Calculation of the Sixfold Integrals of the Ampere Force Law in a Closed Circuit

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Abstract—**The sixfold integrals of the Ampere force law are calculated for a closed circuit, using CERN routines. The theoretical calculations are complementary to the experiments, which were performed using the same pi-frames. The dimensions of the analyzed circuit were chosen to be exactly the same as those of the experimental check. The results show that, taking into account the statistical and systematic errors of the numerical method, the Ampere law is in complete agreement with the experiment, in contradiction to other experiments which state the opposite.**

I. Introduction

THE PURPOSE of this paper is to present the theoretical calculations, for specific pi-frames, using the Ampere force law. The calculations were carried out by employing the RIWIAD and RGAUSS routines (mainly RIWIAD). These theoretical calculations are complementary to the experiments [1], which were performed using the same pi-frames. In the experiments we measured the quantity $F_{e/d}/I^2$ for wires of various diameters. The dimensions of the analyzed circuit were chosen to be exactly the same as those of the experimental check. The theoretical results were required to test whether Ampere's law complies with the experimental results.

Over the last 150 years there have been several publications [2]–[29] and a lot of controversy about the Biot–Savant-Lorentz (B-S-L) and Ampere force laws. This controversy was updated in recent years as a result of both theoretical [21], [23], [25], [26], [29] and experimental [17]–[19], [29] studies on this subject.

Below we shall outline the most representative works concerning this controversy.

In his *Memoire* of 1823 on p. 280, Ampere [2] began his criticism of the B-S-L formula, which gives the magnitude and direction of the force exerted by an element of a circuit on a magnetic pole . . .

Cleveland [4] determined theoretically and experimentally the forces in a rectangular circuit one side of which was mechanically separable from the other three, e.g., he determined the force exerted by a part of a circuit upon another part of the same circuit. He stated: "In order to explain the observed forces by the use of the B-S-L equation, it is necessary to make the doubtful assumption that a part of the rectangular circuit can lift itself." Also he stated: "Maxwell's theory has several recognized weaknesses, among which is the following: it makes action and reaction not equal and opposite for elements of a circuit." Cleveland's experimental results showed that action and reaction were equal and opposite for the mechanically separable parts of the rectangular circuit, thus showing that the B-S-L law was wrong in this instance.

Mathur [7] stated that: "It is not often realized that Newton's law of action and reaction is not generally valid for the mutual forces between current elements. The correct results are obtained when it is recognized that the third law is not applicable to mutual forces between current elements." He calculated, with the help of the B-S-L law, the force on a part of a rectangular circuit arrangement discussed by Dunton [6], assuming, as he stated, the inapplicability of Newton's law. His theoretical results, in this case, agreed with the experimental results of Dunton. He concluded that the B-S-L law, when the inapplicability of Newton's law is recognized, gives for the outward force a value about 20 times larger than that which is computed by applying the B-S-L law, but assuming (wrongly) the validity of Newton's law. Also he said that it was not the B-S-L law that was in error, but it led to wrong results, when the applicability of Newton's third law was not recognized.

Robertson [9] calculated the outward force on one side of Dunton's rectangular circuit, one side of which was mechanically separable from the other three, using Ampere's formula. He obtained a value which was fairly close to Dunton's value.

In the theoretical calculations of all the above papers a wire of zero diameter has been used as well as arbitrary gaps to avoid the infinities which are present in this kind of calculations both for the B-S-L and Ampere force laws. The questions they have raised are going to be discussed in Section IV of the present work.

Graneau [17] among other issues deals with the force distribution along the projectile branch of the railgun accelerator. He employs finite current element analysis and shows that both theories (B-S-L and Ampere's) give approximately the same acceleration force distribution and that the total acceleration force furnished by them agrees well with an experimental check. He also states that relativistic field theory does not conform with Newton's third law. For his last remark we refer to Section IV of the present work.

