameters are purely relative quantities that do not depend on the choice of a frame of reference. However, this does not apply to the parameter $\boldsymbol{u}$, which is a quantity that depends on the point of view of an observer.

The time $\tau < t$ is a moment in the past and is defined by the equation

$$\tau = t - \frac{r}{c}. \tag{32}$$

Importantly, r is the absolute value of $\boldsymbol{r}$ and is a function of τ, because of definition (29). Because the parameter τ therefore occurs on both sides of equation (32), the equation can usually be solved only numerically. However, this aspect does not present a major practical problem, because equation (32) is one-dimensional and can always be solved quickly and unambiguously when the relative speed is below the speed of light, c. The proof that a single unique solution always exists is simple and can be performed by using Banach's fixed point theorem.

We will soon see that the definition of $\boldsymbol{u}$ can no longer be used under the previous approximations and that a correction must be performed. Before we come to this, we need to simplify the formula (28) for the special case of uniform motion.

3. Simplification of the solution for uniformly moving point charges

If all the involved point charges are moving almost uniformly, equation (28) for the electromagnetic force can be simplified. First, we exploit that $\boldsymbol{a} \approx \boldsymbol{0}$, given our assumption of uniform motion. Therefore, equation (25) reduces to

$$\boldsymbol{E} = \frac{q_s}{4\,\pi\,\varepsilon_0} \left(\frac{(\boldsymbol{r}\,c + r\,\boldsymbol{v})\left(c^2 - v^2\right)}{(r\,c + \boldsymbol{r}\cdot\boldsymbol{v})^3} \right). \tag{33}$$

This equation can be further transformed to

$$\boldsymbol{E} = \frac{q_s}{4\,\pi\,\varepsilon_0\,\gamma(v)^2} \frac{\boldsymbol{r} + \frac{r}{c}\,\boldsymbol{v}}{\left(r + \boldsymbol{r}\cdot\frac{\boldsymbol{v}}{c}\right)^3}, \tag{34}$$

where $\gamma(.)$ is the Lorentz factor. This equation can now be substituted for $\boldsymbol{E}$ in the force (28), thus yielding

$$\boldsymbol{F} = \frac{q_s\,q_d}{4\,\pi\,\varepsilon_0\,\gamma(v)^2} \frac{\boldsymbol{r} + \frac{r}{c}\,\boldsymbol{v} + \frac{1}{c^2}\,\boldsymbol{u}\times(\boldsymbol{r}\times\boldsymbol{v})}{\left(r + \boldsymbol{r}\cdot\frac{\boldsymbol{v}}{c}\right)^3}. \tag{35}$$

Because of definition (29), $\boldsymbol{r}$ is a retarded quantity. However, the velocities $\boldsymbol{u}$ and $\boldsymbol{v}$ are not retarded, because the velocities are assumed to be constant. Because retarded variables complicate calculations, expressing equation (35) in terms of the non-retarded distance vector

$$\boldsymbol{s} := \boldsymbol{r}_d(t) - \boldsymbol{r}_s(t) \tag{36}$$

is useful. To remove $\boldsymbol{r}$, we use the fact that, for uniform velocities, $\boldsymbol{v}$, the equation

$$\boldsymbol{s} = \boldsymbol{r} + \boldsymbol{v}\,(t - \tau) \tag{37}$$

applies. Because $t - \tau = r/c$,

$$\boldsymbol{r} = \boldsymbol{s} - \frac{r}{c}\,\boldsymbol{v}. \tag{38}$$

Equation (35) can therefore be simplified to

$$\boldsymbol{F} = \frac{q_s\, q_d}{4\,\pi\,\varepsilon_0\,\gamma(v)^2}\,\frac{\boldsymbol{s} + \frac{1}{c^2}\,\boldsymbol{u} \times (\boldsymbol{s} \times \boldsymbol{v})}{\left(r + \boldsymbol{r} \cdot \frac{\boldsymbol{v}}{c}\right)^3}. \tag{39}$$

If we multiply both sides of equation (38) with $\boldsymbol{r}/r$, we see that

$$r + \boldsymbol{r} \cdot \frac{\boldsymbol{v}}{c} = \boldsymbol{s} \cdot \frac{\boldsymbol{r}}{r}. \tag{40}$$

Inserting equation (38) on the right side of the equation above yields

$$r + \boldsymbol{r} \cdot \frac{\boldsymbol{v}}{c} = \frac{s^2}{r} - \frac{\boldsymbol{s} \cdot \boldsymbol{v}}{c}. \tag{41}$$

On the basis of equation (32), $r = c\,(t - \tau)$. Furthermore, because of equation (37), the relation $r = ||\boldsymbol{s} - \boldsymbol{v}\,(t - \tau)||$ holds. Together, these two equations yield

$$r = ||\boldsymbol{s} - \frac{\boldsymbol{v}}{c}\,r||. \tag{42}$$

This equation can be solved, thus yielding

$$r = \frac{c\,s^2}{\boldsymbol{s} \cdot \boldsymbol{v} + c\,\sqrt{s^2 - \frac{1}{c^2}||\boldsymbol{s} \times \boldsymbol{v}||^2}}. \tag{43}$$

If we substitute this relation for r on the right side of equation (41), we obtain

$$r + \boldsymbol{r} \cdot \frac{\boldsymbol{v}}{c} = \sqrt{s^2 - \frac{1}{c^2}||\boldsymbol{s} \times \boldsymbol{v}||^2}. \tag{44}$$

