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Source: *Isis*, Vol. 59, No. 3 (Autumn, 1968), pp. 300-311
Published by: The University of Chicago Press on behalf of The History of Science Society
Stable URL: http://www.jstor.org/stable/227884
Accessed: 17/10/2013 15:47

The Contributions of Hermann von Helmholtz to Electrodynamics

By *A. E. Woodruff**

I

WHEN HERMANN VON HELMHOLTZ published the first of his major researches into electrodynamics in 1870, no one electrical theory had received universal acceptance. The potential law of F. E. Neumann, Weber's expression for the action at a distance between moving charges, and the aether theory of Maxwell were the most widely cited among the proposed theories which could account for the major electromagnetic phenomena then known. This multiplicity of mutually incompatible expressions resulted from the fact that the only measurements which had been made involved closed circuits. In two papers, published in 1845 and 1848, Neumann developed his electrodynamic potential, in terms of which one may express analytically the induced electromotive force in closed linear (thin wire) circuits.[1] This potential is

$$-\frac{1}{c^2} i_1 i_2 \int\int \frac{d\sigma_1 \cdot d\sigma_2}{r}$$

in modern notation.[2] The negative gradient (rate of variation with respect to changing relative position) of this expression yields the correct ponderomotive forces between closed circuits, even though the force obtained between the current elements themselves disagrees with the force law of Ampère.[3] The electromotive force induced

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[1] *Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin,* 1845, pp. 1-87; 1848, pp. 1-71; reprinted in Wilhelm Ostwald's series *Klassiker der exakten Wissenschaften* (Leipzig: Engelmann) as No. 10 (1889) and No. 36 (1892). The expression for the potential is introduced in the earlier paper, which contains a treatment of induction in nondeforming circuits from a more general viewpoint. The use of the potential to handle all problems of induction and ponderomotive force in closed circuits is found in the second paper. Neumann did not develop his potential as an expression valid between current elements, which is where the major ambiguities arise.

[2] That is, vector notation and the use of c for the speed of light. It should be noted that Weber used the symbol c to denote the ratio between the electrostatic and electrodynamic (not electromagnetic) units of charge, which ratio is $\sqrt{2}$ times the speed of light. The terms i_1, i_2 are the currents; $d\sigma_1$, $d\sigma_2$ are the directed lengths of the elements; r is the distance between the elements. The scalar product $d\sigma_1 \cdot d\sigma_2 = d\sigma_1 \, d\sigma_2 \cos\theta$, where θ is the angle between directions of $d\sigma_1$ and $d\sigma_2$. The expression is integrated around each circuit.

[3] Ampère's force between current elements,

in one circuit by the initiation of a current in the other, or by the approach of the latter from a large distance, is the time derivative of the potential, with the current in the first circuit set equal to unity.

Weber's law, which first appeared in 1846, went further than Neumann's in uniting all known electrical phenomena into one formula.[4] The guiding idea was that a charge acts directly at a distance on any other charge, in an amount which deviates from Coulomb's law by terms depending on the relative motion of the charges. Weber's force between two charges e_1 and e_2, which was assumed to be central, or directed along the line between them, is (again, in modern notation)

$$\frac{e_1 e_2}{r^2}\left[1 - \frac{1}{2c^2}\left(\frac{dr}{dt}\right)^2 + \frac{r}{c^2}\frac{d^2r}{dt^2}\right].$$

The first term is just Coulomb's law. The kinetic terms (the second and third) in this expression were derived from Ampère's force with the assumption that currents (metallic currents, at any rate) consisted of the equal and opposite motion of the two kinds of charge relative to the conductor. The correct results for electromagnetic induction in closed circuits follow, although the consequences of Weber's law differ from Neumann's when applied to current elements. Being a prime example of the Newtonian program of seeking mathematical expressions for the attractions and repulsions of nature, Weber's law represented the type of approach which long retained preference among most Continental physicists.[5]

It is not simply to its late appearance on the field that the widespread reluctance to accept Maxwell's theory must be attributed. Maxwell was not always clear in the manner in which he presented his theory. Whether or not Maxwell's ideas were fundamentally consistent,[6] there is no doubt that they confused his contemporaries. Aside from various expressions of malaise scattered throughout the literature, a clear discussion of some of the difficulties is found in the introduction to Hertz' *Electric Waves*.

