Chapter 14

The entry of physicist Ampère: electricity and magnetism, especially their connections, 1820-1827

I. Grattan-Guinness, *Convolutions in French Mathematics, 1800–1840*

14.1 Prefaces

14.1.1 Plan of the chapter

From Fresnel we move to his landlord Ampère, known already in our pages as a mathematician but now to take his major place for his contributions to the founding of electromagnetism and electrodynamics, in a frenzy of activity carried out between 1820 and 1827. As with Fresnel's Chapter 13 and Fourier in Chapter 9, one figure dominates, and so a fairly close chronology can be followed.

§ 14.2 begins by noting Oersted's announcement in 1820 of the electromagnetic effect between a magnetic needle and a wire carrying an electric current, and then proceeds to Ampère's discovery of the electrodynamic effect between wire and wire, and his adoption of electrical aetheric models up to 1822; but it also takes in the first ideas of Biot, and a contribution from Fresnel. § 14.3 is confined to the following year, with important contributions made by Ampère's followers Savary and Demonferrand. Then § 14.4 records the consolidation of Ampère's theory, between 1824 and 1826, as he extended his theory to wires and surfaces; but it also records further work by Biot, and especially a substantial treatment of magnetism by Poisson, which Ampère had to translate into his own electrical vision. The short § 14.5 describes further contributions to magnetism, by Poisson and Arago. The concluding § 14.6 compares and contrasts the achievements of Ampère and Poisson, and also notes some reactions made by Laplace and Despretz (like Fresnel, once a lodger with Ampère). Before launching upon the story, however, some general remarks need to be passed about Ampère's manner of communicating his ideas.

14.1.2 The chaos of Ampère's publications

As we shall soon see, Ampère reacted quickly to Oersted's discovery in 1820. His current researches were proceeding with convenient speed on differential equations (§ 11.2.1), chemistry and psychology; but now he suddenly switched into the new topic in this his 46th year, with contributions made at times even weekly to *Académie* meetings (which were rather slack at the time, incidentally). His efforts were reported at length, probably by himself, over the following years in the annual notices of the *secrétaires perpétuels* (Delambre, and from 1822 his successor Fourier), and also in the accounts of *Académie* meetings published in various journals; but the main publicity came from a torrent of articles, letters and pamphlets which he published both in France and abroad over the next six years.

All this sounds like typical Paris; but Ampère surpassed all his colleagues in the scale and confusion of his printing and reprinting. Not content with the usual scheme of big paper and summary papers, he wrote a string of short articles, changing them at times quite considerably in their appearances from one journal to another, again when they were made available as supposed offprints, and yet again when including them in collections.

Of this latter form of publication the most substantial is a book entitled *Recueil d'observations électro-dynamiques*, which appeared in at least two versions between the summers of 1822 and of 1823; the definitive version has 378 pages (followed by four mis-numbered pages of contents) and carries the date '1822' on the cover, even though at least two items come from 1823 (one is so dated). It includes versions of several of Ampère's papers, and also some by other authors; and while it carries the name of Crochard (the publisher of Arago's *Annales*) on its title page, several of the papers were taken from the printings of journals put out by other houses, while others were reset. I cite it as Ampère *1823g*.[1])

For the period 1820-1827 Ampère produced around 60 different pieces, but nearly a century of publications (including anonymous pieces). In addition, there is a considerable collection of manuscripts kept in his *Nachlass* (§ 4.2.5) and elsewhere, and an unreliable edition of letters (*Correspondance*) on which to draw. Further, Ampère himself is unreliable in his autobiographical statements, even when they were made soon after the (supposed) events.

Fortunately for my purposes, I can leave much of this morass to historians of physics, with my deepest condolences.[2]) As usual, I concentrate on the mathe-

---

[1]) Some material was taken from the Swiss journal *Bibliothèque universelle*, and it emerges from Ampère's correspondence with the de la Rives (two of its editors) that extra copies of these articles were produced with page numbers appropriate to the *Receuil*, and that copies of the book were prepared in both Paris and Geneva. See *Bibliothèque Publique et Universitaire* (Geneva), ms. fr. 2311, fols. 22-32; ms. fr. 2314, fols. 62-72; and file 'D.O.': the letters are printed in Ampère *Correspondence* (see its index).

[2]) Only very recently has the necessary research been started. Blondel *1982a* is the single most important study: the detailed bibliography on pp. 187-191 includes all items and reprints, apart from pieces in *MU*. On the earliest stages of his thought see also Blondel *1978a*, and the differing interpretation in Williams *1983a*; on his motivation, and the reactions of contemporaries, see Caneva *1980a*. The main manuscripts are held at *AS*, *Ampère*, chs. 156-208 *bis*.

Three collections of papers by Ampère and others are worth mentioning: the *Receuil* (Ampère *1823g*) just described in the text; two volumes of French texts (including translations) prepared in the 1880s (Joubert *Electrodynamics*), with useful notes but rather cavalier treatment of some sources; and the short selection of English translations (Tricker *1965a*) prefaced by a somewhat unreliable introduction. Some other reprints are listed in the bibliography.

matical aspects of the story, and include only some principal features of the physics, leaving out the electrochemistry entirely. Experiments are also treated lightly, even Ampère's. He often conceived them well; in an intelligent strategy, he sought primarily for equilibriate situations for founding his theory (and so never produced tables of data). Some of them were carried out at home, with his lodgers Fresnel and Despretz. His experimental technique was not good: he was handicapped by a weak right arm caused by a poorly healed fracture.[1])

14.2 Ampère's first moves, 1820–1822

14.2.1 Oersted's discovery and its reception, 1820

Oersted's account of 'experiments relative to the effect of a current [*conflictus*] of electricity on a magnetic needle', dated 21 July 1820 and written in Latin, was first published soon afterwards as the pamphlet *1820a* and sent around Europe by its author. The interest in his discovery was neither as instantaneous nor as widespread as is often thought. In particular, in the world's scientific capital of Paris, the discovery of this Danish physicist is said to have been described as 'a German dream'; for hitherto electricity and magnetism had not normally been regarded as intimately linked—the influential view of Coulomb in both areas (§ 7.5) had held them to be separate. However, there was a measure of interest invoked around the world, and various translations of the pamphlet soon appeared. Four of these were in French (listed together as Oersted *1820b*), two in the Paris *Journal de physique* and Arago's *Annales*, and the other two in foreign journals which play a role in this story: the Swiss *Bibliothèque universelle*, and the newly founded Belgian *Annales générales des sciences physiques* (noted in § 10.1.2).

The news was first brought officially to the *Académie* by Arago, at the meeting of 4 September (*PV*, *7*, 83); and from then on he, Ampère and Biot, together with a few followers, were reporting these frequently, even giving experimental de-

There are numerous articles on Ampère's and others' work, although of course few attend to mathematical aspects. Reiff and Sommerfield *1912a*, 9-20 contains a short summary; Hashimoto *1983a* contrasts Ampère's and Biot's approaches. Devons *1978a* contains a succinct summary of various experiments. A full study would need to draw on all the information contained in *PV*, *7–8* for the period.

[1]) Ampère mentioned his handicap in a letter of 23.3.1815 to 'le maire', kept at *Wellcome* but not published in his *Correspondance*.

monstrations there. In particular, Ampère had conceived of the electrodynamic effect between wire and wire, and was able to report it to the *Académie*, only a fortnight later (pp. 94-95).

14.2.2 Ampère's adoption of the electrical aether, 1820-1821

Oersted's initial discovery concerned the relationship between magnetic needle and an electric wire, and much of his and others' work (including Biot's) was concerned with studying it further. The usual view of the effect was to think of the wire influencing the needle; but with his customary gift for the unusual, Ampère allowed himself also the converse interpretation, that the needle detected the presence of the current. This view led him to discover that the current passed through the pile, contrary to the prevailing French theory of its virtual absence there (see for example, Biot *1816c, 2*, 545-546): the finding led him to think that electric currents existed in the earth. Further, when suspending an 'astatic needle' (his expression) about an axis parallel to the earth's magnetic field, he found that it set in the plane normal to a nearby electrical wire. Putting together these two ideas, he then concluded that a magnet in ordinary use was actually detecting electrical activity taking place around the earth. Since these fields had to be (roughly) circular in profile, they would act like the currents in a circular pile; thus, conversely, such a pile would manifest no electrical current directly, although the needle would be effected. Pressing the analogy between the local and the terrestrial, he wondered if his supposed electrical fields were 'the cause of this recently asserted internal heat in deep mines' (*1820d*, 242), in allusion to the work of Laplace and Fourier of a few months previously (§ 12.4.3-4).

Laplace himself came quickly into the story. We recall from § 7.5.3 that he had been interested in electrostatics twenty years earlier: now he urged Ampère to detect effects in the needle from a *long* wire, to rule out the possibility that they were caused by the pile (Ampère *1820d*, 247). He (or Arago) presumably invited Ampère to visit the *Bureau des Longitudes* on 27 September and repeat the 'magnetico-Voltaic experiments' ('*expériences magnético-voltaiques*': *BL, 4*, meeting minutes). The form of the adjective is interesting: within a few weeks Ampère was speaking of the 'electromagnetic experiments' (as in the title of his paper *1820e*). The new word was introduced in Oersted *1820c*.

These words reflected the changed relationship between electricity and magnetism which now had to be envisaged. In line with his own researches and thoughts, Ampère gave electricity his vote: 'any magnet is only an assembly of galvanic currents which lie in planes perpendicular to its axis, and which move following closed and re-entrant curves' (*1820d*, 242). Unfortunately, in faulty logic he often referred later to this reductionist position as 'the *identity* of electricity and magnetism'. He reinforced it with his first famous discovery, also in this September: that the effects could be obtained with wires *alone*, attraction when the currents followed the same sense and repulsion with the opposite sense. In order to distinguish this class of effects from the others, he proposed in 1822 to call it 'electrodynamic action', due to the 'electricity in motion in voltaic conductors': it was distinguished from the 'distribution of the two electric fluids in repose in bodies where they manifest themselves' (and upon which we saw Poisson mathematicise in § 7.5), for which Ampère was to propose 'the name of *electro-static action*' (*1822d*, 200).

Ampère first presented these findings, and various related ones, principally in two papers. One, *1820b*, appeared in two parts in the September and October numbers of Arago's *Annales*, and again (in revised form, of course) as the opening of the *Recueil*: it is a well-known piece. By contrast, the other paper, which contains the early papers read at the *Académie* and so is historically more valuable, came out only once, as *1820d*, and then in the Belgian *Annales*. (As well as Flourens as reporter, an editor was the French soldier and naturalist J. B. Bory de Saint-Vincent, *correspondant* of the *Académie*, and possibly he obtained this scoop.) It is typical of the wayward Ampère that he should put out important stuff abroad; and it is also like him that in placing soon afterwards in the *Journal de physique* a brief account *1820c* of his *Académie* talks, the story told was considerably different. This account was written to supplement the summary of his and Oersted's work published in Hachette *1820a*; we recall from § 7.5.5 that, like Laplace, Hachette had long entertained an interest in electricity, and now it was aroused again.

14.2.3 Ampère's trigonometric expression for electrodynamic action, 1820

The first sign of sums occurred in the paper *1820e* mentioned above, which was written with some help from the elderly mineralogist F. Gillet de Laumont and appeared in the *Annales des mines*; a slightly modified version was to appear in the *Recueil*. Referring to a talk given at the *Académie* on 4 December (*PV*, *7*, 108), he sought 'the analytical expression of the attractions and repulsions of the electric currents' to cover magnetic, electromagnetic and electrodynamic effects. Taking the last case as fundamental, he announced, without proof, that if 'two infinitely small portions of electric current' A and B, at steady intensities g and h, separated by a distance r, set at angles α and β respectively to AB and in directions which created with AB two planes at angle γ apart, then 'the action that they exert the one upon the other' was given by

$$\frac{gh}{r^2}\left(\sin\alpha\sin\beta\sin\gamma+\frac{n}{m}\cos\alpha\cos\beta\right), \qquad (1423.1)$$

where '$\frac{n}{m}$' was his (curious) notation for an unknown constant, which experimental work suggested to be 'very small and must be considered absolutely null'. He noted immediately the analogy between the first term of (1423.1) and the sine law of radiation which we saw in § 12.4.1 had recently exercised Fourier (Ampère *1820e*, 83-86).

For some reason Ampère never published the manuscript that he prepared for the meeting of 4 December, although it clearly states a number of his present concerns. Concluding from his experimental work that an inverse square law of central forces of attraction and repulsion could be adopted, he cautioned that the 'action' was not isotropic. Thus, using a differential model, the action A of an infinitely long straight wire at the point P perpendicularly distant a from one end Q was given by

$$A=\int_0^\infty gf(a/x)/(a^2+x^2)\,dx=g/a, \qquad (1423.2)$$

where g was an electrical constant and f denoted the unknown function of the angle between dx and Q: the evaluation g/a was verified by experimental work (*1820f*, 129-131). (1423.2) was thus an integral equation for f, which he did not attempt to solve. We shall find Laplace's variant of (1423.2) arising in the next sub-section.

Ampère's argument to produce (1423.1) *was* published, in the *Journal de physique*, as a 'Note' *1820g* on the unpublished manuscript: it was revised in a paper *1823f* written for the *Recueil*, as we shall see in § 14.2.5, but the diagrams given there probably apply also to this earlier reasoning. (1423.1) came from the case shown in Figure 1423.1: two infinitesimal elements of current AG and BH at

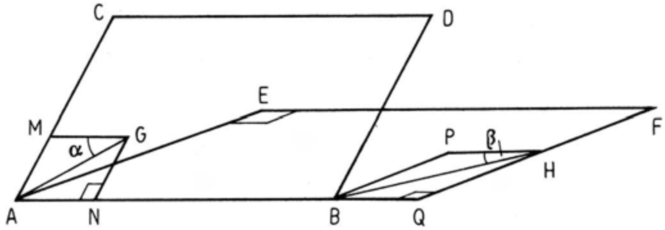


Figure 1423.1 Ampère's diagram for action between current elements (*1823f*, after pl. 6, fig. 22) angles α and β to the line AB joining two ends. He set up rectangles AMGN and BPHQ around these elements as diagonals, for he intended to imitate some principles of the composition and decomposition of forces in mechanics. Accordingly, the cumulative action along AB between AG and BH was the sum of the actions between four pairs: AM, BQ (zero because at right angles), AN, BP (zero), AM, BP (components at angle γ) and AN, BQ (collinear components, and therefore subject to a factor k, independent of all angles and distances, of reduction of their action). Assuming also a product law for the 'intensities' i and i' of current and the inverse square law of attraction and repulsion, the cumulative action was given by

$$[ii'(ds\sin\alpha)(ds'\sin\beta)\cos\gamma/r^2]+[kii'(ds\cos\alpha)(ds'\cos\beta)/r^2]\,; \quad (1423.3)$$

that is, (1423.1) after suitable adjustments were made. Total action between wires could then be calculated as the appropriate integrals.

Given the assumed physical principles, the demonstration was satisfactory enough, mechanics indeed in all but language. But the assignment to zero of $\dfrac{n}{m}$ in (1423.1) (or k in (1423.3)) from experimental evidence was a mistake which blocked some of his progress from 1820 until 1822. Before we note the revision, however, we must take up Biot's work, and a suggestion from Fresnel.

14.2.4 Contributions from Biot and Fresnel, 1820–1821

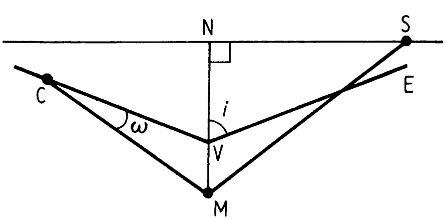
In most of the papers just cited Ampère referred to the experiments carried out by Biot. These were effected with the help of Felix Savart. He was born in 1791 at Mézières to Gerard, a constructor of instruments at the military school there, and then at Metz after the school moved to that town; his uncle Nicolas was a laboratory technician at the school and then at the *Ecole Polytechnique*, where his younger brother, also Nicolas, was a member of the 1808 *promotion* and then a student at the Metz school; he himself studied medicine, but he turned to physics in the late 1810s (see the Savart files in *VT*, X^0 91). His most sustained interest in research was to be in acoustics and vibrations, as will be duly noted below in § 15.5.1; it had already attracted the attention before 1820 of Biot, who then secured his help in the study of the effect of an electric wire upon a magnetic needle.

