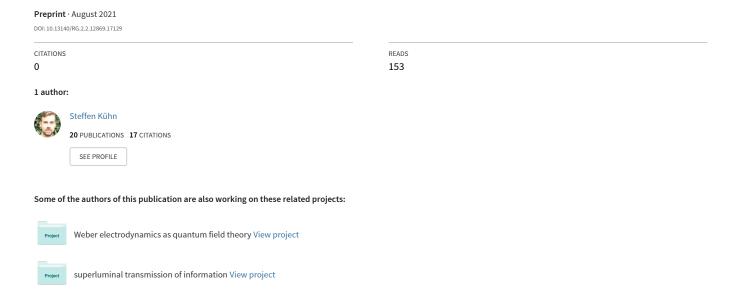
Experimental investigation of an unusual induction effect and its interpretation as a necessary consequence of Weber electrodynamics



Experimental investigation of an unusual induction effect and its interpretation as a necessary consequence of Weber electrodynamics

Steffen Kühn August 3, 2021

The magnetic force acts exclusively perpendicular to the direction of motion of a test charge, whereas the electric force does not depend on the speed of it. The present article provides experimental evidence that, in addition to these two forces, a third electromagnetic force exists which is proportional to the velocity of the test charge and acts parallel, and not perpendicular, to the direction of motion. Such a force cannot be explained by the Maxwell equations and the Lorentz force, since it is incompatible with this framework for principal reasons. However, this force would be well compatible with Weber electrodynamics and Ampère's original force law, since this old form of electrodynamics not only predicts the existence of such a force, but also seems to make it possible to calculate it accurately.

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The electromagnetic force F onto a test charge q with velocity v, so is generally accepted, is fully given by the two fields E and B and the formula of the Lorentz force

I. Introduction

$$\boldsymbol{F} = q\,\boldsymbol{E} + q\,\boldsymbol{v} \times \boldsymbol{B}.\tag{1}$$

As can be easily seen, the formula of the Lorentz force cannot express a force component proportional to qv, because qE is independent of v and the term $v \times B$ is always oriented perpendicular to v, since the scalar product $v \cdot (v \times B)$ equals zero. Thus, should an electromagnetic force be measured which is proportional to the velocity of the test charge and furthermore acts parallel to the direction of motion, so it would be intrinsically incompatible with the Lorentz force (1).

However, the experiment performed in this article provides clear signs that such a force exists. This may seem highly implausible after more than a century of practical experience in electrical engineering, but one must realize that these forces occur only in very special situations. Moreover, only very few people would expect that unexpected effects would be waiting to be discovered at such an ancient theory. However, it is not the first time that such forces are reported [6] and even André-Marie Ampère himself explicitly investigated this kind of force in experiments and consciously included the results in his force formula [5], [3].

To the present theory of electrodynamics, i.e. the Maxwell equations in combination with the Lorentz force, such a force is incompatible. An earlier preceding theory is Weber electrodynamics. However, the term electrodynamics is somewhat inappropriate for this, since Weber electrodynamics consists of only one formula, which resembles Coulomb's law. However, in contrast to Coulomb's law, Weber's formula contains not only the distance between the charges as a parameter, but also the relative velocity and the relative acceleration.

Since Weber electrodynamics is only a force law without fields, it is immediately obvious that it is not suitable for explaining electromagnetic waves. However, and this must be emphasized, this is not a principle-related deficiency but only a result of the fact that Weber electrodynamics was almost completely forgotten due to the success of the Maxwell equations. One could even state that it was simply missed to find the corresponding field equations. Apart from this obvious shortcoming, it is a remarkably simple and powerful theory that can probably explain all quasi-stationary effects [7], [2].

The simplest way to get an understanding of Weber electrodynamics from a modern point of view is to assume that the potential energy between two point charges q_s and q_d at the locations r_s and r_d is given by the formula

$$V = \frac{1}{\gamma(\dot{r})} V_C(r), \tag{2}$$

with

$$V_C(r) = \frac{q_s \, q_d}{4 \, \pi \, \epsilon_0 \, r} \tag{3}$$

being the classical potential energy of two point charges at rest with respect to each other. r := ||r|| in this formula is the distance between the two point charges, i.e. the Euclidean norm of the distance vector

$$r = r_d - r_s. (4)$$

As usual, the point on top of a symbol indicates the derivative with respect to time. Consequently, $\dot{r} = \dot{r} \cdot r/r$ is not the differential velocity $\dot{r} = \dot{r}_d - \dot{r}_s$ nor its Euclidean norm. Instead,

 \dot{r} is the relative velocity, i.e. the velocity with which the two charges approach or move away from each other on their connecting line.

