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Translational Inertial Dragging

Ø. Grøn[1,2] **and E. Eriksen**[2]

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The electrical field inside a uniformly charged, slowly accelerated spherical shell is calculated. The result is used to find the inertial translational dragging field inside a slowly accelerated spherical shell of dust particles, according to the linearized gravitational field equations. The relevance of this effect in connection with Mach's principle and the principle of relativity is discussed.

1. INTRODUCTION

The inertial dragging due to rotating masses was discovered by Lense and Thirring [1–5], who considered solutions of the linearized gravitational field equations inside rotating shells. This rotational inertial dragging was later found also in exact solutions of the complete gravitational field equations, describing rotating black holes [5].

The corresponding *translational inertial dragging* has been considered by some authors [6–11]. In the present article we calculate the dragging field inside a slowly accelerated spherical shell, according to the linearized gravitational field equations. This is performed by first finding the electrical field inside a uniformly charged, slowly accelerated spherical shell and then translating the result to the gravitational case.

The resulting field represents a translational analogue of the Lense–Thirring effect. The significance of the translational inertial dragging field in connection with Mach's principle and Einstein's relativity principle is discussed in the final sections of the article, where a variant of Newton's bucket experiment in which the bucket is uniformly accelerated is considered.

[1] Oslo College of Engineering, Cort Adelers gt. 30, Oslo, Norway.
[2] Institute of Physics, University of Oslo, P.O. Box 1048 Blindern, Oslo, Norway.

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2. THE ELECTRIC FIELD INSIDE A UNIFORMLY CHARGED, SLOWLY ACCELERATED SPHERICAL SHELL

Consider a uniformly charged, spherical shell slowly accelerated in the laboratory system as shown in Fig. 1. We calculate the electromagnetic field at a point P inside the shell. The point K is the position of a surface element $d\sigma$ on the sphere at a moment t. The point $\tilde{K}$ is the retarded position of this element. We assume that $\beta \equiv v/c \ll 1$ and $rg/c^2 \ll 1$, where v is the velocity of the shell, g its acceleration, and r the distance from K to the field point P. The calculations are performed to first order in these quantities. This gives the retarded values:

$$\left.\begin{aligned} \tilde{\beta} &= \beta - gr/c^2, \qquad s = (\beta - gr/2c^2)\, r \\ \tilde{r} &= r - \beta r \cos\theta + (gr^2/2c^2)\cos\theta \end{aligned}\right\} \tag{1}$$

where s is the distance between K and $\tilde{K}$.

The charge of the surface element $d\sigma$ is

$$dQ = (Q/4\pi R^2)\, d\sigma \tag{2}$$

where Q is the charge of the shell, and R its radius. The Lienard–Wiechert potentials at the point P due to this charge-element are to the required order

$$\left.\begin{aligned} d\Phi &= (r - \mathbf{r}\cdot\tilde{\boldsymbol{\beta}})^{-1}\, dQ = dQ/r - (g\, dQ/2c^2)\cos\theta \\ d\mathbf{A} &= \Phi\boldsymbol{\beta} = (dQ/r)\,\boldsymbol{\beta} - (dR/c^2)\,\mathbf{g} \end{aligned}\right\} \tag{3}$$

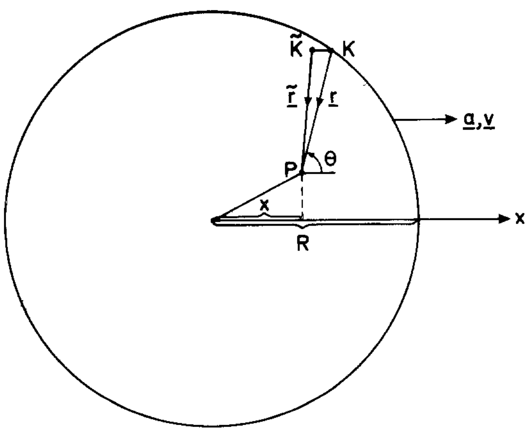

Fig. 1. Uniformly accelerated spherical shell.