Therefore, we can transform equation (39) into

$$\boldsymbol{F}(q_s, q_d, \boldsymbol{s}, \boldsymbol{v}, \boldsymbol{u}) = \frac{q_s\, q_d}{4\,\pi\,\varepsilon_0\,\gamma(v)^2}\,\frac{\boldsymbol{s} + \frac{1}{c^2}\,\boldsymbol{u} \times (\boldsymbol{s} \times \boldsymbol{v})}{\left(s^2 - \frac{1}{c^2}||\boldsymbol{s} \times \boldsymbol{v}||^2\right)^{3/2}}. \tag{45}$$

This equation for the force between two uniformly moving point charges now depends only on the non-retarded distance vector $\boldsymbol{s}$. Parameters that depend on the past time τ are no longer present. We can verify that equation (45) is identical to equation (35) by inserting relation (37) into equation (45).

4. Force exerted by a direct current on a test charge

Equation (45) can now be used, for example, to calculate the force that an infinitely long, straight, direct current exerts on a test charge q_d that is moving uniformly at speed $\boldsymbol{w}$ in the rest frame of an observer. To preserve generality, we assume that both the negative and the positive charge carriers in the electrical conductor are moving at the average drift velocities $\boldsymbol{w}_-$ and $\boldsymbol{w}_+$. According to the definition of the velocity $\boldsymbol{u}$, this should correspond to the velocity $\boldsymbol{w}$, as this is the velocity of the test charge from the observer's point of view. However, since we have neglected special relativity, we expect problems. For this reason, we want to leave the parameter $\boldsymbol{u}$ open to find out whether the problems can be circumvented.

To keep the calculation simple, we assume that the charge carriers in the electrical conductor are always located on the x-axis. For the velocities $\boldsymbol{w}_-$ and $\boldsymbol{w}_+$, we can then define $\boldsymbol{w}_- = w_-\,\boldsymbol{e}_x$ and $\boldsymbol{w}_+ = w_+\,\boldsymbol{e}_x$. Furthermore, we assume that the test charge is at the location $\boldsymbol{r} = r\,\boldsymbol{e}_z$ and has the velocity $\boldsymbol{w} = w_x\,\boldsymbol{e}_x + w_z\,\boldsymbol{e}_z$. Figure 1 shows the configuration schematically.

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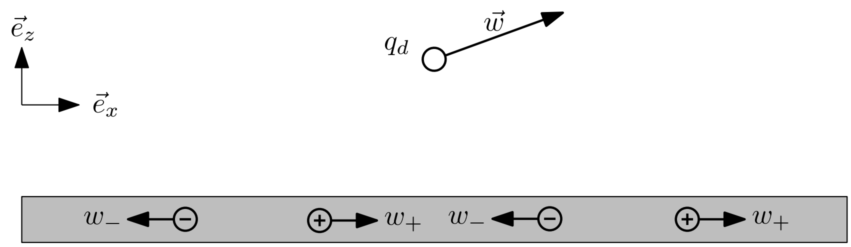

Fig. 1. Schematic representation of a long straight conductor with a direct current and a test charge moving with velocity $\boldsymbol{w}$. In this section, the net force of all charge carriers of the conductor on the test charge and then the net force of the test charge on all charge carriers in the conductor are calculated. Both net forces should be equal and opposite in Newtonian mechanics.

The force $\boldsymbol{F}_{wd}$ on the test charge q_d can then be calculated by integrating over all the charge carriers in the conductor as follows:

$$\begin{aligned}\boldsymbol{F}_{wd} = &\int_{-\infty}^{\infty} \boldsymbol{F}(n\,(-e), q_d, \boldsymbol{r} - x\,\boldsymbol{e}_x, \boldsymbol{w} - \boldsymbol{w}_-, \boldsymbol{u}_-)\,\mathrm{d}x\, + \\ &\int_{-\infty}^{\infty} \boldsymbol{F}(n\,(+e), q_d, \boldsymbol{r} - x\,\boldsymbol{e}_x, \boldsymbol{w} - \boldsymbol{w}_+, \boldsymbol{u}_+)\,\mathrm{d}x.\end{aligned} \tag{46}$$

Here, n is the number of charge carriers per meter of conductor, and e represents the elementary charge. For the velocity $\boldsymbol{u}$, we try $\boldsymbol{u}_- := c_1\,\boldsymbol{w} - c_2\,\boldsymbol{w}_-$ and $\boldsymbol{u}_+ := c_1\,\boldsymbol{w} - c_2\,\boldsymbol{w}_+$, with c_1 and c_2 being constants that we want to determine later. For $c_1 = 1$ and $c_2 = 1$, for example, $\boldsymbol{u}$ would be identical to the corresponding relative velocity. For $c_1 = 1$ and $c_2 = 0$, in contrast, we would have $\boldsymbol{u} = \boldsymbol{w}$, as is the case in magnetostatics, quasistatics and special relativity.

