XXXVIII. *On the Identity of the Vibrations of Light with Electrical Currents.* By L. LORENZ*.

THE science of our century has succeeded in demonstrating so many relations between the various forces (between electricity and magnetism, between heat, light, molecular and chemical actions), that we are in a sense necessarily led to regard them as *manifestations of one and the same force*, which, according to circumstances, occurs under different forms. But though this has been the guiding idea with the greatest inquirers of our time, it has been by no means theoretically established; and though the connexion between the various forces has been demonstrated, it has only been explained in single points. Thus Ampère has theoretically explained the connexion between electricity and magnetism, though he has not furnished a proof of the possibility of the peculiar molecular electrical currents (assumed by him) which in virtue of their own power are continuous; and, in like manner, Melloni was subsequently led step by step to the assumption of an identity of light with radiant heat. These theories are, however, quite isolated members of the great chain; and so far are we from being able to follow out theoretically the idea of the unity of force, that even now, half a century after Œrsted's discovery, the two electricities are regarded as electrical *fluids*, light as vibrations of *æther*, and heat as motions of the *molecules of bodies*.

Yet these physical hypotheses are scarcely reconcileable with the idea of the unity of force; and while the latter has had a signal influence on science, this can by no means be said of the former, which have only been useful inasmuch as they furnish a basis for our imagination. Hence it would probably be best to admit that in the present state of science we can form no conception of the physical reason of forces and of their working in the interior of bodies; and therefore (at present, at all events) we must choose another way, free from all physical hypotheses, in order, if possible, to develope theory step by step in such a manner that the further progress of a future time will not nullify the results obtained.

This idea is at the basis, not only of the present investigation, but also of my earlier researches on the theory of light †; and I am the more moved to adhere to it, that it shows in a remarkable manner how the results which I venture here to develope attach themselves to those I have formerly obtained, and go hand in hand with them. At the same time that I keep the investigation free from all physical hypotheses, I shall endeavour

* Translated from Poggendorff's *Annalen*, June 1867.

† Phil. Mag. S. 4. vol. xxvi. p. 81.

to demonstrate a new member in the chain which connects the various manifestations of the forces; I shall prove that in accordance with the laws for the propagation of electricity under the action of free electricity, and of the electrical currents of the surrounding media, which we can deduce from experiment, periodical electrical currents are possible which in every respect behave like the vibrations of light; from which it indubitably follows that *the vibrations of light are themselves electrical currents.*

We know that light is produced by a wave-motion with very rapid periodical motions which we may call vibrations. It is the peculiarity of these vibrations that they are at right angles to the direction in which the wave of light travels; and we may say that this peculiarity has not found a correct explanation in the theory of elasticity, or in the analogous one of Cauchy; for, apart from the fact that this theory necessitates the assumption of a special medium (the luminous æther, which moreover stands quite isolated and separate from any other observation or demonstrable connexion with other forces), even with this assumption, and the various hypotheses of Cauchy, it is scarcely possible to imagine a medium in which a wave-motion could travel without a trace of longitudinal vibrations. Convinced that this theory cannot give a real, but only a factitious explanation even of the peculiarity of light (the transverse vibrations), I had formerly drawn attention to the fact that variable electrical currents, which induce in closed conductors currents that are parallel with the original ones, are similar to the vibrations of light, which in a certain sense also induce parallel vibrations. But as the laws of induced currents, generally admitted and based on experiment, did not directly lead to the expected result, the question was whether it was not possible so to modify the laws assumed that they would embrace both the experiments on which they rest and the phenomena which belong to the theory of light.

Kirchhoff (Pogg. *Ann.* vol. cii.) has expressed the laws of the motion of electricity in bodies with constant conducting-power by the following equations,

$$\left.\begin{aligned} u &= -2k\left(\frac{d\Omega}{dx} + \frac{4}{c^2}\frac{dU}{dt}\right), \\ v &= -2k\left(\frac{d\Omega}{dy} + \frac{4}{c^2}\frac{dV}{dt}\right), \\ w &= -2k\left(\frac{d\Omega}{dz} + \frac{4}{c^2}\frac{dW}{dt}\right); \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

in which u, v, w are the components of the electrical density of the current in the point $x\ y\ z$, k the constant conducting-power, c a constant, and