Maxwell's fundamental innovation was the introduction of the rate of change of the electric displacement (which was present in the aether in "empty" space as well as in matter) as a current in the equations for the magnetic field. This enabled him to avoid consideration of open circuits—a major problem for the usual electrodynamic theories—because with this additional term all currents became closed. But this introduction of the displacement current, as distinct from the polarization current of a dielectric in the usual Poisson-type formulation (the concept used by Helmholtz), was not an obvious extension of the prevailing ideas. Most Continental

dating from 1820–1822, is a repulsion of magnitude:

$$-\frac{i_1 i_2}{r^2}\left[d\sigma_1 \cdot d\sigma_2 - \frac{3}{2}\frac{\mathbf{r}\cdot d\sigma_1\,\mathbf{r}\cdot d\sigma_2}{r^2}\right].$$

Ampère's investigations in electrodynamics are summarized in his *Thèorie mathèmatique des phénomènes electro-dynamiques* (Paris, 1827). See also my article "Action at a Distance in Nineteenth Century Electrodynamics," *Isis*, 1962, *53*:439-459.

4 *Abhandlungen der Fürstlichen Jablonowskischen Gesellschaft,* 1846, pp. 211–378; *Werke,* Vol. III (Berlin, 1893), pp. 25–214. I have discussed Weber's force in "Action at a Distance."

5 "We have all more or less imbibed with our mother's milk the ideas of magnetic and electric fluids acting directly at a distance." Ludwig Boltzmann, *Vorlesungen über Maxwells Theorie der Elektrizität under des Lichts,* Vol. I (Leipzig, 1891), p. 2.

6 I am indebted to Joan Bromberg and C. W. F. Everitt for a discussion which has at least revealed to me how complex this question is. An article by Dr. Bromberg on the subject has appeared in the *American Journal of Physics,* 1968, *36*:142–151.

theoreticians, being committed to Newton's method of proceeding in a purely mathematical way, "without hypothesis" or at any rate with as few assumptions as possible, were wary of making (or at least of admitting to) free creations of the human mind. The displacement current was first presented as a consequence of a mechanical model in which Maxwell himself reposed no great confidence, and later on it was introduced without any real justification. On the other hand, in the Helmholtz derivation of the Maxwell theory, as we shall see, the action-at-a-distance view is adhered to, with the addition of a pervasive dielectric medium whose infinite polarizability gives to charge the property of behaving like an incompressible fluid, so that all circuits are closed. Maxwell's theory thereby appears as a limiting consequence of accepted ideas. It is small wonder, then, that Maxwell's contemporaries were puzzled by this theory,[7] and that Hertz at first understood by it the conceptually completely different dielectric theory of Helmholtz, which served to interpret Maxwell's equations for Continental physicists. Indeed, what scientists today think of as classical electrodynamics is not Maxwell's theory of stresses in a medium but a combination of the atomic conception of electricity, developed by Weber and championed by Helmholtz,[8] with the field equations of Maxwell—that is, the electron theory of Lorentz.[9] The present paper sketches the part played by Helmholtz in this evolution of ideas.[10]

II

At the commencement of his major electrical researches around the beginning of the year 1870, Hermann von Helmholtz was Professor of Physiology at Heidelberg.[11] His most renowned previous work in physics, the elucidation of the conservation of energy, had followed investigations into the theory of animal heat. Here, too, physiology gave impetus to physics. Helmholtz had long before established the finite rate of propagation of nervous impulses and had recently been continuing this investigation, initiating the impulse by a sudden electric current from an induction apparatus. The interpretation of his results depended on the manner in which currents begin to flow inside extended conductors. This necessitated the consideration of open circuits, in which the charge density alters with time. But for open circuits the different electrodynamic theories gave discordant results. Furthermore, as we will discuss below, Helmholtz' mathematical investigations[12] showed that Weber's law led to unphysical instabilities and could not be regarded as satisfactory.

[7] Thus Boltzmann begins his preface to his work on Maxwell's theory (*Vorlesungen*, p. iii) by paraphrasing Faust:

So soll ich denn mit saurem Schweiss
Euch lehren, was ich selbst nicht weiss.

[8] Helmholtz supported the atomicity of charge in his Faraday Lecture at the Royal Institution in 1881. *Wissenschaftliche Abhandlungen*, 3 vols. (Leipzig, 1882), Vol. III, pp. 52–87.

[9] See Tetu Hirosige, *Japanese Studies in the History of Science*, 1962, *1*:101–110.

[10] A critical discussion of some of the theories dealt with here is to be found in Alfred O'Rahilly's unorthodox *Electromagnetics* (Cork: Cork Univ. Press, 1938). Pierre Duhem's *Les théories électriques de J. Clerk Maxwell* (Paris: Hermann, 1902) criticizes the difficulties in Maxwell's presentation and favors Helmholtz. See also J. J. Thomson's "Report on Electrical Theories," *British Association Report, Aberdeen*, 1885, pp. 97–155; Gustav Wiedemann, *Die Lehre von der Elektricität*, Vol. IV (2nd ed., Braunschweig: Vieweg, 1898), pp. 1015–1203.