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In the experiments reported in [28], again, a zero diameter is used but the infinities which are present are overcome by bending the free ends of the moving pi-frame. However, this bending may cause uncertainties in the current distribution inside the mercury cups and one cannot have rigorous results from both theory and experiment. Apart from this, in [28] three systematic errors (the surface tension of mercury, the restoring force of the moving pi-frame, and the pinch effect) were not taken into account properly (see [1]).

Maxwell was aware that the formulas both of Ampere and of Biot-Savart-Lorentz give the same results for closed circuits. It has been known for a long time and established analytically [11] that both force laws agree identically on the reaction force between two closed circuits.

Recently Ternan [25] and Jolly [26] claim that Ampere's and B-S-L's force laws predict equal and opposite forces on complementary parts of a current distribution. The proofs of Ternan and Jolly are similar and their main difference is in the symbols used. But both of these proofs are incomplete because they do not discuss the convergence of the volume integrals, when the volume element on which the force acts is part of the closed circuit.

Christodoulides [32] has shown analytically the equivalence of the two laws examining, in detail, the convergence of the volume integrals which appear in these laws. In addition, he has shown that the forces predicted by the two laws are the same at each point of the conductor and, consequently, the force distributions predicated by both laws are equal, irrespectively of whether the volume current element is situated outside or inside the distribution exerting the force.

The present paper, along with the experimental one [1], and the theoretical one of B-S-L [30] proves, for the first time, even if they refer to a specific configuration, that the two laws are equivalent between parts belonging to the same circuit and also in agreement with the experiment. We use sixfold integrals to overcome the singularities and infinities involved both in the Ampere and B-S-L laws in this case.

It is well known that infinities which appear with models using conductors with zero cross section are overcome using finite cross section, that is, volume integrals (for a rigorous proof see [32]).

The calculations of our sixfold integrals have been performed using the routine RIWIAD (see [30]), which avoids singularities and infinities due to the nature of the Monte Carlo model-method employed

II. The Form of Ampere Force Law Using Volume Elements and Current Densities

Let dA_i and dl_i be the cross-sectional area and the length of the volume element dV_i, respectively, $\vec{r}_f$ and $\vec{r}_i$ the vector distances of dV_f and dV_i from a common origin of coordinates (see Fig. 1), and $\vec{r}_{fi}$ the distance between the two volume elements. Then, according to the Ampere force law, the differential force $d^6\vec{F}_{fi}$ that the volume element dV_i (where there is a current with density $\vec{J}_i$) exerts on volume elements dV_f (in which there is current with density $\vec{J}_f$), is given by

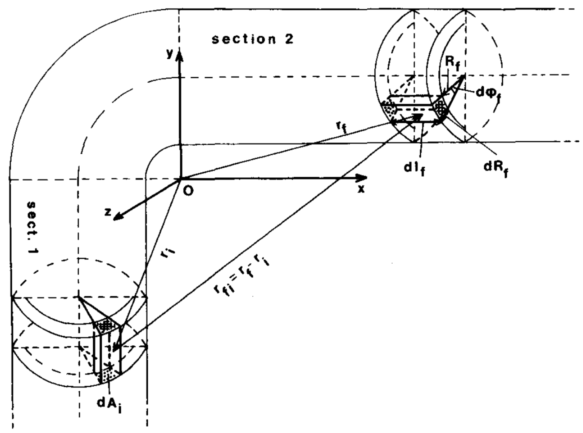

Fig. 1. Details of the evaluation of the force F_{21}. The origin of coordinates is O.

$$d^6\vec{F}_{fi} = \frac{\mu_0}{4\pi} J_i J_f \cdot \left(-\frac{\vec{r}_{fi}}{r_{fi}^3}\right)\left(2\, d\vec{l}_i \cdot d\vec{l}_f - \frac{3}{R_{fi}^3}(d\vec{l}_i \cdot \vec{r}_{fi})(d\vec{l}_f \cdot \vec{r}_{fi})\right) dA_i\, dA_f \quad (1)$$

where

$$\vec{r}_{fi} = \vec{r}_f - \vec{r}_i.$$

The element $d\vec{l}_i$ has the direction of the current density $\vec{J}_i$ and is always perpendicular to the area dA_i ($dA_i = d\varphi_i \cdot dR_i R_i$ (see Fig. 1)).