The integrals in equation (46) can be solved to yield

$$\boldsymbol{F}_{wd} = \frac{e\,n\,q_d\,\left(f_x\,\boldsymbol{e}_x + f_z\,\boldsymbol{e}_z\right)}{2\,c^2\,\varepsilon_0\,\pi\,r\,\sqrt{1 - \frac{w_z^2}{c^2}}} \tag{47}$$

with

$$f_x := (1 + c_1)\,(w_+ - w_-)\,w_z \tag{48}$$

and

$$f_z := (w_+ - w_-)\,(c_2\,(w_+ + w_-) - (c_1 + c_2)\,w_x)\,. \tag{49}$$

For $w_-, w_+, w_z \ll c$, we obtain the first-order approximation

$$\boldsymbol{F}_{wd} \approx \frac{e\,n\,q_d \begin{pmatrix} (1 + c_1)\,(w_+ - w_-)\,w_z \\ 0 \\ -(c_1 + c_2)\,(w_+ - w_-)\,w_x \end{pmatrix}}{2\,c^2\,\varepsilon_0\,\pi\,r}. \tag{50}$$

The force can also be calculated by using classical magnetostatics. The magnetic field $\boldsymbol{B}$ is first calculated and then explicitly substituted into the formula of the Lorentz force (6). This approach yields

$$\boldsymbol{F}_{wd} = -\frac{I\,q_d\,\left(\boldsymbol{w} \times \boldsymbol{e}_y\right)}{2\,c^2\,\varepsilon_0\,\pi\,r}. \tag{51}$$

For the current I, the equation $I = e\,n\,(w_+ - w_-)$ applies. Substituting this definition into the equation above yields

$$\boldsymbol{F}_{wd} = \frac{e\,n\,q_d \begin{pmatrix} (w_+ - w_-)\,w_z \\ 0 \\ -(w_+ - w_-)\,w_x \end{pmatrix}}{2\,c^2\,\varepsilon_0\,\pi\,r}. \tag{52}$$

A comparison of equations (50) and (52) shows that, if the parameters $c_1 = 1$ and $c_2 = 1$ are chosen, the force obtained by electrodynamics would be *twice as large* as that obtained by magnetostatics. Setting $c_1 = 1$ and $c_2 = 0$ likewise does not produce a meaningful result. The only possible choice is $c_1 = 0$ and $c_2 = 1$.

This requirement for c_1 and c_2 means that the parameter $\boldsymbol{u}$ in equation (28) cannot be the relative velocity $\boldsymbol{v} = \dot{\boldsymbol{r}}_d - \dot{\boldsymbol{r}}_s$ or the velocity $\boldsymbol{w} = \dot{\boldsymbol{r}}_d$ of the test charge relative to the observer. Instead, $\boldsymbol{u}$ must be the velocity $-\dot{\boldsymbol{r}}_s$, which is constant in time for uniform velocities. This brings us to formula

$$\boldsymbol{F} = q_d \left(\boldsymbol{E} - \frac{\dot{\boldsymbol{r}}_s}{c} \times \left(\frac{\boldsymbol{r}}{r} \times \boldsymbol{E}\right)\right). \tag{53}$$

However, equation (53) is not applicable in classical mechanics, because it violates the conservation of momentum. Switching the source and receiver of the force does not result in a mere change in sign, as Newton's third law would require of a valid force formula. This problem is well known in the scientific literature and is justified by arguing that the radiation field can also emit and absorb momentum (e.g., [7]).

However, in non-relativistic quasistatic electrodynamics, we expect that the force exerted on the test charge by the long, straight conductor in the previous section would be inversely equal to the force exerted on the conductor by the test charge. To calculate this force, we change the source and receiver of the force in equation (46). In doing so, we must substitute $-\boldsymbol{w}$ for $\boldsymbol{u}$, as required by equation (53). We obtain

$$\begin{aligned} \boldsymbol{F}_{dw} = &\int_{-\infty}^{\infty} \boldsymbol{F}(q_d, n\,(-e), x\,\boldsymbol{e}_x - \boldsymbol{r}, \boldsymbol{w}_- - \boldsymbol{w}, -\boldsymbol{w})\,\mathrm{d}x + \\ &\int_{-\infty}^{\infty} \boldsymbol{F}(q_d, n\,(+e), x\,\boldsymbol{e}_x - \boldsymbol{r}, \boldsymbol{w}_+ - \boldsymbol{w}, -\boldsymbol{w})\,\mathrm{d}x. \end{aligned} \tag{54}$$

For speeds well below the speed of light (i.e., $w_-, w_+, w_z \ll c$), the solution to equation (54) can be simplified to

$$\boldsymbol{F}_{dw} = \frac{e\,n\,q_d \begin{pmatrix} -2\,(w_+ - w_-)\,w_z \\ 0 \\ (w_+ - w_-)\,w_x \end{pmatrix}}{2\,c^2\,\varepsilon_0\,\pi\,r}. \tag{55}$$

A comparison of equations (55) and (52) shows that $\boldsymbol{F}_{wd} \neq -\boldsymbol{F}_{dw}$. Because of this non-equivalence, when equation (6) is used, the conservation of momentum is already violated in simple applications with direct currents and very slow test charges. This violation is, of course, unacceptable in engineering.

5. Solution to the problem

At this point, one might be concerned that the Lorentz transformation can really never be avoided and that it must be applied even for small relative velocities. However, there is a way out. To explain the approach, we go back to the solutions of the Lienard-Wiechert potentials (19) and (20) in the rest frame of the test charge. We now substitute these solutions without transformation into the Lorentz force (6) and find that the velocity $\boldsymbol{u}$ is zero in the rest frame of the test charge. This means that the force in the rest frame reduces to the formula $\boldsymbol{F} = q_d\,\boldsymbol{E}$. In Newtonian mechanics, a force formula, such as Newton's law of gravity, Coulomb's law or the Weber force, does not depend on the frame of reference in which an observer is located, since only distance vectors or, in the case of the Weber force, relative velocities

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are involved. Following the same logic, we now apply the transformations (21), (22), (23) and (24) to the force. In practice, this method does not change the calculations in the previous section, except that we assume from the outset that the velocity $\boldsymbol{u}$ is zero.