They presented two papers to the *Académie* in 30 October and 18 December 1820, under Biot's name (*PV*, *7*, 99, 118); but they published only a short statement *1820a* on the first one in Arago's *Annales* and an even briefer communication *1820b* to the *Société Philomatique*. Biot gave their ideas and results main publicity in 1821: firstly, in an April lecture at a public meeting of the *Institut*, published that month as *1821a* in the *Journal des savants* (of which, we recall from § 10.1.2, he was an editor); and secondly in a new chapter for the second edition of his textbook in physics (*Physics*$_2$ (1821), *2*, 117-128), which came out in June (*PV*, *7*, 201). It is perhaps a sign of his growing (feeling of?) isolation that he used the usual journals so little.

To measure the strength of the 'action', Biot drew on Coulomb's technique that he had used with Humboldt for measuring terrestrial magnetism (§ 7.5.4); noting the rate of oscillation of a magnetic needle, both with and without compensation for the earth's magnetic field, at varying positions from straight and V-shaped wires. Each class of cases led to a major finding, of which the first, given in the October presentation and published in the note *1820a*, was the more durable: that the intensity of the action decreased linearly with the perpendicular distance from a straight wire. They also reported that Laplace had shown them mathematically that this result entailed a (Newtonian) inverse-square law of action between the elements of the wire and the needle. Neither they nor Laplace published this proof, and it is not clear how far he went. In the notations of Figure 1424.1, the mathematical problem was to solve for unknown function $f(R)$ the integral equation

$$\int_{-\infty}^{\infty} \frac{Rf(R)}{(R^2 - a^2)^{1/2}} dR = \frac{K}{a}, \quad \text{with } K \text{ constant;} \qquad (1424.1)$$

presumably he noted, or claimed from dimensional considerations (as Ampère was tardily to note in *1826e*, 203), that an isotropic inverse square law applied for $f(R)$ (with $K = \pi$).[1] However, another solution of (1424.1) for f is aR^{-3}, or $R^{-2}(a/R)$, which includes also the sine law of decomposition of the action; and this result relates to Biot and Savart's second finding, announced in their December presentation. By measuring the action for V-shaped wires CVE for various



MS = R, SN = x, VN = a, ∠ NVE = i,
CM = r, VC = s, VM = b, ∠ VCM = ω

Figure 1424.1 Notations for the straight and V-shaped wires

[1]) In a footnote to his selection from a later presentation by Biot and Savart, Joubert claims to know that Laplace argued from considering the action at M from two parallel wires on either side of it (*Electrodynamics*, *1* (1883), 113). But the argument is not strong, and ignores electrodynamic action between the wires.

values of i, they claimed both this law, and also the proportionality to i of the total action of the wire on M (Biot *Physics*$_2$ (1821), *2*, 123). However, given the law, the total action is easily shown to be

$$\int_0^\infty (\sin \omega / r^2)\, ds, \quad \text{or} \quad \tan \frac{1}{2} i \tag{1424.2}$$

after the evaluation is carried out. This blunder was to be exposed in 1823, as we shall see in § 14.3.2.

Biot and Savart subscribed to separate ontologies for electricity and magnetism, and regarded the electromagnetic phenomenon as a basic effect, a couple-like turning of wire and pole about each other in a direction perpendicular to the direction of the element of the wire and also to the line joining it to the pole. Since Ampère adopted a novel standpoint, reducing magnetism to electricity, he had to reconcile it with more standard views such as theirs. Accordingly, in his paper of 4 December 1820, he showed that his reasoning in terms of rectangular electrical circuits in the magnet could be rephrased as a Biotian action between 'particles of boreal fluid and of austral fluid, placed two by two' parallel to the axis of the magnet, and attracting and repulsing with the conducting wire (*1820f*, 131-132). In this way he tried to extend his theory of largely static situations to dynamic phenomena.

Ampère continued to examine Biot's approach in his presentations to the *Académie* over Christmas and the New Year, but he chose to keep his findings to himself; his published statements covered other matters, principally theory and experimentation relating to his law (1423.3).[1] But apparently Laplace came into the story again, asking Ampère to check if the electrical theory of magnetism could be reconciled with Biot's finding of 18 December for the V-shaped wire. Ampère did not obtain (1424.2), but showed that the total moment of wire bent at angle 2β upon a small magnet placed distance a away from the vertex and turned by angle Θ from equilibrium was proportional to

$$\sin \Theta \int_{a \cos b}^{\infty} \frac{a^4 \sin^4 b + 2au \sin^2 b \cos b + u^2 \cos^2 b}{(a^2 \sin^2 b + u^2)^{3/2}}\, du, \quad \text{or}$$
$$\frac{\sin \Theta \sin b}{a} \left(1 + \frac{1}{1 + \cos b}\right). \tag{1424.3}$$

[1]) These are the meetings of 26.12.1820, and 8, 15.1.1821 (*PV*, *7*, 119, 125, 130): for his statements see Ampère *1820e*, 87-92; *1820h*, 256-258; and *1821a*. On the details about to be summarised, see Blondel *1982a*, 96-101 (but $(1424.3)_2$ below is mis-stated there); and the manuscripts cited (*AS*, *Ampère*, ch. 166). Compare Ampère's published treatment in *1826e*, note 5.

This result confirmed Biot's inverse distance law, but also suggested a change of sign when $\pi < b < 2\pi$ and the pole lay inside the V, contrary to Biot's claim of its *proportionality* to *b*.

The last of Ampère's lectures contained a new theoretical feature, and one in which his lodger and fellow aetherian Fresnel played a role. 'At the time when I was concerned with those ideas' on attraction and repulsion outlined in his *Académie* talk of 6 November, Ampère recalled in the *Recueil*, 'Mr. Fresnel communicated to me his fine researches on light from which he has deduced the laws which determine all the circumstances of the phenomena of optics' (*1823f*, 214). Successfully rewarded for his work on diffraction (§ 13.2.6), engaged in polarisation (§ 13.3.4) and about to seek a theory of double refraction (§ 13.4.2), Fresnel submitted through Ampère a sealed packet to that *Académie* meeting of 6 November 1820 (*PV*, *7*, 100). Ampère asked for it to be opened, and thereby revealed a short essay in which Fresnel reported his attempts to generate electricity in a wire wrapped helically around a cylindrical magnet, the converse of one of Ampère's methods of magnetising a bar. Arago, who presented some of his own results on magnetising metals by means of electrical discharges at the same meeting, published versions of both his and Fresnel's results in his *Annales* (Arago *1820a* and Fresnel *1820a*).

Had Fresnel's experiments been more successful, he might have preempted Faraday's discovery of secondary induction in 1831. But at least his work may have stimulated, or reinforced, a modification of Ampère's theory of magnetism as circulating electrical currents. We recall from § 13.3.4 that in 1816 Ampère had suggested transverse vibrations to Fresnel: now, in return, Fresnel outlined in a manuscript a view in which, 'by analogy with the magnetisation of each annulus of the surface of a steel cylinder placed in a helix traversed by a current [...] a conjoined wire parallel to a steel wire must produce around the molecules of the latter currents in the plane passing through the two wires', the sense of its rotation determining the polarity. These currents were called 'particulate' (*1821j*, 141).[1])

While aware of its speculative character, Ampère was sufficiently struck by Fresnel's view to include it in his paper for the *Académie* on 15 January 1821, and in one of his published accounts he stated about himself that 'Several considerations that [he] has not developed, seem to him to give somewhat more probabilities' to it over the competing theory that the circular currents were disposed concentrically about the axis of the magnet (Ampère *1821a*, 163).

---

[1]) Fresnel wrote two manuscripts on this theory, which were found by Joubert in *AS*, *Ampère* (now at ch. 184) and published by him in his edition: I list them together as Fresnel *1821j*. One is titled but undated, and the other untitled but dated as of 5. 6. 1821; there is no fundamental difference in content between them.

On 2 April 1821, at a public meeting of the *Institut*, both Ampère and Biot spoke. Biot's talk was (presumably a summary of) the paper *1821a* mentioned above, put in the *Journal des savants*, in which his true (and false!) results were presented. Ampère's lecture *1821b*, which was published widely (including in the *Moniteur universel* 13 months late, the delay having been caused by the 'abundance of political material'), concentrated on his and Arago's experiments and stressed his claim for the 'identity' of electricity and magnetism. And he concluded with an unusual piece of worldliness:

> the electrophore and the Leyden bottle being able to serve from now on for navigators as an infallible means of remagnetising to saturation the needles of their compasses [...] I should perhaps have contributed, by my researches, to the improvement of magnetic formulae, intended to render more certain, and to extend by new applications, the use of an instrument, without which the largest part of the earth will still be unknown.

Perhaps in an attempt to accommodate the positivistic inclinations of some of his Parisian colleagues, or to avoid the adoption of hypotheses, Ampère normally wrote on electricity and magnetism in a phenomenological vein, eschewing noumenal questions. But there were exceptions: his acceptance of Fresnel's theory of particulate currents is one, and another example occurred in a letter of 15 May 1821 to the Swiss physicist Gaspard de la Rive, which was published in the recipient's journal *Bibliothèque universelle*. Adopting the two-fluid theory of electricity then prevelant in France (§ 7.5.5), he spoke, rather in passing, of 'the series of decompositions and of recompositions of the fluid formed by the reunion of the two electricities of which one regards electrical currents as composed' (*1821d*, 122: cited from the reprint in the *Recueil*).

Thus at this time Ampère's aetherian framework was based on electric current regarded as de- and recomposition of fluid(s), and magnetism construed in terms of these currents rotating around each magnetic molecule; in addition, and not in very easy harmony with these positions, the earth's magnetic field was supposed to be composed of such currents girdling the earth. He seems to have entertained the philosophy of theories as hypotheses (§ 1.3.5), but he was ready to grant them truthhood if circumstances seemed right. In another letter in the *Bibliothèque universelle*, written in April to the German physicist P. Erman and recalling his discovery that the current passed through the pile (§ 14.2.2), he wrote (*1821c*, 116):

> When one finds such an accord between the facts and the hypothesis whence one started, can one only recognise it as a simple hypothesis? Is it not by contrast a truth founded on incontestable proofs?

14.2.5 Rotations and circulations of currents, 1821-1822

During parts of 1821 Ampère was ill, and his teaching at the *Ecole Polytechnique* was heavy, including even the short course on machines (*EP*, X2c/7, registers for 1820-1821). But in the autumn he had to face a powerful criticism from Faraday, whose paper 'On some new electro-magnetical motions' came out in a French translation *1821a* in Arago's *Annales*, soon after its appearance in a London journal. A seminal paper in Faraday's contributions to the topic, it announced that continuous rotation could occur if a pivoted cylindrical magnet moved around a fixed wire, and also if a pivoted wire moved round a fixed magnet. In October he sent to Ampère and Hachette one of his pieces of apparatus, and Ampère demonstrated its working to the *Académie* in November (*PV*, *7*, 247).

From the theoretical point of view, the chief challenge to Ampère's view was Faraday's conviction that such motions could not be explained by theories based on inter-molecular forces. Faraday's alternative, drawn from this and other experiments, was to give preference to curved 'lines of force'; but Ampère was anxious to preserve his own approach. Accordingly, when the translation was prepared, he had a set of appendicial notes made by a new helper, Felix Savary (who, as was warned in § 1.4.5, is not to be confused with Biot's Felix Savart), *polytechnicien* of the *promotion* of 1815 and thus one of Ampère's old students, and in 1821 principally a geographer by profession. Ampère added his name to these notes to indicate his agreement with them, but I shall cite them as Savary *1821a*.

In his second note Savary rejected Faraday's implicit claim in the paper that the rotatory motion could be taken as a 'primitive fact' in electromagnetic phenomena, and in the next note he showed how that motion could be explained in Ampère's terms. Figure 1425.1 shows the projection A of an element of wire reacting with a molecule c via the rotating electric current bdb′ of electricity. At b the current was moving away from A, but at the image point b′ it was approaching A, so that the actions along bA and b′A worked in opposite directions and thus took a resultant perpendicular to cA. Generalising to a "full" magnet composed of particulate currents around each molecule, observable motion would occur, which which would of course be rotatory if the wire were pivoted about a fixed point.

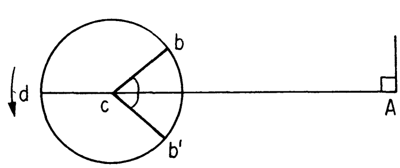


Figure 1425.1 Savary's diagram for rotating motion (*1821a*, pl. 7, fig. 17)

The translator of Faraday's paper was J. Riffault, an elderly chemist who was noted in a footnote to § 13.4.1 as the editor of a supplementary volume of his translation of T. Thomson's *System of chemistry* to which Fresnel contributed an extensive survey article on wave-theory optics. The book also contained an 91-page 'Exposition of new discoveries in magnetism and electricity'. Once again Ampère was co-author, but the writing was done almost entirely by another former student at the *Ecole Polytechnique*: Jacques Babinet, member of the strong 1812 *promotion* mentioned in § 10.1.1, and in February 1822, when the book was published, *Professeur* of physics at one of the leading Paris *collèges*. I shall cite his article as Babinet *1822a*.

As was made clear in the revised title given to the article when the publisher, Méquignon-Marvis, put it out in 1822 as a separate pamphlet, it dealt mainly with the experiments, both Ampère's and others'. Although unfortunately it contained no references, the presentation was the most complete and orderly yet attempted. On the theoretical side, however, the situation was less happy from Ampère's point of view. For example, the electrical theory of a magnet was argued in terms of currents 'disposed around its axis in closed curves' (art. 29), with the theory of particulate currents around each molecule given only at the very end (art. 89: since he cited Ampère's January *Académie* papers, it was probably added on at the last minute). Again, the (non-mathematical) account of the action between two wires was based on Ampère's formula (1423.3) in which the second term was set to zero (arts. 45, 16).

Since this was the version of the formula which Ampère himself had discussed in his writings, Babinet cannot be blamed; but in fact somewhere around this time Ampère realised that the second term could not be dropped. Entering into the book business himself, he produced the first version of his extraordinary *Recueil* described in § 14.1.2, which included several of his *Annales* papers and others from the *Bibliothèque universelle*, Faraday's paper and Savary's comments—and also a reprint from the *Journal de physique* of a lecture *1822d* given to the April 1822 public meeting of the *Institut.*[1]) In this lecture he surveyed experimental work carried out by himself and others since 1821; he also published for the first time the words 'electro-static' and 'electro-dynamic' (as cited in § 14.2.2).

[1]) K. Caneva has found a copy of this version of the *Recueil*; its title begins 'Receuil des mémoires', and it comprises the first 318 pages of the full version of 1823. As it is exceedingly rare, I shall not cite it, and give the reference Ampère *1823f* for the 'notes' about to be discussed.

14.2.6 Ampère's differential expression for electrodynamic action, 1822

Ampère added 'notes' *1823f* to the printing of this lecture in his *Recueil*; and they are important for us, for they included not only Figure 1423.1 for the trigonometric expression (1423.3) for electrodynamic action but also this extension. Assuming for generality an inverse n-th power law, the expression for the mutual action $\mathscr{A}$ between two infinitesimal elements of wire ds and ds' carrying respective steady currents i and i' was given by

$$\mathscr{A} = ii'ds\,ds'(\sin\alpha\sin\beta\cos\gamma + k\cos\alpha\cos\beta)/r^n. \qquad (1426.1)$$

Previously he had set to zero the constant k: now, however, he took (1426.1) as containing *two* unknown constants n and k, and reported that 'I discovered a new case of equilibrium which furnishes me' with a relation between them (*1823f*, 234).