$d^6\vec{F}_{fi}$ is the force acting on element dV_f (in which a current is flowing) due to the current in element dV_i (i stands for initial and f for final, and the element which experiences the force is the final).

We take μ_0 (the permeability constant) $= 4\pi \times 10^{-7}$ Wb/A · m. Then, the force $\vec{F}_{fi}$ is given in newtons when we measure dl and r in meters, dA in square meters, and J in ampers per square meter.

J in (1) is taken as I/(cross-sectional area of the wire), where I is the current in the wire.

III. Results

For a detailed description of the suspension of the frames see [1], [30].

For Ampere's calculations consider Fig. 2. The origin of coordinates in now O'. We have eight sections which give rise to ten integrals. The different parameters, as shown in Fig. 2, have the values: $R_0 = 4.5$ mm, $R = 1.5$ mm, $D_3 = 0.467$ m, $D_2 = 0.468$ m, $D_1 = 0.758$ m. R varies from 0 to 1.5 mm, u from 0 to $\pi/2$, and φ from 0 to 2π. The results for Ampere's ten integrals are shown in Table I (column 6) (for 90-percent confidence level).

Considering 99-percent confidence level, we get Table II (for details see [30]).

TABLE I
RESULTS OF THE EVALUATION OF ALL SIXFOLD INTEGRALS (WITH RIWIAD) OF THE BIOT–SAVART–AMPERE FORCE LAWS
(The results refer to the quantity F/I^2 for the same parameters for both laws.)

		F/I^2 (g · wt/A^2)			
Integral	Integral Multiplicity	Biot–Savart $R = 1.5$ mm, $R_0 = 4.5$ mm	Integral	Integral Multiplicity	Ampere $R = 1.5$ mm, $R_0 = 4.5$ mm
F_{21}	2	0.38156E − 04 ± 0.13166E − 07	F_{21}	2	0.53065E − 04 ± 0.37082E − 07
F_{24}	2	0.10277E − 04 ± 0.14935E − 07	F_{25}	2	0.38819E − 05 ± 0.40910E − 09
F_{44}	2	0.91117E − 05 ± 0.43786E − 06	F_{35}	1	0.27418E − 05 ± 0.13646E − 09
F_{41}	2	0.23132E − 05 ± 0.46524E − 08	F_{41}	2	0.18008E − 05 ± 0.36436E − 09
F_{23}	1	0.14169E − 05 ± 0.41429E − 10	F_{31}	2	0.13387E − 05 ± 0.11540E − 09
F_{61}	2	0.89386E − 07 ± 0.16470E − 10	F_{26}	2	0.60944E − 07 ± 0.10660E − 10
F_{25}	2	0.15647E − 07 ± 0.22097E − 11	F_{46}	2	0.41175E − 07 ± 0.79418E − 11
F_{43}	2	0.13185E − 07 ± 0.19837E − 11	F_{36}	2	0.14773E − 07 ± 0.22012E − 11
F_{45}	2	0.13607E − 09 ± 0.33533E − 13	F_{76}	2	0.40928E − 09 ± 0.77556E − 13
F_{65}	2	0.15327E − 09 ± 0.33860E − 13	F_{86}	2	0.33385E − 09 ± 0.73258E − 13
F_{64}	2	0.91896E − 09 ± 0.22583E − 12			
Total sum and its error		1.21372E − 04 ± 8.7667E − 07	Total sum and its error		1.23150E − 04 ± 7.4173E − 08