If we repeat the calculations of section 4 with $\boldsymbol{u} = \boldsymbol{0}$, we find that although the conservation of momentum would now apply, the force does not correspond to our expectations from magnetostatics and quasistatics. For this reason, we now introduce an *ad hoc assumption* and postulate that the force formula is

$$\boldsymbol{F} = q_d\, \gamma(v)\, \boldsymbol{E}. \tag{56}$$

Since the speed v in $\gamma(v)$ is a relative quantity, the force now only depends on relative quantities. Note that this is a small correction, because $\gamma(v)$ is practically equal to one for $v \ll c$.

Given this alternative definition of the force, we find that equation (45) becomes

$$\boldsymbol{F}(q_s, q_d, \boldsymbol{s}, \boldsymbol{v}) = \frac{q_s\, q_d}{4\,\pi\,\varepsilon_0\,\gamma(v)}\,\frac{\boldsymbol{s}}{\left(s^2 - \frac{1}{c^2}\|\boldsymbol{s}\times\boldsymbol{v}\|^2\right)^{3/2}}. \tag{57}$$

As a consequence, all conservation laws of classical mechanics are now satisfied [8]. Furthermore, we can verify that equations (46) and (54) produce the correct results when the revised force formula (57) is used[3]. This means that the Lorentz force is now correctly included.

Another strong argument that the ad hoc assumption is reasonable is that if we expand the right side of equation (57) with respect to $\boldsymbol{v}$ into a Taylor series, we obtain the second-order approximation

$$\boldsymbol{F}(q_s, q_d, \boldsymbol{s}, \boldsymbol{v}) \approx \frac{q_s\, q_d\, \boldsymbol{s}}{4\,\pi\,\varepsilon_0\, s^3}\left(1 + \frac{v^2}{c^2} - \frac{3}{2}\left(\frac{\boldsymbol{s}}{s}\cdot\frac{\boldsymbol{v}}{c}\right)^2\right). \tag{58}$$

This force formula corresponds to that of C. F. Gauss [9, p. 617] from 1835, which W. Weber used as the starting point for Weber electrodynamics and which is directly compatible with Ampère's original force law [10]. The specialist literature on Weber electrodynamics outlines how the electrodynamics of Ampère, Gauss and Weber is capable of correctly reproducing the entire electrostatics and magnetostatics (e.g., [11]). Specifically, Gauss's formula contains the physical Lorentz force without having to add it explicitly as a supplementary formula. Notably, even J. C. Maxwell favored Ampère's original force law and praised it as the best possible law for quasistatics [12, p. 161].

In summary, replacing (6) with (56) and using the solution (25) results in the following equation

$$\boxed{\boldsymbol{F} = \frac{q_s\, q_d\, \gamma(v)}{4\,\pi\,\varepsilon_0}\left(\frac{(\boldsymbol{r}\, c + r\,\boldsymbol{v})\left(c^2 - v^2 - \boldsymbol{r}\cdot\boldsymbol{a}\right)}{(r\, c + \boldsymbol{r}\cdot\boldsymbol{v})^3} + \frac{r\,\boldsymbol{a}}{(r\, c + \boldsymbol{r}\cdot\boldsymbol{v})^2}\right),} \tag{59}$$

taking into account definitions (29), (30), (31) and (32). This formula contains all aspects of classical non-relativistic electrodynamics, including Lorentz force, electromagnetic induction, electromagnetic waves and the Doppler effect. The formula is also highly suitable for computer simulations, because no differential equations must be solved, and calculations can therefore be performed very efficiently. The following section provides several illustrative examples.

6. Applications and explanations

Formula (59) is clearly particularly suitable for calculating the electromagnetic fields generated by accelerated point charges. To illustrate this aspect, let us consider a cyclotron, i.e., a simple particle accelerator for generating non-relativistic ions [13]. With formula (59), the calculation of the field is fairly straightforward.

First we need the trajectory of the accelerated ion. A typical feature of the cyclotron is that the radius increases proportionally to the speed of the ion. We therefore model the spiral trajectory approximately with the formula

$$\boldsymbol{r}_s(t) = s\, t\left(\boldsymbol{e}_x\, \sin(2\,\pi\, f\, t) + \boldsymbol{e}_y\, \cos(2\,\pi\, f\, t)\right). \tag{60}$$

[3]The verification can be found in the Mathematica script.

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The parameter s determines how rapidly the spiral increases per unit of time. The parameter f is the cyclotron frequency.

This trajectory can be inserted into formula (29). Furthermore, the trajectory $\boldsymbol{r}_d(t)$ of the test charge is required in formula (29). Importantly, the test charge need not be a real charge and can instead be purely virtual. In that case, we would want to determine the force that would be measured if a test charge were present at a certain location $\boldsymbol{r}_0$ at time t. The trajectory of such a virtual test charge at rest is $\boldsymbol{r}_d(t) = \boldsymbol{r}_0$. If the parameter $\boldsymbol{r}_0$ is left variable, the time-dependent force $\boldsymbol{F}(t)$ becomes a time-dependent field $\boldsymbol{F}(\boldsymbol{r}_0, t)$.