The detailed account of the argument was presented to the *Académie* on 10 June 1822, with a short addition made a fortnight later (*PV*, *7*, 336, 344): the two papers were published in Arago's *Annales* as *1822a* and *1822b*, and again in the *Recueil*. A mathematical advance was involved: (1426.1) had been established for a contiguous sequence of rectilinear elements (*1823f*, 232-233), but now it took a new form, and for curved wires also, thanks to the calculus of curves.

Figure 1426.1_1 shows two curved wires BM and B'M', of arc lengths s and s' at points $M(x, y, z)$ and $M'(x', y', z')$ respectively in the (left-handed) axis system AXYZ. Ampère took infinitesimal elements Mm and M'm', of lengths ds and ds' respectively, and drew on elementary differential geometry to reformulate the trigonometric expression (1426.1) in calculus form. Allowing for the modes of change (increase or decrease) of the variables,

$$\begin{aligned}&\cos\alpha = dr/ds \quad\text{and}\quad \cos\beta = -dr/ds', \quad\text{so that}\\ &\cos\alpha\cos\beta = -(dr/ds)(dr/ds'),\end{aligned} \qquad (1426.2)$$

thus producing the second trigonometric term in (1426.1). For the first, he s-differentiated $(1426.2)_2$ to obtain

$$\sin\beta\, d\beta/ds = d^2r/ds\,ds', \qquad (1426.3)$$

and then considered the increment mM on BM (*1822a*, 304-306).

My Figure 1426.1_2 may help to explain Ampère's rather cryptic account of his procedure. The move from M to m altered the orientation of T'M' (the tangent line to B'M' at M') from β to $(\beta + d\beta)$, and this angle was given by the projection of ∠MM'm onto the plane of ΔT'M'M. Hence, again taking account of the direc-

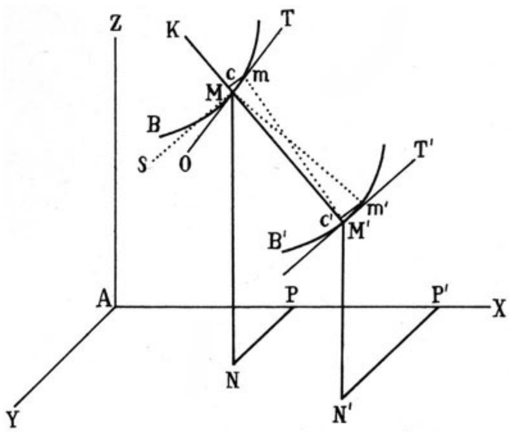
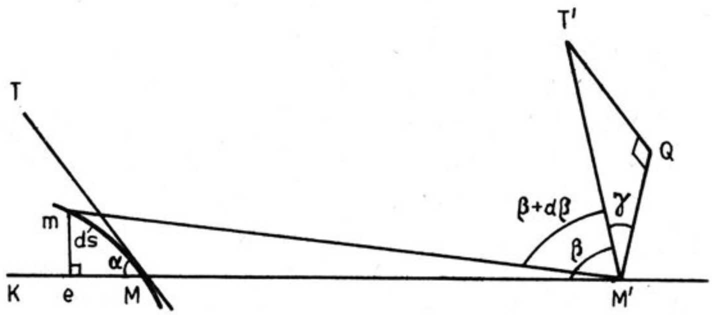


Figure 1426.1 Ampère's diagram for the action between curved wires (*1822a*, his pl. 8, fig. 14), together with notations for ΔmMM′ and T′M′



M′Q is projection of M′T onto plane of ΔmMM′.
MM′ = r,
∠ TMK = α,
∠ T′M′K = β

tion of change,

$$d\beta = -\angle\text{MM'm}\cos\gamma, \tag{1426.4}$$

where γ was the angle between the planes of ΔmMM and ΔTMM′. Now, relying on infinitesimals to endorse the ensuing equations,

$$\angle\text{MM'm} = \tan\text{MM'm} = \frac{\text{me}}{\text{eM'}} = \frac{\text{me}}{\text{MM'}} = \frac{ds\sin\alpha}{r}. \tag{1426.5}$$

So, using (1426.3-5), the first term of (1426.1) was obtained:

$$\sin\alpha\sin\beta\cos\gamma = (\sin\alpha\cos\gamma)\sin\beta = -rd^2r/ds\,ds'. \tag{1426.6}$$

Thus, from (1426.3, 6),

$$\begin{aligned}\sin\alpha\sin\beta\cos\gamma + k\cos\alpha\cos\beta &= -\left(r\frac{d^2r}{ds\,ds'} + k\frac{dr}{ds}\frac{dr}{ds'}\right)\\ &= -r^{1-k}d\left(r^k\frac{dr}{ds'}\right)\Big/ds.\end{aligned} \tag{1426.7}$$

Hence (1426.1) could finally be written

$$\begin{aligned}\mathscr{A} &= -ii'\, r^{1-k-n}(d(r^k\, dr/ds')/ds)\; ds\, ds' \\ &= -ii'\, r^{1-k-n}\, d(r^k\, d'r),\end{aligned} \tag{1426.8}$$

where (following Poisson *Mechanics*$_1$ (1811), *1*, 245), he set 'd' and 'd'' for the partial differentials of r with respect to s and s' respectively (Ampère *1822a*, 306-310).

Ampère now described the new 'case of equilibrium' from which the relation (1426.2) between k and n was obtained. In the notation of his Figure 1426.1$_1$, if B'M' were an element of a fixed horizontal circular wire $\mathscr{C}$ of radius a and centre A, and BM belonged to a wire $\mathscr{S}$ of "any" shape,[1]) free to rotate about the axis AZ, to points of which its ends were attached, then in fact $\mathscr{C}$ could not induce any continuous rotation in $\mathscr{S}$. Thus the moment about AZ of the mutual action between BM and B'M' would be zero, yielding the desired relation between k and n.

To calculate this moment, Ampère converted from Cartesian to cylindrical polar coordinates (z, u, t) about A for M, where AN = u and $\angle$PAN = t, and values (z', u', t') for M'. Thus

$$r^2 = z^2 + a^2 + u^2 - 2au\cos(t' - t); \tag{1426.9}$$

$$\therefore\; r d'r = au\sin(t' - t)\; dt', \tag{1426.10}$$

so that, in (1426.8),

$$\mathscr{A} = -ii'ar^{1-n-k}\, d't'd(r^{k-1}\, u\sin(t' - t)). \tag{1426.11}$$

Now this action lay along MM', of course; hence its component $\mathscr{T}$ in the tangential direction MO was expressed as

$$\mathscr{T} = \mathscr{A}\cos\angle\mathrm{M'MO} = \mathscr{A}\,(\mathrm{AN'}\sin\angle\mathrm{NAN'})/\mathrm{MM'} = (\mathscr{A}\, a\sin(t' - t))/r. \tag{1426.12}$$

Thus the moment $\mathscr{M}$ of $\mathscr{T}$ was given, via (1426.11-12), on p. 314 as

$$\mathscr{M} = \mathscr{T} \times \mathrm{MQ} = Tu = (\mathscr{A}\, au\sin(t' - t))/r \tag{1426.13}$$

$$= -ii'\, a^2\, d'\, t'\, r^{-n-k}\, u\sin(t' - t)\; d(r^{k-1}\, u\sin(t' - t)). \tag{1426.14}$$

According to the experimental finding, the d'-integral of (1426.14) along the path of $\mathscr{S}$ would be zero. A full treatment of the resulting line integral would have been a sophisticated task for the analysis of Ampère's time: in effect he in-

[1]) The principal problem concerning generality concerns whether the curves were planar or not. Ampère was often not clear on this point; however, when he referred to a 'closed' circuit he definitely had in mind a wire which came back to its point of departure, not the circuit including the battery or pile.

voked a variant of the fundamental theorem of the calculus of variations to say that, since the value of the integral was to be independent of the relation between r, s and t applicable to $\mathcal{S}$, then (1426.14) should be an exact differential with respect to d; and a necessary condition for this was given by equating the powers in the two terms for r there:

$$k-1=n-k, \quad \text{or} \quad n+2k=1. \tag{1426.15}$$

'Such is the relation that experience shows to exist between k and n', as he put it; and assuming the inverse square law given by $n=2$, he deduced that $k=-\frac{1}{2}$ instead of the zero value mistakenly proposed in 1820 at (1423.3) (*1822a*, 311-315).[1])

The brief addition *1822b*, read a fortnight later, was equally noteworthy. Assuming (1426.15), and resolving the action $\mathcal{A}$ along the lower element ds' in the general case, Ampère found from (1426.2$_2$, 8$_3$) this expression for the component $\mathcal{E}$ of action along ds':

$$\begin{aligned}\mathcal{E} &:= \mathcal{A}\cos\beta = (-ii'\,r^k\,d(r^k\,d'r))\,(-d'r/d's') \\ &= ii'\,r^k d'r\,d(r^k\,d'r)/d's'.\end{aligned} \tag{1426.16}$$

Its integral along the upper wire was therefore given by d-integration of $\mathcal{E}$:

$$I = ii'(r^k\,d'r)^2/2d's' + C = \tfrac{1}{2}\,ii'\,r^{2k}\,d's'\cos^2\beta + C, \tag{1426.17}$$

where C was a constant of integration. Now if this wire 'forms a completely closed circuit', then $I=0$, 'from which it follows that the resultant of all the actions exerted by a closed circuit upon a small portion of [the other] conductor is always perpendicular to the direction of this little portion'. Further, 'it would be the same for any assembly of closed circuits, and in consequence for a magnet, when one considers it as such'.

The transition from the trigonometry of (1423.1) to the calculus used here marks an increase in mathematicisation which we have noted elsewhere in this book. The greater capacity of the calculus to handle the *continuous* phenomena involved paid off, since now Ampère not only had an argument for the value of but also, in the short addition, a circulation theorem. His further mathematical studies of electrodynamics were to continue this line of thought.

[1]) Ampère could have proved that $n=2$ from a variant on the argument to find the action (1423.2) of an infinitely long wire upon (in this case) a parallel element of length h and distant a away along the perpendicular. Experiment shows that this action was proportional to h/a; calculation via the inverse n-th law produces an expression proportional to h/a^{n-1}. He did use this approach in his course in physics at the *CF* in 1826-1827 (*AS*, *Ampère*, ch. 213: see Lützen *Liouville*, ch. 7).

14.3 Ampère's supporters, 1823

14.3.1 The impact of Savary: Ampère's 'methodical exposition'

> When $n = 2$, one has $k = -\frac{1}{2}$, but however be the force of the analogies which carry [one] to think that n is in fact equal to 2, one has no proof of it deduced directly from experience, since all the experiments made on this matter have been of letting a Voltaic conductor act upon a magnet, and in consequence apply only by an extension[,] which one cannot regard as a complete demonstration, to the mutual action of two infinitely small portions of electric currents.
>
> Ampère *1822a*, 315

Ampère made this fine judgement of his position, both theoretical and experimental, immediately after producing (1426.15) in *1822a*, as just described. While he was clearly aware of his difficulties, nevertheless he had achieved enough to summarise his findings in a general way. Accordingly, somewhere near the end of that year he wrote a paper called 'Methodical exposition of electrodynamic phenomena and of the laws of these phenomena' (*1822c*). He published it first with the *Société Philomatique*; in an extended form in the *Journal de physique*; with further additions for his 1823 *Recueil*; and then in 1823 as a pamphlet with Bachelier, with a print-run of 300 copies (*AN*, F^{18} 104, no.522). To add to the complications, he put out in the following year 16 extra pages of notes to append to the pamphlet version; I shall discuss this item in § 14.4.1. The variety of versions of the 'Methodical exposition' make its title something of an irony, but at least the footnotes attached to all the versions gave valuable page-referenced guidance to the reader around the chaotic *Recueil*.

Interestingly, Ampère gave pride of place to one of his newest results; for he began with a second deduction made in the addition *1822b* to *1822a*, and which was verified experimentally in September while visiting Gaspard de la Rive in Geneva. If two collinear elements ds and ds' were taken, then in (1426.1) $\alpha = 0$ and $\beta = 0$ (or π); hence, since (1426.15) showed that $k = -\frac{1}{2}$, then $\mathscr{A} < 0$, so exhibiting a repulsion between them.

In the rest of the 'Exposition' Ampère ran through the principal findings made earlier, concerning the attraction and/or repulsion between various arrangements of straight, circular, helical and curved wires. The only mathematics represented was the formula (1426.8) for the action $\mathscr{A}$ between ds and ds', incorporating the relation (1426.15) between k and n.

The main cause of the additions made to the 'Exposition', and of the lengthy notes for the pamphlet version, was two papers submitted to the *Académie* at its meeting of 3 February 1823 (*PV*, 7, 441). The author of one was the 25-year-old Savary; the other came from Jean Demonferrand, four years his senior, like Babinet (§ 14.2.5) a member of the distinguished *promotion* of 1812 at the *Ecole Polytechnique*, and at the time *Professeur* of physics at the *Collège* at Versailles. (In 1825 he was to succeed Fresnel as graduation examiner for descriptive geometry and physics at the *Ecole Polytechnique*.) In a piece of doubtless deliberate timing, the *Académie* meeting three weeks later heard Fourier's report *1823a* recommending Savary's paper to the *Savants étrangers* and Ampère *1823d* to act similarly over Demonferrand's; in addition, Ampère presented a copy of the pamphlet version of the 'Exposition' (*PV*, 7, 451-453).

However, the fates of the two submissions were different. Savary's quickly came out in the *Journal de physique* as *1823a*, and then as a pamphlet from Bachelier, with a brief addition at the end; an extract *1823c* also appeared in Arago's *Annales*, and again in Ampère's *Recueil*. However, for some reason Demonferrand's paper was not published, and became known at first only from Fourier's report (which, as *secrétaire perpétuel*, he placed in his notice of the work of the *Académie* for 1823), and in an anonymous summary of both Demonferrand's and Savary's papers published in the *Bibliothèque universelle* and excerpted for the *Société Philomatique*. I cite these two items as Ampère *1823b* and *1823a* respectively; he seems essentially to have been their author, as they contain passages from the *Recueil* version of the 'Exposé' (*1823g*, 342-344).

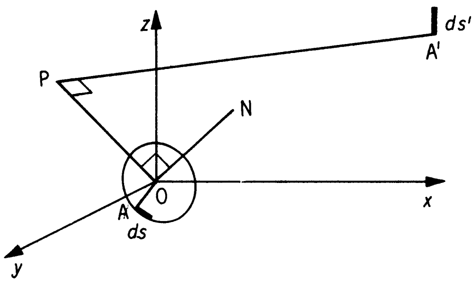
In addition, it is worth recording that a file in Ampère's *Nachlass* shows that he rewrote large parts of Savary's main paper *1823a* on proof![1]) Nevertheless, I shall treat Savary as its author; and as it is the only one of these writings to contain the mathematics, I shall draw on it solely for the account which now follows.

14.3.2 Savary on the action between element and circle

Impressed by the ubiquity of circles in Ampère's work—magnets containing particulate electric currents, experiments and results with circular and helical wires, and so on—Savary took the action between circles and infinitesimal elements as his concern. I present his main result, and a few of the cases to which he applied it.

Savary set up a (left-handed) Cartesian axis system $Oxyz$ as shown in Figure 1432.1 in such a way that O lay at the centre of the circle $\mathscr{C}$, of radius a and normal ON, and Oz was parallel to the element ds' at A′. Dropping the perpendi-

[1]) See *AS*, *Ampère*, ch. 172. According to *AN*, F^{18} 87, no. 220, Ampère paid for the printing of 500 copies of Savary's paper in pamphlet form. His letters of the time to Savary are printed in *Correspondence*, 931-935.