$$U=\iiint\frac{dx'\,dy'\,dz'}{r^3}(x-x')\left[u'(x-x')+v'(y-y')+w'(z-z')\right],$$

$$V=\iiint\frac{dx'\,dy'\,dz'}{r^3}(y-y')\left[u'(x-x')+v'(y-y')+w'(z-z')\right],$$

$$W=\iiint\frac{dx'\,dy'\,dz'}{r^3}(z-z')\left[u'(x-x')+v'(y-y')+w'(z-z')\right],$$

$$\Omega=\iiint\frac{dx'\,dy'\,dz'}{r}\epsilon'+\int\frac{ds'}{r}e',$$

in which u', v', w' are the components of the density of the current in the point $x'\,y'\,z'$, ϵ' the density of the free electricity in this point, e' the density upon the element of surface ds', and r the distance of the points $x\,y\,z$ and $x'\,y'\,z'$.

These formulæ express that the components of the electromotive force in $x\,y\,z$, which according to Ohm's law are $\frac{u}{k}, \frac{v}{k}, \frac{w}{k}$, are a sum of two components of electromotive force,—one arising from the inducing action of free electricity, the other from the inducing action of the variable intensities of the current in all the elements of the body.

Kirchhoff has further expressed the relations between the components of the current and the free electricity by the two equations

$$\left.\begin{aligned}\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}&=-\frac{1}{2}\frac{d\epsilon}{dt},\\ u\cos\lambda+v\cos\mu+w\cos\nu&=-\frac{1}{2}\frac{de}{dt},\end{aligned}\right\}\quad . \quad . \quad . \quad (2)$$

in which λ, μ, and ν are the angles which the normal to the surface, directed inward, makes with the coordinate axes.

It is at once obvious that the equations (1), which are deduced in a purely empirical manner, are not necessarily the exact expression of the actual law; and it will always be permissible to add several members, or to give the equations another form, always provided these changes acquire no perceptible influence on the results which are established by experiment. We shall begin by considering the two members on the right side of the equations *as the first members of a series.*

By the equation

$$\bar{\Omega}=\iiint\frac{dx'\,dy'\,dz'}{r}\epsilon'\left(t-\frac{r}{a}\right)+\int\frac{ds'}{r}e'\left(t-\frac{r}{a}\right)$$

let a new function $\overline{\Omega}$ be defined, in which by the relations $\epsilon'\left(t-\frac{r}{a}\right)$ and $e'\left(t-\frac{r}{a}\right)$, where a is a constant, it shall be expressed that these are the same functions of $\left(t-\frac{r}{a}\right)$ as ϵ' and e' are of t in the above expression Ω. Now by the development of the series we have

$$\epsilon'\left(t-\frac{r}{a}\right)=\epsilon'-\frac{d\epsilon'}{dt}\cdot\frac{r}{a}+\frac{d^2\epsilon'}{dt^2}\cdot\frac{r^2}{a^2}\cdot\frac{1}{1.2}-\ldots,$$

$$e'\left(t-\frac{r}{a}\right)=e'-\frac{de'}{dt}\cdot\frac{r}{a}+\frac{d^2e'}{dt^2}\cdot\frac{r^2}{a^2}\cdot\frac{1}{1.2}-\ldots,$$

which series are inserted in the above equation, and this is then differentiated with respect to x. There is thus obtained

$$\frac{d\overline{\Omega}}{dx}\frac{d\Omega}{dx}+\frac{1}{2a^2}\frac{d'}{dt^2}\left[\iiint\frac{dx'\,dy'\,dz'}{r}(x-x')\epsilon'+\int\frac{ds'}{r}(x-x')e'\right]-\ldots;$$

and if in this equation for $\frac{d\epsilon'}{dt}$ and $\frac{de'}{dt}$ the values given by equations (2) are substituted, we obtain by partial integration

$$\frac{d\overline{\Omega}}{dx}=\frac{d\Omega}{dx}-\frac{1}{a^2}\frac{d}{dt}\iiint\frac{dx'\,dy'\,dz'}{r}u'+\frac{1}{a^2}\frac{dU}{dt}-\ldots, \quad (3)$$

U having here its previous meaning. Hence we may put

$$\frac{d\overline{\Omega}}{dx}+\frac{1}{a^2}\frac{d}{dt}\iiint\frac{dx'\,dy'\,dz'}{r}u'\left(t-\frac{r}{a}\right)=\frac{d\Omega}{dx}+\frac{1}{a^2}\frac{dU}{dt}, \quad . \quad (4)$$

where it is indicated by $u'\left(t-\frac{r}{a}\right)$ that u' is here a function of $\left(t-\frac{r}{a}\right)$ instead of t alone.

