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Weyl's Philosophy of Physics: From Apriorism to Holism (1918-1927)

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Résumé : Dans cet article, j'entends décrire comment évolue la philosophie de la physique de Weyl au cours de la période 1918-1927. Je rappellerai en particulier qu'il développe différentes formes d'« apriorisme » entre 1918 et 1923 : un apriorisme « spéculatif » avec sa théorie unifiée des champs (1918-1921), puis une conception des connaissances *a priori* largement inspirée de la *Wesensanalyse* de Husserl dans ses travaux sur le problème de l'espace (1921-1923). Je montrerai par ailleurs que le holisme de Weyl, i.e., la thèse selon laquelle seule une théorie physique envisagée comme un tout peut faire l'objet de tests empiriques, occupe le devant de la scène dès 1924-1925 dans des textes qui portent sur les fondements des mathématiques. Ce holisme est étroitement lié à sa conception de la théorie physique comme construction symbolique de la réalité. Au bout du compte je caractériserai le point de vue holiste de Weyl en le comparant à certaines réflexions développées par Cassirer, Einstein et Hilbert à peu près au même moment.

Abstract: In this paper, I intend to describe the evolution of Weyl's philosophy of physics during the period 1918-1927. In particular, I will recall that he developed different versions of "apriorism" between 1918 and 1923: first a so-called "speculative" apriorism associated with his unified field theory (1918-1921) and second a conception of a priori knowledge mainly inspired by Husserl's *Wesensanalyse* with his work on the problem of space (1921-1923). I will also show that Weyl's "holism"—i.e., the thesis according to which only a physical theory as a whole can be subject to empirical tests—came to the forefront in 1924-1925 in texts on the foundations of mathematics. This holism is closely related to his definition of a physical theory as a symbolic construction of reality. Finally, I will characterize Weyl's holistic viewpoint

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by comparing it to the thoughts developed by Cassirer, Einstein and Hilbert around the same time.

1 Introduction

From his early work on general relativity (1917-1918) to his main papers on quantum mechanics (1927-1928), Weyl made substantial changes in his philosophy of physics. E. Scholz has pointed out, in [Scholz 2004, 2011], that Weyl's publications devoted to his purely infinitesimal geometry and his problem of space were related to different versions of "apriorism", whereas his later texts in quantum mechanics expressed a kind of "empirical turn". In this article, I would like first to offer a better understanding of Weyl's apriorism between 1918 and 1923. To this end, I will refer to a broader philosophical controversy, which opposed at that particular time Arthur Erich Haas and Weyl to Hans Reichenbach and Erwin Freundlich. To my knowledge, this controversy has been studied only partially by T. Ryckman in [Ryckman 2005]. Moreover, contrary to M. Friedman [Friedman 1999], I will show that Kant's transcendental idealism as such represents a marginal source of inspiration in Weyl's work on his unified field theory as well as on his problem of space, although he uses a Kantian vocabulary in both cases.

Then, I intend to underline the importance of an intermediate step between Weyl's early apriorism and his later empirical turn: his clear adhesion to a kind of confirmational holism which came to the forefront in the mid-1920s. From 1924 onward, Weyl published a series of texts on the Brouwer-Hilbert controversy;[1] in those papers, he claimed that only a physical theory as a whole can be empirically tested. Moreover, it appears that Ernst Cassirer, Albert Einstein, David Hilbert and Weyl shared this holistic viewpoint during the 1920s. So, I would like to single out the characteristic features of Weyl's holism, by relating it to the thoughts developed by Cassirer, Einstein and Hilbert on physical theories. Already in [Sieroka 2010], we can find a very precise comparison between Cassirer's and Weyl's respective conceptions of a physical theory. I would like to go one step further by also referring to Einstein and Hilbert.

In section 1, I will describe Weyl's early "speculative" apriorism,[2] which characterizes his thoughts in 1918-1921, and I will study very carefully the so-called Freundlich-Haas-Reichenbach-Weyl controversy on the role played by geometry and a priori knowledge in general relativity. In section 2, I will review the changes encountered by Weyl's philosophy of physics in his work on the so-called problem of space. He developed in particular a new kind of

1. See in particular [Weyl 1924, 1925].
2. See also [Scholz 2011, 185–192].

apriorism based on a specific interpretation of Husserl's *Wesensanalyse* that differed significantly from Carnap's.[3] In the last section, I will progressively characterize Weyl's holism, which became central in his thoughts from 1924 onward and which is intrinsically related to his definition of a physical theory as a symbolic construction of reality.

2 Weyl's speculative apriorism under debate (1918-1921)

In a recent paper, E. Scholz has used the expression "speculative-a priori validity claim" [i.e., *spekulativ-apriorischer Geltungsanspruch*] to describe Weyl's philosophical attitude toward geometry and physics when working on his project of a unified field theory (1918-1921) [Scholz 2011, 185–192]. I will go in depth into E. Scholz's analysis in two ways: first, it is necessary to elaborate more extensively why the expression "speculative apriorism" is particularly relevant in order to characterize Weyl's early philosophy of physics; second, I will focus on an important philosophical controversy opposing Haas and Weyl to Freundlich and Reichenbach in 1919-1921.

2.1 A characterization of Weyl's early "apriorism"

Before analyzing Weyl's philosophical views from 1918 to 1921, I would like to emphasize that "apriorism" can refer indistinctly to philosophical interpretations of special and general relativity that are quite different, or even opposed. For instance, Cassirer and Reichenbach developed almost simultaneously two opposite versions of a so-called "relativized" apriorism. On the one hand, in his book entitled *Zur Einsteinschen Relativitätstheorie*, Cassirer interpreted general covariance as a "regulative idea[4] of all fundamental physical objects interacting through dynamical laws completely without reference to a background space-time" [Ryckman 2005, 46]. As a consequence, this principle "is no longer constitutively a priori in Kant's sense" [Ryckman 2005, 46]. On the other hand, in his book entitled *Relativitätstheorie und Erkenntnis a priori*, Reichenbach temporarily built up a weaker form of apriorism, based on the distinction between two different meanings of a priori knowledge: (a) constitutive of the concept of an object; (b) apodictically true. He rejected the second meaning before interpreting coordination principles in physics as a priori statements in a weak sense.