TABLE II
CUMULATIVE RESULTS OF B–S (FOR ALLCASES) AND AMPERE FORCE LAWS TAKING INTO ACCOUNT THE DUPLICATION OF THE ERRORS DUE TO THE INCREASE OF CL TO 99.9 PERCENT
(Five largest integrals = 5 li)

B–S, $R = 1.5$ mm, $R_0 = 4.5$ mm 5 largest integrals 1.21133E − 04 ± 1.7534E − 06	B–S, 11 integrals $R = 1.5$ mm, $R_0 = 4.5$ mm 1.21372E − 04 ± 1.7534E − 06	B–S, 5 li $R = 1.5$ mm, $R_0 = 2$ mm 1.22393E − 04 ± 5.3844E − 07	B–S, 5 li $R = 1.0$ mm, $R_0 = 4.5$ mm 1.28660E − 04 ± 3.0209E − 06	B–S, 5 i φ from 175° to 185° 1.3516E − 04 ± 2.2472E − 05
B–S, 5 li φ from −5° to 5° 1.4395E − 04 ± 1.2985E − 05	B–S, 5 li $J = A \cdot r$ 1.1848E − 04 ± 2.0341E − 06	B–S, 5 li $J = A\sqrt{R^2 - r^2}$ 1.2278E − 04 ± 1.9198E − 06	B–S, 5 li 7 filaments 1.2105E − 04 ± 1.9174E − 06	Ampere 10 li $R = 1.5$ mm, $R_0 =$ 4.5 mm 1.2315E − 04 ± 1.4835E − 07

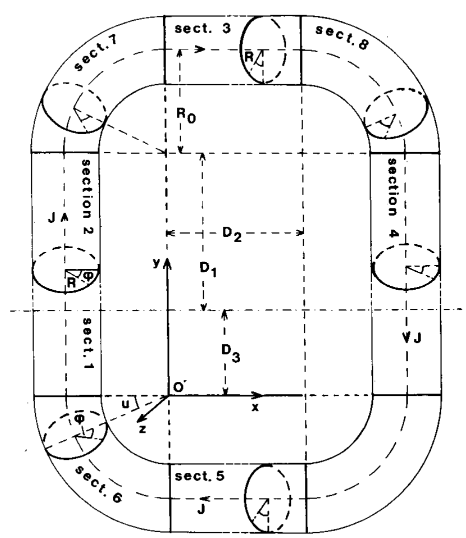

Fig. 2. The wire configuration is divided into eight sections for the case of Ampere force law.

The equations expressing the Ampere force law, in differential form, among the various sections, are given explicitly in Appendix I (only for the five largest integrals).

IV. AN INTERESTING NOTE

a) In [31] we note "... Considering these interparticle forces to be newtonian, we understand that for the mutual interaction of any two particles the forces obey $\vec{F}_{ij} = -\vec{F}_{ji}$, where $\vec{F}_{ij}$ represents the force exerted upon particle i by particle j and vice versa ...".

b) In Fig. 3, *abcd* is the stationary part (pi-frame) and *a'fed'* is the moving part (pi-frame). If F and I are the measured force on the moving part and the current in the circuit, respectively, we find that the experimental value of F/I^2 is [1]

$$F_e/I^2 = 1.2298E - 04 \pm 1.5E - 0.6.$$

The theoretical value of F/I^2, considering the forces on side *fe* (moving part) by the sides *ab*, *bc*, and *cd* of the stationary part (because all other forces on other sides of the moving part cancel out), using the B–S–L law and the same pi-frames as above, is

$$F_{t1}/I^2 = 2.43E - 06 \pm 1.04E - 10.$$

F_{t1}/I^2, which is called by some an external force, is 50 times smaller than F_e/I^2.

