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On the Anisotropy of Inertia.

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1. – Recently there has been considerable interest [1-3] in the possibility of an anisotropy of inertia, and an upper limit has been placed on the magnitude of such an anisotropy [3]. However this upper limit has been determined by assuming a specific model, and it is therefore of interest to examine the « model dependence » of such estimates.

In particular it has been assumed that the anisotropy appears only in the kinetic energy terms in the Hamiltonian. However it would seem plausible that the potential energy terms should then also be anisotropic since they are produced by the exchange of quanta, now presumed to have anisotropic inertial properties. Indeed in the next section we will sketch a simple model in which, though the kinetic energy terms and potential energy terms are separately anisotropic, there is an exact cancellation and the total Hamiltonian exhibits no anisotropy. Thus depending on what one assumes concerning the potential energy terms, it would seem that one can get a great variety of predictions.

2. – We start with the Dirac Hamiltonian $\alpha_i p_i + m\beta$ (here we have put $c=1$ and we sum over repeated indices $i, j, \ldots$, the indices running from 1 to 3). A natural generalization to include a possible anisotropy is

$$\alpha_i \Omega_{ij} p_j + m\beta \,, \qquad \Omega_{ij} = \Omega_{ji} \,.$$

Squaring this we obtain as the generalization of the Klein-Gordon operator

$$p_i \Omega_{ij} \Omega_{jk} p_k + m^2 \,, \tag{1}$$

from which the non-relativistic expression for the kinetic energy becomes

$$(p_i \Omega_{ij} \Omega_{jk} p_k)/2m \,,$$

which is the form suggested by COCCONI and SALPETER [1].

However (1) further suggests that forces mediated by quanta of mass μ should be derived from the Klein-Gordon operator

$$p_i \Omega_{ij} \Omega_{jk} p_k + \mu^2 \,, \tag{2}$$

(1) G. COCCONI and E. SALPETER: *Nuovo Cimento*, **10**, 646 (1958).

(2) A. CARRELLI: *Nuovo Cimento*, **13**, 853 (1959).

(3) G. COCCONI and E. SALPETER: *Phys. Rev. Lett.*, **4**, 176 (1960).

and hence we are led to the potential (the Green's function for (2))

$$V \sim \int d^3p \frac{\exp[ip_j R_j]}{p_j \Omega_{sk} \Omega_{kL} p_L + \mu^2},$$

which, by the change of variables $\Omega_{kL} p_L \to p_k$ is seen to be of the form $V = V(\Omega_{ij}^{-1} R_j)$.

Putting all this together we have for the N-particle Hamiltonian of our model

$$\sum_{\alpha=1}^{N} \frac{p_i^{(\alpha)} \Omega_{ij} \Omega_{jk} p_k^{(\alpha)}}{2m_\alpha} + \\ + \sum_{\alpha,\beta=1}^{N} V_{\alpha\beta}(\Omega_{ij}^{-1} R_j^\alpha - \Omega_{ij}^{-1} R_j^\beta),$$

which would appear to be extremely anisotropic. However in fact it is not since by the unitary transformation

$$\Omega_{ik} p_k^{(\alpha)} \to p_i^{(\alpha)}, \qquad \pi_i^{(\alpha)} \to \Omega_{ik} R_k^{(\alpha)},$$

it is transformed into

$$\sum_{\alpha=1}^{N} \frac{p_i^{(\alpha)} p_i^{(\alpha)}}{2m_\alpha} + \sum_{\alpha,\beta=1}^{N} V_{\alpha\beta}(R_i^{(\alpha)} - R_i^{(\beta)}),$$

which is clearly isotropic q.e.d.

We conclude with the following remarks:

(i) Since our N particles could include those of any external sources which may be present, our model implicitly contains the possibility of external fields. However because it is only meant as an illustrative model we will not attempt to refine it by introducing vector (magnetic field) interactions.

(ii) We chose the same Ω_{ij} for the quanta as for the particles, and the same Ω_{ij} for each kind of particle. Any deviation from equality would, of course, yield deviations from exact cancellation

(iii) We have neglected any possible spatial dependence of Ω_{ij}.