3. See [Carnap 1922] and [Friedman 1999] for a commentary on this point.

4. According to Cassirer, general covariance plays a central role in securing the systematic unity of empirical knowledge and it can thus be considered as a regulative idea.

"Apriorism" is also a category employed by the actors of the time in the context of philosophical controversies. For instance, in a famous conference entitled "*Die philosophische Bedeutung der Relativitätstheorie*", the philosopher Moritz Geiger, a former student of Husserl, opposed "apriorism" to "positivism" and "empiricist realism". He strongly criticized a version of positivism inherited from Ernst Mach and represented by Joseph Petzoldt. He also rejected a so-called empiricist realism embodied by Hermann von Helmholtz. Finally, Geiger's conference turned into a *plaidoyer* for "apriorism" [Geiger 1921, 25].

I would like to offer now a threefold characterization of Weyl's early apriorism. In the first place, contrary to Cassirer and Reichenbach, Weyl did not introduce a kind of relativized (synthetic) a priori knowledge in his early work on special and general relativity. Moreover, until 1923, he was convinced that a clear demarcation line between a priori principles and empirical statements could be drawn in physics.

In the second place, Weyl did not focus on Kant's transcendental idealism as such in his philosophical reflections on general relativity. His early apriorism depended upon various sources, mainly Johann Gottlieb Fichte and Edmund Husserl, to whom he referred almost simultaneously.[5]

In the third place, Weyl's "speculative" apriorism was based above all on his conviction that geometry prevails over physics. This idea reflects his close links with the Göttingen School of mathematical physics around Hilbert, although the latter would harshly criticize Weyl's project of a unified field theory in 1919-1920.[6] Indeed, already in the first edition of *Raum, Zeit, Materie*, Weyl drew a parallel between Riemannian geometry and action-by-contact physics [Weyl 1918b, 81]. Far from being a marginal remark, this thought completely determined Weyl's investigations in mathematical physics at that time: since he is convinced that there exists a well-coordinated development of infinitesimal geometry and action-by-contact physics, he strongly believes that his generalization of Riemannian geometry can naturally receive a meaningful interpretation in physics.

Weyl already recognized in 1918 that this way of deducing a physical theory from his so-called purely infinitesimal geometry belongs to "pure speculation". At that particular time, such an expression implied that his theory was *awaiting an empirical validation*. For instance, in [Weyl 1918a, 38], he uses the expression "in pure speculation" [*im rein Spekulativen*], while trying to find a physical ground for his theory. Moreover, in his response to Einstein's main objection—according to which, the length of a standard rod and the speed of a standard clock would depend on their "prehistory" [*Vorgeschichte*] in the

5. See in particular [Scholz 2005] and [Ryckman 2005].

6. For a careful analysis of this point, see in particular [Scholz 2011] and [Eckes 2014].

framework of Weyl's unified field theory[7]—, he concedes that there is still no empirical ground for his theory, before employing the term "speculation" to describe his approach: "we are dealing here with pure speculation; comparison with experiment is an understood requirement".[8] Despite this concession, Weyl rejects Einstein's objection by supposing that the metric introduced in his geometry is not bound to direct observation [Weyl 1918a, 42].

At this point, I would tend to underline a provisional tension in Weyl's argument: on the one hand, he makes a clear distinction between "the mathematical *ideal* of vector-transfer" and "the *real* situation regarding the movement of a clock"; on the other hand, he believes that the (mathematical) superiority of his geometry over Riemannian geometry must be reflected in nature. After 1921, he remained convinced that his geometry was mathematically more relevant than Riemannian geometry, while admitting that his early project of a unified field theory belonged to "pure speculation". Accordingly, this expression will take a new meaning: his theory *has definitely no empirical ground*.[9]

2.2 The Freundlich-Haas-Reichenbach-Weyl controversy

Einstein's objections against Weyl's unified field theory represent the first step of a broader philosophical controversy that opposed Weyl and Haas to Freundlich and Reichenbach. Weyl's speculative approach lays at the very core of this controversy. Let us recall briefly the chronology of this dispute.

The Austrian physicist Arthur Erich Haas argued in favor of Weyl's conception of geometry and physics in an article entitled "Die Physik als geometrische Notwendigkeit" [Physics as geometric necessity] [Haas 1920]. According to Haas, the recent developments in special and general relativity were based on purely geometric ideas. To this end, he referred in particular to Hermann Minkowski's conference on space and time [Haas 1920, 122], before claiming arguably that

> *Einstein* just needed to apply Riemann's knowledge, which is valid for an arbitrarily given geometric manifold, to Minkowski's universe [...], in order to build up a *theory of gravitation* on a purely geometric ground.[10]

7. The first trace of this objection can be found in Einstein's letter to Weyl from April 15, 1918 [Einstein 1998, 720–721]. Einstein formulates again this objection in a series of remarks at the end of [Weyl 1918a, 40].

8. [Weyl 1918a, 42]: "Es handelt sich hier wirklich um reine Spekulation; der Vergleich mit der Erfahrung ist selbstverständliches Erfordernis." English translation in [O'Raifeartaigh 1997, 36].