Now, if we consider the theoretical value of F/I^2 exerted on side *fe* by the sides *ab*, *bc*, and *cd*, plus the forces exerted on *fe* by the sides *a'f* and *d'e* (the latter is called

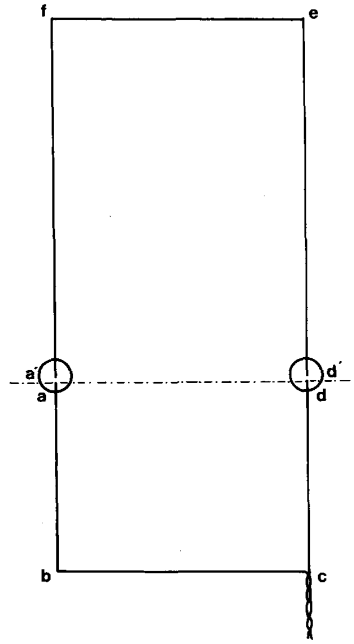

Fig. 3. The moving and stationary pi-frames.

by some internal forces), we find [30] using the B–S–L law and the same pi-frames

$$F_{t2}/I^2 = 1.2137E - 04 \pm 1.75E - 06.$$

That is, F_{t2}/I^2 is in agreement with F_e/I^2, within the error.

At this point we should note that for the corresponding case of the Ampere force law it is immaterial if one considers the external forces only or the external plus internal forces because the theoretical results are identical in both cases (external only or external plus internal). The reason is that the internal forces cancel out in the Ampere force law.

c) In the above consideration it is not correct to discuss the internal and external forces because we do not have two isolated systems. The distinction between "moving part" and "stationary part" as two isolated systems is not correct. These two systems communicate through the cups and all we have is one circuit. So, if we want to find the total force on the moving part, with the Biot-Savart-Lorentz force law, we have to consider the sum of the following forces:

$$\vec{F}_{fe,bc} + \vec{F}_{fe,ab} + \vec{F}_{fe,dc} + \vec{F}_{a'f,bc} + \vec{F}_{d'e,bc} + \vec{F}_{ed',ab} + \vec{F}_{fa',dc} + \vec{F}_{fe,a'f} + \vec{F}_{fe,d'e} - (\vec{F}_{fa',fe} + \vec{F}_{ed',fe} + \vec{F}_{dc,fe} + \vec{F}_{ab,fe})$$

which reduces to $\vec{F}_{fe,fb} + \vec{F}_{fe,bc} + \vec{F}_{fe,ec}$, because all the other forces cancel out. So, Newton's third law, for the case of the B–S–L force law, is not violated. The Ampere's force law presents no problem as stated in part b) of this section.

Finally, there is no obvious way on how one could employ the field momentum to explain the discrepancy between parts a) and b) of this section. In addition, the field momentum cannot account for the 50 times larger force. Moreover, we cannot consider radiation of the field because the current is constant.

V. Conclusions

From Table II we see that the B–S–L [30] and Ampere force laws are equivalent within the error, for 99-percent confidence level. (See [30] for comments on the confidence level.) Considering Table II and [1] we see that the Ampere's force law is in complete agreement within the error limits with the experiment, in contradiction to other experiments and theories which state the opposite.

Appendix I

When dl_i, $\vec{r}_i$ belong to section 1 of Fig. 2, and $d\vec{l}_f$, $\vec{r}_f$ belong to section 2 of Fig. 2, (1) takes the form

$$d^6F_{21} = K\left(\frac{1}{r^3}\right)\left(\frac{3}{r^2}(D_3 + y_f - y_i)^2 - 2\right) \cdot (D_3 + y_f - y_i)Z_0$$

where

$$K = \frac{\mu_0}{4\pi} J_i J_f$$

$$Z_0 = dy_i\, dy_f\, d\varphi_i\, d\varphi_f\, dR_i\, dR_f \cdot R_i R_f$$

$$\vec{r} = \vec{r}_f - \vec{r}_i$$

$$r^2 = (R_f \cos\varphi_f - R_i \cos\varphi_i)^2 + (D_3 + y_f - y_i)^2 + (R_f \sin\varphi_f - R_i \sin\varphi_i)^2$$