A'P = p PO = q
∠ POy = ψ
∠ NOz = δ
∠ BOy = ω

Figure 1432.1 Diagram for Savary's analysis of the action of a circle and an element (after *1823a*, fig. 1)

cular A'P from A' to the plane of $\mathscr{C}$, he set up in effect spherical polar coordinates also, with q, ψ and ω. Thus the coordinates of A' (x', y', z') and of an element A (x, y, z) of $\mathscr{C}$ were given by

$$x' = -a \sin\omega \sin\delta, \quad y' = a\cos\omega \quad \text{and} \quad z' = a\sin\omega\cos\delta; \qquad (1432.1)$$

$$x = p\cos\delta - q\sin\delta\sin\psi, \quad y = q\cos\psi \text{ and}$$
$$z = p\sin\delta + q\cos\delta\sin\psi; \qquad (1432.2)$$

$$\therefore\ r := \mathrm{AA'} = (p^2 + q^2 + a^2 - 2aq\cos(\omega - \psi))^{1/2}. \qquad (1432.3)$$

As for the elements of current,

$$ds = a\,d\omega \quad \text{and} \quad ds' = dz, \quad \text{and he set } i = i' = 1 \qquad (1432.4)$$

for convenience; so Ampère's basic formula (1426.8) for action between ds and ds', with both k and n present, took the form

$$\mathscr{A} = -\frac{x^{1-k-n}}{(1+k)\,a}\,\frac{d^2(r^{1+k})}{d\omega\,dz}. \qquad (1432.5)$$

Thus the cumulative action of ds on $\mathscr{C}$, which lay along AA', could be resolved into its Cartesian components

$$(X, Y, Z)\,dz, \quad \text{where} \quad X = \int_0^{2\pi} (\mathscr{A}a/r)\,(x + a\sin\delta\sin\omega)\,d\omega, \qquad (1432.6)$$

$$Y = \int_0^{2\pi} (\mathscr{A}a/r)\,(y - a\cos\omega)\,d\omega,$$
$$Z = \int_0^{2\pi} (\mathscr{A}a/r)\,(z - a\cos\delta\sin\omega)\,d\omega. \qquad (1432.7)$$

The evaluations of these integrals, aided by (1432.1-2) and some differentiations with respect to z and ω, was routine passage-work. It produced the results

$$X = (1-k)\, a^2 py \int_0^{2\pi} \sin^2(\omega - \psi)/r^{n+3}\, d\omega, \qquad Z = 0, \tag{1432.8}$$

$$\begin{aligned} Y = (1-k)\, a^2[(y^2+z^2)\cos\delta - xz\sin\delta] \\ \times \left(\int_0^{2\pi} \sin^2(\omega-\psi)/r^{n+3}\, d\omega\right) - \tfrac{1}{2}a^2\cos\delta \int_0^{2\pi} 1/r^{n+1}\,\delta\omega. \end{aligned} \tag{1432.9}$$

$\left(\text{Savary } \textit{1823a}, \text{ 344-348, with a few changes of notation, and my use of } \int_a^{b}\right)$.

In the rest of his paper Savary applied and adapted these evaluations to a septet of cases, several of which were suggested by Ampère's and others' experiments. The most striking case was his derivation of a second equation to accompany Ampère's (1426.15) in the determination of n and k: as with Ampère, he drew on an experiment which established a case of equilibrium. Apparently Gay-Lussac, in collaboration with his older chemist colleague J. Welter, had wrapped a wire helically around a steel annulus, and passed a current through it; and they found that, while breaking the annulus revealed that magnetisation had occurred, it exercised no action on a current element when in its unbroken state, whatever was the orientation of the element to the annulus.

Although they did not publish these experiments, this discovery was known to Ampère and Savary, and it gave Savary the chance to apply his formulae to the case where, by Ampère's theory of the magnet, plane $\mathscr{C}$ of Figure 1432.1 became the tiny circular section of the annulus (not shown in the Figure). First of all, he applied (1432.8-9) to the case where $a << r$ and found that

$$X \doteq (1-k)\, py/r^{n+3} + O((a/r)^2)$$

and $$Y = -(kr^2\cos\delta + (1-k)\, px)/r^{n+3} + O((a/r)^2), \tag{1432.10}$$

where he absorbed a constant factor, πa^2, into X and Y (p. 348). Then he assumed the annulus to be horizontal (his fig. 2: imagine the annulus to be a huge doughnut coming out of the page of Figure 1432.1), so that the little circle carrying ds was vertical, and moreover set at the angle φ to a given horizontal coordinate axis direction. He took three cases for setting the isolated element ds': vertical; and horizontal and pointing towards, or normal to, the axis of the annulus. In each case he integrated both expressions of (1432.10) over $[0, 2\pi]$ of φ and thereby calculated the components of the total action between the annulus and ds'. According to the experiments of Gay-Lussac and Welter, all of these expressions had to be set to zero; and calculation showed them all either to be zero anyway or of the form

$$-(kn+1)\,c\varrho\int_0^{2\pi}\sin^2\varphi/r^{n+3}\,d\varphi, \tag{1432.11}$$

where c was the projection of OA′ upon the Oxy-plane and ϱ was the radius of the annulus. Now, clearly the integrand of (1432.11), c and ϱ were positive; so the nullity had to be produced by the other factor. Thus Savary obtained the desired equation

$$kn+1=0, \quad \text{to join Ampère's} \quad n+2k=1. \tag{1432.12}$$

These equations took as roots

$$k=-\tfrac{1}{2}, \quad n=2, \quad \text{and} \quad k=1, \quad n=-1; \tag{1432.13}$$

but since k was known by experimental evidence to be negative, the second pair of roots could be ignored. This left the first pair, which established inverse squarism (pp. 349–352) in a way for which Ampère had hoped.

Some of Savary's cases involved an 'electrodynamic cylinder', Ampère's name for a wire wrapped in a thin helix with a curved or straight axis A, and then returned back along A to cancel out the axal component of the action of the helix. He set it horizontally so that p of Figure 1432.1 served as the variable for the axis (in this case straight, and parallel to AP). Integrating the appropriate versions of (1432.10) with respect to p, he found that, if the ends of the cylinder were located at points $(x', y', 0)$ and $(x'', y'', 0)$, distant r' and r'' respectively from the centre O of coordinates, then the cumulative force was given in components by

$$\frac{1}{2}\left(-\frac{y'}{r'^3}+\frac{y''}{r''^3},\ \frac{x'}{r'^3}-\frac{x''}{r''^3},\ 0\right) \tag{1432.14}$$

(pp. 352–353). He also deduced the property obtained by the experiments of Biot and Savart: that the action of a magnet varied inversely with perpendicular distance from a wire, and was the algebraic sum of attractions and repulsions to its ends (pp. 356–357). In calculating the 'oscillations of a cylinder subject to the action of an indefinite angular conductor', he refuted Biot's claim of its proportionality to the angle 2β of a V-shaped conductor, producing instead the evaluation $\tan\frac{1}{2}\beta$ which was given earlier at (1424.2). In a nice touch of diplomacy, he (or his ghost-writer Ampère) observed that this expression 'differs little from the result that Mr. Biot had obtained by experiment' (p. 364).

Savary's last case, the 'mutual action of two cylinders', included the case of magnetic dip; and for the purpose he treated the earth as a *thin* cylinder of electric currents around the magnetic axis, a 'supposition that one must consider, at the very most, only as an approximation more or less removed from that which really happens in the interior of our globe' (p. 367). Drawing on formulae derived from (1432.8-10), he found that if the magnet was placed at latitude l and inclined at angle i to the local horizontal, then the moment of rotation about its centre c was proportional to

$$(\sin(i + l) - 3\cos i \sin l)/r^3, \qquad (1432.15)$$

where r was the distance between c and the centre of the magnetic axis. Hence in equilibrium

$$\sin(i + l) = 3\cos i \sin l, \quad \text{that is,} \quad \tan i = 2\tan l, \qquad (1432.16)$$

which was Bowditch's simplified form (754.8) of Biot's formula $(754.7)_2$ for magnetic dip (pp. 364-370). Interestingly, Savary mentioned Bowditch by name, and must have seen his paper *1818a* presenting the equation.

Ampère was obviously excited by Savary's paper; not only did he prepare the unsigned piece *1823b* for the *Bibliothèque universelle* mentioned above but also added a paper *1823c*, and put both items in his *Recueil*. In addition, and in a collaboration which may surprise, he and Rodrigues wrote a review *1823a* of the pamphlet version of Savary's paper for the original form of Ferrusac's *Bulletin* (§ 11.1.1). In these writings Ampère stressed how all of Savary's results drew on the form (1432.5) of his theory of inter-elemental action, and that they included both Coulomb's law of inverse-square action between the poles of a magnet (§ 7.5.2) and Biot's findings for electromagnetism.

Savary extended one result in an 'Addition' which Ampère communicated to the *Académie* on 28 July and explained at the meeting of the following week (*PV*, 7, 520-521). The paper does not seem to have been formally submitted, but it quickly appeared in the *Journal de physique* as Savary *1823b*. He took the annulus again but calculated the action which a *segment* of it exerted on the element. Drawing on suitable versions of (1432.6-7), he showed that the expressions depended only on the positions of the ends of the segment, corroborating Ampère's experimental findings that certain actions were independent of the shape of a wire between fixed points (pp. 296-298). He also cited from Ampère *1823f*, 21 the claim that the same property held of a portion of an 'electrodynamic cylinder' and set up the differential expressions for action between element and cylinder from which the claim would be proved (*1823b*, 298-303).

14.3.3 Demonferrand's 'manual of dynamical electricity'

According to Ampère's report *1823d*, Demonferrand's February manuscript contained some of the same calculations as Savary's: its new results concerned the action of a long straight wire on a very short element, and on a V-shaped helix, and Ampère devoted only a couple of paragraphs to them (*1823g*, 344 (the *Recueil* version of his 'Exposition'); *1823a*, 62). But its unlucky author made some impact in this summer, for he published the first textbook on the new subject: a *Manuel d'électricité dynamique*, a 218-page volume (Demonferrand *1823a*). It was put out by Bachelier in time for Ampère to present a copy to the *Académie* on 21 July (*PV*, 7, 513), the meeting before those involved in Savary's 'Addition'. The *Conseil Royal de l'Instruction Publique* recommended it for teaching in the quality schools (such as the *Collège* where Demonferrand himself taught); but only 1,000 copies were printed (*AN*, F^{18} 87, no. 215), and only this one printing was produced, thus exemplifying the patchy French interest in the topic which was mentioned in § 14.2.1. In some compensation, translations were soon published: in German (prepared by the young G. Fechner), Italian (by S. Gherardi, one of Ampère's contacts there) and English (by J. Cumming, Professor of Chemistry at Cambridge).

Apart from the chaotic organisation of Plates II-IV, Demonferrand *1823a* gave a generally clear survey of the state of knowledge. His scope was broader than that of Babinet's survey of the previous year (§ 14.2.5), for he covered much of the theory and the principal formulae as well as the main experiments. As he followed Ampère's position in all respects, he covered the whole range of actions between wires and magnets, and Ampère's theory of magnetism. Although he gave few detailed references, he indicated on the contents page the discoverer of each effect and the creator of each idea.

Demonferrand may well have discussed the contents of the book with Ampère, for it contains some details which were not then well known: for example, Ampère's theory of currents (§ 14.2.4) as created by the successive de- and recompositions of the two electric fluids (p. 7). He gave the mathematical parts of electrodynamics in the second part of the book, beginning with the trigonometric and the calculus forms of the basic formulae for inter-elemental action, proceeding to the two equations for n and k, and so on (pp. 43-65). He completed the part with a largely prosodic summary of the results contained in his and Savary's February papers, from which it emerges that he had found a few more things than Fourier or Ampère had given him credit. For one of these he provided the mathematical details (a straightforward case of taking components of the resultant actions involved): that if a straight wire were fixed at one point, which lay on the axis of an electrodynamic cylinder, then it took up positions of equilibrium or set itself into continuous rotation about this axis according to the relative positions of axis, wire and cylinder (pp. 76-80).

Ampère must have felt encouraged during the summer of 1823 by these valuable contributions from these two disciples; the state of truths, of which he had written to Erman two years earlier (§ 14.2.4), was increasing. He wrote to Erman again in August, surveying many of the ideas and results obtained and sending copies of his *Recueil* and Savary's first paper. On the latter he opined: 'I regard it as one of the finest works on this subject [...]'.[1])

14.4 The consolidation of Ampère's theory, 1824-1826

14.4.1 Ampère's line integral for electrodynamic action, 1824

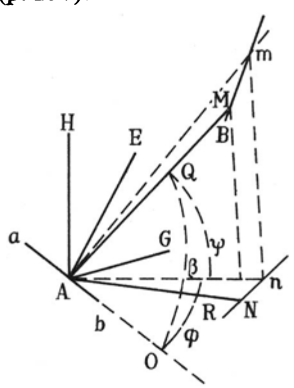
At the turn of 1823 and 1824 Ampère presented to three successive meetings of the *Académie* some developments of his own and Savary's ideas (*PV*, *7*, 605, 607; *8*, 5). His paper appeared in two parts as *1824a*, in the June and July issues of Arago's *Annales*; then, in characteristic style, he rewrote a few passages, added 24 pages of new notes, and published the whole as a 67-page pamphlet *1824c* through the houses of Crochard and Bachelier. Carrying as main title *Précis de la théorie des phénomènes électro-dynamiques*, a sub-title indicated that it served as a supplement to his *Recueil* and Demonferrand's *Manuel*. To add to the chronological mystery, he placed an 'Extract' *1824e* of the first *Académie* paper in the June 1824 issues of the *Bulletin* of the *Société Philomatique* and of Ferrusac's *Bulletin*, in which the *pamphlet* version was announced as just published; but he presented a copy to the *Académie* only in September (*PV*, *8*, 132, 133), and it may well have come out around then.[2])

Ampère began by introducing a new term: an ensemble of continuous tiny circular currents, each one perpendicular to the curve passing through the centres, was now to be known 'under the name of *solenoid*, from the Greek word σωληνοειδής, derived from σωλήν, canal, and which signifies precisely what has the form of a canal' (*1824a*, 135), in the sense of 'canal' as a narrow channel then used in fluid theory. He then proceeded to calculate five cases of action: element/cylinder, element/closed circuit, element/solenoid, solenoid/cylinder, and solenoid/solenoid. He largely followed Savary's methods, but introduced important touches of his own.

[1]) This letter is held at *Darmstaedter*, and is transcribed in § 20.6. It is not to be found in Ampère's *Correspondence*, although an earlier letter was reprinted (on pp. 914-918) from that archive.

[2]) Joubert printed the manuscript presented by Ampère on 22.12.1823 (*Electrodynamics*, *1* (1885), 395-410); it is now kept at *AS*, *Ampère*, ch. 173. Another manuscript, dealing mostly with electrochemical aspects, appeared posthumously in Moigno *1849a*, 224-240, and is reprinted in Blondel *1982a*, 177-186.

The main result occurred in the first case, of which Ampère gave an exceedingly confusing account. In Figure 1441.1 an element ab, of length ds' and carrying current i' acted upon a ds-long piece mM of a 'system of closed currents' $\mathscr{C}$ bearing current i. The action between ab and mM lay along AM, and could be resolved into components along Ab and along the perpendicular AG to Ab in the plane bAM. Drawing on his formula (1426.11) of 24 June 1822 for the component of the action of ds' on ds in the direction of ds, and assuming an inverse nth-power law for generality, he used the integral (1426.17) around $\mathscr{C}$ to show that the component of the cumulative action $\mathcal{F}$ along Ab was zero. Hence $\mathcal{F}$ lay perpendicular to Ab; but as $\mathscr{C}$ was arbitrary, the direction of $\mathcal{F}$ was not known further. He took it in two components: $\mathcal{F}_1$ (say) in the plane bAG (but now AG was *not* specified within the plane) and $\mathcal{F}_2$ perpendicular to $\mathcal{F}_1$ and to Ab (p. 137).



ab = ds'
mM = ds
AM = r

∠ bAM = β
∠ bAG = τ

Figure 1441.1 Ampère's representation of action between an element and a solenoid (*1824a*, his fig. 1)

To find an expression for $\mathcal{F}_1$, Ampère seems have gained some inspiration from Savary's 'Addition' *1823b*. From his own integral (1426.17) he obtained the form

$$\mathcal{F}_1 = -ii'\,ds' \int_{\mathscr{C}} (\tfrac{1}{2}\,\mathrm{AN}^2/r^{n+1})\,d\varphi, \qquad (1441.1)$$

where AN was the projection of AM onto the plane bAG. He also observed that $\frac{1}{2}\,\mathrm{AN}^2\,d\varphi$ was the area of the projection NAn of the differential radial sector MAm onto that plane, and that $\mathcal{F}_1$ was independent of the direction of ds' (pp. 136–138). Now thinking in terms of projections, he defined the line integral

$$U := \int_{\mathscr{C}} (\mathrm{AN}^2/r^{n+1})\,d\varphi \qquad (1441.2)$$

and set up Cartesian axes to which the normal (AZ, say) of plane bAG made angles ξ, η and ζ respectively, and on whose planes the expressions corresponding to U took the values A, B and C. (Another use for 'A', I fear!) Then, by a standard theorem,

$$U = A\cos\xi + B\cos\eta + C\cos\zeta. \tag{1441.3}$$

He now in effect lay off lengths A, B and C along the respective axes, and took the line AD (not shown in Figure 1441.1) of their resultant D, set at angles ξ', η', and ζ' to their components and ψ' to its own projection onto bAG. (1441.3) then became

$$U = D(\cos\xi'\cos\xi + \cos\eta'\cos\eta + \cos\zeta'\cos\zeta) = D\sin\psi'; \tag{1441.4}$$

hence (1441.1) reduced to

$$\mathcal{F}_1 = -\tfrac{1}{2}\, ii'\, Dds' \sin\psi' \tag{1441.5}$$

(pp. 138–142). Since AG was in effect arbitrary, a similar formula would apply for the other component $\mathcal{F}_2$; for some reason he did not say so, but still drew conclusions from (1441.5) about 'the total resultant [$\mathcal{F}$] of the actions exerted' on ds by $\mathscr{C}$.