The right hand of this last equation is a series of which only the first two members are retained, and whose following members proceed by increasing powers of $\frac{r}{a}$. If a be assumed $=\frac{c}{2}$, both these members become the same as the expression between the parentheses in the first of equation (4); but now, according to Weber's determination,

$$c=284736 \text{ miles},$$

while the greatest value of r in the experiments has only exceeded a few feet; hence $\frac{r}{a}$ is an infinitely small magnitude. Hence the following members of the above series are everywhere quite inappreciable, provided only that the differential quotients of the components of the current of the second and third order be not very great as compared with the time, which comes into play here also.

Hence the equations for the propagation of electricity, as regards the experiments on which they rest, are just as valid as equations (1), if, by the aid of the equation (4) and its two analogous equations, the following form be assigned to them,

$$\left.\begin{aligned} u &= -2k\left(\frac{d\overline{\Omega}}{dx}+\frac{4}{c^2}\frac{d\alpha}{dt}\right), \\ v &= -2k\left(\frac{d\overline{\Omega}}{dy}+\frac{4}{c^2}\frac{d\beta}{dt}\right), \\ w &= -2k\left(\frac{d\overline{\Omega}}{dz}+\frac{4}{c^2}\frac{d\gamma}{dt}\right), \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (A)$$

where, for brevity's sake, we put

$$\alpha = \iiint \frac{dx'\,dy'\,dz'}{r}\,u'\left(t-\frac{r}{a}\right),$$

$$\beta = \iiint \frac{dx'\,dy'\,dz'}{r}\,v'\left(t-\frac{r}{a}\right),$$

$$\gamma = \iiint \frac{dx'\,dy'\,dz'}{r}\,w'\left(t-\frac{r}{a}\right).$$

These equations are distinguished from equations (1) by containing, instead of U, V, W, the somewhat less complicated members α, β, γ; and they express further that the entire action between the free electricity and the electrical currents *requires time to propagate itself*—an assumption not strange in science, and which may in itself be assumed to have a certain degree of probability. For in accordance with the formulæ found, the action in the point $x\ y\ z$ at the moment t does not depend on the *simultaneous* condition in the point $x'\ y'\ z'$, but on the condition in which it was at the moment $t-\frac{r}{a}$; that is, so much time in advance as is required to traverse the distance r with the constant velocity a.

The constant a which enters into equations (A) should, from the foregoing, be made equal $\frac{c}{2}$; closer investigations, however, will show that other values are also possible. The first equation (A) may also be written in the following manner:—

$$u=-2k\left(\frac{d\Omega}{dx}+\left(\frac{4}{c^2}-\frac{1}{a^2}\right)\frac{d}{dt}\iiint\frac{dx'\,dy'\,dz'}{r}u'+\frac{1}{a^2}\frac{dU}{dt}\cdots\right),$$

which expression in the case of $a=\frac{c}{2}$ leads us back to the first equation (1), whereas, if a were assumed to be infinitely great, it would obtain just the form which would result from Neumann's electrodynamical theory. And as this theory also agrees with experiment, it is obvious that a is not defined by it, and must for the present be regarded as an indeterminate magnitude. Yet it must be very great, of the same order as c, to allow the following members of the series to be considered infinitely small. If, for instance, $a=\frac{c}{\sqrt{2}}$, the above equation will represent a mean between Weber's and Neumann's theories.

It now becomes necessary to obtain, in another manner, a determination of these undefined constants, and, if possible, seek a confirmation or correction of the results found. It might then be attempted, by using the indication of the formula, that electrical actions require time for their propagation, to find a probable hypothesis of the mode of action of dynamical electricity, by which results might be obtained similar to those already found. I have found that this may be effected in *several* ways; this method, however, quite loses its value, because its significance would entirely depend on finding an hypothesis which in and for itself is more probable than all others. After careful investigation of this point, I have completely given up the idea of getting any good from physical hypotheses; and we can only develope the consequences from the results found, and inquire whether this does not furnish an indication towards answering the question.