[11] The standard biography is Leo Koenigsberger, *Hermann von Helmholtz*, trans. (and abridged) by F. A. Welby (Oxford: Clarendon Press, 1906).

[12] *Journal für die reine und angewandte Mathematik*, 1870, *72*:57–129; *Wissenschaftliche Abhandlungen*, Vol. I, pp. 545–628.

Helmholtz' method of attack was to generalize the electrodynamic potential between linear current elements to

$$-\frac{1}{2c^2}\frac{i_1 i_2}{r}\left[(1+k)\,d\sigma_1\cdot d\sigma_2+(1-k)\frac{\mathbf{r}\cdot d\sigma_1\,\mathbf{r}\cdot d\sigma_2}{r^2}\right]$$

(in modern notation) where k is an undetermined constant. For $k=1$, this reduces to the integrand in Neumann's potential; the additional term which appears when $k\neq 1$ drops out on integrating over a closed circuit. Neumann had only treated closed circuits, so that his form of the potential was (and remains) perfectly sufficient for his purposes. The potential between current elements which is derived from Weber's law is identical to the above with $k=1$, while the results of Maxwell's theory can be obtained by setting $k=0$ and introducing a dielectric medium.

The next step was to develop the equations of motion of electricity in extended, three-dimensional conductors on the basis of this potential, in the fashion of the derivation of similar equations from Weber's law by Kirchhoff.[13] The potential for linear currents is readily extended to volume currents; the electrodynamic potential per unit volume at the position x at the time t may be written in the form

$$-\frac{1}{c^2}\mathbf{j}(\mathbf{x},t)\cdot\mathbf{U}(\mathbf{x},t).$$

Here $\mathbf{j}(\mathbf{x}, t)$ is the current density (electric current per unit of volume) at the position $\mathbf{x}$ at time t, and $\mathbf{U}(\mathbf{x}, t)$ represents the effect on this current of all the currents in the surrounding space. $\mathbf{U}(\mathbf{x}, t)$ differs from the modern vector potential by a factor c. Helmholtz' potential law, generalized to extended currents, yields the following expressions for $\mathbf{U}(\mathbf{x}, t)$:

$$\mathbf{U}(\mathbf{x},t)=\int\left[\frac{1+k}{2}\frac{\mathbf{j}(\mathbf{x}',t)}{r}+\frac{1-k}{2}\frac{\mathbf{r}\,\mathbf{r}\cdot\mathbf{j}(\mathbf{x}',t)}{r^3}\right]d^3\mathbf{x}'$$

integrated over all space, where $\mathbf{r}=\mathbf{x}-\mathbf{x}'$, $\mathbf{j}(\mathbf{x},t)$ is the current density. Kirchhoff's starting point was this expression with $k=-1$. The resulting differential equations satisfied by $\mathbf{U}$ are (suppressing the arguments of the functions $\mathbf{U}$, $\mathbf{j}$, and φ):

1. Within the conductor:

$$\Delta\mathbf{U}-(1-k)\nabla\frac{\partial\varphi}{\partial t}=\frac{4\pi}{\varkappa}\left[\nabla\varphi+\frac{1}{c^2}\frac{\partial\mathbf{U}}{\partial t}\right],$$

where φ is the electrostatic "potential function of the free electricity," and $\varkappa$ is the resistivity of the conductor.[14] The left-hand side of this equation is shown by Helm-

[13] *Annalen der Physik*, 1857, *102*:529–544.

[14] The symbol ∇ represents the gradient operator, so that $\nabla\varphi$, a vector, is the gradient of φ, while Δ represents the Laplacian, so that

$$\Delta\mathbf{U}\equiv\frac{\partial^2\mathbf{U}}{\partial x^2}+\frac{\partial^2\mathbf{U}}{\partial y^2}+\frac{\partial^2\mathbf{U}}{\partial z^2}.$$

$\Delta\mathbf{U}$ could be characterized as the "lumpiness" in the space distribution of $\mathbf{U}$.

holtz' analysis of the preceding equation for $\mathbf{U}$ to have the value $-4\pi\,\mathbf{j}$, as in the next equation, while on the right-hand side the expression in brackets is the negative of the electric force field, so that this equation embodies Ohm's law.

2. Elsewhere, with a given current density $\mathbf{j}$:

$$\Delta\mathbf{U} - (1-k)\nabla\frac{\partial\varphi}{\partial t} = -4\pi\,\mathbf{j}.$$

3. Additionally, everywhere:

$$\nabla\cdot\mathbf{U} = -k\frac{\partial\varphi}{\partial t}.$$

4. The boundary conditions at the surface of the conductor require that:

$$\mathbf{U},\ \varphi,\ \frac{\partial\mathbf{U}}{\partial n}$$

be each continuous across the boundary.