The integrals over the spherical surface are performed by the introduction of spherical coordinates with origin at the center of the sphere. Using the addition theorem for Legendre functions, we then obtain

$$\begin{aligned} \Phi &= Q/R - (Qg/3Rc^2)x \\ \mathbf{A} &= (Q/R)\,\boldsymbol{\beta} - (Q/c^2)\,\mathbf{g} \end{aligned} \tag{4}$$

where x is the x-component of P's distance from the origin (see Fig. 1). We see that $\mathbf{A}$ is homogeneous inside the sphere and, consequently, that there is no magnetic field in this region. Note, however, that $\mathbf{A}$ is time dependent, since the motion of the shell is accelerated.

The electrical field strength $\mathbf{E}$ is given by

$$\mathbf{E} = \nabla\phi - \partial\mathbf{A}/\partial t = -(2Q/3Rc^2)\,\mathbf{g} + (Q/c^3)\,\dot{\mathbf{g}} \tag{5}$$

With $d\mathbf{g}/dt = 0$ we get

$$\mathbf{E} = -(2Q/3Rc^2)\,\mathbf{g} \tag{6}$$

To first order in β and rg/c^2 there is a homogeneous electrical field inside the sphere. The field strength is proportional to the acceleration of the shell, and the electrical field is directed oppositely to the acceleration if the shell is positively charged. In this case a test particle with negative charge $-q$ and mass m will get an acceleration

$$a_q = \tfrac{2}{3}[(Qq/R)/mc^2]\,\mathbf{g} \tag{7}$$

in the same direction that the shell accelerates, the proportionality factor being essentially the ratio between the particle's binding energy and its rest energy.

3. TRANSLATIONAL INERTIAL DRAGGING INSIDE AN ACCELERATED MASSIVE SHELL

We now consider space-time inside an accelerated (i.e., with an *observed* acceleration), nonrotating, and uncharged massive shell with radius R and mass M. Since space-time inside a static spherical shell is flat, the deviation from Minkowski metric inside an accelerated shell will vanish in the limit of zero acceleration. Thus, even for a shell with an arbitrarily large mass, the metric is close to the Minkowski metric, $\eta_{\mu\nu}$, if the acceleration is sufficiently small. The relevant assumption is $Rg/c^2 \ll 1$.

The metric is written

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}, \qquad |h_{\mu\nu}|\ll 1 \tag{8}$$

Using harmonic coordinates, the field equations take the form

$$\Box^2 h_{\mu\nu}=-16\pi G S_{\mu\nu}, \qquad S_{\mu\nu}\equiv T_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}T^{\lambda}{}_{\lambda} \tag{9}$$

where $T^{\mu\nu}$ are the components of the energy-momentum tensor.

The solution of Eq. (9) inside the shell is found from the retarded potentials

$$h_{\mu\nu}=(4G/c^2)\int [S_{\mu\nu}/r]_{\text{ret}}\, dV \tag{10}$$

To first order in v/c and rg/c^2, radial collapse of the shell does not affect the form of the metric inside it. We may therefore, without loss of generality, assume that the shell consists of dust particles. Then the quantities $S_{\mu\nu}$ in the shell are given by

$$S^{\mu}{}_{\mu}=\rho/2, \qquad S_{01}=-\rho v \tag{11}$$

where ρ is the proper density of the dust.

For a slowly moving shell, $\beta\ll 1$, we can now write the potentials at a point P inside the shell directly from our knowledge of the electromagnetic potentials that were found in Section 2. The only nonvanishing components of $h_{\mu\nu}$ are

$$\left.\begin{aligned} h_{00}&=(2GM/c^2R)(1-gx/3c^2)\\ h_{01}&=-(4GM/c^2)(v/R-g/c)\end{aligned}\right\} \tag{12}$$

where M is the mass of the shell. The corresponding line-element has the form

$$\begin{aligned} ds^2=&-\left[1-\frac{2GM}{c^2R}\left(1-\frac{gx}{3c^2}\right)\right]c^2\,dt^2-\frac{8GM}{c^2R}\left(v-\frac{Rg}{c}\right)dt\,dx\\ &+\left[1+\frac{2GM}{c^2R}\left(1-\frac{gx}{3c^2}\right)\right](dx^2+dy^2+dz^2) \end{aligned} \tag{13}$$

In the static case $g=v=0$ and the metric can be brought to the Minkowski form by a constant adjustment of the coordinates, for shells with an arbitrarily large mass.