For a given function ϕ, provided the point $x\ y\ z$ is within the limits of the integral, we have

$$\left.\begin{array}{l}\left(\Delta_2-\frac{d^2}{a^2\,dt^2}\right)\iiint\frac{dx'\,dy'\,dz'}{r}\phi\left(t-\frac{r}{a},x',y',z'\right)\\ =-4\pi\phi(t,x,y,z),\quad.\quad.\quad.\quad.\quad.\quad.\quad.\end{array}\right\}\quad.\quad(5)$$

where

$$\Delta_2 \text{ is written for } \frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}.$$

The proof of this theorem, which moreover is not difficult to see, is found in my paper in Crelle's Journal, vol. lviii. By its aid the equations (A) are transformed into the following differential equations:—

$$\Delta_2 u - \frac{1}{a^2}\frac{d^2u}{dt^2} = 8\pi k\left(\frac{d\epsilon}{dx} + \frac{4}{c^2}\frac{du}{dt}\right),$$

$$\Delta_2 v - \frac{1}{a^2}\frac{d^2v}{dt^2} = 8\pi k\left(\frac{d\epsilon}{dy} + \frac{4}{c^2}\frac{dv}{dt}\right),$$

$$\Delta_2 w - \frac{1}{a^2}\frac{d^2w}{dt^2} = 8\pi k\left(\frac{d\epsilon}{dz} + \frac{4}{c^2}\frac{dw}{dt}\right),$$

with which is connected by (2) the equation

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = -\frac{1}{2}\frac{d\epsilon}{dt}.$$

These equations are satisfied, for instance, by

$$u = e^{-hz}\cos p(\omega t - z), \quad v = 0, \quad w = 0, \quad . \quad . \quad . \quad (6)$$

where h, p, ω are constants between which the relations prevail,

$$h^2a^2 = p^2(a^2 - \omega^2) \text{ and } hc^2 = 16\pi k\omega. \quad . \quad . \quad . \quad . \quad (7)$$

From this preliminary treatment of equations (A), it is clear that *periodical* electrical currents are possible, that such ones travel like a *wave-motion* with the velocity ω, and, like light, make vibrations which are at right angles to the direction of propagation. If we assume thence that the vibrations of light themselves are electrical currents, ω expresses the velocity of light, while a is the velocity with which electrical action is propagated through space. It is manifest, further, from the latter equation, that, when the electrical conductivity k of the body is very small, the two velocities tend to become equal to one another.

The velocity with which in Weber's electrodynamical experiments the electrical action at a distance has passed from one conductor to another through the air, is according to this result the same as the velocity of light in air. But now, according to Weber's determination, $c = 284736$ miles, and therefore

$$\frac{c}{\sqrt{2}} = 201360,$$

a magnitude which remarkably agrees with the various determinations of the velocity of light; for they lie both above and below this value in such a manner that the present may be regarded as a new determination of the velocity of light, and not necessarily inferior in accuracy to any other. We have therefore some reason for taking $a=\frac{c}{\sqrt{2}}$; and if this value $a\sqrt{2}$ be substituted for c in equations (A), the accuracy of this assumption is confirmed by the circumstance that the equations assume now a very simple form, and lead to exactly the same differential equations as those which I formerly deduced for the vibrations of light, with the addition of only a single member.

For, in accordance with equations (2), we have

$$\frac{d\epsilon'\left(t-\frac{r}{a}\right)}{dt}=-2\left[\frac{\delta u'\left(t-\frac{r}{a}\right)}{\delta x'}+\frac{\delta v'\left(t-\frac{r}{a}\right)}{dy'}+\frac{\delta w'\left(t-\frac{r}{a}\right)}{\delta z'}\right],$$

where the differentiation in reference to x', y', and z' must be carried out in such a manner that r will be considered constant, and

$$\frac{de'\left(t-\frac{r}{a}\right)}{dt}=-2\left[u'\left(t-\frac{r}{a}\right)\cos\lambda+v'\left(t-\frac{r}{a}\right)\cos\mu+w'\left(t-\frac{r}{a}\right)\cos\nu\right].$$