 γ is the Lorentz factor known from special relativity. If the relative velocity \dot{r} between the two point charges is zero, the Lorentz factor is one and the usual formula (3) of the potential energy of a resting point charge in the field of another resting point charge is obtained.

For small relative velocities \dot{r} , as can be verified by calculating the Taylor series, the approximation

$$V \approx \left(1 - \frac{\dot{r}^2}{2c^2}\right) \frac{q_s q_d}{4\pi \epsilon_0 r} \tag{5}$$

can be obtained. This formula appears for the first time in 1848 in the publications of Wilhelm Weber [2]. The corresponding force formula for this potential energy is

$$F = \frac{q_s q_d}{4\pi \epsilon_0} \frac{r}{r^3} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r \ddot{r}}{c^2} \right), \tag{6}$$

and dates back to 1846 [2]. Between Weber force (6) and potential energy (5) exists, as can be verified in a few steps, the relation

$$-\dot{V} = \mathbf{F} \cdot \dot{\mathbf{r}},\tag{7}$$

which is an alternative representation of the law of energy conservation, since the term on the right side represents the time derivative of the kinetic energy.

The formula (6) expresses the force between two electric point charges. In the macroscopic world, however, one has to deal only seldom with point charges, but much more often with electric currents. An electric current is a multiparticle phenomenon, for example consists a current in a metal wire of resting positively charged ions and negatively charged electrons, which have a average velocity different from zero.

The special feature of Weber's law of force is that one can derive Ampère's force law in its original form without additional assumptions [1]. This means that the Weber force is a microscopic explanation for the magnetic forces between arbitrarily shaped conductor loops. This is really remarkable, considering the simplicity of the expression of the potential energy (2) and realizing that one can work completely without vector potential, magnetic field and Lorentz force (1).

It should also be mentioned that the Weber force satisfies the conservation laws of momentum, angular momentum and energy. The Liénard-Schwarzschild force (equation (8) in [2]), i.e. the counterpart following from Maxwell's equations, violates these and seems little plausible.

The next section describes an experiment in which an unusual aspect of Weber electrodynamics comes into play. The succeeding section examines the effect then by means of a detailed theoretical analysis.

II. Experiment

A. Concept

The basic concept of the experiment is to determine whether charge carriers moving very fast sideways past the plate of a

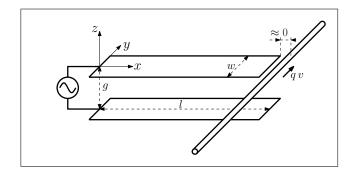


Figure 1. Principle of the experiment: It is to be determined whether during charging and discharging of the capacitor (left) a force is generated on fast moving charge carriers in a tube (right), which is proportional to the velocity and acts in the direction of motion.

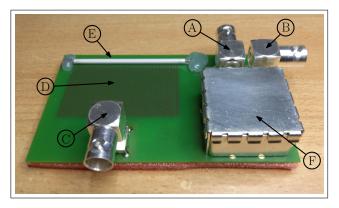


Figure 2. Measuring board: (A) Connection for a DC high voltage to operate the tube, (B) Measuring connector, (C) Input feed to the transmit antenna, (D) Capacitor as transmit antenna, (E) Receiver tube, (F) Shielded receiver circuit

capacitor perceive a force in their direction of motion when the plate capacitor is charged and discharged by an alternating current. The figure 1 shows the principle of the experiment.

As the sketch suggests, a long tube is placed close to one of the plates of a plate capacitor, in which fast electrons are moving. It is obvious that the capacitor on the left side of the figure acts as an antenna and radiates an electromagnetic wave. However, the exact shape of the electromagnetic wave is irrelevant for the experiment, because neither the magnetic nor the electric field are able to generate a force or voltage in the tube, which would be proportional to the velocity of the charge carriers in the tube.

B. Implementation

The experiment was performed by means of a $6.0\,\mathrm{cm}\,\mathrm{x}\,10.5\,\mathrm{cm}$ double-layer printed circuit board with a thickness of 1 mm made of FR4. The figure (2) shows a photo.

As the figure 2 shows, there are three BNC connectors on the board, where (A) is for connecting a 900 V DC voltage to operate the tube (E). The BNC socket (B) is the connection for the oscilloscope. Socket (C) is for connecting a waveform generator to capacitor (D), which is comb-shaped both on top and bottom of the board and has a capacitance of mathematically 37 pF. The reasons for choosing this particular shape of capacitor will become clear in the theory section of this article.