The form of (1441.5) disclosed various properties. In particular, $\mathcal{F}_1$ was zero when AZ lay in the plane bAD, for then $\psi' = 0$; thus it was perpendicular to both Ab and AD. But (1441.3) showed that the direction of AD was specified *in*dependently of that of Ab; hence, as Ab varied in direction in space, $\mathcal{F}$, changed its direction *only* in the plane normal to AD, 'to which I shall give the name of *director plane*'. From (1441.5), the value of $\mathcal{F}_1$ varied between zero for $\psi' = 0$ or π (when Ab ∥ AD) to $-\tfrac{1}{2}\, ii'\, Dds'$ (when Ab ⊥ AD) (pp. 142–144). Later he called AD the 'directrix' (*1826e*, 43).

Of the various cases which Ampère then studied, I consider the first, for both the main result and one feature of the proof will be of use in § 14.4.6. Figure 1441.2 shows the example, where $\mathscr{C}$ was a circle: the coordinate axes were so arranged that AXZ contained two centres (O for the circle $\mathscr{C}$; and A for the midpoint of the element ab, not itself in this plane) and AXY was parallel to the plane of $\mathscr{C}$. The coordinates (x, y, z) of an arbitrary point M of $\mathscr{C}$, of radius m, were given by

$$x = p - m\cos\omega, \quad y = m\sin\omega, \quad z = q, \quad \text{with} \quad r^2 = x^2 + y^2 + z^2. \tag{1441.6}$$

Then, by the definition of A given after (1441.2),

$$A = \int_{\mathscr{C}} r^{-n-1}\,(ydz - zdy) = -mq \int_0^{2\pi} (\cos\omega / r^{n+1})\, d\omega, \tag{1441.7}$$

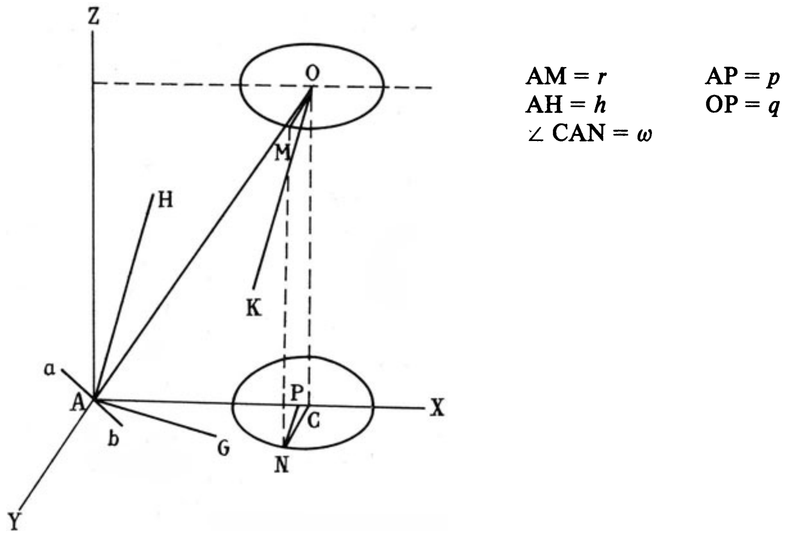


Figure 1441.2 Ampère's diagram for action between an element and a circle (*1824a*, his fig. 7)

with similar expressions for B and C. Thus (1441.3) became

$$U = m^2\left\{[(n+1)(p^2+q^2)\cos\zeta - hq]\int_0^{2\pi}(\sin^2\omega/r^{n+3})\,d\omega - \cos\zeta\int_0^{2\pi} r^{-n-1}\,d\omega\right\}, \tag{1441.8}$$

where h was the length of the perpendicular OK dropped from O to the director plane GAH (another role for 'G'!), given by

$$h = q\cos\zeta + p\cos\xi \tag{1441.9}$$

(pp. 148-149). He did not proceed to evaluate U further, presumably because eliminating ξ, η and ζ from (1441.7) and its companions would give some messy expressions; instead he took the case where $\mathscr{C}$ was a circle of *tiny* radius m, and found

$$U = \pi m^2[(n-1)\cos\zeta/l^{n+1} - (n+1)\,hq/l^{n+3}] + O(m^3), \quad \text{where} \quad l := \text{AO} \tag{1441.10}$$

(pp. 149-150). In the rest of the paper he broadly imitated Savary's calculations of the actions solenoid/element, solenoid/circuit and solenoid/solenoid; the novelties lay in the role of the line integral (1441.1) for inter-elemental action, and the use of the director plane. For example, Savary's expression (1432.14) of the Ox-component of the action between an element and the solenoid with ends distant l' and l'' from O and x-coordinates x' and x'' respectively was obtained, via integration by parts, as

$$A = \frac{\pi m^2}{g} \int_{\mathscr{C}} \left(\frac{dx}{l^3} - \frac{3x\,dl}{l^4}\right) = \frac{\pi m^2}{g}\left(\frac{x'}{l'^3} - \frac{x''}{l''^3}\right), \tag{1441.11}$$

where g was a physical constant indicating the density of circuits in the solenoid (pp. 152-154). In one of the notes added to the *Précis* version of the paper, he also gave this and the other components in terms of Cartesian coordinates as

$$A = \tfrac{1}{2}\, ii' \int_{\mathscr{C}} ((y - y')\, dz - (z - z')\, dy)/r^{n+1} \quad (\text{and } n = 2), \tag{1441.12}$$

and then calculated the moments of these forces about the axes (*1824f*, 44-48).

Ampère must have seen his work reaching some cumulation. To the pamphlet version of his 'Methodical exposition' he wrote a 16-page 'Addition' *1824b*, summarising in prosodic form all the main results from Savary's first paper up to this latest one of his own. And in the *Précis* pamphlet version of his *1824a*, one of the new passages was a note delineating his view of the electrical view of the phenomena, including magnetism (*1824c*, 49-63). 'It is this mania to multiply, as one says, entities without necessity', he wrote in reductionistic vein (p. 60),

> which has let admit into physics, for some time, a luminous fluid distinct from that to which one would attribute the phenomena of heat; it is that [mania] which still inclines [one] to suppose two magnetic fluids different from two electric fluids, although it be shown that electricity, in moving around particles of magnetised bodies precisely as it moves in the Voltaic conductor, and in exerting the same action in consequence, must necessarily produce effects completely identical to those which one attributes to that which one calls *molecules of austral fluid and of boreal fluid.*

14.4.2 Biot and Savart on electromagnetism, 1824

A principal target of Ampère's remarks was, of course, Biot; indeed, earlier in the note he had referred to Biot's *Académie* papers of October and December 1820 and quoted from the second (1821) edition of Biot's *Précis* (§ 14.2.4). In fact, during 1824 Biot and Savart at last made an extended statement of their position and experimental results in a 70-page article appended to the *third* edition of the *Précis*. I cite it as Biot and Savart *1824a*.

Their title, 'On the magnetisation impressed on metals by electricity in motion', shows that they had confined themselves to electromagnetism. They described Oersted's and their own experiments, and from the latter produced tables of data, usually concerning the time N of oscillation of the magnetic needle. One table compared pairs of 'observed forces', as represented by N^2, as a function of the distance D of the straight line from the needle, according to the formula (754.1) for oscillation

$$N_1/N_2 = D_1^2/D_2^2 ; \tag{1442.1}$$

but the ratios are in error to a few per cent. The next table compared the observed values of N_2 against the values calculated from (1442.1) using the data for N_1, D_1 and D_2 (Biot and Savart *1824a*, 719-720).

They confined their mathematical account to a long footnote (pp. 726-730). Figure 1442.1 (which they missed out of the plates) shows the needle AB turning in the plane through the perpendicular to the wire of section F under the forces AD and BD′ acting at the poles perpendicular to FA and FB respectively.[1]) Acc-

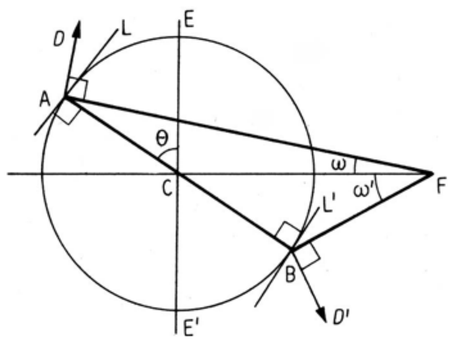


Figure 1442.1 Biot and Savart on the rotation of a magnet AD under influence from a wire F (*1824a*, after fig. 14)

ording to the inverse distance law, which they had established experimentally in 1820, these forces were given by K/R and K/R' respectively (they actually wrote 'Δ' and 'Δ'' for these distances), where K was a magnetic constant. Then, by proceeding trigonometrically in the way that Biot and Humboldt had used in 1804 for terrestrial magnetism (Figure 754.1), they found that the components A' and

[1]) We recall from § 14.2.4 that Biot's theory of electromagnetic action was different from Ampère's, in which action was central (along AF and BF). In the paper just described, Ampère had outlined the derivation of (1441.1) given a construal of a 'complicated phenomenon' which happened to be theirs! (*1824a*, 143-144).

B' of these forces along the tangents AL and BL′ were given by

$$A' = \frac{K(r - D\sin\Theta)}{D^2 + r^2 - 2Dr\sin\Theta} \quad \text{and} \quad B' = \frac{K(r + D\sin\Theta)}{D^2 + r^2 + 2Dr\sin\Theta}. \qquad (1442.2)$$

They went on to discuss their form, especially their variation about the common mean value $(Kr/(D^2 + r^2))$, corresponding to the equilibrium position ECE′ of the needle (where $\Theta = 0$).

One point of difficulty concerned their false law claiming that the total action of a V-shaped wire at the point on the exterior axis was proportional to the angle $2i$ between the branches. We saw at (1424.3) and after (1432.14) that Ampère and especially Savary had criticised this law, Savary producing the correct evaluation (1424.2) of $\tan\frac{1}{2}i$. Biot recalled that 'some first tries' had suggested to him the proportionality law: 'But, due to the imperfection of the experiments, other laws could equally be admitted, and one would be able, for example, to substitute [...] $\tan\frac{1}{2}i$' (p. 742). So, by a lucky shot, he came to try the unmentioned Savary's evaluation of his own integral as an alternative theory, and reported good correlations with data (pp. 744-746).

Biot and Savart's essay was of value for putting some numerical data forth from Paris in the growing international discussion of electromagnetism and electrodynamics; it also gave a much needed account of their efforts. As a contribution to theory as such, however, it was a modest affair.

In the autumn of 1824 Ampère came to an ironic professional relationship with Biot; for in the curious election described in § 13.6.2, he was appointed to the chair in experimental and general physics at the *Collège de France*, where Biot had long held the other physics chair. (Ampère's teaching course was to cover many parts of physics, in a two-year cycle, dealing both with "visible" and intermolecular effects.[1]) His situation there became still more curious during the following year when, while he was absent from Paris on a *Université* tour of inspection, Biot's follower Savart was appointed *préparateur*, or laboratory technician, for physics at the *Collège* (following a family tradition outlined in § 14.2.4!); and he ran a rigid system in the use of the laboratories.[2]) In addition, Ampère was being pressed urgently by repeated requests from the *Ecole Polytechnique* to write up his calculus lectures, as we saw in § 11.6.2. Understandably, his next major researches in electrodynamics did not come to fulfilment until 1825; and his chief new challenge came not from Biot but from Poisson, who produced in 1824 important studies in magnetism.

---

[1]) See the report in *Le globe*, 4 (1826), 240; and *AS*, *Ampère*, esp. chs. 211-213 (used already in the footnote after (1426.15)). Ampère's *CF* course is noted further in § 18.2.7.

[2]) See Colladon's reminiscences in *1893a*, 139-140; and compare his letter to Comberousse of 24. 6. 1887, where Savart was described as 'un ours mal teché, dans toute la force du terme' (*Bibliothèque Publique et Universitaire* (Geneva), ms. 3743, fols. 261-266).

14.4.3 Poisson on magnetism, 1824

Poisson's interest in magnetism seems to have begun around August 1823, for he presented his results at the *Bureau des Longitudes* quite regularly from then on (*BL*, *5*, meeting minutes). Then he wrote them up in the form of two papers which he laid before the *Académie* in February and December 1824 (*PV*, *8*, 14, 28, 165): they both appeared in the *Mémoires* volume for 1821-1822 (*sic*) as *1826b* and *1826c*, 138 pages in all. Lengthy prosodic summaries of each were placed in Arago's *Annales* as *1824c* and *1825a* respectively; much of the first came out also in Ferrusac's *Bulletin* as *1824e*, while the second was itself summarised there as *1825c*. In addition, Francoeur *1824b* described the first paper to the *Société Philomatique*. Thus Poisson gained once again the degree of publicity to which he was accustomed.

Keeping his motivations to himself, as usual, Poisson began with a survey of current knowledge of electricity and magnetism. Without naming anyone, he spoke against Ampère in asserting that 'the identity of the magnetic fluid and of the electric fluid does not necessarily follow from the important facts which have recently been discovered. Happily the solution of this question does not at all bear upon the objective of this Memoir', of which the 'aim is simply to determine the resultant [forces] of their attractions and repulsions, and, if it is possible, how they are distributed in bodies' (*1824c*, 250).

In other words, Poisson attempted to emulate for magnetism his exercises on electrostatics of a dozen years earlier (§ 7.6). However, the difference between the two classes of phenomena precluded him from carrying out a close imitation. For him a magnetisable body $\mathscr{A}$ contained infinitesimally small '*magnetic elements*' $\mathscr{E}$ separated ordinarily by small 'intervals inaccessible to magnetism' and accumulating layers of boreal and/or austral fluid when the '*coercive* force' of magnetisation was at work (pp. 262-263). Accordingly, setting up a Cartesian axis system Oxyz in space, he considered the action on a point M(x, y, z) inside or outside $\mathscr{A}$. He then took a point C(x', y', z') inside some $\mathscr{E}$ and distant ϱ from M. If the volume of $\mathscr{E}$ were h^3, it could be replaced by a cube $\mathscr{E}$ of side h oriented along the axis system; at any point M′ of its surface, given coordinates $(h\chi, h\xi, h\zeta)$ relative to C, was to be found a 'magnetic layer' of depth ε which was small relative even to h. 'Let us call h^2ds the differential element of this surface' there; then '$h^2\varepsilon\, ds$ will be the element of volume of the magnetic layer in this point M″' (p. 264), despite its five spatial dimensions!