9. See the last paragraph of the fifth edition of *Raum, Zeit, Materie* (1923), quoted in [Scholz 2011, 189].

10. [Haas 1920, 124, my translation]: "*Einstein* brauchte nur die Erkenntnis *Riemanns*, die für eine beliebige geometrische Mannigfaltigkeit [...] gilt, auf die

According to Haas, Weyl's unified field theory perfectly illustrates his general thesis that "the foundations of physics are identical with those of mathematics".[11]

Haas' viewpoint was firmly criticized by Erwin Freundlich in a short response, also published in *Die Naturwissenschaft* [Freundlich 1920]. Contrary to Haas, Freundlich claimed that special relativity is based above all on a "purely physical knowledge" [*rein physikalischen Erkenntnis*] which consists in unifying the postulate of relativity and the principle of lightspeed constancy [Freundlich 1920, 234]. He also disproved Haas' assumption according to which Minkowski's work might have foreshadowed Einstein's later contributions in general relativity. In particular, Freundlich considered that Einstein's gravitation theory cannot be interpreted as a mere extension of Minkowski's geometric ideas within a Riemannian framework [Freundlich 1920, 235].

Haas and Freundlich offer here two opposite, but also biased, views on the historical development of special and general relativity. Haas focuses almost exclusively on the geometric ideas governing these two theories, whereas Freundlich insists mainly on their empirical grounds. Finally, he shows that Haas favors almost exclusively the "mathematical-aesthetic aspect" [*mathematisch-ästhetischen Gesichtspunkt*] of general relativity over its physical content. According to Freundlich, Haas' erroneous interpretation of special and general relativity is based on the lack of any explicit distinction between the physical grounds of these theories, their analytic representations and the epistemological issues they raise.

In this short note, Freundlich does not refer to Weyl's unified field theory. Nevertheless, this first dispute extends very soon to Reichenbach and Weyl: in his 1920 book on relativity theory, Reichenbach claims that Weyl and Haas defend exactly the same unfounded point of view on mathematics and physics:

> So it is entirely false if one would conclude, as. e.g., Weyl and also Haas, that mathematics and physics merge into a single discipline.[12]

Reichenbach mainly refers here to [Haas 1920] and to the concluding remarks of [Weyl 1918b]. Indeed, in the last paragraph of [Weyl 1918b], Weyl considers that geometry and physics have exactly the same character before drawing a parallel between physics and formal logic. In this perspective, a physical theory is viewed as a mere formal interpretation of reality. But contrary to Reichenbach's claim, Weyl does not identify purely and simply

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Minkowskiwelt anzuwenden, um (1915) eine *Theorie der Gravitation auf rein geometrischer Grundlage* aufstellen zu können."

11. [Haas 1920, 127, my translation]: "Die Grundlagen der Physik sind dieselben wie die der Mathematik."

12. [Reichenbach 1920, 73]: "Ganz falsch ist es aber, wenn man daraus, wie z.B. Weyl und auch Haas, wieder den Schluss ziehen will, dass Mathematik und Physik zu einer einzigen Disziplin zusammenwachsen." English translation in [Reichenbach 1965, 76].

mathematics and physics.[13] Reichenbach is also well aware of Einstein's main objection against Weyl's theory and he points out a more general epistemological inconsistency in Weyl's reasoning, based on a confusion between the mathematical significance of his purely infinitesimal geometry and its (questionable) effectiveness in physics [Reichenbach 1920, 73]. If we keep in mind Reichenbach's general project of relativizing Kant's conception of a priori knowledge, his final criticism against Haas and Weyl is particularly harsh: Reichenbach accuses them of defending a pre-Kantian viewpoint:

> Physics is not a "geometrical necessity"; whoever asserts this returns to the pre-Kantian point of view where it was a necessity given by reason.[14]

Weyl advocates for the very last time for his unified field theory in [Weyl 1921b], which contains an interesting response to Freundlich and Reichenbach. Indeed, Weyl sums up very briefly the main epistemological objection against his theory:

> From different sides, it has been objected against my theory that it contains things derived a priori from pure speculation, while they can only be settled by experiment.[15]

Then, he underlines the importance of an "a priori element" in the constitution of any physical theory:

> However, one should not forget that, in every knowledge of reality, besides the collection of typical empirical facts, the a priori element, [that is] the shaping of suitable intuitions and concepts, whereby facts can only be interpreted, plays a non negligible role.[16]

In other words, he is not convinced by Freundlich's empiricist views or even by Reichenbach's way of relativizing the meaning and the scope of a priori knowledge. Nevertheless, Weyl does not define this so-called "a priori

13. See Weyl's letter to Reichenbach, February 02, 1921, Reichenbach archives, Pittsburgh University, HR 015-68-04, with a partial transcription in [Ryckman 2005, 262].

14. [Reichenbach 1920, 73–74]: "Die Physik ist eben keine 'geometrische Notwendigkeit'; wer das behauptet, kehrt auf den vorkantischen Standpunkt zurück wo sie eine vernunftgegebene Notwendigkeit war." English translation in [Reichenbach 1965, 77].

15. [Weyl 1921b, 231, my translation]: "Von verschiedenen Seiten ist gegen meine Theorie eingewendet worden, dass in ihr aus reiner Spekulation Dinge a priori demonstriert würden, über welche nur die Erfahrung entscheiden kann." He mentions especially [Freundlich 1920] and [Reichenbach 1920] in a footnote.

16. [Weyl 1921b, 236, my translation]: "Man darf aber nicht vergessen, dass in aller Wirklichkeitserkenntnis neben dem Sammeln typischer Erfahrungstatsachen das apriorische Element, die Bildung von angemessenen Anschauungen und Begriffen, mit Hilfe deren die Tatsachen zu deuten sind, eine nicht zu vernachlässigende Rolle spielt."

element", which covers "intuitions" [*Anschauungen*] and "concepts" [*Begriffe*]. He would clarify this point in several writings published between 1921 and 1923 on his problem of space. Indeed, he had already alluded to this problem in [Weyl 1921b]. So, we might consider at least partially his philosophical reflections on the problem of space as a final response to Freundlich's and Reichenbach's criticisms.

3 A refinement of Weyl's apriorism between 1921 and 1923

In this second part, I will show first that Kant's theory of knowledge plays a minor role in Weyl's philosophical reflections on the problem of space. Second, it seems that the *mathematical* problem of space—as a group-theoretical characterization of Pythagorean manifolds in a generalized sense—prevails in his writings over the *philosophical* problem of space, which consists in studying the interconnections between space, its metrical structure and its material content. This might explain his very specific interpretation of Husserl's *Wesensanalyse* that has been rightly underlined by M. Friedman in [Friedman 1999].