If these values be substituted in

$$\frac{d\overline{\Omega}}{dt}=\iiint\frac{dx'\,dy'\,dz'}{r}\frac{d\epsilon'\left(t-\frac{r}{a}\right)}{dt}+\int\frac{ds'}{r}\frac{de'\left(t-\frac{r}{a}\right)}{dt},$$

by partial integration and introduction of the designations α, β, γ we obtain

$$\frac{d\overline{\Omega}}{dt}=-2\left(\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}\right).$$

Moreover from (5),

$$\frac{1}{a^2}\frac{d^2\alpha}{dt^2}=\Delta_2\alpha+4\pi u,$$

and in like manner for β, γ. If now these values be substituted in the equations (A), after they have been differentiated in reference to t, and if $c=a\sqrt{2}$, we get

$$\left.\begin{aligned}\frac{1}{4k}\frac{du}{dt}+4\pi u&=\frac{d}{dz}\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right)-\frac{d}{dy}\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right),\\ \frac{1}{4k}\frac{dv}{dt}+4\pi v&=\frac{d}{dx}\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right)-\frac{d}{dz}\left(\frac{d\beta}{dz}-\frac{d\gamma}{dy}\right),\\ \frac{1}{4k}\frac{dw}{dt}+4\pi w&=\frac{d}{dy}\left(\frac{d\beta}{dz}-\frac{d\gamma}{dy}\right)-\frac{d}{dx}\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right).\end{aligned}\right\}\quad .\quad (8)$$

Moreover we obtain directly from equations (A)

$$\left.\begin{aligned}\frac{dv}{dz}-\frac{dw}{dy}&=-\frac{4k}{a^2}\frac{d}{dt}\left(\frac{d\beta}{dz}-\frac{d\gamma}{dy}\right),\\ \frac{dw}{dx}-\frac{du}{dz}&=-\frac{4k}{a^2}\frac{d}{dt}\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right),\\ \frac{du}{dy}-\frac{d}{dx}&=-\frac{4k}{a^2}\frac{d}{dt}\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right),\end{aligned}\right\}\quad\ldots\ldots\quad(9)$$

by which equations α, β, γ may be eliminated from the previous equations (8) after they have been differentiated in reference to t. In this way the following equations are obtained:—

$$\left.\begin{aligned}\frac{d}{dy}\left(\frac{du}{dy}-\frac{dv}{dx}\right)-\frac{d}{dz}\left(\frac{dw}{dx}-\frac{du}{dz}\right)&=\frac{1}{a^2}\frac{d^2u}{dt^2}+\frac{16\pi k}{a^2}\frac{du}{dt},\\ \frac{d}{dz}\left(\frac{dv}{dz}-\frac{dw}{dy}\right)-\frac{d}{dx}\left(\frac{du}{dy}-\frac{dv}{dx}\right)&=\frac{1}{a^2}\frac{d^2v}{dt^2}+\frac{16\pi k}{a^2}\frac{dv}{dt},\\ \frac{d}{dx}\left(\frac{dw}{dx}-\frac{du}{dz}\right)-\frac{d}{dy}\left(\frac{dv}{dz}-\frac{dw}{dy}\right)&=\frac{1}{a^2}\frac{d^2w}{dt^2}+\frac{16\pi k}{a^2}\frac{dw}{dt}.\end{aligned}\right\}\quad.\ (B)$$

These equations for the components of the electrical current agree fully with those which I have already found for the components of light up to the last member, into which the electrical conductivity k enters. This member indicates an absorption which will be greater the greater the electrical conductivity, and which is defined by the constant h in the equations (6) if in these $c=a\sqrt{2}$.

If k is *very large* as regards pa, equations (7) give

$$h=p=\frac{2\pi}{\lambda},$$

if λ denotes the wave-length of light; from which it follows that the amplitude of a ray of light which, for instance, has passed through a layer of a good conductor of electricity of the thickness of half a wave-length, is from (6) e^{π} times lessened, and the intensity reckoned proportional to the square of the amplitude, $e^{2\pi}$ or 535 times. This will be the case with all metals; for, according to Weber, the conductivity of copper is $\frac{1}{274100}$ in magnetic measurement, taking the millimetre and the second as units of time and length, and therefore $\frac{c^2}{8}\times\frac{1}{274100}$, or $283433a$ in mechanical measurement, a magnitude which is great as compared with $\frac{2\pi}{\lambda}a$. It is clear, however, that this result can only be considered approximately correct, especially as a perfectly constant conductivity is presupposed, a homogeneity which does not in fact exist. The chief result, however, that *all good conductors of electricity absorb light to a great extent,* is in marked accordance with experiment.