To first order in $\boldsymbol{\beta}$ and Rg/c^2 the geodesic equation may be written.

$$\mathbf{a}=\tfrac{1}{2}c^2\nabla h_{00}-\partial\mathbf{h}/\partial t \tag{14}$$

where $\mathbf{h} \equiv h_{0i}\mathbf{e}_i$ (with summation) and $\mathbf{e}_i$ are spatial unit vectors. Here $\mathbf{a}$ is the coordinate acceleration of a free particle. Inserting the metric (12) and performing the differentiations give an acceleration in the same direction as that of the shell, with a magnitude

$$a = \frac{11}{3}\frac{GMm/R}{mc^2}\,g = \frac{11}{6}\frac{R_s}{R}\,g \tag{15}$$

where $R_s = 2GM/c^2$ is the Schwarzschild radius of the shell.

Equation (15) is an expression of the translational inertial dragging inside a slowly accelerated massive shell. This dragging field is homogeneous, which indicates the absence of tidal acceleration fields as far as the linearized calculation goes. This is confirmed by calculating the components of the Riemann curvature tensor from the line-element (13). The calculation gives the result that the components are proportional to $(g/c^2)^2$.

The acceleration due to the translational inertial dragging effect may suitably be termed inertial acceleration. Equation (15) tells that the inertial acceleration inside an accelerated spherical shell is proportional to the ratio between the shell's gravitational binding energy and its rest energy or, equivalently, to the ratio between its Schwarzchild radius and its radius.

4. PHYSICAL SIGNIFICANCE OF THE TRANSLATIONAL INERTIAL DRAGGING EFFECT

In the following sections we investigate the physical significance of the translational inertial dragging effect in relation to Mach's principle and the general principle of relativity. We consider the limit of very massive shells with a mass equal to the mass of the universe. Two problems that appear in this limit are discussed in this section: (A) Why do we perturb the Minkowski metric and not, for example, the Rindler metric? and (B) Is the translational inertial dragging effect a physical (observable) effect, in the limit of a cosmic shell, or only a nonphysical coordinate effect?

It is useful to start with a brief look at the corresponding rotational effect.

4.1. Rotational Inertial Dragging

The rotational inertial dragging effect, which was discovered by Lense and Thirring [1–3], was later investigated by Cohen and Brill [12, 13] and by Orwig [14]. It was found that in the limit of a spherical shell with a radius equal to its Schwarzschild radius, the interior inertial frames are dragged around rigidly with the same angular velocity as that of the shell. In this case of "perfect dragging" the motion of the inertial frames is completely determined by the shell.

The Machian character of this result was noted by Brill and Cohen, who write [12]:

> A shell of matter of radius equal to its Schwarschild radius has often been taken as an idealized cosmological model of our universe. Our result shows that in such a model there cannot be a rotation of the local inertial frame in the center relative to large masses in the universe. In this sense our result explains why the "fixed stars" are indeed fixed in our inertial frame, and in this sense the result is consistent with Mach's principle.

It should be noted that the solution of the field equations is found by imposing a condition of asymptotic Minkowski metric far from the shell. This indicates that the cosmological application of the results represents an extrapolation that rests on somewhat unsecure ground.

4.2. Translational Inertial Dragging

The situation concerning our deduction of the translational inertial dragging is similar to that of the rotational case.

The deduction of the solution inside the shell is based on a perturbation of the Minkowski metric. The shell accelerates relative to the chosen reference frame. This means that in the limit of a cosmic mass-shell, the reference frame is accelerated relative to the inertial frames. If we consider a homogeneous universe model, the inertial frames follow the cosmic shell. Assume that the observed acceleration of the shell is uniform. It has been suggested to us that, in this case, one should perturb the Rindler metric of a uniformly accelerated reference frame, and not the Minkowski metric.

However, according to a *Machian interpretation* of general relativity, an acceleration field must be due to some source (nonvanishing energy-momentum tensor) generating it. The acceleration field in the Rindler metric does not have this property. It exists in an empty universe. The point of the present article is to investigate if an acceleration field is produced by an accelerating mass (the shell), making any sourceless acceleration field superfluous. In a homogeneous universe the unperturbed solution describes space-time inside a cosmic shell that is observed to be static. This shell does not influence the space-time geometry inside it, which is thus Minkowskian.