3.1 A non-Kantian use of Kant's vocabulary

In Weyl's work on the problem of space, we can find many traces reminding us of Kant's vocabulary. For instance in [Weyl 1923], Weyl distinguishes between *a priori* and *a posteriori* knowledge, but also between *analytic a priori* and *synthetic a priori* statements. Nevertheless, he does not define most of these expressions in a Kantian perspective. Moreover, he harshly criticizes Kant's conception of space in many of his writings related to general relativity. In other words, Weyl does not rehabilitate Kant's conception of space at all. Besides, we have to bear in mind that the expression "form of phenomena" does not have exactly the same meaning in Weyl's early writings on his purely infinitesimal geometry and in his later texts on the problem of space. To our knowledge, this change has not been taken into account in [Bernard 2015].

Indeed, at the beginning of [Weyl 1918c], Weyl gives his first own mathematical interpretation of space-time as "a form of appearances" [*Form der Erscheinungen*] [Weyl 1918c, 2]. Under this expression, he means a completely amorphous four-dimensional continuum in the sense of Analysis situs, i.e., a four-dimensional topological manifold. This definition is not inspired by Kant for three main reasons. First, Weyl does not separate space and time. Second, he distinguishes between the "form of appearances" and "matter" as a physical entity; in other words, he never employs the expression "matter of appearances",[17] which corresponds to sensations in a Kantian perspective. Third, he interprets this distinction in terms of mathematical properties ascribed to a four-dimensional manifold: its topological (and differentiable) structure which can be described fully *a priori*, as opposed to its metrical structure which is determined only a posteriori because it depends on the distribution of matter.

The use of Kant's vocabulary is even more explicit in Weyl's writings on his problem of space. For instance, at the beginning of [Weyl 1922a], Weyl divides reality according to its form (the spatiotemporal extensive medium) and its qualitative content. Then, he summarizes in a few words a so-called "pre-relativistic conception of space" [*vorrelativistische Auffassung des Raumes*] [Weyl 1922a, 212], in which the metric is conceived independently from any material content. Kant's transcendental aesthetic can precisely serve as an example of this misleading conception. As a consequence, Weyl draws a clear opposition between Kant's and Einstein's points of view on space and time. Accordingly, Weyl's apriorism cannot be considered as an attempt to make Kantianism compatible with special and general relativity. I disagree here with M. Friedman's interpretation [Friedman 1999, 51]: Friedman overestimates by far the importance of a so-called "Kantian context" in his interpretation of Weyl's writings from the period 1921-1923.

In fact, Weyl discusses Kant's transcendental aesthetic—in particular the definition of space as a pure form of intuition [A 22-A 23 and B 36-B 37]—in light of general relativity in [Weyl 1923, 43–47]:

> According to Einstein, the metric structure of the universe is not homogeneous. How is it possible, since space and time are forms of phenomena? Only because *the metric structure is not a priori fixed, on the contrary it is a field of physical reality which is causally dependent on the state of matter.*[18]

According to J. Bernard [Bernard 2015, 263], who refers precisely to this passage, Weyl tries to rehabilitate in a certain manner Kant's conception of space and time as forms of phenomena by keeping a "homogeneity that is weak [...] and infinitesimal". This interpretation is not completely convincing for several reasons. First, Weyl never defines homogeneity in a weak sense in [Weyl 1923, 44–45]. Moreover, the question he raises—i.e., "How is it possible [that the metric structure of the universe is not homogeneous], since space and time are forms of phenomena?"—is purely rhetorical. Unquestionably,

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17. Kant distinguishes between the matter [*Materie*] and the form of an appearance [*Erscheinung*]. See in particular Kant's *Kritik der reinen Vernunft*, A 20-A 21 (for the first edition) and B 34-B 35 (for the second edition).

18. [Weyl 1923, 43–44]: "Nach Einstein ist die metrische Struktur der Welt nicht homogen. Wie ist das möglich, da doch Raum und Zeit Formen der Erscheinungen sind? Allein dadurch, dass *die metrische Struktur nicht a priori fest gegeben ist, sondern ein Zustandsfeld von physikalischer Realität, das in kausaler Abhängigkeit steht vom Zustand der Materie*."

Weyl does not take Kant's conception of space and time for granted. He rather underlines the opposition between Kant's fixed dogmas on space and Einstein's recent achievements in general relativity. Weyl is even more explicit in [Weyl 1988]—which he wrote "in 1925 for the edition of Lobatschefskij's works" [Weyl 1988, III]:

> the metric structure becomes a field which depends on matter, it no longer belongs to a stationary homogeneous form of phenomena, on the contrary, it depends on a changeful material becoming.[19]

Then, Weyl recalls in [Weyl 1923, 46] that the property of metric homogeneity is assumed in the case of the *classical* problem of space, i.e., the Riemann-Helmholtz problem of space, which provides a group-theoretical characterization of Euclidean geometry and non-Euclidean geometries. By contrast, this property no longer holds globally in his generalized problem of space [Weyl 1923, 46]. Accordingly, he redefines dramatically the expression "form of phenomena" in a way that is not Kantian at all. By this expression, he finally means the set of all properties characterizing "the a priori essence of space" [*das apriorische Wesen des Raumes*].[20] Moreover, he proves that the Pythagorean nature of the metric can be determined independently from any experience. As a consequence, he must abandon his 1918 mathematical definition of space as a "form of phenomena", since the nature of the metric also belongs to the a priori essence of space [Weyl 1922b, 266].