with

$$\vec{r}_i = -(R_0 - R_i\cos\varphi_i)\hat{x} + y_i\hat{y} + R_i \sin\varphi_i \hat{z}$$

$$\vec{r}_f = -(R_0 - R_f\cos\varphi_f)\hat{x} + (D_3 + y_f)\hat{y} + R_f \sin\varphi_f \hat{z}$$

$$d\vec{l}_i = dy_i\hat{y}$$

$$d\vec{l}_f = dy_f\hat{y}.$$

When $d\vec{l}_i$, $\vec{r}_i$ belong to section 5 of Fig. 2, and $d\vec{l}_f$, $\vec{r}_f$ belong to section 2 of Fig. 2, (1) takes the form

$$d^6F_{25} = K(1/r^3)((3/r^2)(D_3 + y_f + R_0 + R_i\cos\varphi_i)^2 \cdot (x_i + R_0 - R_f\cos\varphi_f)Z_0$$

where

$$K = \frac{\mu_0}{4\pi} J_i J_f$$

$$Z_0 = dx_i\, dy_f\, d\varphi_i\, d\varphi_f\, dR_i\, dR_f\, R_i R_f$$

$$\vec{r} = \vec{r}_f - \vec{r}_i$$

$$r^2 = (R_f\cos\varphi_f - R_0 - x_i)^2 + (D_3 + R_0 + y_f + R_i\cos\varphi_i)^2 + (R_f\sin\varphi_f - R_i\sin\varphi_i)^2$$

with

$$\vec{r}_i = x_i\hat{x} - (R_0 - R_i\cos\varphi_i)\hat{y} + R_i\sin\varphi_i\hat{z}$$

$$\vec{r}_f = -(R_0 - R_f\cos\varphi_f)\hat{x} + (D_3 + Y_f)\hat{y} + R_f\sin\varphi_f\hat{z}$$

$$d\vec{l}_i = -dx_i\hat{x}$$

$$d\vec{l}_f = dy_f\hat{y}.$$

When $d\vec{l}_i$, $\vec{r}_i$ belong to section 5 of Fig. 2, and $d\vec{l}_f$, $\vec{r}_f$ belong to section 3 of Fig. 2, (1) takes the form

$$d^6F_{35} = K(1/r^3)(2 - (3/r^2)(x_f - x_i)^2) \cdot (D_3 + D_1 + 2R_0 + R_i\cos\varphi_i - R_f\cos\varphi_f)Z_0$$

where

$$K = \frac{\mu_0}{4\pi}J_iJ_f$$

$$Z_0 = dx_i\,dx_f\,d\varphi_i\,d\varphi_f\,dR_i\,dR_fR_iR_f$$

$$\vec{r} = \vec{r}_f - \vec{r}_i$$

$$r^2 = (x_f - x_i)^2 + (D_3 + D_1 + 2R_0 + R_i\cos\varphi_i - R_f\cos\varphi_f)^2 + (R_f\sin\varphi_f - R_i\sin\varphi_i)^2$$

with

$$\vec{r}_i = x_i\hat{x} - (R_0 - R_i\cos\varphi_i)\hat{y} + R_i\sin\varphi_i\hat{z}$$

$$\vec{r}_f = x_f\hat{x} + (D_3 + D_1 + R_0 - R_f\cos\varphi_f)\hat{y} + R_f\sin\varphi_f\hat{z}$$

$$d\vec{l}_i = -dx_i\hat{x}$$

$$d\vec{l}_f = dx_f\hat{x}.$$

When $d\vec{l}_i$, $\vec{r}_i$ belong to section 1 of Fig. 2, and $d\vec{l}_f$, $\vec{r}_f$ belong to section 4 of Fig. 2, (1) takes the form

$$d^6F_{41} = K\cdot\left(\frac{1}{r^3}\right)\left(2 - \frac{3}{r^2}\right)\cdot(D_3 + y_f - y_i)^2) \cdot (D_3 + y_f - y_i)Z_0$$