5. At infinite distances:

$$\mathbf{U} = \varphi = 0.$$

These equations agree with those presently accepted when $k = 0$ (the "Coulomb gauge," which was used by Maxwell), except for the absence of the term containing $\partial^2\mathbf{U}/\partial t^2$ in (1) and (2), which is essential for the existence of electromagnetic radiation, and which is supplied further on in Helmholtz' derivation by the effects of the dielectric.

The argument continued by demonstrating the inadequacy of Weber's law on theoretical grounds. Soon after it was first propounded, this law was thought to violate the principle of conservation of energy, since Helmholtz had shown in his 1847 paper that such was true of velocity-dependent forces.[15] But Weber had vindicated his law and had given an expression for the potential energy of two charges,[16] namely

$$\frac{e_1e_2}{r}\left[1-\frac{1}{2c^2}\left(\frac{dr}{dt}\right)^2\right].$$

(Again, the first term is the Coulomb potential and the second the additional, motion-dependent part.) Helmholtz had not considered forces like Weber's which depended on the accelerations as well as on the velocities. Now Helmholtz showed that the flow of electricity in extended conductors would be subject to unnatural instabilities if Weber's law held. Starting from the generalized potential, the energy of the system can be expressed as

$$\frac{1}{8\pi}\int\left[(\nabla\varphi)^2+\frac{1}{c^2}(\nabla\times\mathbf{U})^2+\frac{k}{c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2\right]d^3\mathbf{x}.$$

[15] *Ueber die Erhaltung der Kraft* (Berlin, 1847); *Wissenschaftliche Abhandlungen*, Vol. I, pp. 12–75.

[16] Already in 1848 Weber had given this expression as the potential from which the force may be derived (*Annalen der Physik und Chemie*, 1848, *73*:193–240; *Werke*, Vol. III, pp. 215–254; see p. 245). I inadvertently omitted this reference in my earlier article "Action at a Distance." A discussion of the potential in connection with the conservation of energy appeared in *Abhandlungen der Königlichen Sächsischen Gesellschaft*, 1871, *10*:1–61; *Werke*, Vol. IV, pp. 247–299.

The first term represents the electrostatic, and the remainder the electrodynamic energy. The final, k-dependent term has the same sign as k. If k is negative, some nonequilibrium states will have a smaller energy than the state of rest, and static equilibrium states may not be stable.

It still had to be demonstrated that the predicted instabilities could be evoked by physically available means. Helmholtz proved the instability (for $k < 0$) of the radial motion of electricity in a conducting sphere under the influence of the contraction and expansion of a concentric charged spherical shell. This instability would appear equally well whenever any charged body was moved near the sphere. Indeed, even with a charged particle of mass m approaching a fixed like charge under the influence of Weber's force, the velocity will become infinite in a finite distance if initially

$$\frac{e_1e_2}{r} < mc^2 < E,$$

where E is the energy of the system.[17] Weber replied that the initial velocity in such a case would have to be inordinately large (larger than $\sqrt{2}\,c$) and the critical separation at which the velocity becomes infinite would be of molecular dimensions, so that molecular forces could alter the situation and prevent instability.[18] Helmholtz showed in his next paper in 1873 that these objections were incorrect or could apparently be evaded by including the effect of nonelectrical forces; for example, the charged particle would move ever faster through a frictional medium under the influence of a charged sphere, which could initially be at a great distance away.[19] Weber and his followers made several further attempts to defend his law, but an appendix added by Helmholtz to his paper eight years later, in which he deplored the sometimes acrimonious dispute,[20] represents pretty much the end of a rather sterile debate.

[17] The energy of the system in the situation mentioned is

$$E = \frac{1}{2}mv^2 + \frac{e_1e_2}{r}\left[1 - \frac{v^2}{2c^2}\right]$$
$$= \frac{1}{2}\left(mc^2 - \frac{e_1e_2}{r}\right)\frac{v^2}{c^2} + \frac{e_1e_2}{r}$$

so that as r decreases, the term multiplying v^2 decreases and vanishes at the critical distance $r = e_1e_2/mc^2$. To maintain the energy E constant, v^2 must become infinite, since it is still true at this distance that $E > e_1e_2/r$.

[18] *Abh. Fürstl. Jablon. Ges.*, 1871, *10*:1–61; *Werke*, Vol. IV, pp. 247–299.