3.2 An explanation for Weyl's constructivist interpretation of Husserl's *Wesensanalyse*

In the concluding remarks of [Weyl 1923], Weyl underlines that philosophy and mathematics are closely intertwined in the (exceptional) case of his problem of space [Weyl 1923, 61]. But it does not imply that the mathematical and the philosophical part of this problem play equivalent roles in Weyl's writings. In fact, it seems obvious that the *mathematical* analysis of the problem of space prevails over its philosophical meaning for Weyl. This point is already suggested by the fact that he generally interprets mathematically most of the philosophical concepts he uses.

Moreover, at the beginning of [Weyl 1922a], which is reproduced without any significant change in [Weyl 1923], he expounds with a certain distance a so-called philosophical point of view on space and time, mainly inspired by Kant. Accordingly, he defines the philosophical problem of space as follows: it

19. [Weyl 1988, 36, my translation]: "die metrische Struktur wird zu einem von der Materie abhängigen physikalischen Zustandsfeld, sie gehört nicht mehr zur ruhenden homogenen Form der Erscheinungen, sondern zum wechselvollen materiellen Geschehen."

20. See in particular [Weyl 1922b, 265].

consists in studying the interconnections between 1. space (or more generally an extensive medium of the external world), 2. its metrical structure, 3. its material content. Then, he adopts the viewpoint of the mathematician in order to underline that the resolution of the philosophical problem of space must be guided by a *theoretical construction* based on mathematical principles. From these principles, Weyl deduces a group-theoretical characterization of infinitesimally Pythagorean manifolds, which leads to the philosophical conclusion that the nature of the metric is not a mere convention; on the contrary, it belongs to the a priori essence of space. Weyl is even more explicit at the end of § 18 of [Weyl 1952], criticizing "the impatience of those philosophers who believe it possible to describe adequately the mode of existence on the basis of a single act of typical presentation".[21] In other words, only a theoretical construction offers a progressive access to the essence of space. Weyl's own writings published between 1918 and 1923 illustrate this fact: initially convinced that space-time as such is a four-dimensional topological manifold, he became aware that the Pythagorean nature of the metric is a characteristic feature of the universe.

From these remarks, it is possible to get a better understanding of Weyl's apriorism during the period 1921-1923. In the first section, I have already underlined that Weyl was strongly opposed to any kind of empiricist conception of geometry and physics. In the previous subsection, I have shown that this rejection of empiricism does not lead in any way to a rehabilitation of Kantianism, although we can find some traces of Kant's vocabulary especially in [Weyl 1923]. Finally, the importance granted by Weyl to the mathematical analysis of the problem of space gives rise to a *constructivist* interpretation of Husserl's *Wesensanalyse*, which is rightly underlined by M. Friedman [Friedman 1999, 53]. Moreover, following E. Scholz, I should add that Weyl mainly refers to Husserl's phenomenology in order to criticize a strictly conventionalist conception of our knowledge of space [Weyl 1921a, 133–134], [Scholz 2016, 269].

It is necessary to recall here that Weyl's peculiar interpretation of Husserl's *Wesensanalyse* appeared in the fourth edition of *Raum, Zeit, Materie* dated 1921, i.e., before his discussions with Carnap and Oskar Becker on the problem of space. Indeed, Carnap's dissertation was published in 1922 and almost immediately reviewed by Weyl [Weyl 1922c]; the correspondence between Becker and Weyl on geometry and physics consisted of four letters of Becker to Weyl written between April 1923 and October 1924, namely after the publication of Weyl's main writings on the problem of space. As pointed out in [Mancosu & Ryckman 2002, 206], it is chronologically implausible to say that "Weyl conceives his own analysis of the problem of space as a generalization and refinement of Becker's" [Friedman 1999, 54]. However, M. Friedman

21. [Weyl 1921a, 133]: "die Ungeduld der Philosophen [...] die da glauben, auf Grund eines einzigen Aktes exemplarischer Vergegenwärtigung das Wesen adäquat beschreiben zu können." See [Weyl 1952, 148] for an English translation.

convincingly claims in [Friedman 1999, 47–54] that Weyl and Carnap interpret Husserl's *Wesensanalyse* from two very different perspectives. Carnap's dissertation entitled *Der Raum* [Carnap 1922] begins with a distinction between formal space, intuitive space and physical space. His inquiry is based on "Husserl's concept of "essential insight" [*Wesenserschauung*]" [Friedman 1999, 47], which provides the a priori laws of intuitive space, in particular the property of being infinitesimally (but not globally) Euclidean [Friedman 1999, 48].

We might think for the moment that Carnap's and Weyl's points of view are very close, except that Weyl works on Pythagorean manifolds in a generalized sense, not only on Riemannian manifolds. However, Weyl publishes a harsh review of Carnap's dissertation in 1922 [Weyl 1922c]. According to Weyl, Carnap does not make any explicit link between intuitive space and physical space. It seems easier to understand this objection if we take into account that they propose two diverging interpretations of Husserl's *Wesensanalyse*. By this expression, Carnap means an immediate grasp of the a priori laws of *intuitive* space, whereas it is a phenomenological constitution of *physical* space for Weyl, based on a *theoretical* construction. In other words, Weyl conceives Husserl's *Wesensanalyse* in a constructivist perspective [Friedman 1999, 53]. For instance, at the end of the § 18 of [Weyl 1952], Weyl describes:

> [t]he historical development of the problem of space [that] teaches how difficult it is for us human beings entangled in external reality to reach a definite conclusion.[22]

In a nutshell, between 1921 and 1923, Weyl's apriorism can be fully characterized as follows: (a) the mathematical analysis of the problem of space provides the a priori laws governing physical space (as an extensive medium of external reality) which are not mere conventions; (b) as a consequence, the Pythagorean nature of the metric, which can be determined purely mathematically, belongs to the a priori essence of space; (c) philosophically, Weyl interprets this mathematical analysis in terms of Husserl's *Wesensanalyse*, considering the latter as a non-ending process implying a theoretical construction.