4.3. Observability of Translational Inertial Dragging

The existence of a translational inertial dragging effect in general relativity has been pointed out by Einstein in connection with Machian effects in general relativity [6]. He also discussed a related effect, the possible dependence of the inertial mass of a particle upon the mass of the universe. It was later shown by Brans [15, 16] that there is no such measurable effect in general relativity. The effect which seemed to be apparent in an approximate 3-vector version of the geodesic equation was proved to be only a coordinate effect.

One may wonder if the translational inertial dragging as given in Eq. (15) is also a nonmeasurable coordinate effect. The following example shows that this effect is, in principle, measurable. Consider a laboratory fixed at the surface of the Earth. The laboratory is surrounded by a movable shell, which is small enough that the motion of the shell does not influence the Earth's motion.

In the laboratory we have a horizontal spring weight. With the surrounding shell at rest, the equilibrium position of the spring weight, which is constructed to move along the x axis, is x_0.

Assume now that the shell is accelerating in the x direction as seen from the laboratory. The translational inertial dragging effect then forces the spring weight to a new equilibrium position x. The distance $x - x_0$ is proportional to the observed acceleration of the shell.

In order to estimate the magnitude of this effect, we consider a shell with mass $M = 1000$ kg and radius $R = 10$ m. Then, according to Eq. (15) the acceleration field in the laboratory is of an order of magnitude $a = 10^{-25}$ g, where g is the acceleration of the shell. This effect is, in principle, measurable but extremely small, which leads to practical difficulties in obtaining an experimental test of it.

Let the mass and radius of the shell be increased, so that they approach the total mass and radius of the universe. Then it seems that the observability of the translational inertial dragging effect vanishes, since the cosmic dragging field will now act on the Earth, and thus on the laboratory, in just the same way that it acts on the spring weight. Thus, the mass of the spring weight will remain at the position x_0. One is therefore tempted to conclude that the translational cosmic dragging field due to a cosmic shell is not observable.

It is now shown that if $a = g$ in Eq. (15) for the dragging field due to a cosmic shell, then the above argument and conclusion are not valid. As the mass (and radius) of the shell increases, there will be an increasing inertial dragging, so that the relative acceleration between the Earth and the shell decreases. In the limit of a cosmic shell, the Earth will be at rest relative to the shell (neglecting the effect of the sun).

The cosmic shell is then observed to be static. So we can only conclude that, as measured in a reference frame in which the cosmic shell is static, it does not induce any translational dragging field.

The translational inertial dragging effect appears only in reference frames where the cosmic shell is observed to accelerate. The observed state of motion of the cosmic shell can be defined by measuring the angular distribution of temperature for the cosmic background radiation, with equipment fixed in the laboratory. If one also measures, for example, by means of a spring weight, an acceleration field in a laboratory and finds that it is proportional to the acceleration of the cosmic shell, as inferred from measurements of the cosmic background radiation, then one has observed an instance of the translational inertial dragging effect.

We now consider a laboratory in which such an effect can be measured, Einstein's lift. Assume that it falls freely. An observer fixed on the Earth observes that the lift accelerates downward, and the cosmic mass is at rest. In a Machian spirit, and in accordance with the general principle of relativity, an observer in the lift may consider himself and the lift as at rest. He observes that the Earth and the cosmic mass accelerate in the same direction (upward) and with equal acceleration. [assuming that $a = g$ in Eq. (15) for the dragging field due to a cosmic shell]. Also, he calculates the gravitational acceleration field due to the Earth and finds that it is of equal magnitude to the acceleration of the cosmic mass but oppositely directed (i.e., downward).

The translational inertial dragging effect is essential when this observer is going to explain his observations. He finds that the Earth falls freely in the cosmic dragging field and that the lift is at rest because it is in equilibrium, acted upon by two equally large, oppositely directed gravitational acceleration fields: that due to the Earth and the dragging field due to the accelerated cosmic mass.