However, we could also keep further parameters variable. For example, we could assume that the virtual test charge is moving along the trajectory $\boldsymbol{r}_d(t) = \boldsymbol{r}_0 + \boldsymbol{v}_0\, t$. In this case, the force would be a field that also depends on the velocity $\boldsymbol{v}_0$. Force fields that depend on the velocity are not unusual; e.g., the Lorentz force (6) is a field that depends on both the location and the velocity of the test charge. Therefore, formula (59) can also represent fields, although it might initially appear to be a direct force between two point charges.

Another characteristic is the inherent retardation. To calculate the force at time t, a past moment τ is required, which must be calculated with the formula (32). In most cases, equation (32) does not have an algebraic solution. However, a numerical solution is always possible, because equation (32) is scalar and has exactly a single unique solution for non-relativistic point charges. Furthermore, equations (30) and (31) are required, which can be easily calculated by differentiating equation (29).

The example illustrates that the calculation is simple and easy to understand. Figure 2 shows the direction and strength of the electromagnetic force that would be experienced by test charges at rest at these locations. Because the virtual test charges do not move, the depicted field corresponds to the electric field.

By assigning a speed to the virtual test charges, the force that would act on a segment of a metallic conductor loop with direct or alternating current can also be calculated. This force is equal to the sum of two forces, i.e., the force on the stationary metal ions and the force on the electrons. Because the electrons are moving, but the ions are not, the sum does not necessarily equal zero.

This residual is usually referred to as the magnetic force. In fact, as Ampère has already shown, small permanent magnets can also be interpreted as small direct current loops. Thus, the magnetic force is contained in equation (59), although the magnetic field is no longer included. Instead, the magnetic and electric forces now represent a unity and, depending on how the charges and test charges are moving, sometimes the magnetic and sometimes the electrical aspects can be more pronounced.

Furthermore, voltages can be easily calculated. For this purpose, only the electromagnetic force (59) must be integrated along a path of virtual test charges. In this way, the voltage that would be induced in an antenna or conductor can be calculated, and whether these are dipole or loop antennas is irrelevant.

A further advantage of the described model is that the Doppler effect is included. The Hertzian dipole serves as a simple example. A Hertzian dipole is a model for an elementary transmit antenna, and consists of two opposite equal charges, $+q_s$ and $-q_s$, which oscillate against each other with the trajectories $+\boldsymbol{s}(t)$ and $-\boldsymbol{s}(t)$, and whose center of gravity follows a common trajectory $\boldsymbol{r}_c(t)$. If the center of gravity of the Hertzian dipole is at rest, the calculation of the field with equation (59) is trivial, because we only need to insert the trajectories into equations (29), (30), (31) and (32). However, the calculation is also relatively simple if the center of gravity is moving uniformly. Figure 3 shows the field for the special case of a sinusoidal oscillation with and without Doppler effect. Because $\boldsymbol{s}(t)$ can be an almost arbitrary function, many types of moving transmit antennas can be represented.

As has become clear, performing correct relativistic calculations as much as possible is advantageous, even when working with non-relativistic charge carriers. As demonstrated, this process need not be complicated, and the Lorentz transformation can frequently be avoided. Another important advantage is that the electrodynamics presented herein ensures the conservation of momentum under all circumstances [8]. Therefore, the force that the ion being accelerated in the cyclotron exerts on the cyclotron corresponds exactly to the force that the cyclotron exerts on the ion. This aspect does not play a practical role in this specific case but is often highly relevant in other applications.

Because it is important for the basic understanding, it is pointed out that the conservation of momentum only applies if all components of an isolated system are considered. If, for example, only the transmitting antenna is considered, then the momentum of this antenna is generally not a conserved quantity and it appears as if the radiated wave transports the missing momentum. Consequently, there is no contradiction

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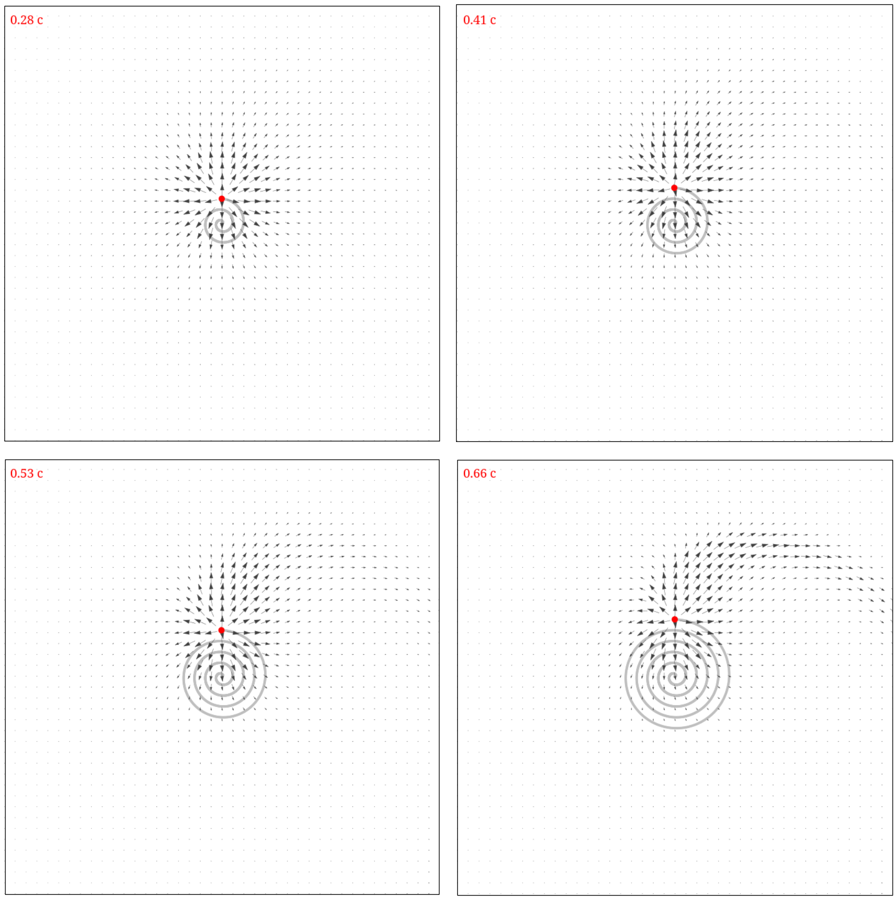