When the electrical conductivity is very small, equations (7) give

$$h=8\pi\frac{k}{a}.$$

Now in the case of copper, whose conductivity has been given above, $\frac{k}{a}$ has been found equal to 283433; but for all transparent bodies the conductivity is millions of times as small as that of copper; and liquids form a marked exception, where the chemical activity and the mobility of the particles exert so great an influence on the determination of the conductivity in the proper sense that it becomes in fact impossible; we thus find that the conductivity of all other transparent media is *so many* millions of times as small as that of the metals that the coefficient of absorption h, as well as the last member of equations (B), will disappear, by which the latter become quite identical with the equations of light. Just as we can infer their opacity from the good conductivity of the metals, so from the very small transparency of a body we may conclude that, as compared with metals, it is *an extremely bad conductor* of the electrical current, a result which experiment has also fully confirmed.

The vibrations arising from equations (B) are transversal; and even if the member containing R be retained, longitudinal vibrations will not be possible. By differentiating the three equations (8) in reference to x, y, and z, and addition, we obtain

$$\frac{d\theta}{dt}+16\pi k\theta=0,$$

if

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=\theta.$$

It is manfest from this that θ cannot be a periodical function of the time, from which it follows that longitudinal vibrations cannot take place. As, moreover, this equation shows that the value of θ diminishes as the time increases, and is independent of the components of all surrounding points, we are compelled to assume generally that $\theta=0$; from which it follows, since

$$\theta=-\frac{1}{2}\frac{d\epsilon}{dt},$$

that in the interior of a body with constant conducting-power no development of free electricity is possible. This result is different from that which Kirchhoff has deduced from the original equations (1)—that is, that in the interior of a conductor there is in general free electricity; but from the whole of the present investigation it will be clear that at all events this conclusion cannot be drawn with any degree of certainty.

Thus, after it has been proved that from equations (A), which embrace the laws of electrical currents that are in accordance with experiment, the differential equations (B) can be deduced, which show that electrical currents behave in every respect like the vibrations of light, the question arises whether, on the other hand, the laws of electrical currents can be deduced from the known laws of light. I shall now show that this is in fact possible, in such a manner that equations (A) can be again deduced from equations (B), provided the conditions be introduced into the latter which must be fulfilled at the limit of the body, and which we must know in order to deduce from the differential equations such others, which in a certain sense are their integrals. At the same time it will be seen that these limiting conditions are just the same as those I have already found (Pogg. *Ann.* vol. cxviii. p. 126; Phil. Mag. S. 4. vol. xxvi. p. 93) for the components of light; so that for this calculation we need make no other assumptions than just those which the theory of light gives.

For an element of the surface which is at right angles to the axis x, I have found that the magnitudes

$$u, \quad v, \quad \frac{du}{dy}-\frac{d}{dx}, \quad \frac{dw}{dx}-\frac{du}{dz}$$

on both sides of the element are equal; from this the limiting conditions for all other elements of the surface will be found, because the direction of the coordinate axis is arbitrarily chosen. These conditions are deduced from the differential equations of the components of light, which was possible in this case, because they held generally for all heterogeneous media, and they remained the same even after the members from the equations (B) containing the factor k had been added to the equations, a circumstance which is now seen to be necessary.

For a body whose conductivity is constant, which is surrounded by absolute non-conductors (no matter whether they really exist or not), the above-named magnitudes become zero on the surface of the body, since any electrical current is impossible in the entire insulating surface which surrounds the body.

We introduce now into equations (B), instead of u, v, w, x, y, z, the notation u', v', w', x', y', x'; and first suppose $t-\frac{r}{a}$ substituted for t, where r denotes the distance of a fixed point $x\ y\ z$

from the point $x' y' z'$. The equations thus altered will hold good, provided the differentiation indicated on the left side be considered partial in such a manner that it is not effected as regards r. Both sides are afterwards multiplied by $\frac{dx' dy' dz'}{r}$, and the equations are integrated over the entire space of the body. By partial integration, for instance, the first member of the first equation with the previous notation becomes

$$\iiint \frac{dx' dy' dz'}{r} \frac{\delta^2 u'\left(t-\frac{r}{a}\right)}{\delta y'^2} = -\int \frac{ds'}{r} \frac{\delta u'\left(t-\frac{r}{a}\right)}{\delta y'} \cos\mu$$

$$+ \frac{d}{dy}\iiint \frac{dx' dy' dz'}{r} \frac{\delta u'\left(t-\frac{r}{a}\right)}{\delta y'},$$

in which the last member by repeated partial integration passes into

$$-\frac{d}{dy}\int \frac{ds'}{r} u'\left(t-\frac{r}{a}\right)\cos\mu + \frac{d^2\alpha}{dy^2}.$$