In this respect, the role played by Husserl's philosophy in Weyl's work must be significantly minimized. First, Weyl's explicit references to Husserl remain very localized and they serve very specific goals; in particular, Weyl mentions Husserl's *Wesensanalyse* in order to criticize empiricism as well as conventionalism. Therefore, it is impossible to generalize the present statement about space to other writings, especially his 1918 articles on his purely infinitesimal geometry. Second, Weyl provides a very specific interpretation of Husserl's *Wesensanalyse*, which is consistent with his assumption according to which the *mathematical* analysis of the problem of space plays a guiding

22. [Weyl 1921a, 133] and [Weyl 1952, 147] for the English translation.

role in order to solve the philosophical problem of space. Third, Fichte's idealism remains Weyl's main source of inspiration in the development of his constructivism in mathematics as well as in theoretical physics. Indeed, Weyl finally criticizes "the passive Husserlian viewing of essences" [Sieroka 2009, 93]. It seems now necessary to give a more accurate picture of Weyl's "constructivism" in theoretical physics, to which he began to give its final shape in a series of writings published between 1924 and 1927.

4 Weyl's holism and physical theories as symbolic constructions of reality

In this last section, I will focus on Weyl's epistemological holism and his underlying constructivism which became a characteristic feature of his reflections on physical theories around 1924-1925. I will first explain why he fully develops this philosophical thesis in a series of texts on the foundations of mathematics. Second, I will show that Weyl is well aware of the philosophical consequences linked with holism: in particular, under the assumption that a physical theory must be treated as a whole, it seems impossible to make a clear distinction between a priori principles and empirical statements. Third, I will underline the fact that his holistic viewpoint is closely related to his definition of a physical theory as a symbolic construction of reality. This might suggest some similarities between Cassirer and Weyl which should not be carried too far.

4.1 Some important concessions to Hilbert in theoretical physics

The very first trace of Weyl's holism can be found in the first edition of *Raum, Zeit, Materie* [Weyl 1918b, 60]. However, this kind of thought plays a very marginal role in Weyl's writings on general relativity. By contrast, in [Weyl 1924] and [Weyl 1925], holism becomes central and it is closely related to his new definition of a physical theory as a symbolic construction of reality.[23] Under this expression, he means that a physical theory is not based on any intuitive "evidence". It must rather be conceived as a mathematical construction, which is not directly linked with empirical data. In other words, a physical theory *represents* reality through a system of interrelated symbols. Yet, in these two writings, Weyl discusses mainly Hilbert's and Brouwer's points of view on the foundations of mathematics. As a consequence, Weyl's constructivist and holistic viewpoint in theoretical physics reflects the evolution of his thoughts on the foundations of mathematics around 1924-1925.

23. See in particular [Weyl 1924, 1925].

As underlined by P. Mancosu, N. Sieroka and R. Tieszen, Weyl was no longer satisfied with a strictly intuitionist viewpoint on the foundations of mathematics around 1924-1925, mainly for pragmatic reasons [Mancosu 1998, 80–81], [Sieroka 2009, 89] and [Tieszen 2000, 274].[24] But it does not mean that he turned away from intuitionism in favor of formalism. In fact, he "attempts to take a middle stand between Hilbert and Brouwer" [Mancosu 1998, 80]. Weyl's position on the foundations of mathematics seems quite ambivalent: "he feels that there is a fundamental gap between the intuitive and the theoretical or conceptual in mathematics" [Tieszen 2000, 295]; but they also represent two complementary aspects of mathematics. In this respect, he distinguishes between "intuitive mathematics" which provides a set of individual propositions based on intuition and "symbolic mathematics" which characterizes "the parts of mathematics that cannot be founded on intuition" [Weyl 1925, 541], [Tieszen 2000, 295].

Accordingly, Weyl uses the term "construction" in two very different ways. On the one hand, he refers to *constructive procedures*, as opposed to axiomatic procedures, in the proof of existence theorems. In this regard, "construction" is a characteristic feature of *intuitive* mathematics [Weyl 1927, 41]. In particular, he was still convinced after 1924-1925 that mathematics would be meaningless without any explicit construction of mathematical objects. On the other hand, he also defines the term "construction" as a *symbolic representation*, which gives rise to mathematical theories that exceed any intuition. This idea is tightly related to the very notion of *symbolic* mathematics. Now, in which way can we characterize Weyl's constructivism in *theoretical physics*?

We might answer this question by referring to the end of [Weyl 1924] where he claims that theoretical physics provides a kind of knowledge, which strongly differs from intuitive or phenomenal knowledge, since no single proposition can receive an (intuitive) meaning by itself in that case. In particular, a physical law must be regarded as being a part of a whole theory conceived as a *symbolic construction* of reality. As a consequence, in his reflections on the foundations of mathematics, Weyl mentions theoretical physics only when dealing with *symbolic mathematics*. Moreover, between 1927 and 1928, he referred to theoretical physics in order to clarify his position in the debate opposing Brouwer to Hilbert. In fact, Weyl shows that intuitionism is not consistent with his holistic conception of a physical theory and he claims that Hilbert shares this holistic viewpoint with him [Weyl 1927, 49]. Indeed, in his 1927 conference on the foundations of mathematics held in Hamburg, Hilbert said unambiguously that it is impossible to test a physical theory hypothesis by hypothesis [Hilbert 1928, 79]. To our knowledge, this is the very first expression of Hilbert's holism in physics. In his commentary, Weyl fully agrees with him on this particular point [Weyl 1928, 87–88].

24. [Sieroka 2009, 89]: "[A]s a practicing mathematician and theoretical physicist he retreated from the general foundational ambitions of intuitionism."