Fig. 2. The drawings show an ion being accelerated in a cyclotron. The arrows represent the strength and direction of the force that would be experienced by a test charge at rest at the respective location. The gray spiral symbolizes the path that the ion has already traveled. At the top left, the speed of the ion is shown. The electric field already noticeably differs from the Coulomb field at approximately 50 percent of the speed of light. Note that at such high speeds there is no guarantee that the model represents a valid approximation. However, the results are in line with expectations and show the tendencies that result from high speeds. (f = 60 MHz, $s = 0.02\,c$, plot area: 4x4 m)

to the empirical fact that electromagnetic waves appear to transport momentum if considered independently.

The electrodynamics presented herein can also be used for applications involving interactions. For example, we could define a second point charge that reacts to the primary wave by changing its speed in accordance with Newton's laws. Because of this acceleration, the second point charge also generates a wave. Not only two, but also several or even many point charges can be simulated. Figure 4 shows a very simple example of 22 point charges that together form 11 Hertzian dipoles, which are arranged in a row with a spacing of 10 cm and oscillate at a frequency of 1 GHz (green dots). Together they generate an electromagnetic wave, which is shown in Figure 4 on the left.

If such a wave falls on a metallic surface, the wave is reflected. Each surface element behaves as a dipole, which is pulled apart by the incident wave. Such a dipole inevitably generates a secondary wave that weakens the incident field at the location of the dipole. Therefore a reflection can be easily modeled by placing discrete dipoles at the location of the metallic surface and modeling their motion by using Newton's laws.

Figure 4 shows the field of such a modeled metal surface in the middle. To better illustrate the complexity of the resulting effects, the metal surface has two openings. Each dipole of the metal surface is marked with a blue dot. The metal surface clearly radiates in both directions. Both waves interfere both in front

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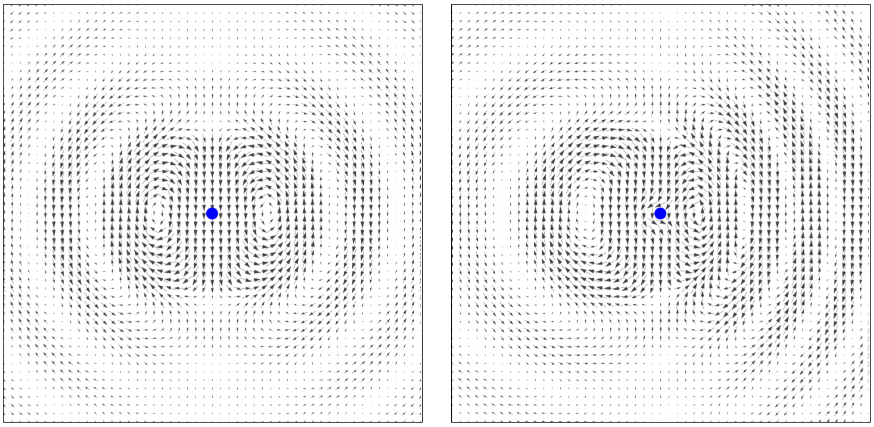

Fig. 3. Field of a resting and moving Hertzian dipole for $\boldsymbol{s}(t) = \boldsymbol{e}_z\, a \sin(2\pi f t)$ and $\boldsymbol{r}_c(t) = \boldsymbol{e}_x\, v\, t$ ($a = 10$ nm, $f = 1$ GHz, $t = 0$). The field for $v = 0$ is shown on the left. The plot on the right shows the field for $v = 0.3\,c$. The plot area has a size of 1x1 m.