[19] *J. reine angew. Math.*, 1873, *75*:35–66; *Wissenschaftliche Abhandlungen*, Vol. I, pp. 647–687. The example appears in a preliminary account in the *Monatsberichte der Berliner Akademie*, 18 April 1872, pp. 247–256; *Wissenschaftliche Abhandlungen*, Vol. I, pp. 636–646.

[20] Weber in *Abh. Kgl. Sächs. Ges.*, 1878, *11*:641–696; *Werke*, Vol. IV, pp. 361–412. C. Neumann in *Ber. Kgl. Sächs. Ges.*, 1871. Helmholtz in the Appendix (1881) to the article in n. 19, *Wissenschaftliche Abhandlungen*, Vol. I, pp. 685–687. I call the dispute "sterile" because the ultimate decision in favor of Maxwell's theory over Weber's did not depend on it, but on Hertz' demonstration of the propagation of electromagnetic waves. It might be noted that relations between Weber and Helmholtz were not particularly cordial, as Helmholtz himself remarked in a letter to his wife in 1851 (see Koenigsberger, *Helmholtz*, p. 81). Besides the disputes mentioned in the text, Helmholtz participated in the rejection of the proposed electrical current unit "weber" in favor of "ampere" at the First International Congress on Electricity in Paris, 1881 (*Elektrotechnisches Zeitschrift*, 1881, *2*:391, cited in the dissertation of K. H. Wiederkehr, *Wilhelm Webers Stellung in der Entwicklung der Elektrizitätslehre*, Hamburg: Institut für Geschichte der Naturwissenschaften, 1960). Weber's followers, particularly K. F. Zöllner, tended to exacerbate the dispute.

III

In order to arrive at results similar to those of Maxwell's theory, Helmholtz introduced in his first paper a dielectric and diamagnetic aether.[21] Assuming that only electrostatic forces are present, the polarization, or "electric moment" of the medium, **P**, measuring the displacement of charge within the medium, is proportional to the electrostatic field $-\nabla\varphi$:

$$\mathbf{P} = -\varepsilon\nabla\varphi,$$

where ε is known as the electric susceptibility of the medium. The current density $\mathbf{j}$ is presumed to contain, besides the ordinary current, the polarization term $\partial\mathbf{P}/\partial t$, since "the act of polarization forms . . . a kind of electric motion." The magnetic moment **I** of the medium is proportional to the magnetic field:

$$\mathbf{I} = \theta\left[-\nabla\chi + \frac{1}{c}\nabla\times\mathbf{U}\right],$$

where θ is the magnetic susceptibility and χ the magnetic potential. Finally, if electrodynamic as well as static forces are assumed to polarize the medium, the full equation for the polarization from electrostatic and electrodynamic forces is

$$\frac{1}{\varepsilon}\mathbf{P} = -\nabla\varphi - \frac{1}{c^2}\frac{\partial\mathbf{U}}{\partial t} + \frac{1}{c}\frac{\partial}{\partial t}\nabla\times\int\frac{\mathbf{I}}{r}d^3\mathbf{x}.$$

Combining these new conceptions with his earlier results, Helmholtz arrived at the following wave equations for **P** and **I** in a homogeneous dielectric medium:

$$\Delta\mathbf{P} = 4\pi\varepsilon\,(1+4\pi\theta)\frac{1}{c^2}\frac{\partial^2\mathbf{P}}{\partial t^2} + \left[1 - \frac{(1+4\pi\theta)(1+4\pi\varepsilon)}{k}\right]\nabla\nabla\cdot\mathbf{P}$$

$$\Delta\mathbf{I} = 4\pi\varepsilon\,(1+4\pi\theta)\frac{1}{c^2}\frac{\partial^2\mathbf{I}}{\partial t^2}$$

$$\nabla\cdot\mathbf{I} = 0.$$

The first equation has for its solutions transverse waves of electric polarization propagating with the speed

$$\frac{c}{\sqrt{4\pi\varepsilon(1+4\pi\theta)}}$$

and the longitudinal waves with the speed

$$c\sqrt{\frac{(1+4\pi\varepsilon)}{4\pi\varepsilon k}}.$$

[21] *Wissenschaftliche Abhandlungen,* Vol. I, pp. 611–628.