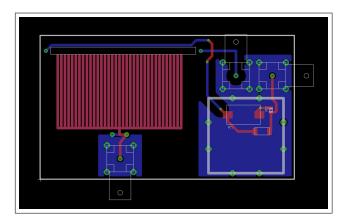


Figure 3. Printed circuit board with receiver and transmit capacitor. The top side is shown in red, the bottom side in blue

The metal case (F) in the figure below on the right contains a few components which are shielded against electromagnetic interference. The ground of the housing is connected with that of the sockets (A) and (B), but not with that of socket (C).

The tube (E) is a CCFL (Cold-cathode fluorescent lamp) of the type BF2661-24B from the manufacturer JKL, which emits UV radiation of the wavelength $\lambda = 253.7\,\mathrm{nm}$ [8]. A CCFL is a type of tube in which the electrons are drawn from the cathode only by the high intensity of the electric field. Since there is no vacuum in the tube but a gas, the velocity v of the electrons is not proportional to the applied voltage and can only be estimated to about 0.76 percent of the vacuum light speed c with equation

$$\frac{1}{2}m_e v^2 = h\frac{c}{\lambda} \tag{8}$$

 $(m_e$ - mass of the electron, h - Planck constant). This type of tube was chosen because the external dimensions imposed tight constraints and a sufficiently thin tube with a Wehnelt cylinder and without gas filling was not available as a component.

Figure 3 shows the layout of the two-layer PCB, with the top layer shown in red and the bottom layer in blue. The corresponding circuit without the capacitor (D) serving as transmit antenna is shown in figure 4. It consists only of a load resistor R1 which limits the current through the tube to 1.39 mA, and a passive high-pass filter consisting of a high-voltage capacitor C1 and a resistor R2 which decouples the measurement connector (B) from the high voltage and filters out frequencies below of about 1 MHz.

The components R1, R2 and C1 are located under a shielded metal housing and above a ground plane on the bottom side of the PCB. The traces outside the housing were designed to be as short as possible. Furthermore, care was taken to minimize the area of the receiving antenna's conductor loop. However, some compromises had to be made here, since the high voltage imposed minimum distances.

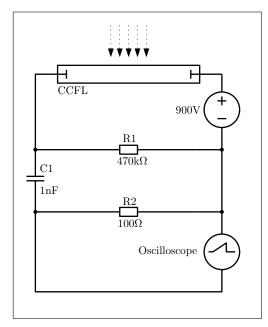


Figure 4. Circuit of the receiver without transmit capacitor

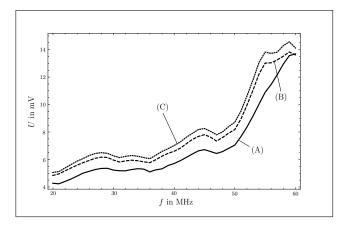


Figure 5. Measured amplitudes as function of the transmission frequency: (A) without tube, (B) with tube but without high voltage, (C) with tube and high voltage switched on.

C. Results

The first measurement of the experiment was performed without the tube soldered in to determine how strongly the feed line of the tube would act as an electrical antenna for parasitic longitudinal electric fields. For this purpose, a sinusoidal voltage with a amplitude of 2.5 V in the range between 20 and 60 MHz was applied to the BNC socket (C). The result is shown in figure 5 as a black solid line (A). It could be observed that it did not matter whether the high voltage was switched on or off, since the measured curve was almost identical in both cases. It should be mentioned that the amplitude of the measured signal frequency was two to three orders of magnitude higher than that of interfering frequencies and was therefore clearly distinguishable.

After installation of the tube, the experiment was repeated again with the high voltage source switched on and off. The results are shown as the curves (B) and (C) in Figure 5, where

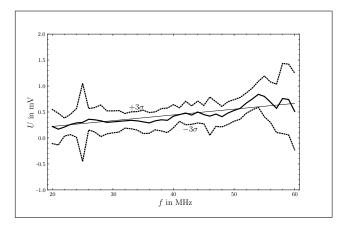


Figure 6. The measured voltage difference between the high voltage switched off and on. The dashed curves indicate the 3σ confidence interval. The thin line is a linear fit of the measured curve. The slope corresponds to $11.1\,\mathrm{pV/Hz}$.

curve (B) was measured with the high voltage switched off. As can be seen, even with the tube turned off, the amplitude generally increased by about 1.5 mV compared to curve (A). This is quite in line with expectations, since the gas in the tube can be polarized and consequently reacts to parasitic electric fields in the longitudinal direction of the tube.