The effect of the translational dragging field due to accelerated *cosmic* mass is not a small one. In fact we are quite used to it. But with reference to Newtonian mechanics we talk of inertial force fields in accelerated reference frames. However, according to the general principle of relativity, we may consider the laboratory as at rest. We then talk of gravitational dragging (acceleration) fields. The concept of "inertial forces," which may be regarded as a sort of trick in Newtonian mechanics, is thereby made superfluous.

5. DOES THE GENERAL THEORY OF RELATIVITY CONTAIN THE GENERAL PRINCIPLE OF RELATIVITY?

The consequences of the above results in connection with the principle of relativity are now discussed.

5.1. Einstein's Position: General Relativity Includes the General Principle of Relativity

In the following discussion we defend Einstein's view, that the principle of relativity is contained in the general theory of relativity. This view has been expressed succintly by Møller in his standard textbook on general relativity [17]:

> Einstein advocated a new interpretation of the fictitious forces in accelerated systems of reference. The "fictitious" forces were treated as real forces on the same footing as any other force of nature. The reason for the occurrence in accelerated systems of reference of such peculiar forces should, according to this new idea, be sought in the circumstance that the distant masses of the fixed stars are accelerated relative to these systems of reference. The "fictitious forces" are thus treated as a kind of gravitational force, the acceleration of the distant masses causing a "field of gravitation" in the system of reference considered. Only when we work in special systems of reference, viz. systems of inertia, it is not necessary to include the distant masses in our considerations, and this is the only point which distinguishes the systems of inertia from other systems of reference. *It can, however, be assumed that all systems of reference are equivalent with respect to the formulation of the fundamental laws of physics.* This is the so-called *general principle of relativity.*

Note that the effect of local mass distributions has not been taken into consideration in the paragraph cited. The "systems of inertia" mentioned by Møller are those of a homogeneous universe. In general the term "system of inertia" should be replaced by "frames of reference in which the cosmic mass has no observed rotation or translational acceleration." In the following such frames of reference are called "cosmic frames." The significance of the cosmic frames is that observers at rest in a cosmic frame experience no gravitational dragging, neither rotational nor translational. A much-used cosmic frame is the one associated with the "comoving coordinate systems" which are employed to describe homogeneous, isotropic, nonrotating, and expanding universe models by the Robertson–Walker line-element.

In order to prepare for a thorough discussion of the general principle of relativity, we now define precisely the concepts involved.

5.2. Definition of the Concept "Gravitational Field"

We start by defining the concepts "coordinate system" and "reference frame." A *coordinate system* K is a set of four variables x^μ such that each event in that part of space-time which is covered by K corresponds to one set of numbers (x^0, x^1, x^2, x^3), and all events have different sets of numbers. A *frame of reference* R is a set of noncrossing time-like curves in space-time. These curves are world lines of a set of "fundamental particles" or "fundamental observers" defining R. Roughly, one can say that a reference frame is a set of fundamental particles with given motion.

A change of coordinates inside a given reference frame is called an *internal coordinate transformation.* It has the form

$$x'^i = x'^i(x^j), \qquad x'^0 = x'^0(x^\mu), \qquad i, j = 1, 2, 3, \qquad \mu = 0, 1, 2, 3 \quad (16)$$

The Christoffel symbols of the second kind transform according to

$$\Gamma'^\lambda{}_{\mu\nu} = \frac{\partial x'^\lambda}{\partial x^\tau} \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x}{\partial x'^\nu} \Gamma^\tau{}_{\alpha\beta} + \frac{\partial x'^\lambda}{\partial x^\tau} \frac{\partial^2 x^\tau}{\partial x'^\mu \, \partial x'^\nu} \quad (17)$$

where the first term represents the tensor transformation and the second, inhomogeneous term makes $\Gamma^\lambda{}_{\mu\nu}$, in general, a nontensor. It follows that $\Gamma^\lambda{}_{\mu\nu}$ transforms as a tensor under every linear transformation, for example, under a Lorentz transformation.

The particular Christoffel symbols $\Gamma^i{}_{\mu 0} = \Gamma^i{}_{0\mu}$ transform tensorially under internal coordinate transformations. They are called the *physical* Christoffel symbols.