Upon this dimensional marsh of points, surfaces and volumes Poisson built his theory of inter-elemental magnetic action. The difference between the quantities of boreal and austral fluids at M′ was represented by a positive or negative dimensionless factor μ; in the absence of a coercive force there was fluid balance over $\mathscr{C}$, a condition expressed by the integral over the surface $\mathscr{S}$ of $\mathscr{C}$:

$$\int_{\mathscr{S}} \mu\varepsilon \, ds = 0 \tag{1443.1}$$

(p. 264: I follow him in writing the surface integral as a single integral). Further, assuming Coulomb's inverse-square law (§ 7.5.2), the component of the action between M and M′ along (for example) Ox was given by

$$\mu h^2 \varepsilon \, ds (1/\varrho^1)_x, \quad \text{where} \quad \varrho_1 := \mathrm{MM}'; \tag{1443.2}$$

and since M′ was near to C, then by Taylor's theorem

$$1/\varrho_1 = 1/\varrho + \sum_{x'} h\chi\,(1/\varrho)_{x'} + O(h^2). \tag{1443.3}$$

Multiplying (1443.3) by $\mu h^2 \varepsilon ds$ and integrating over $\mathscr{C}$ gave for the component λ of the action between M and $\mathscr{C}$ the expression

$$\lambda := \int_{\mathscr{S}} \mu h^2 \varepsilon (1/\varrho_1)_{x'} \, ds = h^3 q_x, \tag{1443.4}$$

where the "potential" q was defined via auxiliary constants $\{\alpha'\}$ by

$$q := \sum_{x'} \alpha' \,(1/\varrho)_{x'}, \quad \text{with} \quad \alpha' := \int_{\mathscr{S}} \chi\mu\varepsilon \, ds,$$
$$\beta' := \int_{\mathscr{S}} \xi\mu\varepsilon \, ds \quad \text{and} \quad \gamma' := \int_{\mathscr{S}} \zeta\mu\varepsilon \, ds. \tag{1443.5}$$

Similar expressions to $(1443.4)_3$ pertained to the components along Oy and Oz (pp. 264–266).

Poisson now in effect sought the resultant of the quantities in $(1443.5)_{2\text{-}4}$ by rotating the axes so that two of them (β' and γ', say) were zero, leaving α' as the resultant δ (boreal or austral in nature). If its direction made angles a, b and c with the original axes, then

$$\alpha' = \delta \cos a, \beta' = \delta \cos b, \gamma' = \delta \cos c \quad \text{and} \quad \delta^2 = \alpha'^2 + \beta'^2 + \gamma'^2. \tag{1443.6}$$

Similarly, if CM made angles l, l' and l'' with these axes, then

$$x - x' = \varrho \cos l, \quad y - y' = \varrho \cos l' \quad \text{and} \quad z - z' = \varrho \cos l'', \tag{1443.7}$$

$$\text{where} \quad \varrho^2 = (x - x')^2 + (y - y')^2 + (z - z')^2. \tag{1443.8}$$

Then, by the standard differentiations of (1443.8) with respect to $\{x\}$ and $\{x'\}$

and using (1443.5), (1443.4) became

$$\lambda = h^3 \left[\sum_{x'} \delta \cos a\,(1/\varrho)_{x'}\right]_x = \left[(h^3\delta/\varrho^3) \sum_{x'} (x - x') \cos a\right]_x \tag{1443.9}$$

$$= -(h^3\delta/\varrho^3)\,(3 \cos i \cos l - \cos a), \tag{1443.10}$$

where i was the dihedral angle between the planes defined by (1443.6-7):

$$\cos i = \cos a \cos l + \cos b \cos l' + \cos c \cos l''. \tag{1443.11}$$

Similar expressions applied in the directions Oy and Oz, using angles l' and b, and l'' and c, for l and a in (1443.10) (pp. 266-268).

The last stages of Poisson's reasoning are strongly reminiscent of Ampère's use of the director plane at (1441.4). However, although Ampère's paper was presented to the *Académie* several weeks earlier it was not published, even in short form, until after Poisson's summary *1824c* was out in Arago's *Annales*, and one cannot assume that Poisson knew of its contents: further, Poisson was of course quite capable of thinking out such moves for himself. We shall see Ampère's reaction to (1443.10) in § 14.4.6; first, a summary of Poisson's main concerns is in order.

The rest of this first part ran through the potential theory and related results appropriate to the theory of magnetism proposed. If M lay outside $\mathscr{A}$, then the 'action' of $\mathscr{A}$ on M was given by volume integrals over $\mathscr{A}$ of (1443.4) and its companions; if M lay inside an $\mathscr{E}$ in $\mathscr{A}$, then these forces were supplemented by surface integrals of $\mathscr{E}$ on M; if M lay in $\mathscr{A}$ but outside all $\mathscr{E}$s, then it was in equilibrium (pp. 270-276). To find conditions of equilibrium of the fluids in $\mathscr{A}$ when under both internal and external magnetic influence, he found the principal axes of the quadric surface representing the action (pp. 278-283) by methods similar to those used to find the principal axes of inertia of a solid body (and presented in his own *Mechanics*$_1$ (1811), *2*, 94-97, for example). He then expressed these actions, and related expressions, in various forms by using touches of potential theory and properties of the Legendre functions (citing his recent *1823h* (§ 11.4.6) for the latter). In broad terms, viewed against the programme of Laplacian physics—which for him was still alive—his results resembled those of his papers on electrostatics of a dozen years earlier (§ 7.6): while he could not produce a mathematical theory of inter-molecular activity, at least he used a molecularist theory of the constitution of magnetic matter.

14.4.4 Poisson's 'simplification' of his formulae: a general divergence theorem

The second part of Poisson's *1826b*, called 'Simplification of the preceding Formulae', contained a major result—indeed, one of greater significance than the title of the part in which it was presented would suggest. The formulae involved as the starting-point were the volume integrals of (1443.4), when taken to express the action of $\mathscr{A}$ on an exterior point M. Taking the components of the force as x-, y- and z-derivatives of

$$Q := \iiint_{\mathscr{A}} [(1/\varrho)_{x'}\, \alpha' + (1/\varrho)_{y'}\, \beta' + (1/\varrho)_{z'}\, \gamma']\, k'\, dx'\, dy'\, dz' \qquad (1444.1)$$

(p. 294), where k' was the density of 'magnetic elements' around (x', y', z') (p. 269), he invoked integration by parts to convert Q to

$$Q = \iiint_{\mathscr{A}} \sum_{x'} (\alpha' k'/\varrho)_{x'}\, dx'\, dy'\, dz' - \iiint_{\mathscr{A}} \left(\sum_{x'} (\alpha' k')_{x'}/\varrho\right) dx'\, dy'\, dz'. \quad (1444.2)$$

He now simplified the first term by taking $\mathscr{A}$ to be a convex body and dividing it into two parts by the largest section parallel to the Oxy-plane. He considered an arbitrary vertical line in $\mathscr{A}$, which cut its surface at points U and L, where the normal to $\mathscr{A}$ made angle n' with the upward vertical (acute at U, obtuse at L). Thus the surface element $d\omega'$ there was related to its Oxy projection by the equation

$$\text{'}dx'\, dy' = \pm \cos n'\, d\omega'\text{'}, \qquad (1444.3)$$

and the z' term in the first term of (1444.2) was converted as follows:

$$\iiint_{\mathscr{A}} (\gamma' k'/\varrho)_{z'}\, dx'\, dy'\, dz' = \iint_{\mathscr{S}} [\gamma' k'/\varrho]_{z_L}^{z_U}\, dx'\, dy' = \iint_{\mathscr{S}} ((\gamma' k' \cos n')/\varrho)\, d\omega', \quad (1444.4)$$

where z_U and z_L were the values of z at U and L respectively.[1]) The other two parts of the first term could be treated similarly, producing on p. 296 the equation

$$\begin{aligned} &\iiint_{\mathscr{A}} [(\alpha' k'/\varrho')_{x'} + (\beta' k'/\varrho')_{y'} + (\gamma' k'/\varrho')_{z'}]\, dx\, dy\, dz \\ &= \iint_{\mathscr{S}} (\alpha' \cos l' + \beta' \cos m' + \gamma' \cos n')\, (k'/\varrho)\, d\omega'. \end{aligned} \qquad (1444.5)$$

[1]) These are my notations (with 'U' for 'upper' and 'L' for 'lower'): Poisson wrote the values of the functions as '$\left[\frac{\gamma' k'}{\varrho}\right]$' and '$\left(\frac{\gamma' k'}{\varrho}\right)$' respectively. These symbols recall his notations for the Lagrange and Poisson brackets (§ 6.3), although of course they carry a completely different reference. '$\mathscr{S}$' is now the surface of $\mathscr{A}$.

This is an early and important case of a general divergence theorem, and one wonders as to its possible sources. Two precedents come to mind: Lagrange's conversion of volume to surface integrals in connection with solving the equations of three-dimensional sound propagation (*1762e*, art. 45) and Gauss's proof of the zero value of a surface integral of form $(1444.5)_2$ in a study of the attraction of spheroids (*1813b*, arts. 3-4). However, neither of these results drew on integration by parts, and of course Poisson was capable of obtaining (1444.5) by his own efforts. The nearest (but still rather distant) result in his earlier work was his 1820 transformation (1042.1) of double integrals into single integrals. Saigey *1830a* appreciated (1444.5) in Ferrusac's *Bulletin.*

Poisson did not raise (1444.5) to the central place in his theory that our hindsight of potential theory would suggest to us. But he noted in passing that it could apply if $\mathscr{A}$ admitted more than two points of intersection with straight lines (p. 294), and he sought at once the analogue to (1444.2) when M lay within $\mathscr{A}$. Curiously, he did not invoke his distinction between points within and without an 'element' $\mathscr{E}$ of $\mathscr{A}$ or his double integrals found on p. 276 for the former case, but instead worked on volume integrals and modified them along the lines of his 1813 derivation of Poisson's equation (651.8) by setting a tiny sphere around M and showing that the action (X, Y, Z) were given now not as derivatives of Q but by

$$X = Q_x - 4\pi k\alpha/3\,, \qquad (1444.6)$$

with similar expressions for Y and Z, where k was the value of k' at M (pp. 296-298).

From then on Poisson could proceed to study magnetic equilibrium in terms of differential equations of the type used in mechanics and elsewhere (Laplace's equation, Legendre functions and the like): (1444.5) was used at times to ease the integration of some expressions. As in his first paper on electrostatics (§ 7.6.4), he applied his results to spheres. On p. 317 he cited his own recent short paper *1824b* on the electrostatics of thin hollow spheres,[1]) for a comparison between magnetic and electrical action on an exterior point of bodies placed inside a hollow sphere. In his second, December, paper he imitated other parts of his

---

[1]) While Poisson's main paper *1826b* on magnetism is dated as of February 1824, this piece *1824b* on electrostatics was sent to the *SP* in April. Most of it was reprinted as *1824f* in Ferrusac's *Bulletin.*

In July 1824 Poisson examined, with Cauchy, Lacroix and others, a *PF* doctoral thesis on electrostatics by the *normalien* H. (Veron-)Vernier (*AN*, AJ[16] 5323, no. 99). Vernier *1824a* made some modest extensions to Poisson's integral solutions (764.24) for two spheres by studying distribution across three spheres. At this time Vernier was *Professeur* of mathematics at the *Collège Royal* at Caen: soon afterwards he moved to the *Collège Louis-le-Grand* in Paris, where one of his students was Galois (§ 18.2.3). Later he became a popular writer of mathematics textbooks.

first paper on electrostatics by repeating for spheroids his results just obtained for spheres (*1826c*, 492-521).

The rest of both of these papers related to the earth's magnetic field: they seem to have been partly inspired by the publication in 1823 of a study of magnetism by P. Barlow, professor at the Royal Military College at Greenwich.[1]) One of Barlow's concerns was the correction of compasses in iron ships by the placing of metal spheres in their vicinity, and Poisson applied his calculations to this case (*1826b*, 331-338; *1826c*, 521-533). The question of ship magnetism was to arise for him again, although not for another 12 years, as we note in § 18.3.2. In the meantime, we must describe Ampère's reactions to these results found for magnetism in general.

14.4.5 Ampère's monograph on 'electrodynamic phenomena', 1826

My three great things, the attractions, the projections, the areas.

Ampère (*AS*, *Ampère*, ch. 172:
quoted in Joubert *Electrodynamics*, *2* (1887), 37).

Ampère did not return to publishing on electrodynamics until the autumn of 1825. He presented a paper to the *Académie* in September (*PV*, *8*, 282), which appeared in two parts in Arago's *Annales* as *1825c*. He reported a new equilibrium experiment, in which a circular are AA′ remained stationary when current passed through it and down the radii to the centre G of the arc and then in an arbitrary closed circuit, thus showing that the arc received no tangential component of force from the circuit. This experiment replaced an earlier one on the null action between two curved wires, in that its mathematicisation produced the same first relation (1426.15) between n and k (p. 387); and Ampère felt happier with this one, despite several difficulties concerning friction and balance of the equipment. He then went on to a more detailed analysis than had been executed hitherto on the action on an open arc, drawing on integrals of his differential formula (1426.17) and finding evaluations in terms of the coordinates of the end points, with corollaries for closed circuits. Many of them repeated or extended results found by him and Savary in earlier work; but in a short paper presented to the

[1]) Barlow *1823a*: Poisson cited it in *1826b*, 331, and mentioned its results in *1826c*, 489-491. He may well have come across the book when Dupin presented a copy to the *AS* on 24.2.1823 (*PV*, *7*, 450). This was the meeting at which the reports on Savary and Demonferrand (§ 14.3.1) were communicated.

Académie in November (*PV*, *8*, 309) and published in a Belgian journal as *1826b*, he gave a prosodic account of some new consequences. The most interesting one stated that if three similar closed circuits, of dimensions in harmonic proportion ($1{:}p{:}p^2$) with each other, were set in the same plane in projective alignment, then the middle one would remain stationary only if inverse square attraction obtained. '[I]t is much to be hoped that this experiment be made with an instrument susceptible of all the precision that one can desire', he mused (p. 206); it was to be an unfulfilled wish.

By now Ampère must have felt that he had enough material to attempt a definitive statement of his theory, including the pertaining mathematics. The ensuing statement is his best remembered writing, a long essay which came out in November 1826 as a book from Méquignon-Marvis (the publisher of the Riffault volume containing Babinet's essay described in § 14.2.5) and a few months later in the *Mémoires* of the *Académie*. The main text was reproduced unchanged, but the four appendicial notes of the *Mémoires* version had been revised and expanded into five for the book (which appeared first). Out of respect for this double publication, I shall refer to the work as a monograph, and cite it as Ampère *1826e*.

Table 1445.1 outlines its contents, my division into sections closely following the contents list which Ampère put into the book version. It shows well his inability to follow Cauchy or Poisson, say (or his follower Babinet, for that matter) and give a systematic presentation, divided into numbered sections; instead a rather impromptu sequence of ideas and results was culled out of his earlier writings. In his title he mentioned six of his *Académie* presentations,[1] but other papers (and manuscripts) were involved in it too. The table shows that many of its parts had already been worked out in essentials in earlier papers (a few passages are in fact almost *verbatim* transcriptions); and while the new details and cases are often interesting, there is space here only to note some principal novelties and features.

[1]) Typically, two dates are wrong: 26. (not 20.) 12. 1820; and 28. (not 21.) 11. 1825. The closest relevant publications to the dates given are *1820g*, *1820c*, *1822a*, *1824a*, *1825c* and *1826b*.

I have to cite the monograph by page numbers, and I use those of the book version: for the *Mémoires* printing of the main text, add 172. In 1883 the *Mémoires* version was reset and published by Hermann, and in 1958 photoreprinted by Blanchard (without indication of source or version). It repeats the original text to the extent even of preserving some of the *Mémoires* page numbers in cross-references, and includes the original plates at the end: its page numbers are found roughly by taking $\frac{3}{4}$ of the original numbers. In 1887 Joubert reprinted the book version in his collection, incorporating into the text redrawn (and occasionally variant) versions of the figures: his page numbers are around $\frac{9}{10}$ of the originals.