However, it is remarkable that the amplitudes were further increased when the high voltage was switched on, i.e. when the electrons in the tube were moving with a speed of about $0.0076\,c$. This cannot be explained with the longitudinal electric field component, since the electric force does not depend on the velocity of the charge carriers and a magnetic field cannot accelerate charge carriers in the direction of motion.

Moreover, if we calculate the difference of the curves (C) and (B), we obtain the curve of figure 6, in which we can see a linear frequency dependence. This is consistent with Weber electrodynamics, as will become evident in the theory section.

III. THEORY

A. Current in the transmitter

To analyze the experiment, we first need the current i(x,t) in the transmit capacitor (D) in figure 2. A single tooth of the comb-shaped capacitor can be interpreted schematically, as shown in figure 1, as a biplanar microstrip with a sinusoidal AC voltage $u(t) = u(x = 0, t) = U_0 e^{i\omega t}$ with amplitude U_0 and angular frequency ω applied to its input at x = 0.

For the calculation of the current, the microstrip is considered as an unterminated transmission line. Therefore, the telegrapher's equations apply and it is possible to use equation (15) from [9] for the calculation of the transfer function. Since the line is unterminated in this specific case, the termination impedance is $Z_T \rightarrow \infty$. Hence, equation (15) from [9] simplifies to

$$H = \frac{\cosh\left((l-x)\sqrt{\frac{Z_L'}{Z_Q'}}\right)}{\cosh\left(l\sqrt{\frac{Z_L'}{Z_Q'}}\right)}.$$
 (9)

In this,

$$Z_L' = i \omega L' \tag{10}$$

is the series impedance with L' being the inductance per meter and

$$Z_Q' = \frac{1}{\mathrm{i}\,\omega\,C'},\tag{11}$$

with C' as the capacitance per meter. The series resistance is neglected as irrelevant.

Equation (16) in [9] provides the voltage

$$u(x,t) = H U_0 e^{i\omega t} \tag{12}$$

along the microstrip. Equation (20) gives the current

$$i(x,t) = -\frac{1}{Z_I'} \frac{\partial H}{\partial x} U_0 e^{i\omega t}.$$
 (13)

Substituting the equation (9) gives

$$u(x,t) = \frac{\cos\left(\sqrt{L'C'}\,\omega\,(l-x)\right)}{\cos\left(\sqrt{L'C'}\,\omega\,l\right)}\,U_0\,e^{\mathrm{i}\,\omega\,t} \tag{14}$$

and

$$i(x,t) = \sqrt{\frac{C'}{L'}} \frac{\sin\left(\sqrt{L'C'}\,\omega\,(l-x)\right)}{\cos\left(\sqrt{L'C'}\,\omega\,l\right)} U_0 e^{\mathrm{i}\left(\omega\,t + \frac{\pi}{2}\right)}. \tag{15}$$

The maximum frequency in the experiment was $60\,\mathrm{MHz}$, which corresponds to a wavelength of 5 m. Since this was very large compared to the length $l=3\,\mathrm{cm}$ of the capacitor, voltage (14) and current (15) can be approximated by the first order Taylor series with respect to ω . This simplifies the two equations to

$$u(x,t) \approx u(t) = U_0 e^{i\omega t}$$
 (16)

and

$$i(x,t) \approx C' \,\omega \,(l-x) \,U_0 \,e^{\mathrm{i} \left(\omega \,t + \frac{\pi}{2}\right)}. \tag{17}$$

As can be seen, the voltage in the capacitor is everywhere exactly equal to the input voltage. However, as expected, the current decreases linearly and disappears at the end of the line. Furthermore, it can be seen that the current precedes the voltage by 90° and that the inductance per meter L' has no effect. For the capacitance per meter C' applies

$$C' = \frac{\epsilon_r \, \epsilon_0 \, w}{g},\tag{18}$$

since it is a simple plate capacitor, with ϵ_r being the relative permittivity of the medium between the plates. Using the parameters of the experiment $\epsilon_r = 4$, w = 1 mm (width of one of the 35 teeth) and g = 1 mm, we obtain C' = 35.4 pF/m.