To sum up, in [Weyl 1924] and in [Weyl 1925], Weyl mainly refers to theoretical physics in order to distance himself from a strictly intuitionist point of view. But in [Weyl 1927] and in [Weyl 1928], he goes one step further by underlining the *irrelevance* of intuitionism in theoretical physics. We can measure now to which extent Weyl's thoughts on physical theories were related to his broader reflections on intuitive and symbolic mathematics during the period 1924-1928. In particular, it seems obvious that Hilbert and Weyl share the idea according to which a physical theory is a set of interrelated symbols. However, Weyl's epistemological holism is not inspired by Hilbert for chronological reasons: it already appears in [Weyl 1918b] and in [Weyl 1924]. Moreover, in his 1927 conference on the foundations of mathematics, Hilbert only claimed that propositions in physics are logically interdependent. By contrast, Weyl defines physical theories as symbolic representations "of the transcendent" [Weyl 1925, 540], with a clear reference to Fichte at the end of [Weyl 1924]. A better characterization of Weyl's constructivism is thus needed.

4.2 Duhem, Einstein, Weyl and the relativization of apriorism

In fact, Weyl's conception of a physical theory as a symbolic construction of reality and his corresponding holism are not entirely new in the philosophical literature on physics. In particular, there are some similarities between his and Duhem's epistemological point of view, which is fully developed in [Duhem 1906]. Indeed, the idea according to which only a whole theory can be empirically tested is formulated for the very first time by Duhem in a series of articles published at the end of the nineteenth century. He provides then a detailed account on holism and its philosophical consequences in [Duhem 1906] part II, chapter VI. This book was translated into German by the socialist activist Friedrich Adler very shortly after,[25] making it accessible to German-speaking scientists and philosophers. Weyl only refers once to *La Théorie physique* in the bibliography of [Weyl 1927, § 21]. In this respect, it seems very difficult to suppose that Weyl's constructivism is mainly and directly influenced by Duhem. Nevertheless we would like to put forward three main points of convergence between Duhem's and Weyl's reflections on theoretical physics.

First, both of them make a clear distinction between direct observation and experiment which always involves a theoretical interpretation.[26] Second, Weyl's idea according to which a physical theory is a symbolic construction of reality is repeatedly expressed in a very similar way by Duhem in [Duhem 1906, part II, chap. V]. Duhem points out, for instance, that physical laws "are not only abstract; they are, in addition, symbolic, and the symbols

25. [Duhem 1908]. The German edition of [Duhem 1906] is prefaced by Ernst Mach.
26. See in particular [Duhem 1906, 205], [Weyl 1927, 49], [Weyl 1928, 88] and [Sieroka 2010, 298] for a commentary on this particular point.

assume meaning only by grace of the physical theories".[27] Third, just like Duhem, Weyl considers that only a theoretical whole can be empirically tested, rejecting in particular the possibility of crucial experiments in physics [Sieroka 2010, 303], [Duhem 1906, 262–263].

Moreover, such a holistic point of view is also shared by Einstein. The latter might have been aware of the existence of [Duhem 1906] in the early 1910s. At that particular time, Einstein was close to Adler, i.e., the translator of [Duhem 1906] [Howard 1990, 264]. Indeed, there are several traces indicating that Einstein fully embraces such a conception of physical theories. We might refer first to his 1910-1911 lecture notes on electricity and magnetism; Einstein's holism is also explicitly expressed in his review of Alfred Elsbach's 1924 book entitled *Kant und Einstein*, [Howard 1990, 371–372]. Assuming that a physical theory consists of closely interrelated propositions, Einstein points out the *arbitrariness* of any distinction between a priori and a posteriori knowledge within a physical theory. In other words, he considers that holism contradicts the possibility of isolating once and for all a priori principles in physics.

We would like to show now that Weyl is fully aware of this epistemological consequence. Therefore, his holistic point of view in physics is a central ingredient in order to explain why he progressively tempered his previous apriorism. Indeed, in [Weyl 1927, § 21], Weyl claims unambiguously that it is impossible in practice to make a clear distinction between a priori and a posteriori knowledge in physics:

> Thus in the practice of scientific research the clear-cut division into a priori and a posteriori in the Kantian sense is absent, and in its place we have a rich scale of gradations of stability.[28]

This argument reminds us of Einstein's thoughts in his review of Elsbach's book. By contrast, Weyl still repeats in [Weyl 1927, 95–100] the conclusions of his philosophical analysis of the problem of space which lead to "a clear-cut division into a priori and a posteriori" statements. As a consequence, there is apparently a contradiction in Weyl's thoughts. In fact, we need to recall that he does not aim at building up a perfectly consistent philosophical system in [Weyl 1927]: different steps in the development of his philosophical reflections seem to coexist and each of these steps is related to a specific topic.

As underlined in the previous subsection, holism in theoretical physics is a central argument used by Weyl in order to distance himself from intuitionism

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27. [Duhem 1906, 234]: "[les idées qui composent une loi physique] ne sont pas seulement abstraites, elles sont, de plus, symboliques, et les symboles qu'elles constituent ne prennent un sens que grâce aux théories physiques." English translation in [Duhem 1954, 166].

28. [Weyl 1927, 114]: "So existiert in der Praxis der naturwissenschaftlichen Arbeit nicht der schroffe Gegensatz von a priori und a posteriori im Kantischen Sinne, sondern eine reiche Stufenskala der Festigkeit." English translation in [Weyl 1949, 153–154].

and to make a series of concessions to Hilbert. Moreover, this holistic point of view is also inspired by Einstein and it leads to a relativization of Weyl's previous apriorism. Nevertheless, the very notion of symbolic construction, which is central in Weyl's conception of a physical theory, does not appear in any of Einstein's writings. Therefore, it seems necessary to compare it to Cassirer's philosophy of symbolic forms, in order to fully characterize Weyl's constructivism.