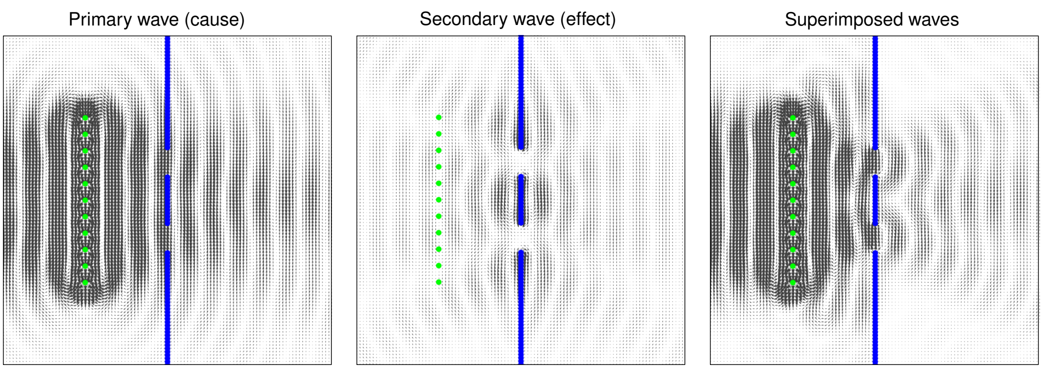

Fig. 4. An antenna (green) generates an electromagnetic wave that falls on a surface with two openings (blue). The nuclei and electron shells of the atoms on the surface react to the incident force wave by being accelerated in opposite directions. As a result, the surface itself becomes the transmitter of a secondary wave, which is shown at the center. Both waves, primary and secondary, superimpose. The resulting field is shown at the right. The example illustrates that the model also includes optical effects.

of and behind the metal surface. The sum of the two fields is shown at right. The fields behind the double slit are seen to form an interference pattern. However, there are also interesting effects in front of the metal surface, because the secondary wave acts back on the source of the primary wave, thereby creating an interaction between the transmitter (green) and itself. The resulting effects resemble effects that are known from quantum mechanics. Further details can be found in reference [14].

This example demonstrates that formula (59) can also be used to simulate and interpret phenomena of optics and quantum mechanics. Of note, the simulation could also be applied to moving transmitters and reflectors, because formula (59) satisfies the principle of relativity. The great importance of this principle is next illustrated through a simple example from quasistatics. A conductor loop with direct current is well known to induce a voltage in a conductor loop in the vicinity if both conductor loops are moving relative to each other. In practice, which of the two conductor loops is in motion does not matter. The only important aspect is that a relative speed is present.

With formula (59), this effect can be directly attributed to the motion of the individual charge carriers in the conductor loop. To provide a demonstration, we define the trajectory of a single point charge moving

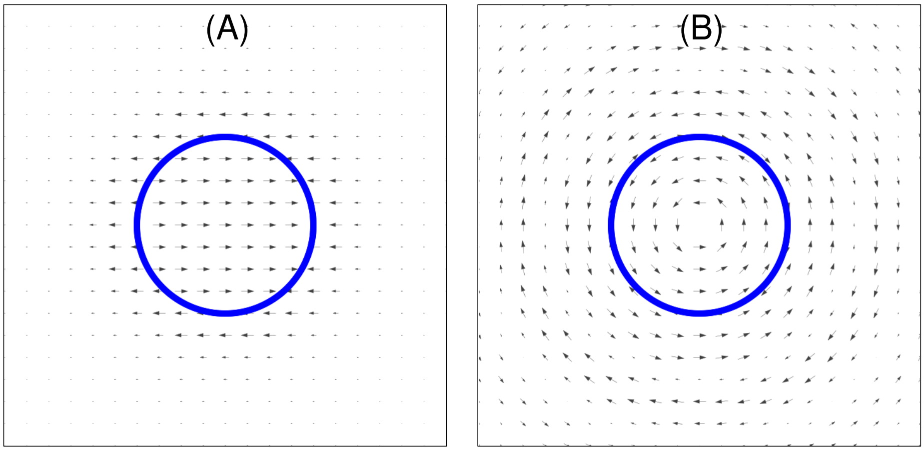

Fig. 5. (A) The field of a round conductor loop with a direct current flowing clockwise and moving upward at a constant speed. The field corresponds exactly to the field that would be obtained if the conductor loop were at rest, and all virtual test charges were moving downward at constant speed. The force is traditionally referred to as an electric force in the first case and as a Lorentz force in the second case. (B) In this case, the conductor loop is at rest, and an alternating current is flowing (1 GHz, plot area 1x1 m). The wave trains are clearly visible. If a second conductor loop were added, an alternating current would be induced in it.

at a constant angular velocity ω on a circular path of radius R:

$$\boldsymbol{r}_s(t) := R\left(\boldsymbol{e}_x\,\sin(\omega\,t+\phi)+\boldsymbol{e}_y\,\cos(\omega\,t+\phi)\right). \tag{61}$$

The electromagnetic field of this point charge can be obtained by inserting it into equations (29), (30), (31) and (32), and finally into equation (59). However, these equations also contain the trajectory of the test charge. If we assume that the test charge is moving with trajectory $\boldsymbol{r}_d(t) := \boldsymbol{r}_0 + \boldsymbol{v}\,t$, we obtain

$$\boldsymbol{r} = \boldsymbol{r}_d(\tau) - \boldsymbol{r}_s(\tau) = \boldsymbol{r}_0 - (\boldsymbol{r}_s(\tau) - \boldsymbol{v}\,\tau)\,. \tag{62}$$

However, the term $\boldsymbol{v}\,\tau$ with the constant velocity $\boldsymbol{v}$ could also be attributed to the trajectory $\boldsymbol{r}_s(t)$. In this case, the field-generating point charge would additionally move linearly to the circular motion. The test charge would instead be at rest. Both cases are completely equivalent. Consequently, the generated fields are also identical. Therefore, whether the test charges are moving, or we assume completely stationary test charges and only interpret the sources to be moving, makes no difference. As seen, the classical principle of relativity is fulfilled, because the derivation of the formula was based not on a Lorentz transformation but on a Galilean transformation.