B. Force caused by a current element

It is well known that the current in a metallic wire consists of electrons moving with a drift velocity u. The metal ions, on the other hand, which compensate the negative charge of the electrons towards the outside, are at rest. The force of a short segment of the wire of length ζ at the origin of the coordinate system onto a test charge q is then the sum of the Weber forces of all resting metal ions and all moving electrons which are contained in this piece of wire.

This force shall now be calculated. For this purpose, first the formula of the Weber force (6) is converted into a better practicable form. Let $\mathbf{v} := \dot{\mathbf{r}}$ be the first time derivative of the distance vector (4) and $\ddot{\mathbf{r}} := \mathbf{a} = \dot{\mathbf{v}}$ the second derivative. This then allows us to set up the equations

$$\dot{r} = \frac{\mathrm{d}}{\mathrm{d}t} \sqrt{r \cdot r} = \frac{r \cdot v}{r} \tag{19}$$

and

$$\ddot{r} = \frac{\mathrm{d}}{\mathrm{d}t}\dot{r} = \frac{v^2}{r} + \frac{r \cdot a}{r} - \frac{(r \cdot v)^2}{r^3}.$$
 (20)

Substituting this into the Weber force (6), we obtain for $a \approx 0$ the force formula in vector notation:

$$F(q_s, q_d, \mathbf{r}, \mathbf{v}) = \left(1 + \frac{v^2}{c^2} - \frac{3}{2} \left(\frac{\mathbf{r}}{r} \frac{\mathbf{v}}{c}\right)^2\right) \frac{q_s q_d}{4\pi \epsilon_0} \frac{\mathbf{r}}{r^3}.$$
 (21)

It is now assumed that there are n electrons moving in the piece of wire. The total force F_T of the wire segment onto the test charge q moving with the velocity v at location r is then

$$F_{T} = F(-ne, q, r, v - u) + F(ne, q, r, v)$$

$$= \frac{e n q}{8 c^{2} \epsilon_{0} \pi} \frac{r}{r^{3}} \left(3 \frac{(u r)^{2}}{r^{2}} - 2 u^{2} + 4 u v - 6 \frac{(u r) (v r)}{r^{2}} \right).$$
(22)

Since the drift velocities \boldsymbol{u} in metallic conductors are very small, all terms of order $O(u^2)$ can be neglected and, using the relation $\mu_0 = 1/(c^2 \epsilon_0)$ and due to $\boldsymbol{i} := -n e \boldsymbol{u}/\zeta$, the approximation

$$F_T(\mathbf{r}, \mathbf{i}) \approx \frac{q \mu_0 \zeta}{4 \pi} \frac{\mathbf{r}}{r^3} \left(3 \frac{(\mathbf{i}\mathbf{r})(\mathbf{v}\mathbf{r})}{r^2} - 2 \mathbf{i}\mathbf{v} \right).$$
 (23)

is obtained. This corresponds to Ampère's original force law of 1822 [3], but not the Biot-Savart law in combination with the Lorentz force. [6].

C. Induced voltage in the tube

The previously derived formula (23) can now be used to calculate the force that the total current (17) in the two strip lines of figure 1 produces on the test charge q in the tube next to the capacitor.

For simplicity, it is assumed that the microstrips are so narrow that the current can be considered as a line current. In this case, the force F_u of the upper microstrip on a charge at location r is

$$\boldsymbol{F}_{u} = \frac{1}{\zeta} \int_{0}^{t} \boldsymbol{F}_{T}(\boldsymbol{r} - x \boldsymbol{e}_{x}, i(x) \boldsymbol{e}_{x}) dx.$$
 (24)

However, since the test charges are only inside of the tube, it holds that $\mathbf{r} = l \mathbf{e}_x + y \mathbf{e}_y$, with y being the only variable parameter. The force \mathbf{F}_l of the lower microstrip can be calculated analogously and we get

$$\boldsymbol{F}_{l} = \frac{1}{\zeta} \int_{0}^{l} \boldsymbol{F}_{T}(\boldsymbol{r} - (x \boldsymbol{e}_{x} - g \boldsymbol{e}_{z}), -i(x) \boldsymbol{e}_{x}) dx.$$
 (25)

It should be noted that the lower microstrip is not only shifted downward by g, but that the current also flows in the opposite direction.