where

$$K = \frac{\mu_0}{4\pi}J_iJ_f$$

$$Z_0 = dy_i\,dy_f\,d\varphi_i\,d\varphi_f\,dR_i\,dR_fR_iR_f$$

$$\vec{r} = \vec{r}_f - \vec{r}_i$$

$$r^2 = (D_2 + 2R_0 + R_f\cos\varphi_f - R_i\cos\varphi_i)^2 + (D_3 + y_f - y_i)^2 + (R_f\sin\varphi_f - R_i\sin\varphi_i)^2$$

with

$$\vec{r}_i = -(R_0 - R_i\cos\varphi_i)\hat{x} + y_i\hat{y} + R_i\sin\varphi_i\hat{z}$$

$$\vec{r}_f = (D_2 + R_0 + R_f\cos\varphi_f)\hat{x} + (D_3 + y_f)\hat{y} + R_f\sin\varphi_f\hat{z}$$

$$d\vec{l}_i = dy_i\hat{y}$$

$$d\vec{l}_f = -dy_f\hat{y}.$$

When $d\vec{l}_i$, $\vec{r}_i$ belong to section 1 of Fig. 2, and $d\vec{l}_f$, $\vec{r}_f$ belong to section 3 of Fig. 2, (1) takes the form

$$d^6F_{31} = K\cdot\left(\frac{1}{r^3}\right)\left(\frac{3}{r^2}\right)(D_3 + D_1 + R_0 - R_f\cos\varphi_f - y_i)^2(x_f + R_0 - R_i\cos\varphi_i)Z_0$$

where

$$K = \frac{\mu_0}{4\pi}J_iJ_f$$

$$Z_0 = dx_f\,dy_i\,d\varphi_i\,d\varphi_f\,dR_i\,dR_fR_iR_f$$

$$\vec{r} = \vec{r}_f - \vec{r}_i$$

$$r^2 = (x_f + R_0 - R_i\cos\varphi_i)^2 + (D_3 + D_1 + R_0 - R_f\cos\varphi_f - y_i)^2 + (R_f\sin\varphi_f - R_i\sin\varphi_i)^2$$

with

$$\vec{r}_i = -(R_0 - R_i\cos\varphi_i)\hat{x} + y_i\hat{y} + R_i\sin\varphi_i\hat{z}$$

$$\vec{r}_f = x_f\hat{x} + (D_3 + D_1 + R_0 - R_f\cos\varphi_f)\hat{y} + R_f\sin\varphi_f\hat{z}$$

$$d\vec{l}_i = dy_i\hat{y}$$

$$d\vec{l}_f = dx_f\hat{x}.$$

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REFERENCES

[1] P. G. Moyssides, "Experimental verification of the Biot–Savart–Lorentz and Ampere force laws in a closed circuit, revisited," this issue, pp. 4313–4321.
[2] J. C. Maxwell, *A Treatise on Electricity and Magnetism*, vol. 2. Oxford, UK: Oxford Univ. Press, 1873.
[3] A. M. Ampere, *Memoires sur Electrodynamic*, vol. 1, XXV. Paris, France: Gauthier-Villars, 1885.
[4] F. F. Cleveland, "Magnetic forces in a rectangular circuit," *Phil. Mag., 57*, vol. 21, p. 416, 1936.
[5] A. E. Clayton, "Validity of laws of electrodynamics," *Nature*, vol. 140, p. 246, 1937.
[6] W. F. Dunton, "Validity of laws of electrodynamics," *Nature*, vol. 140, p. 245, 1937.
[7] S. B. L. Mathur, "Biot-Savart law and Newton's third law of motion," *Phil. Mag.*, vol. 32, p. 171, 1941.
[8] J. M. Keller, Newton's third law and electrodynamics," *Amer. J. Phys.*, vol. X, p. 302, 1942.
[9] I. A. Robertson, "An historical note on a paradox in electrodynamics," *Phil. Mag.*, vol. 36, p. 32, 1945.
[10] L. Page and N. I. Adams, Jr., "Action and reaction between moving charges," *Amer. J. Phys.*, vol. 13, p. 141, 1945.
[11] R. C. Leness, "The equivalence of Ampere's electrodynamic law and that of Biot and Savart," *Contemp. Phys.*, vol. 4, p. 453, 1963.
[12] W. G. V. Rosser, "Electromagnetism as a second order effect," *Contemp. Phys.*, vol. 3, p. 28, 1961.
[13] E. Charles, "Mechanical forces on current-carrying conductors," *Proc. Inst. Elec. Eng.*, vol. 110, no. 9, p. 1671, 1963.