The transverse waves of magnetic polarization have the same speed as the electric transverse waves, while the longitudinal magnetic waves have infinite speed (i.e., do not exist). However, with the introduction of the aether, c no longer represents the observed ratio between the electrical units, because it is introduced theoretically as the ratio between these units in an absolute (aetherless) vacuum. Since, for example, the aether is susceptible to a dielectric polarization, the charge actually observed in an experiment to determine the electrostatic unit of charge is the "bare" charge present together with the opposite charge induced in the immediately surrounding aether.[22] To account for these unremovable effects arising from the the presence of the aether, Helmholtz demonstrates that we must introduce a "renormalized" constant c_{obs} representing the abstract experimental ratio, which is related to the "true" but unobservable ratio c by the equation

$$c = c_{obs} \sqrt{1 + 4\pi\varepsilon_0} \sqrt{1 + 4\pi\theta_0},$$

where ε_0 and θ_0 are the susceptibilities of the (actual, aether-filled) vacuum. Then, in vacuo, the speeds are

longitudinal (electric):

$$c_{obs} (1 + 4\pi\varepsilon_0) \sqrt{\frac{1 + 4\pi\theta_0}{4\pi\varepsilon_0 k}}$$

transverse (electric and magnetic):

$$c_{obs} \sqrt{\frac{1 + 4\pi\varepsilon_0}{4\pi\varepsilon_0}}.$$

Helmholtz characterizes the Maxwellian limit by putting $k = 0$, in which case the longitudinal wave disappears, while he lets ε_0 be infinitely large compared to $1/4\pi$ (so that **P**, the immediate action of the local polarization, overwhelms the electric field from distant charges $-\nabla\varphi$), whence it follows that the transverse waves have the speed c_{obs} as in Maxwell's theory. However, the single condition $\varepsilon_0 \rightarrow \infty$ suffices to remove the longitudinal wave whatever the (finite non-negative) value of k. As Poincaré remarked,[23] the Maxwellian limit of Helmholtz' theory should not depend on k since in this limit only closed circuits appear. Helmholtz further notes that equations of the dielectric theory give the correct boundary conditions for reflection and refraction at the surface between two transparent media, a problem which Maxwell had not considered.

Helmholtz was fully aware of the conceptual differences between his dielectric theory and Maxwell's:

> The two theories are opposed to each other in a certain sense, since according to the theory of magnetic induction originating with Poisson, which can be carried through in a fully corresponding way for the theory of dielectric polarization of insulators, the action at a distance is diminished by the polarization, while according to Maxwell's theory on the other hand the action at a distance is exactly replaced

[22] A similar sort of problem arises in modern quantum electrodynamics, from the jargon of which the words in quotation marks have been introduced.

[23] *Electricité et Optique*, Vol. II (Paris, 1871), p. 112. See also the discussion in O'Rahilly, *Electromagnetics*.

by the polarization. . . . It follows. . . from these investigations that the remarkable analogy between the motion of electricity in a dielectric and that of the light aether does not depend on the particular form of Maxwell's hypotheses, but results also in a basically similar fashion if we maintain the older viewpoint about electrical action at a distance.[24]

IV

The theoretical investigations of Helmholtz could not resolve the problem of which electrical theory, if any, was correct. Weber's law appeared to be ruled out by the instabilities which were implied. But the constant k could have any non-negative value, and the dielectric aether might or might not be included, without affecting the established experimental results. New experiments would have to be devised to decide among the alternatives or against all of them. Not only was the value of k irrelevant for closed circuits, but so long as it was not much greater than unity, determination of its value would require research involving conductors of sufficient size and time measurements of sufficient accuracy to permit the observation of effects propagating with the speed of light. Even a long wire would not suffice if the propagating electric waves had wavelengths much greater than its diameter; in this case the equations reduced to those which Kirchhoff derived from Weber's law.

Despite this pessimistic conclusion, Helmholtz devoted most of his later electrical research to the attempt to find decisive experiments. Three contending viewpoints were to be distinguished: the potential theories, with whatever value of k, but no dielectric aether; Ampère's law for the action-at-a-distance force between circuits; and the Faraday-Maxwell theory, or equivalently the extended potential theory with the aether. A paper of 1874 was devoted to drawing the distinction between the potential theories and Ampère's law.[25] The latter consists only of a force between pairs of current elements, while the potential expression, if it is assumed to be valid for current elements, also yields forces between a current element and the end of an open circuit, and between two ends of open circuits.

Experiments designed to distinguish between these theories were carried out by Helmholtz and by N. Schiller at the University of Berlin, where Helmholtz had become Professor of Physics in 1871. In Schiller's research, performed in the summer of 1874, a magnetized steel ring was suspended in a metal housing and a static electric generator was discharged through a nearby point.[26] If the point is regarded as the end of an open circuit, the electrical force exerted by this end on the molecular currents in the magnet would cause a detectable turning of the ring. No deflection was noted, so that either the potential theory was wrong or it was incomplete, in the sense that the convective motion of electricity in the air (or the polarization of the air) also exerts electrodynamic forces. This problem was resolved shortly afterward by Henry Rowland in Helmholtz' laboratory with an experiment demonstrating the magnetic effects of convective electricity.[27]

[24] *Wissenschaftliche Abhandlungen*, Vol. I, pp. 556–558.