We also need to make a distinction between the acceleration of a particle relative to a chosen frame of reference and the acceleration of a particle relative to an observer *in free fall.* The first type of acceleration is frame dependent. It is called *observed acceleration.* The observed acceleration is represented by a 3-vector. It is the ordinary acceleration of a body, as measured by standard measuring sticks and clocks fixed in the reference frame of the observer.

The second type of acceleration is frame independent. It is called *cosmic acceleration.* The cosmic acceleration is represented by a space-like 4-vector, the four-acceleration. It is the acceleration of a body, as measured by standard measuring sticks and clocks fixed in the successive instantaneous inertial rest frames of the body.

A local *inertial reference frame* is rotation-free and freely falling. The fundamental particles of an inertial reference frame have vanishing four-acceleration. They have no cosmic acceleration. In addition, the four-velocity field of these particles has no rotation.

The world lines of particles with vanishing four-acceleration are described by the geodesic equation. According to this equation the observed acceleration of a free particle in a frame with nonvanishing cosmic acceleration, i.e., a noninertial frame, arises entirely from the nonvanishing physical Christoffel symbols, $\Gamma^i{}_{\mu 0}$ [18, 19].

We are now able to give a definition of the concept "gravitational field" which takes into account both Newtonian and Einsteinian properties of gravity. This is performed by including both the Newtonian nontidal component of gravity and the relativistic tidal component in an extended definition of gravitational field.

There is a *nontidal* component of a gravitational field present at a point of space-time, if and only if at least one of the Christoffel symbols $\Gamma^i{}_{\mu 0}$ is nonvanishing at this point. There is a *tidal* gravitational field present at a point of space-time, if and only if the Riemann curvature tensor is nonvanishing at this point. There is a *gravitational field* present at a point of space-time, if and only if the Riemann curvature tensor *or* at least one of the Christoffel symbols $\Gamma^i{}_{\mu 0}$ is nonvanishing at this point.

The nontidal component of a gravitational field is calculated from Einstein's field equations, for a given choice of coordinate system. In order to find the nontidal component of a gravitational field, the equations are solved for $\Gamma^i{}_{\mu 0}$. The tidal component is found by solving the equations for the Riemann curvature tensor.

The tidal component is due to mass energy, localized or not. Even if the tidal component of the gravitational field in, for example, the Schwarzschild space-time is source-free, in the sense that it represents a vacuum solution of Einstein's field equations, one may identify a localized spherical mass distribution as "cause" of this field.

In general a nontidal gravitational field has two contributions. There is a contribution associated with localized matter distributions. It is proportional to the mass of the system and inversely proportional to the square of the distance from its center, in the Newtonian limit. There is also a cosmic contribution to the nontidal gravitational field. This is the inertial dragging field. It is proportional to the mass of the cosmic matter and its observed rotation or acceleration.

The nontidal component of a gravitational field is not a propagating field in the usual (wave) sense. The tidal component propagates as gravitational waves. If, say, a galaxy explodes, gravitational waves are emitted and the mass of the galaxy decreases. Then the region in which the decrease in mass may be (electromagnetically) observed is extending with the velocity of light. This is the same as the spreading of a nontidal gravitational field, since $\Gamma^r{}_{00}$ is proportional to the mass of the galaxy in the Schwarzschild space-time surrounding it. It is interesting to note that gravitational waves have to move with the velocity of light in order that the tidal and nontidal components of a gravitational field shall spread together.

From the geodesic equation one finds that the Christoffel symbols $\Gamma^i{}_{\mu 0}$ have the following physical significance. $\Gamma^i{}_{00}$ gives freely moving particles a translational acceleration. $\Gamma^i{}_{j0}$ gives freely moving particles a Coriolis acceleration. Thus it is natural to separate a nontidal gravitational field into two components: a translational component and a rotational one, defined as follows.

There is a *translational* component of a gravitational field present at a point of space-time, as measured in a given reference frame, if and only if at least one of the Christoffel symbols $\Gamma^i{}_{00}$ is nonvanishing at this point. There is a *rotational* component of a gravitational field present at a point of space-time, in a given reference frame, if and only if at least one of the Christoffel symbols $\Gamma^i{}_{j0}$ is nonvanishing at this point.