If now all the members on the left side of the equation in question be treated in the same manner, it will be found that, if all integrals are to vanish in reference to the surface of a body, we must have

$$\left(\frac{du'}{dy'} - \frac{dv'}{dx'}\right)\cos\mu - \left(\frac{dw'}{dx'} - \frac{du'}{dz'}\right)\cos\nu = 0,$$

$$u'\cos\mu - v'\cos\lambda = 0, \quad u'\cos\nu - w'\cos\lambda = 0,$$

in which equations we suppose t again introduced instead of $t-\frac{r}{a}$, which is permissible, since the equations are valid for all values of t, and the differentiation is not to be effected in reference to r.

For an element at right angles to the x-axis (that is, for

$$\cos\mu = 0, \quad \cos\nu = 0),$$

these equations give

$$v' = 0 \text{ and } w' = 0,$$

and the corresponding equations, which are obtained from the two other equations (B), and can be deduced from the above equations by changing the letters, give

$$\frac{du'}{dy'} - \frac{dv'}{dx'} = 0 \text{ and } \frac{dw'}{dx'} - \frac{du'}{dz'} = 0.$$

Hence, if the integrals in reference to the surface of the body are to disappear, they must for an element at right angles to the

x-axis fulfil just the very conditions which are deduced from the theory of light for this element; and since the direction of the axis is arbitrarily chosen, the same must hold good for all elements of the surface.

Thus, as, assuming these limiting conditions, the integrals to the surface disappear, by the above calculation, from the first equation (B) there results

$$\frac{d}{dy}\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right)-\frac{d}{dz}\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right)=\frac{1}{a^2}\frac{d^2\alpha}{dt^2}+\frac{16\pi k}{a^2}\frac{d\alpha}{dt}.$$

If, in accordance with the earlier notation, we put

$$\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}=-\frac{1}{2}\frac{d\bar{\Omega}}{dt}$$

and further, on the right hand, in accordance with the general theorem (5),

$$\frac{1}{a^2}\frac{d^2\alpha}{dt^2}=\Delta_2\alpha+4\pi u,$$

the equation takes this form—

$$\frac{1}{2}\frac{d^2\bar{\Omega}}{dx\,dt}=4\pi u+\frac{16\pi k}{a^2}\frac{d\alpha}{dt}.$$

In an analogous manner the two other equations (B) give

$$\frac{1}{2}\frac{d^2\bar{\Omega}}{dy\,dt}=4\pi v+\frac{16\pi k}{a^2}\frac{d\beta}{dt},$$

$$\frac{1}{2}\frac{d^2\bar{\Omega}}{dz\,dt}=4\pi w+\frac{16\pi k}{a^2}\frac{d\gamma}{dt},$$

from the latter of which we obtain by the elimination of $\bar{\Omega}$,

$$\frac{dv}{dz}-\frac{dw}{dy}=-\frac{4k}{a^2}\frac{d}{dt}\left(\frac{d\beta}{dz}-\frac{d\gamma}{dy}\right),$$

an equation which is identical with the first equation (9); and both the others can be formed in a corresponding manner.

If now, by the aid of these equations,

$$\frac{dv}{dz}-\frac{dw}{dy},\quad \frac{dw}{dx}-\frac{du}{dz},\quad \frac{du}{dy}-\frac{dv}{dx}$$

be eliminated from equations (B), the first of these, after integrating in respect to t, will give

$$\frac{d}{dy}\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right)-\frac{d}{dz}\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right)=-\frac{1}{4k}\frac{du}{dt}-4\pi u,$$

which is identical with the first equation (8); and if here the last member, in accordance with (5), be put

$$-4\pi u=\Delta_2\alpha-\frac{1}{a^2}\frac{d^2\alpha}{dt^2},$$

and the designation $\overline{\Omega}$ be introduced, after another integration in reference to t we have

$$u = -2k\left(\frac{d\overline{\Omega}}{dx} + \frac{2}{a^2}\frac{d\alpha}{dt}\right).$$

Since we kave $a\sqrt{2}=c$, we have returned to the first equation (A); and the two others may by analogy be deduced from this. The constants which should have been added in the two integrations in reference to t have been here omitted, because it is clear that such arbitrary constants would have no significance in the present case.