[25] *J. reine angew. Math.*, 1874, *78*:273–324; *Wissenschaftliche Abhandlungen*, Vol. I, pp. 702–762.

[26] Schiller later had this experiment performed with greater care at the Moscow University by his colleagues A. Stoletow and R. Colley. At Kiev, he carried through an experiment which had been suggested in 1873 by Helmholtz (*Wissenschaftliche Abhandlungen*, Vol. I, pp. 688–701), to decide between Ampère's law and the dielectric current. Although Schiller's results supported the latter, Helmholtz was evidently dissatisfied with the sensitivity of this experiment (see the note added to his *Wissenschaftliche Abhandlungen*, Vol. I, p. 701, in 1881). Schiller reported his work in *Ann. Phys.*, 1876, *159*:456–473, 537–553.

[27] *Ann. Phys.*, 1876, *158*:487–493; Helmholtz, *Wissenschaftliche Abhandlungen*, Vol. I, pp. 791–797.

The experiment carried out by Helmholtz himself involved a cylindrical capacitor whose plates extended over two opposed quadrants about the central axis *a* (see Fig. 1).[28] The inner pair of plates *bb* could be rotated about this axis and were grounded. (The plates are seen end-on in the figure.) The outer plates *cc* were connected by a commutator alternately with the ground when the inner plates were aligned as in *A*, and with one plate of a Kohlrausch capacitor, the other plate of which was grounded, when the inner plates were oriented as in *B*. The apparatus was placed in a strong axially symmetrical magnetic field normal to the plane of

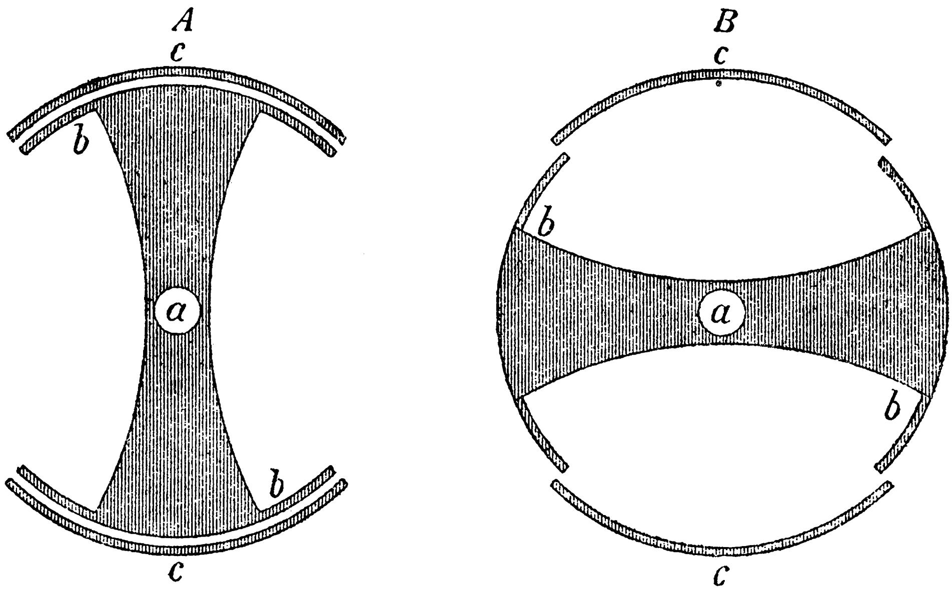

FIGURE 1. Orientations of the capacitor plates in the experiment of Helmholtz.

the figure, and the inner plates were rotated. Since the field is symmetric, there should be, according to the potential law (without the aether), no change in the potential of the inner plates on rotation, and so the Kohlrausch capacitor should not become charged. On the other hand, if the circuit is closed by sliding contacts between the plates as they pass each other, the potential law attributes the induced electromotive force exclusively to the sliding contact itself. In fact, the Kohlrausch capacitor became charged in both cases, and indeed within the experimental error to the same extent in each. The potential law was therefore falsified, whereas the experimental results agreed with the law of electromagnetic induction (not his potential law) which Neumann had derived in 1845 from Ampère's action-at-a-distance force,[29] or equally well with Maxwell's theory or equivalently the potential

[28] *Ann. Phys.*, 1876, *158*:87–105; *Wissenschaftliche Abhandlungen*, Vol. I, pp. 774–790.