Note that the translational component of a gravitational field is generally not homogeneous (i.e., position independent). Also, a homogeneous field can have nonvanishing tidal components (as in the Friedmann cosmological models).

The existence of a nontidal gravitational field is frame dependent. The existence of a tidal gravitational field is frame independent. Our extended definition of the concept gravitational field gives meaning to statements such as, "A uniform gravitational field is indistinguishable from a uniform acceleration of a reference frame" [20, 21], where "uniform" is understood in the sense "translational," as defined above; and "Locally (in a region of space-time not too large) one cannot *in principle* distinguish between the action of a gravitational field and an acceleration" [22]. A related statement is that one can transform away a gravitational field locally by going into a local inertial frame [23–25]. The precise meaning of the word "local" in this context is that the observations are to be temporally and spatially restricted so that tidal forces cannot be measured for a specified measuring accuracy. Thus the statements above concern the *nontidal* components of gravitational fields.

We now proceed to a discussion of the two usual versions of the principle of equivalence and the general principle of relativity.

5.3. The Fundamental Principles

According to the *weak principle of equivalence* the inertial mass of a particle is proportional to its gravitational mass.

The *strong principle of equivalence* may be formulated as follows: given a certain measuring accuracy, then there exists a local inertial system J_{GR}, so that to every physical process P_1 in J_{GR}, there exists a physical process P_2 in an inertial system J_{SR} in flat space-time, with the property that P_2 is observed in J_{SR} just like P_1 is observed in J_{GR}, assuming that P_1 and P_2 are sufficiently restricted in space and time.

The strong principle of equivalence concerns the existence and physical equivalence of inertial systems in arbitrary regions of space-time. However, if we compare observations in noninertial and inertial reference frames, we find differences. Light, for example, is deflected in a noninertial reference frame but moves along a straight line in an inertial frame. There are generally several ways, both mechanically and optically, to discover if one is in an inertial or a noninertial reference frame.

In spite of this, Einstein generalized the special principle of relativity to a general principle of relativity, encompassing noninertial reference frames. Even if one restricts oneself to using inertial frames, the chain of events considered in a process will depend not only upon the laws of nature, but also upon the boundary conditions. If arbitrary reference frames are employed, the process will also depend upon the metric. If the laws of nature are formulated in a metric-independent way, one may state *the general principle of relativity* as follows: *the laws of nature may be stated in the same way in every frame of reference.* As for the motion of light, for example, the law is that light follows null geodesic curves. From this law it follows that light is deflected in noninertial frames and follows straight paths in inertial ones.

Actually, there exist two versions of the general principle of relativity. The above one, which is equivalent to the formulation of Møller cited above, is henceforth referred to as "the weak principle of relativity." It concerns all types of physical phenomena.

The second version of the general principle of relativity, which is *not* equivalent to the first one, is loosely referred to as "the principle of relativity of all motion" and may be stated: as far as gravitational phenomena are concerned, every observer may consider his laboratory as at rest. This is referred to as "the strong principle of relativity." The expression "gravitational phenomena" means that the observed bodies are neutral with respect to all types of charge (except mass).

The weak principle of relativity concerns our formulations of the laws of nature. The strong principle of relativity, on the other hand, concerns the *consequences* of these laws, as to observable phenomena. It says that the gravitational laws must, among others, have as a consequence that all gravitational phenomena observed in an arbitrary laboratory can be explained, while considering the laboratory as at rest. As shown below, the translational inertial dragging effect is of vital importance in this respect.

The special principle of relativity says that there is no absolute velocity. This applies to all types of phenomena. The strong principle of relativity says that, with respect to gravitational phenomena, there exist no absolute acceleration. One may note that Newtonian dynamics and gravitational theory obey the special principle of relativity but not the strong principle of relativity.

As an illustration of the role of inertial dragging for the validity of the strong principle of relativity, we consider the Moon orbiting the Earth. As seen by an observer on the Moon both the Moon and the Earth are at rest (disregarding the observed spin of the Earth, which is of no concern here). If the observer solves Einstein's field equations for the vacuum space-time outside the Earth, he might come up with the Schwarzschild solution and conclude that the Moon should fall toward the Earth, which it does not. So it seems impossible to consider the Moon as at rest, which would imply that the strong principle of relativity is not valid.