[14] J. R. Tessman, "Maxwell-out of Newton, Coulomb, and Einstein," *Amer. J. Phys.*, vol. 34, p. 1048, 1966.
[15] S. Rashleigh and R. Marshall, "Electromagnetic acceleration of macroparticles to high velocities," *J. Appl. Phys.*, vol. 49, p. 2540, 1978.
[16] I. R. McNab, "Electromagnetic macroparticle acceleration by a high pressure plasma," *J. Appl. Phys.*, vol. 51, p. 2549, 1980.
[17] P. Graneau, "Application of Ampere's force law to railgun accelerators," *J. Appl. Phys.*, vol. 53, p. 6648, 1982.
[18] P. Graneau, "Electromagnetic jet-propulsion in the direction of current flow," *Nature*, vol. 295, p. 311, 1982.
[19] P. T. Pappas, "The original Ampere force and Biot-Savart and Lorentz forces," *Nuovo Cimento*, vol. 76B, p. 189, 1983.
[20] P. Graneau, "First indication of Ampere tension in solid electric conductors," *Phys. Lett.*, vol. 97A, p. 253, 1983.
[21] —, "Compatibility of the Ampere and Lorentz force laws with the virtual-work concept," *Nuovo Cimento*, vol. 78B, p. 213, 1983.
[22] —, "Longitudinal magnet forces?" *J. Appl. Phys.*, vol. 55, p. 2598, 1984.
[23] —, "Ampere tension in electric conductors," *IEEE Trans. Magn.*, vol. MAG-20, p. 444, 1984.
[24] P. Graneau and P. N. Graneau, "Electrodynamic explosions in liquids," *Appl. Phys. Lett.*, vol. 46, no. 5, p. 468, 1985.
[25] J. C. Ternan, "Equivalence of the Lorentz and Ampere force laws in magnetostatics," *J. Appl. Phys.*, vol. 57, no. 5, p. 1743, 1985.
[26] D. C. Jolly, "Identify of the Ampere and Biot-Savart electromagnetic force laws," *Phys. Lett.*, vol. 107A, p. 231, 1985.
[27] P. T. Pappas and P. G. Moyssides, "On the fundamental laws of electrodynamics," *Phys. Lett.*, vol. 111A, no. 4, p. 193, 1985.
[28] P. G. Moyssides and P. T. Pappas, "Rigorous quantitative test of Biot-Savart-Lorentz forces," *J. Appl. Phys.*, vol. 59, no. 1, p. 19, 1986.
[29] P. Graneau and P. N. Graneau, "The electromagnetic impulse pendulum and momentum conservation," *Nuovo Cimento*, vol. 7D, no. 1, pp. 31-45, 1986.
[30] P. G. Moyssides, "Calculation of the sixfold integrals of the Biot-Savart-Lorentz force law in a closed circuit," this issue, pp. 4298-4306.
[31] Berkeley Physics Course, *Mechanics Volume 1*, 2nd ed. New York, NY: McGraw-Hill, 1973, p. 174.
[32] C. Christodoulides, "Equivalence of the Ampere and Biot-Savart force laws in magnetostatics," *J. Phys. A: Math. Gen.*, vol. 20, pp. 1-6, 1987.

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