[29] *Abh. Kgl. Akad. Wiss. Berlin*, 1845, p. 1. This is the law

$$i\,\mathbf{E} \cdot d\sigma \propto -\mathbf{v} \cdot \mathbf{F}_A\, d\sigma$$

giving the electromotive force $\mathbf{E} \cdot d\sigma$ in a current element $d\sigma$ traversed by the current *i*, moving with the velocity **v**, and acted on by the Ampèrian force $\mathbf{F}_A$ from closed circuits in the neighborhood; this law directly relates the results of Faraday and Lenz on induction to Ampère's ponderomotive force, given in differential form in n. 3 above.

law with the dielectric aether. The decision between these, Helmholtz concluded, required the investigation of the electrodynamic action of dielectrics.

V

The problem posed by Helmholtz in the last of his major investigations into the foundations of electrodynamics became a prize problem set by the Berlin Academy of Science in 1879: "to establish experimentally any relation between electromagnetic forces and the dielectric polarization of insulators."[30] Helmholtz urged his student Heinrich Hertz to take up the problem, but Hertz' preliminary calculations discouraged him from tackling it just then.

In 1886 Hertz began research on electrical oscillations with a higher frequency (100 megacycles) than had hitherto been investigated.[31] His initial attempt to detect an electromagnetic effect of changing dielectric polarization involved placing a large block of insulator between capacitor plates in the primary circuit and a secondary loop containing a spark gap next to the block in order to pick up the inductive effects. The attempt was frustrated by the appearance of strong sparks in the spark gap whether or not the block was in place, even though the remainder of the primary circuit was relatively distant. Hertz realized then that his assumption that the electrical forces from the capacitor plates could not give rise to sparks in the nearly closed secondary circuit was no longer valid when such rapid oscillations were involved, and he set out to discover what he could about the new phenomena. He found that the action extended over unexpectedly large distances and that there were certain positions of the secondary loop for which the presence or absence of the block made the sparks in the gap appear or not, thus solving the problem set by the Academy.

As we saw, Helmholtz' extension of the potential theory to arrive at Maxwell's results involved three major hypotheses: that a changing dielectric polarization creates the same electromagnetic forces that would be created by an equivalent conduction current; that electromagnetic forces can produce dielectric polarization; and that the vacuum is a dielectric. Hertz had given evidence for the first and now turned to the last, which he felt "contained the gist and special significance of Faraday's, and therefore of Maxwell's view."[32] The three hypotheses together implied the finite rate of propagation of electric action and the existence of electromagnetic waves in air, which is for these purposes practically the same as the vacuum. This set the goal for Hertz' further research, and on 13 December 1888 he reported the culminating results to the Berlin Academy in his "On Electric Radiation," the paper which decisively confirmed the Maxwell theory by demonstrating the existence of electromagnetic radiation.[33]

VI

In retrospect, the significance of the work of Helmholtz in electrodynamics is that of making Maxwell's theory intelligible to the German physicists and of inspiring the experimental research of Hertz which confirmed it. Led to his investigations by a problem arising in his physiological researches, Helmholtz endeavored to place the major conflicting theories into a unified framework, making

[30] Hertz, *Electric Waves*, trans. D. E. Jones (London, 1893), p. 1.

[31] *Ann. Phys.*, 1887, *31*:421; *Electric Waves*, pp. 29–53. The frequency is important because, other things being equal, the power output of an electric oscillator is proportional to the fourth power of the frequency.

[32] *Electric Waves*, p. 7.

[33] *Ann. Phys.*, 1889, *36*:769; *Electric Waves*, pp. 172–185.

it easier to distinguish between them experimentally. The framework did violence to Maxwell's ideas, as distinct from his results, but these ideas had not always been clearly expressed. "Notwithstanding the greatest admiration for Maxwell's mathematical conceptions," Hertz wrote later,[34] "I have not always felt quite certain of having grasped the physical significance of his statements. Hence it was not possible for me to be guided in my experiments directly by Maxwell's book [the *Treatise*]. I have rather been guided by Helmholtz's work." He continued: "But unfortunately, in the special limiting case of Helmholtz's theory which leads to Maxwell's equations, and to which the experiments pointed, the physical basis of Helmholtz's theory disappears, as indeed it always does, as soon as action-at-a-distance is disregarded."

Therefore, Hertz was driven to consider the meaning of Maxwell's theory on his own, coming to the modern view in his famous pronouncement: "To the question, 'What is Maxwell's theory?' I know of no shorter or more definite answer than the following—Maxwell's theory is Maxwell's system of equations."[35]

The conceptual models of dielectric aethers have disappeared in favor of the more abstract approach. But the role they played in the development of the theoretical ideas of Maxwell and, in a mathematical way, of Helmholtz should not be forgotten.

[34] *Electric Waves*, p. 20.

[35] *Ibid.*, p. 21.