4.3 Cassirer and Weyl on symbolic constructions

Within the general framework of his philosophy of symbolic forms, Cassirer conceived mathematics and natural sciences as symbolic constructions, reminding us of Weyl's definition of a physical theory. We will see below that this argument needs to be balanced. But we would like first to put forward the role played by Cassirer in the promotion of Duhem's epistemological holism among German-speaking philosophers and scientists. It is highly plausible that Weyl discovered [Duhem 1906] through Cassirer. Indeed, in § 21 of [Weyl 1927], Weyl mentions almost simultaneously [Duhem 1906], [Cassirer 1910, 1921]. Cassirer referred precisely to Duhem's distinction between physical laws and phenomenal knowledge in *Substanzbegriff und Funktionsbegriff* [Cassirer 1910, 190]. Cassirer even extended Duhem's thoughts to special and general relativity in [Cassirer 1921, 10]. Only two years after the publication of Weyl's *Philosophie der Mathematik und Naturwissenschaft*, Cassirer mentioned again Duhem in the third volume of his *Philosophie der symbolischen Formen* [Cassirer 1929]. At that particular time, he made an explicit link between holism and symbolic constructions. The similarities between Cassirer's and Weyl's reflections become now very obvious [Sieroka 2010, 312–314]. Although they agree to a certain extent on the definition and the structure of a physical theory, there are no mutual influences between them during the 1920s.

For sure, Weyl was invited in February 1928 at the University of Hamburg and met Cassirer on this occasion. More specifically, at the end of 1927, Weyl received a first invitation from his friend Erich Hecke, a specialist in number theory, to give a talk on group theory and quantum mechanics in Hamburg University.[29] At the very beginning of 1928, Weyl received a second invitation from Cassirer to address the Philosophical Society of Hamburg on *Philosophie der Mathematik und Naturwissenschaft*.[30] At that time, Cassirer was about to publish the last volume of his *Philosophie der symbolischen Formen*, in which he discusses very precisely Weyl's arguments on the foundations of mathematics, as they are developed in [Weyl 1925, 1927]. In particular, Cassirer criticizes Weyl's opposition between intuition and theory, as it is

29. See Hecke's letter to Weyl dated November 28, 1927, *ETH-Bibliothek*, Hs 91: 595. For an analysis of the Hecke-Weyl correspondence, see in particular [Eckes 2016].

30. See Cassirer's letter to Weyl from January 18, 1928, *ETH-Bibliothek*, Hs 91: 506. Cassirer was well aware of Hecke's first invitation.

expressed in [Weyl 1925, 541]. According to Cassirer, Weyl's reflections are based on old metaphysical dualisms. Cassirer disputes more specifically the existence of a theoretical "need" [*Bedürfnis*] which would lead to "a symbolic representation of the transcendent" [*symbolische Darstellung des Transzendenten*] [Weyl 1925, 542]. Indeed, in his philosophy of symbolic forms, Cassirer rejects any reference to a so-called "transcendent" beyond symbols. In fact, as pointed out by N. Sieroka, Weyl's constructivism rests on a particular interpretation of Fichtean philosophy, mainly inspired by his colleague Fritz Medicus at the *Eidgenössische Technische Hochschule* [Sieroka 2009, 91]. Indeed, Weyl explicitly refers to Fichte in [Weyl 1925, 540]. Moreover, Weyl uses expressions such as "lively theoretical need" [*lebendig[es] theoretisches Bedürfnis*] or "mind's vital process" [*Lebensprozess des Geistes*] which suggest to the reader that his reflections belong to a kind of "*Lebensphilosophie*". By contrast, Cassirer's philosophy of symbolic forms is dominated by the concept of spirit [*Geist*], with a clear reference to phenomenology in an Hegelian sense at the very beginning of [Cassirer 1929]. N. Sieroka has rightly pointed out that these differences between Cassirer and Weyl reflect a broader opposition between two forms of neo-Kantian philosophy; the Marburg School and the Southwest (German) School [Sieroka 2010, 324]. This opposition leads to a full characterization of Weyl's constructivist and holistic viewpoint in physics, which is clearly inspired by a certain interpretation of Fichte's philosophy, not by Cassirer's philosophy of symbolic forms.

5 Concluding remarks

To sum up, I have identified three interrelated steps in the development of Weyl's philosophy of physics between 1918 and 1927. First, in 1918-1921, Weyl's reflections can be characterized as a kind of speculative apriorism for the following reason: he strongly believes that a suitable generalization of Riemannian geometry must lead to an empirically consistent theory. His unified field theory and his underlying speculative approach are precisely at the core of the so-called Freundlich-Haas-Reichenbach-Weyl controversy. Weyl rejects Freundlich's strictly empiricist views; neither is he convinced by Reichenbach's relativized apriorism. However, Reichenbach's strong criticisms against the "pre-Kantian" views of Haas and Weyl might partly explain why the latter finally abandons his early speculative approach around 1921.

Second, Weyl recognizes at that particular time that his unified field theory is not empirically founded and that it is not possible to deduce a priori a physical theory from a geometric framework. However, in his writings on the problem of space, which were mainly published between 1921 and 1923, he claims that any physical theory is based on an "a priori element". The mathematical analysis of the problem of space can be considered as a characterization of this element: the Pythagorean nature of the metric depends upon purely mathematical principles; as a consequence, it belongs to "the a priori essence of space". This expression reminds us of Husserl's phenomenology. More precisely, following M. Friedman, I have tried to explain that Weyl develops a constructivist interpretation of Husserl's *Wesensanalyse*. Nevertheless, I strongly disagree with M. Friedman when he claims that Weyl's apriorism is also deeply influenced by Kant. I have shown that it is not the case, although Weyl repeatedly used Kant's vocabulary in 1918 and later on in 1921-1923 as well.

Third, Weyl's philosophy of physics took its final shape around the years 1924-1925 in the context of the controversy opposing Brouwer to Hilbert on the foundations of mathematics. Weyl finally defines a physical theory as a symbolic construction of reality and he shows that only a theory as a whole can be empirically tested. In other words, his philosophy of physics can be characterized as a constructivism and, accordingly, as a holism. These views play a central role in his broader reflections on physics as well as on mathematics: first, it partly explains why he distanced himself from intuitionism around 1924-1925; second, why it led to give his previous apriorism a milder drift and foreshadowed his later empirical turn which would occur at the end of the 1920s within his work on quantum mechanics; third, it shows that Fichtean philosophy still remained an important source of inspiration for Weyl in the mid-1920s.

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