Tricker *1965a*, 155-200 contains a (rather free) translation of pp. 3-37, 94-110, 113-118 *passim*, 131-132. However, not all the breaks in translation are indicated, Ampère's footnote of pp. 13-14 is silently transferred to his p. 107, and a passage on pp. 197-198 is quietly made into a footnote to p. 132!

Table 1445.1 Schematic outline of Ampère's monograph *1826e*

The third column indicates the *first* earlier paper(s), if any, containing similar text: the presentation in the monograph is often more detailed in some way or another.

Page	Description	Textual remarks
3	Phenomena, '*expérience*', hypothesis	*1822a*, 293-300 *passim*
13	Four cases of equilibrium	*1822a*, 300-303; *1825c*, 381-384; *1825a*; *1826b*
27	Two elements: trigonometic and calculus forms (1423.1), (1426.2); (1426.15) between n and k	*1820g*; *1822a*, 305-315; *1822b*
40	Closed circuit, director plane, etc.	*1824a*, 136-148 *passim*
50	Element/small circle: force component (1441.10), etc.	*1824a*, 148-150
57	Two coplanar circuits: $k = -\frac{1}{2}$	*1825a*, 208-210
65	Two straight wires: integral (1426.17) for element/circuit, etc.	*1822c*, art. 11; *1825c*, 389-402, 36-40 [from each part]
94	Solenoid/element and -/solenoid	Reworking of *1824a*, 151-159, 246-247
105	Solenoids as magnets: Biot, Oersted	*1824a*, 247-250; *1824c*, 49-57, 63-67; *1824a*, 255-258
131	Pole/circuit and -/shell	Reworking of *1825e*
151	Three hypotheses on electromagnetism	Further developed in *1827b* (§ 14.4.7)
157	Impossibility of perpetual motion: Biot's errors, Faraday's experiments, etc.	
188	Magnetic shell: Biot and Savart, Poisson	
196	Arago's wheel experiment	See § 14.5.2
198	Aether, fluids, modes of action	
202	Note 1: derivation of (1426.2)	*1822a*, 305-308; *1822b*
206	Note 2: simplification of (1426.2)	
207	Note 3: use of note 2 for two straight wires	Only in book version
214	Note 4: properties of directix; (1441.10)	
216	Note 5: Pole/V- and -rhombic wires: tan $\frac{1}{2}i$ result (1424.2), etc.	
223	Table of contents	Only in book version: ends on p. 226

The main clause of the title is worth noting. Firstly, the *Mémoires* version was called 'Mathematical theory of electrodynamic phenomena solely deduced from experience [*expérience*]', but the word 'mathematical' was omitted from the book title. Secondly, the word '*expérience*' in French is ambiguous between 'experience' and 'experiment' (see the quotation from Ampère at the head of § 14.3.1 for a good example). Ampère stressed experience, for he gave a phenomenological account of the phenomena (pp. 3–12, 198); in making an analogy, he even claimed that 'The theory of heat really rests upon general facts given immediately by observation' (p. 4), which, apart from the optimistic 'immediately', would have suited Fourier (§ 12.5.3). He also described some of his experiments, but confessed at the end (p. 201) that 'I have not yet had the time to construct the instruments' required for the three-circuits experiment proposed in 1825 (and described on pp. 25–26 here). His philosophy of theories (§ 1.3.5) was to treat them as truths rather than hypotheses (recall the quotation at the end of § 14.2.6); in this view experiments take a relatively minor role, as confirmations rather than tests. His later book on philosophy (§ 18.2.7) affirms this viewpoint.

In contrast to the concurrent researches of those genuine empiricists Coriolis and Poncelet (§ 16.4), Ampère made some but not much use of the conservation of *forces vives* (see p. 193), and only spoke in passing of 'the energy of the pile' (p. 50). Instead, he backed his position by strong appeals to (his conception of) Newton's philosophy of physics, which also served as inspiration for his theory of the action (and reaction) of inter-elemental forces. However, his empiricism was tempered in various ways; for example, he saw these forces as transmitted by means of an inertia-free aether, a fluid which, according to his conception of the de- and recomposition of the two electric fluids (p. 128), 'can only be that which results from the combination of the two electricities' (p. 113). This is the closest that he ever came to an explanation of how the central action took place between infinitesimal elements.

Ampère's loyalty to Newtonian principles unfortunately forbad him from following Poinsot (§ 6.2.1) and admitting couples in his theory; indeed, he regarded 'a primitive couple' in electrodynamics as a 'singular hypothesis' which was 'absolutely useless for the explanation of the observed facts' (p. 123: compare p. 179), 'a false interpretation of these facts' that had been adopted by certain unnamed physicists (p. 124). Biot was his principal target; in various places he criticised Biot's theory, and even wrote sarcastically of Biot's error (1424.2) over the action on a pole of a V-shaped wire (p. 217).

14.4.6 Ampère's translation of Poisson's theory of magnetism

By contrast, Ampère took very seriously Poisson's recent essay on magnetism, for he had to express its conclusions within his own electrical theory of magnetism. In an early footnote he noted briefly the concordance of his and Poisson's views (p. 13), and near the end recorded some of the details (p. 194–196). To effect this agreement, he presented in between some remarkable results on line and surface integrals, in general kinship with Poisson's divergence theorem (1444.5) relating surface and volume integrals.

Ampère's first theorem constituted 'a new manner of considering the action of plane circuits of any form and size'. He partitioned the interior of this circuit $\mathscr{C}$ into a lattice of infinitesimal rectangles $\mathscr{R}$ and regarded a current around $\mathscr{C}$ as equivalent to the sum of the currents around each $\mathscr{R}$ (p. 56). To us this suggests theorems on complex integration, and thus Cauchy's booklet *1825b* of the previous year comes to mind; yet we saw in § 11.5.7 that Cauchy did not speak of contours and usually avoided the geometrical *Denkweise* altogether, while here Ampère never used complex variables or even numbers, so that influence from one *professeur* at the *Ecole Polytechnique* to the other does not seem to apply. Indeed, Ampère himself did not perceive the generality of his approach, for he applied it only to the special cases of the action (1441.7) of a circuit on an element, and between two tiny coplanar circuits (pp. 57-58).

A still more interesting theorem occurred later when Ampère extended this latter case to two arbitrary plane circuits s and s'. After finding contour integrals of the form (1441.1) for the Cartesian components of one circuit on an element of the other (pp. 133-139), 'Let us now imagine two surfaces taken at will σ, σ', bounded by the two contours s, s' [...] and on these surfaces infinitely thin layers of the same magnetic fluid which is held there by a coercive force sufficient for it not to be displaced from there' (p. 139). Taking elements of surface $d^2\sigma$ and $d^2\sigma'$ distant r apart and with respective coordinates (x, y, z) and (x', y', z') and depths ε and ε' of fluid, the action between them was given by

$$-\mu\varepsilon\varepsilon'\, d^2\sigma\, d^2\sigma'/r^2, \tag{1446.1}$$

where μ was a scaling constant clarifiying the units and dimensions of the expression (contrast Poisson before (1443.1)). He now took a new surface (σ_1, say) lying very close to σ, and also bounded by s and containing a deposit of the magnetic fluid opposite to that on σ. By a transformation similar to Poisson's at (1443.9), he found that the Ox component of the action between $d^2\sigma'$ at m$'$ and $d^2\sigma$ at m, from σ to σ_1, was given by

$$\mu\mu' d^2\sigma\, d^2\sigma'\, h\cos\xi\left(\frac{3(x-x')}{r^4}\frac{\delta r}{\delta x}-\frac{1}{r^3}\right)+O(h^2), \tag{1446.2}$$

where 'δ' indicated the variation caused by passing from σ to σ_1 and h was the (small) length of the normal to σ_1 from σ at m, making an angle ξ with Ox (pp. 139-141). Similar expressions applied for Oy and Oz.

'We are now going to determine the form of the position of the element $d^2\sigma$' (p. 141). For the purpose Ampère offered Figure 1446.1, in which $d^2\sigma$ and $d^2\sigma'$ were located respectively at m and m$'$, and the first surface σ was divided into 'an

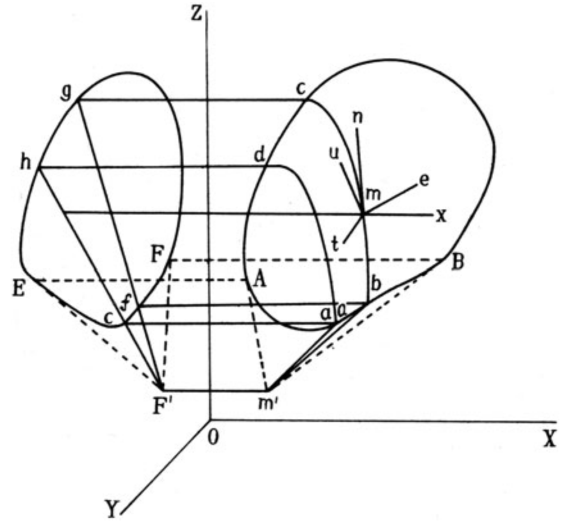


Figure 1446.1 Ampère's diagram for his surface-contour theorem (*1826e*, his fig. 42)

infinity of infinitely narrow zones' abcd by planes normal to be Oyz-plane and containing m′p′. Perhaps by generalising upon his projection of the circle in Figure 1441.1, he then projected abcd onto the Oyz-plane, producing there the sector cfgk. Setting up polar coordinates (r, u, φ) for m relative to p′ as local origin and m′p′ as polar line, he found that

$$d^2\sigma = u\, du\, d\varphi; \tag{1446.3}$$

and by straightforward calculus he obtained relations such as

$$r\,\delta r = (x - x')\,\delta x + u\,\delta u. \tag{1446.4}$$

Thus the form of (1446.2) was transformed on p. 143 as follows:

$$\mu h\varepsilon\varepsilon' du\, d\varphi \left(\frac{3(r-u)}{r^4} - \frac{1}{r^3}\right) = \mu h\varepsilon\varepsilon'\, d\varphi\, d\left(\frac{u^2}{r^3}\right). \tag{1446.5}$$

Equation (1446.5) was the key to Ampère's analysis, since the left hand side had led to the Poissonian (1446.2) while the right hand side contained the integrand of his line integral (1441.1) for action between element and wire; and various integrations developed the link. For example, carrying out the (u^2/r^3)-integration gave the Ox-component of the force acting between $d^2\sigma'$ and the portion of the shell between σ and σ_1 bounded by planes mad and mbc containing the angle $d\varphi$; thus φ-integration gare the Ox-component of the total force X exerted upon $d^2\sigma'$ at m′ by the shell:

$$X = \mu g \varepsilon d^2\sigma' \int (u^2/r^3)\, d\varphi, \quad \text{where} \quad g := h\varepsilon, \tag{1446.6}$$

the assumedly constant volume of the electrical fluid on the surface, and where the limits of integration were taken on the contour s. If the other components were Y and Z (and determined as above by projecting σ onto the Oxz- and Oxy-planes respectively), then the Ox-component of the resultant force of the shell upon m′ was given by $(Yz' - Zy')$ (pp. 145-147).

While Ampère referred in this passage to the applicability of this analysis to (Poissonian) talk of boreal and austral fluids on σ and σ_1 respectively, for some reason he did not fully address Poisson's competing theory until a later section of his monograph. After recalling the form of (1446.6), he repeated his dissection of the shell on a second shell bounded by surfaces σ' and σ_1' with common contour s', with m on the first one serving as local origin. Taking polar coordinates (u, ψ) and (w, χ) in the mxz- and mxy-planes respectively, like (u, φ) in (1446.3), he evaluated the components of force of the second shell upon an element ds of the contour s of the first one. That for Ox read

$$\mu\, gg' \left[dy \int_{s'} (w^2/r^3)\, d'\psi - dz \int_{s'} (v^2/r^3)\, d'\chi \right], \quad \text{where} \quad g' := h'\varepsilon', \tag{1446.7}$$

the constant corresponding to g in $(1446.6)_2$, and 'd'' indicated differentiation with respect to variables on the second shell (pp. 188-193): the full component was obtained by integrating over s. But he had obtained $(1446.7)_1$ *already*, as a version in polar coordinates of his expression (1441.12) for the Ox-component of the action between an electrical element and a closed circuit (p. 139); here $\mu gg'$ served for the product $\frac{1}{2}\, ii'$ there of two steady currents i and i'.

Thus the link between Poisson's magnetic elements and Ampère's solenoids was basically made, and for two cases: 'that of a series of magnetic elements of the same intensity, uniformly distributed along a straight or curved line which surround all the little circuits of the solenoid'; and also when 'a solid and closed Voltaic circuit' was compared with 'magnetic elements of the same intensity, uniformly distributed on any surface bounded by this circuit, when the axes of the magnetic elements are everywhere normal to that surface' (pp. 194-195, where he also explicitly cited Poisson's analysis of a magnetic body (§ 14.4.3)). In a related derivation, given in note 4 on the directrix, he emulated Savary at (1432.15) by producing Bowditch's formula for magnetic dip.

14.4.7 Ampère on three theories of electricity and magnetism, 1826

This reconciliation of different physical models was of great importance to Ampère at this time. In the monograph he discussed three theories for electromagnetic action between an element AB and a pole M: forces acting along AM and BM; both forces passing through the midpoint O of AB; and a couple acting in the direction perpendicular to AB and to OM (pp. 151-156). In a long paper on this matter presented to the Belgium Academy in October 1826 and published by them as his *1827b*, he extended the discussion into a general appraisal of three hypotheses for electromagnetism. 'The first consists in admitting the existence of two fluids called austral and boreal', as thought Poisson (for example); 'But some vague and general insights should not suffice, for the explanation of the facts which comprise this new branch of Physics' (p. 225). 'The second is that by which I have given an account of the observed phenomena', where the magnet was treated as a solenoid (pp. 225-226). 'Finally the third hypothesis', supported by Biot, assumed 'a primitive elementary action', in which magnet and pole tended to turn each other, thus 'forming what Mr. Poinsot has called a couple'; while for Ampère this was apparently 'directly contrary to the first principles of Dynamics', it needed examination (p. 226). In order to further his comparison he presented again his surface-circuit theorem (1446.5) (pp. 227-231), and related the expressions both to Poisson's formula (1443.10) for magnetic action (pp. 231-233) and to his own (1441.1) for element/solenoid action (pp. 236-241). In the rest of the paper (pp. 248-274), which also appeared as *1828a* in Arago's *Annales*, he considered various cases of magnet/circuit action, noting all hypotheses but keeping his preference clearly evident.[1])

In this way Ampère brought to a close his main sequence of research publications on electricity and magnetism (although he continued to lecture on the subject, and demonstrate experiments, at the *Collège de France*). From the simple trigonometry of inter-elemental action to the calculus version, and extensions to circles, circuits and solenoids, he covered an impressive range of ideas and results, including an attempt to embrace magnetism under an electrical theory. While Parisian staples such as differential equations and potential theory were absent from his thought, and couples were banned for dubious reasons, he had still produced another major contribution to the mathematicisation of physics. And he had done so in a town not notably interested in his work: his chief fol-

[1]) Ampère's ideas on the character of electric currents themselves went through various changes at this time: from rectilinear to toroidal solenoids to the assumption of pre-existent currents. These changes naturally affected the form of his view of magnetism. On these developments, see Blondel *1982a*, 157-163.

lowers Babinet, Demonferrand and Savary were not major French figures. Further, like a rotational current, his course of publication followed a circle, from Brussels to Brussels: from *1820d* in the *Annales générales* to *1827b* with the Belgium Academy.

14.5 Poisson's further studies of magnetism, 1825–1826

14.5.1 Poisson on terrestrial magnetism

§ 14.4.5 began with a paper presented by Ampère to the *Académie* in November 1825: at the same meeting Poisson handed in his latest results on terrestrial magnetism (*PV*, *8*, 309). A prosodic version, Poisson *1825f*, appeared with Arago's *Annales* (with a note added by Arago) and with the *Société Philomatique*, who also badly printed a selection *1826a* of the sums, the full treatment appearing as a (short) paper *1825g* in the *Connaissance des temps*. Cournot wrote an extensive review *1826a* for Ferrusac's *Bulletin*.