The field of a complete conductor loop is obtained by varying the parameters ϕ and ω, so that a complete conductor loop with positive and negative charge carriers is obtained. The sum of the individual fields then yields the complete field of the moving conductor loop. Analysis of this field indicates that it corresponds exactly to quasistatics predictions in both strength and direction. Figure 5 (A) shows an example of the field of a round conductor loop that moves upward at a constant speed, and in which the current flows in a clockwise direction. However, the figure can also be interpreted as showing the conductor loop at rest and all virtual test charges moving downward at a constant speed. The force would traditionally be referred to as an electric force in the first case but as a Lorentz force in the second case. However, the difference does not exist, because both cases are fully equivalent, owing to the Galilean principle of relativity.

The example illustrates that electrodynamics based on equation (59) both (i) complies with the classical principle of relativity and (ii) contains the Lorentz force, although the typical cross product term and the magnetic field no longer appear. Both (i) and (ii) also apply for electromagnetic waves. If the conductor loop in Figure 5 (A) were allowed to oscillate faster and faster while reducing the maximum displacement[4], wave trains could eventually be recognized. Therefore, magnetostatics gradually turns into electrodynamics without any change in the mathematics. This aspect can also be seen in the field of Figure 5 (B), where

[4]Otherwise, the speed of the charge carriers in the conductor loop would eventually become greater than the speed of light

the conductor loop is at rest, and an alternating current is flowing. This special case can also be easily simulated with formula (61). The linear term ωt need only be replaced with a sinusoidal term.

The two examples in Figures 5 (A) and (B) illustrate that equation (59) can explain both Lorentz force and electromagnetic induction in all its forms. The example in Figure 4 shows that optical effects can also be simulated, and the example in Figure 2 is related to a particle accelerator. These examples illustrate the versatility of formula (59). In fact, all electromagnetic effects generated by non-relativistic point charges[5] can be modeled. Formula (59) ensures that two ostensibly contradictory requirements are met, because on the one hand, Newtonian mechanics and the Galilean principle of relativity apply, and on the other hand, the electromagnetic force for *each* test charge is ensured to move at the speed of light regardless of its own speed. This principle is important and should also be satisfied by a good non-relativistic approximation.

The model is particularly suitable for simulations, because in all applications, the same formula (59) can be used. However, it is also well suited for didactic purposes, because the electrodynamics does not require differential equations, and everything can be explained with a comparatively simple two-point interaction. Many additional examples can be found in the documentation of the software *OpenWME* [15].

However, some limitations arise from the use of the Galilean transformation and, most importantly, from the use of Newtonian mechanics. The electrodynamics presented herein is formally unsuitable for particles that are moving at speeds very close to the speed of light. In particular, the electrodynamics presented herein does not provide an answer to the question of why particles cannot be accelerated to superluminal speeds. The disproportionately strong increase in kinetic energy for particle speeds close to the speed of light is likewise not included in the presented model. An experiment demonstrating the limitations is that of Bertozzi [16].

A further limitation arises because the model was designed for point charges in a vacuum: if one is interested in the propagation of electromagnetic waves in media, one would need to model the medium with a grid of atoms and/or freely moving charge carriers. This process would be too complicated for engineering. For materials science, however, the model could be interesting precisely for this reason, because it is not empirical and is very close to the physical foundations. The slower speeds of light in media are then a consequence of the Ewald-Oseen extinction theorem [17].

7. Conclusions

As shown herein, Maxwell's equations can be solved relatively easily for arbitrarily moving source charges and stationary test charges. However, if the solution is generalized to arbitrarily moving test charges by means of a Galilean transformation, a problem arises in which the resulting electrodynamics violates the conservation of momentum and, particularly concerningly, is not compatible with magnetostatics.

A solution might be to use the Lorentz transformation and the laws of relativistic mechanics instead of the Galilean transformation. However, doing so would be far too laborious for typical engineering tasks. In addition, the relativistically correct Maxwell electrodynamics for point charges is also afflicted by problems that ultimately led to the development of quantum electrodynamics (QED), which itself is not free of contradictions and problems [18].

Fortunately, in almost all engineering tasks, working with the theory of relativity is unnecessary. Instead, the familiar Galilean transformation can be used. However, as this article has explained, formula (56) should then be used instead of the Lorentz force (6). Otherwise, numerous problems would result. The lack of requirement for the magnetic field might initially be somewhat surprising but can be explained by the Lorentz force's being already almost correctly contained in the electric field, because of to the use of Liénard-Wiechert potentials.

In summary, the small and plausible modification described herein provides an electrodynamic model for non-relativistic point charges that is inherently compatible with Newtonian mechanics and, as illustrated through several examples, is highly versatile. A further advantage is that the electrodynamics consists of only the two equations (32) and (59), thus eliminating the need to solve differential equations or to use the special theory of relativity. Nevertheless, Einstein's postulates are fulfilled, because formulas (32) and (59) have the same form in all inertial and non-inertial frames of reference, and the force propagates simultaneously at the speed of light for all stationary and moving test charges.

[5] Although non-relativistic point charges can be fast, their speed is still markedly below the speed of light.

Furthermore, effects such as the Doppler effect are included, and the conservation of momentum can be demonstrated to always be fulfilled in all circumstances. This aspect is particularly important for simulations, because it ensures that objects composed of charges do not suddenly start to move for no reason. Coverage of all effects of magnetostatics and quasistatics, including the Lorentz force, is ensured by the calibration with Ampère's original force law, which J. C. Maxwell favored as the best possible law for quasistatics.

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