This result is a new proof of the identity of the vibrations of light with electrical currents; for it is clear now, not only that the laws of light can be deduced from those of electrical currents, but that the converse way may be pursued, provided the same limiting conditions are added which the theory of light requires. Thus we are in a position to deduce by calculation alone the inducing action of free electricity as defined by Kirchhoff's equations (2), as well as the inducing action of variable electrical currents, borh of which are contained in equations (A), by simply starting from those facts which are necessary to deduce the laws of light, and afterwards adding a single member to the differential equations found between the so-called components of light. This member expresses in a correct manner the absorption of light in good conductors of electricity, and disappears for perfectly transparent bodies.

Without dwelling more minutely on the consequences of the results obtained here, which manifestly lead us a step further towards developing the idea of the unity of force, and open a fresh field for future inquiries, I shall in conclusion call attention to the conclusions which we are entitled to draw with some degree of probability as to the mode of action of light, and how we are placed as regards the physical hypotheses concerning light.

Were we to attempt to represent the laws of electrical currents in such a manner that they would be generally valid for given *heterogeneous* bodies, and not merely for homogeneous bodies with constant conducting-power, this would appear to be best effected by starting from the differential equations, and regarding a and k as variable magnitudes. This would be more especially in agreement with the general equations found in the theory of light for heterogeneous media; besides, those limiting conditions which must be fulfilled for homogeneous bodies would then be contained in the differential equations and could be deduced from them. In this manner, however, a form corresponding in simplicity to the equations (A) could not be attained for heterogeneous bodies; and this must then be considered a special case obtaining alone for homogeneous bodies, while the differential equations would remain the original and only valid ones on which the physical explanation would have to depend. The theoretically important conclusion would thence follow which has been already indicated, that electrical forces require time to travel, and that these forces only apparently act at a distance (as would follow from equations (A) if they were regarded as the fundamental equations), and that every action of electricity and of electrical currents does in fact only depend on the electrical condition of the *immediately surrounding* elements, in the manner indicated by the differential equations (B). This is well known to be an idea indicated by Ampère, and which several physicists, more particularly Faraday, have defended.

The present general opinion regards light as consisting of backward and forward motions of particles of æther. If this were the case, the electrical current would be a progressive motion of the æther in the direction of the (positive or negative) electrical currents. But it is impossible that the same equations which theory deduces for very small displacements from equilibrium should hold good for all kinds of displacements whatever; and it just follows from the whole of this investigation that the same equations hold for both cases. Light cannot, therefore, consist of vibrations of the kind hitherto assumed; and this last consequence of the theory of æther makes it untenable.

There is, on the other hand, another conception of the nature of light-vibrations, to which I have already adverted*, and which perhaps now becomes more probable. For if we suppose light to consist of *rotating* vibrations in the interior of bodies, about axes which, according to the theory of electricity, we regard as directions of vibration, the electrical current is no translatory motion, but a rotation continued in one direction, and the axis of rotation becomes then the direction of the current. This rotation will only be continuous in good conductors, and the motion travel there in the direction of the axis, whereas it becomes periodical in bad conductors, and is propagated by what in electricity we call induction, in a direction at right angles to the axis of rotation. In this idea there is scarcely any reason for adhering to the hypothesis of an æther; for it may well be assumed that in the so-called vacuum there is sufficient matter to form an adequate substratum for the motion.

This hypothesis as to the nature of light and of electrical currents will probably, as science progresses, either assume a new form, or be totally rejected. But the result of the present investigation, that the vibrations of light are electrical currents, has been obtained without the assumption of a physical hypothesis, and will therefore be independent of one.

* Pogg. *Ann.* vol. cxviii. p. 113; Phil. Mag. S. 4. vol. xxvi. p. 82.