Concerned to mathematicise the known variations in the strength of the magnetic field, Poisson proposed to suspend in equilibrium two saturated steel needles A and B such that their centres of gravity lay parallel to the line of magnetic inclination at the place on the earth in question, and to oscillate each one in the presence of the other and also on its own. This typically Poissonian exercise in parameters involved seven known quantities: for A, the periods t and Θ of oscillation on its own and near B respectively, and its moment of inertia m about the vertical; for B, the corresponding t', Θ' and m'; and the distance r between their centres. The unknowns were the densities μ and μ' of 'free fluid' (the excess of one magnetic fluid over the other, we recall from § 7.6.2) in A and B respectively; and f, a scaling constant of magnetic action, independent even of the materials and temperature(s) of A and B.

Poisson formed equations by treating the oscillations as those of a pendulum. When swung on its own, the standard expression ((652.1) with $f(\alpha) = 1$) gave

$$t = \pi\sqrt{m/(\varphi h)}\,, \quad \text{where} \quad h := \int_A \mu x\, dx, \tag{1451.1}$$

the magnetic moment of A, and φ the strength of the earth's magnetic field; when A swung in the presence of B, (1451.1) was modified to

$$\Theta = \pi\sqrt{m/(\varphi h + fq)}\,, \tag{1451.2}$$

where q was the total moment of magnetic action of B on A at the centre of gravity of A, given by the double integral

$$q = \int_{\mathrm{B}} \int_{\mathrm{A}} \frac{\mu\mu' x}{(r + x - x')^2} \, dx \, dx' . \tag{1451.3}$$

Eliminating h, $(1451.1_2$-$2)$ gave

$$fq = \frac{\pi^2}{m} \left(\frac{1}{\Theta^2} - \frac{1}{t^2} \right). \tag{1451.4}$$

By symmetry of A and B, the analysis could be repeated for parameters t', Θ', h' and q', the last two defined as the analogues for B of h and q (*1825g*, 322-324).

As μ and μ' were not known, q and q' could not be evaluated directly; so Poisson proposed to expand them in a power series in r^{-1}. Noting that the assumed symmetry of magnetism of A would eliminate each even power of r^{-1}, and that the constant term also vanished because of zero net fluid over A (as in (1443.1)), he found an expansion for q (which I render in summation notation) and converted (1451.4) to

$$f\left(hh' + \sum_{j=1}^{\infty} k_j r^{-2j} \right) = m\pi^2 r^3 \, (t^2 - \Theta^2)/2t^2\Theta^2 . \tag{1451.5}$$

A similar equation in $\{k_j'\}$ (for q'), m', t' and Θ' applied for B, leaving the constants on the left hand side as unknown. Now each of them, like m, m', t and t', depended on the magnets *as wholes*, and so as many of them could be calculated as was desired from experiments for different values of r. He took as example the case where r was large enough to work only with the first term fhh' on the left hand side; then $(1451.5_1, 1)$, and their mate equations for B, gave enough scope for elimination to obtain an expression for φ. Alternatively, taking fhh' as known from experiment to have the value ϱ^2, he multiplied $(1451.5)_1$ by its companion for B and obtained

$$\varphi^2 = F^2 f, \quad \text{where} \quad F := \pi^2 \sqrt{mm'}/(tt'\varrho) , \tag{1451.6}$$

a quantity which he held to be independent of A and B themselves (since $m \propto t^2$ and $m' \propto t'^2$: pp. 324-326). Hence the variation or constancy of φ could be determined by repeating experiments at the same place and at different times.

14.5.2 Poisson on Arago's wheel and the process of magnetisation

Poisson's ideas were suggestive; and in the brief note appended to the printing of Poisson *1825f* in his *Annales*, Arago *1825a* reported that he had proposed to the

Bureau des Longitudes on 16 November 1825 a method of giving steel needles a common degree of magnetisation. 'This procedure is based on the property which a magnetised needle possesses, when it is placed in the vicinity of a metallic plate turning about itself', he explained, '[... I] would not be able to indicate in *numbers*, with what precision the method is susceptible; but I will make it the subject of a special Paper if, as I hope, the experiments with which I am concerned give favourable results'.

Arago was referring to an experimental effect which is now known as 'Arago's wheel'. He told the *Académie* of it in July 1826 (*PV*, *8*, 399),[1]) when he also published as account *1826b* in his *Annales*: a short statement *1826a* was put out by the *Société Philomatique*. His discovery attracted much attention:[2]) in particular, Poisson was already interested, and performed his usual service at the next meeting of the *Académie* (*PV*, *8*, 402), with yet another enormous paper. It appeared as *1827d*, 130 pages of the *Mémoires* (in the same volume as Ampère's monograph, and in between Navier *1827b* on the motion of fluids (§ 15.2.7) and his own *1827e* on numerical integration (§ 11.6.5)); Arago's *Annales* took an 18-page prosodic summary *1826e*, which also appeared in the *Bibliothèque universelle*; Ferrusac's *Bulletin* put out a seven-page selection *1826f*; and a partly identical *résumé 1826g* was published by the *Société Philomatique*. This last version was prefaced by a few admiring remarks from Francoeur, the current mathematics editor, for whom the work was 'so new and of such a high importance that we must devote several articles to it', and belonged to 'the works of that nature, where there are such fundamental truths and so few words'.

The fresh mathematical task facing Poisson was to describe the process of magnetisation in moving bodies: thus it was a natural successor to his 1824 study of magnetic equilibrium (§ 14.4.3). He used the same model: discrete 'magnetic elements' in the body, the density k' of their presence at any given point, and μ' of excess of one fluid over another, the variation in both caused by the change in the 'coercive force' of magnetisation, and so on. He began his paper by recalling the principal features and equations of this theory (*1827d*, 446–454) but then made an interesting modification. In the previous study he had regarded as negligi-

[1]) I note here an *AS* event three weeks later, when Savary submitted a paper on magnetism (*PV*, *8*, 411). Arago, Ampère and Dulong were asked to report, but never did; however, the paper soon appeared in Arago's *Annales* as Savary *1827a*, with various summaries and translations elsewhere. He started out from Arago's discovery that magnetisation could be produced by electrical discharge (§ 14.2.4) and studied especially the degree of magnetisation of a needle as a function of its length and diameter, its distance from the discharging (straight or helical) wire, and other parameters (the mathematical aspect was presented on p. 8).

[2]) During 1825 Arago was awarded the Copley medal of the Royal Society for his studies of the magnetism of non-ferrous substances. On his work of this period, and Fresnel's interest in it, see Chappert *1978a*, 55–62.

ble the attraction of an 'magnetic element' 𝒟 of the body 𝒜 on a point M interior to it (p. 454), but now he found it to be of the same order of magnitude as the attractions of the rest of 𝒜 (p. 458). To prove this he applied his divergence theorem (1444.5) over 𝒟; and straightaway he treated this theorem, and the related potential theory, somewhat more systematically than before, defining the magnetic potential (as we would call it) of 𝒜 at M and showing that it satisfied

$$\nabla^2 V = 0, \quad \text{or} \quad -2k\pi, \quad \text{or} \quad -4k\pi \tag{1452.1}$$

(where k was the assumed constant value of k'), 'according as the point M will be situated outside, on the surface or within' 𝒜. '[I] was led to the third value, several years ago', he remarked, in recall of his 1813 derivation of Poisson's equation (651.8): 'I adjoin now the second', which he had missed at that time (p. 463).

Poisson's chief concern was the processes with these magnetic elements 𝒟 and the consequences for the magnetic profile of 𝒜: the 'magnetism in motion' of his title referred to its motion in these 𝒟s, not just of 𝒜 itself. For the purpose he proposed what I call a magnetisation function $f(t)$ of time t, 'which will be null when $t = 0$ and which will acquire a constant value after a certain interval of time' when saturation occurred. 'This time will be very short', but 'could be very different for these diverse substances; and it will depend, as will the form of *ft* [*sic*] of the material and of the temperature of [𝒜] at point M' (p. 467). Accordingly, he incorporated a term, or terms, involving $f(t)$ into the various basic equations of magnetic action and equilibrium.

The resulting calculations are typical Poisson: complicated in form, clever in passage work, confusing in notations,[1] interesting as mathematics. Some mathematical details reflected recent concerns: for example, as in his first paper on magnetism (*1826b*, 271), the negligible difference between infinitesimal sums and integrals (*1827d*, 452, 469), now treated in his recent paper on numerical integration (§ 11.6.5) published in the same volume; the convergence and divergence of power series, as in his studies of Fourier series (§ 11.4.1) and elsewhere; and the use of '∇' as a differential operator (p. 515) reminiscent even to the notation of Brisson (§ 4.3.5), over whose work he and Cauchy had quarrelled the year before (§ 11.5.7).

As happens sometimes with Poisson, his procedures are too complicated, even prolix, for easy summary. As an example, here briefly is his analysis of a hollow

[1] For example, the prime was used to denote both subscripts and differentiation (compare '$t' = dt$' and '$\frac{dft}{dt} = f't$' on pp. 469–470, for one case). As in *1826b*, Poisson used round and square brackets (*1827d*, 506); but he now employed '$\int\limits_a^b$' (p. 470).

homogeneous sphere $\mathcal{S}$ rotating uniformly about a horizontal axis (pp. 487-506). Taking the earth's magnetic field to be constant over the (short) time of magnetisation involved (p, 489), expressing the points M of attraction and M′ of magnetic activity in spherical polar coordinates, and expanding $(MM')^{-1}$ in a series of Legendre functions, he found on p. 489 a power-series expansion of the double integral form of the magnetic potential (1444.1) provided by the divergence theorem (p. 483); then he obtained an expression for the components of force caused by $\mathcal{S}$ at M. For experimental evidence he appealed once again to Barlow, this time Barlow's study *1825a* of the rotation of solid iron sphere (pp. 497-500), to found some corroborations there. Once again he showed his conscience for relating his formulae to known experimental data. Indeed, he reported that Barlow's work had motivated his researches (p. 442).

On 30 August 1826, according to his own date given there, Ampère noted Arago's wheel experiment near the end of his monograph on electrodynamics (*1826e*, 196). He recalled that, with the help of the young Swiss emigré Colladon (see Colladon *1893a*, 124-127) he had found the same effect of induction took place if the wheel rotated before a helical coil of wire: during September he told the *Académie* about it (*PV*, *8*, 429), and published a paper (Ampere *1826c*) in Ferrusac's *Bulletin*. 'This experiment completes the identity of effects produced, either by magnets, or by assemblies of solid and closed Voltaic circuits', he judged in his monograph (Ampere *1826e*, 196-197); however, he made no attempt to convert these calculations of Poisson (which had been presented to the *Académie* six weeks earlier) to his own approach. Thus the two traditions emanating from Poisson and from Ampère continued: electricity and magnetism doubtless had more intimate connections than had previously been thought, but their claimed 'identity' had not been demonstrated, least of all by Ampère!

14.6 Concluding comments

14.6.1 The isolated achievements of Ampère and Poisson

For the fourth time in this book we have arrived at the mid 1820s, and once again we find that the major figures have not only produced major work but also put it out in print. In this case Ampère has been by far the dominant personality, with a sudden entry into experimental physics (where he had no experience) and mathematicisation of the phenomena encountered. This latter aspect demanded of him skills in the calculus and vision of analogies from mechanics—and his earlier mathematical endeavours had trained him well enough to bring him triumph here. From the trigonometric expression (1423.1) for electrodynamic action through the differential form (1426.2) to the line integral (1441.1), and finally the shell-circuit theorem (Figure 1446.1), he showed his ability to obtain general expressions from which as host of special cases of interest could be developed; and in the course of this work he was something of a pioneer in the use of line and surface integrals, still rather unusual objects in the expanding world of the calculus. We can also give Poisson credit here, for his general divergence theorem (1444.5), even though he saw it only as a mathematical tool, a 'simplification of the preceding formulae'.

As physicists, the two men were far apart. Ampère's aetherian theory of electricity, and its status as the basis of magnetism, found no place in the approach of Poisson, who dealt only with magnetism (and electrostatics a dozen years earlier (§ 7.6)). Indeed, he never wrote a word on electrodynamics or electromagnetism—a strange fact about a man who wrote so much on almost everything else—but presumably he took friend Biot's position on the primitivity of the action between magnet and wire. On these various positions, see Bellone *1980a*, ch. 5.

Mention of Biot points to another feature common to this and our earlier stories: although he and Ampère had worked largely in different parts of this whole new range of phenomena, once again the non-Laplacian had triumphed in the competition. Ampère, indeed, had done as well as his lodger Fresnel before him.

Yet there is a difference between this story and its predecessors on optics, fluids and heat: while those topics attracted widespread attention among the Paris mathematicians and physicists, electricity and magnetism enjoyed far less esteem. As was noted in § 14.2.1, only Ampère took a *sustained* interest in the consequences of Oersted's discovery; while Poisson's work on magnetism was rather isolated within the corpus of his researches and did not attract even his own further interest for more than a decade (§ 18.3.2). So while both men had much in print by 1826 in these areas, did they gain many compatriot readers?

Some indications of attention at the educational level can be mentioned. As in § 12.6 for heat theory and § 13.6.1 for physical optics, a good guide is provided by the textbook on physics by Ampère's lodger Despretz, in its second edition of 1827. The total account of electricity and magnetism was just over 200 pages (*Physics*$_2$ (1827), 321-526); but the first 130 pages covered electricity in its various forms, especially the construction of piles and other devices, and some basic properties of electrostatics. Then followed 40 pages on magnetism, ending on pp. 485-489 with Arago's wheel and a reference to Poisson *1826e* (§ 14.5.2). 'Electrodynamic phenomena' provided the final 35 pages or so, with some emphasis on Ampère's principal discoveries and experiments but also describing discoveries of Oersted, Faraday and others—but not Biot or Savart. None of Ampère's mathematics was given; and earlier in the book he had judged 'the resolution of these equations' required in Poisson's calculations in electrostatics as 'often surpass[ing] the resources of analysis' (p. 373).

Despretz was then *répétiteur* in chemistry at the *Ecole Polytechnique*, where Dulong was giving the physics course. We saw in § 13.6.1 that Dulong gradually brought some parts of Fresnel's wave theory of light into his teaching there; similarly, he increased the treatment of electricity and magnetism by degrees, and brought in some aspects of the new subject, for the second-year students. By the mid 1820s he was even presenting Ampère's trigonometric and differential expressions for electrodynamic action, and within a couple of years was also talking about helices (see the registers in *EP*, X2c/7). Thus, at least at this school, and also down the road at the *Collège de France* with Ampère himself, the new subject was gaining some circulation in the educational scene in Paris.

14.6.2 Laplace on problems in physics, 1825

For the third, and last, time in this book we have seen the innovation of a branch of mathematical physics which basically lay outside mechanics. So this chapter can end appropriately with a notice of an occasion at the *Académie* late in 1825, when Laplace showed his latest perceptions of the expanding discipline of physics which he had done so much to encourage.

At the meeting of 28 November 1825 Laplace announced that four topics concerning the physics of the earth needed special attention; the 'intensity of terrestrial magnetism', continuing the work of Arago and Poisson (who had presented at this meeting his paper described in § 14.5.1): the proportion of hydrogen and oxygen in the atmosphere; the pressure of air at sea level; and the temperatures at various depths inside the earth. He asked for a commission to be formed to encourage experimental work on these topics, and was himself duly appointed to it, along with Arago, Fourier, Gay-Lussac, Poisson and Thenard. Legendre preferred that prizes be announced for these topics, and asked Laplace to write up his proposal. True to form, Poisson told the meeting that no new methods were required for the researches; existing ones would suffice.[1]) Unfortunately, no major initiative seems to have been taken at the *Académie* following Laplace's proposal; but the fact that it was made shows that the great man, then 76 years old, was still pretty well abreast of problems in physics.

[1]) Most reports of this meeting, including *PV*, *8*, 309, briefly record the establishment of the commission. However, a more detailed account was given in *Le globe*, *2* (1825), 994.