The total force F onto the test charge is the sum of the two forces. Inserting the equation of the current (17) and solving the integrals gives the equation

$$F e_y = \frac{1}{4\pi} C' l^3 \mu_0 q U_0 v \omega \alpha(y) e^{i(\omega t + \frac{\pi}{2})}$$
 (26)

for the y-component of the total force. The auxiliary function $\alpha(y)$ was introduced for readability and is defined by the equation

$$\alpha(y) := \frac{1}{(l^2 + y^2)^{3/2}} - \frac{y^2}{(g^2 + y^2)(g^2 + l^2 + y^2)^{3/2}}.$$
 (27)

If a charge q is now thought to be guided along the tube with velocity v, work is performed on this charge. Electric voltage U is defined as work per charge, i.e. it can be calculated by solving the integral

$$U = \frac{1}{q} \int_{-\infty}^{+\infty} \mathbf{F} \, \mathbf{e}_y \, \mathrm{d}y. \tag{28}$$

The integration limits of $-\infty$ to $+\infty$ can be motivated by the fact that work is performed only in the vicinity of the microstrip, and integration at a greater distance does not contribute.

By substituting the equation (26) follows

$$U = \frac{1}{4\pi} C' l^3 \mu_0 U_0 v \omega e^{i(\omega t + \frac{\pi}{2})} \int_{-\infty}^{+\infty} \alpha(y) dy.$$
 (29)

The calculation of the integral is straightforward and we obtain

$$\int_{-\infty}^{+\infty} \alpha(y) \, \mathrm{d}y = \frac{2 \, g}{l^3} \arccos\left(\frac{g}{\sqrt{g^2 + l^2}}\right) \approx \frac{g \, \pi}{l^3},\tag{30}$$

with the approximation being valid when g is significantly smaller than l, which is the case in the experiment.

If we now substitute this into equation (29) and compute the absolute value, so we get for the induced voltage an amplitude of

$$\hat{U} = \frac{1}{4} C' g \mu_0 U_0 v \omega. \tag{31}$$

This means the following: If Ampère's force law is valid in its original form, then in addition to the high DC voltage which accelerates the electrons in the tube, there must also be a small AC voltage with amplitude \hat{U} which is induced by the influence of the capacitor and which depends linearly

on the velocity v of the electrons and linearly on the angular frequency $\omega = 2 \pi f$ of the transmitter. But if the Lorentz force (1) is valid, then this voltage must not exist.

In total, the capacitor had 35 of such teeth. Taking this into account and substituting the other parameters

- $C' = 35.4 \,\mathrm{pF/m}$
- $q = 1 \, \text{mm}$
- $U_0 = 2.5 \,\mathrm{V}$
- $v \approx 0.0076 c \approx 2280000 \,\text{m/s}$

of the experiment into the equation (31), we find that the tube acts like an additional voltage source with an amplitude of about 13.9 pV/Hz. At a frequency of 20 MHz, this corresponds to a voltage of 0.28 mV. At 60 MHz, on the other hand, a voltage of 0.83 mV can be expected.

Considering the approximations and uncertainties regarding the properties of the used CCFL tube, this agrees surprisingly well with the measurement results of the experiment, because the function that was estimated from the measured data was $\hat{U}(f) \approx 11.1 \, \mathrm{pV/Hz} \cdot f$. This is very close to the theoretically estimated value and represents a relative error of only 0.2. This error is reduced even further if the length of the tube and the lateral displacements of the microstrips relative to the tube are also taken into account.

IV. SUMMARY AND CONCLUSION

Two things became apparent in this article:

The first point is purely theoretical and consists once more in the conclusion that Ampère's force law in its original form is by no means compatible in all aspects with the Biot-Savart law and with modern electrodynamics, since the latter, for structural reasons, cannot describe a force component which is both proportional to the velocity of the test charge and parallel to the direction of motion [6]. However, Ampère's original force law does contain such force components, as has been shown mathematically. This means that the statement that the Biot-Savart law would be equivalent together with the Lorentz force must be clearly rejected already for purely formal reasons.

The second insight of this article arises from the measurement results of the performed experiment, because they fit remarkably well with the predictions of Weber electrodynamics and Ampère's original force law. However, the experiment should be repeated with a type of tube, that allows to adjust the velocity of the electrons. But as limited as the experiment was, it clearly shows that it is necessary to critically question modern electrodynamics and to design near-field experiments that can distinguish Weber electrodynamics from the modern version

If it should be finally confirmed in future experiments that in the near field the Weber electrodynamics applies – and there was recently another indication [4] –, this would be of enormous consequence, since the Maxwell equations represent the foundation of modern physics. To which extent directly or indirectly derived statements would keep their validity should

then be a point for intensive research, as well as the finding of the field equations which are valid for both the near and the far field.

V. Acknowledgement

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