This problem has the following solution. As observed from the Moon the cosmic mass rotates. The rotating cosmic mass has to be included when the Moon observer solves Einstein's field equations. Doing this he finds that the rotating cosmic mass induces the rotational nontidal gravitational field which is interpreted as the centrifugal field in Newtonian theory. This field explains to him why the Moon does not fall toward the Earth.

As we have shown above, corresponding results are valid for observers with accelerated translational motion.

5.4. The General Principle of Relativity Is Not Included in the General Theory of Relativity: An Argument

An argument against the extension of the principle of relativity to accelerated motion has recently been given, by considering a variant of Newton's bucket experiment in which the bucket is uniformly accelerated [4, 26]. In order to make our discussion of this important question as self-contained as possible, we cite the whole argument given in Ref. 4.

> Mach considered all motion to be relative. In rejecting the notion of absolute space Mach had predecessors in Leibniz and Berkeley, among others. If only relative motion has significance, the inertial frames must be determined by matter. To give these vague ideas a more definite formulation, one may extend the principle of relativity to accelerated motion and postulate that inertial forces are due to the gravitational *field* generated by all matter in the universe. According to Einstein's relativistic theory of gravitation (which has observational support for macroscopic phenomena), however, these notions must be rejected since they imply the *global equivalence* of inertial and certain gravitational forces in contrast to Einstein's principle of equivalence which is purely local. To illustrate this point, consider a variant of Newton's bucket experiment in which the bucket is uniformly accelerated. Other than forces of electromagnetic origin (such as viscosity), the fluid in the bucket is also subject to a *uniform* inertial force field (relative to the bucket). A contradiction arises, however, if the bucket is now treated as freely falling in the gravitational field generated by all the matter in the universe in accelerated motion, since according to Einstein's theory the only external gravitational forces that affect the motion of the fluid relative to the bucket are *tidal* forces.

5.5. Nonvalidity of the Argument

This argument is now discussed in light of the results above.

In the following the term "accelerated" or "acceleration" means "(with) *observed* acceleration."

The calculations in Sections 2 and 3 gave the result that a *freely falling* object in the field generated by an accelerated cosmic mass will accelerate in the same direction as the cosmic mass [see Eq. (15)]. The mass density of the universe is near the critical density. This indicates that the proportionality constant between the acceleration of the free test body and the acceleration of the cosmic mass is of order unity. We assume that the mass and radius of the universe have magnitudes such that $a = g$.

Consider now the argument cited above. One first talks about a uniformly accelerated bucket and then points out that a contradiction arises if the bucket is treated as moving freely in the gravitational field generated by all the matter in the universe in accelerated motion. But according to our results and the assumption above, concerning the mass and radius of the universe, a bucket that moves freely in a horizontal direction will have vanishing acceleration relative to the cosmic mass.

In the argument cited in Section 5.4. the uniform acceleration of the bucket is a "cosmic" acceleration. Therefore, in order to give the bucket this acceleration, a *nongravitational* force must act on it. The bucket may, for example, be acted upon by a rocket. The two situations referred to above are dynamically different. A contradiction would have been present if the argument of Ref. 4 referred to one and the same situation as described from different frames of reference. That is not the case.

5.6. Nonvalidity of the Strong Principle of Relativity Extended to Encompass Electromagnetic Phenomena

In Section 2 we found, with reference to an inertial reference frame, that there was an electric field inside an accelerated charged shell. The same situation can, of course, be described from the uniformly accelerated rest frame of the shell (see Appendix).

Below we consider a variant of Newton's bucket experiment in which a bucket with water, at the surface of the Earth, is given a horizontal cosmic acceleration by, for example, a rocket.[3]

In the electromagnetic case this corresponds to a situation where a test-charge accelerates inside a static charged shell. As described from the inertial rest frame of the shell, it is obvious that the electromagnetic field

[3] The rotation of the Earth is of no relevance for our considerations and is therefore neglected.

tensor vanishes inside the shell. Thus the test-charge does not experience any electrical force.

This leads to an argument in which the general principle of relativity seems to defy our result given in Eq. (13), which predicts the existence of an electrical field inside a charged, uniformly accelerated spherical shell. Assuming relativity of all motion, the electrical field determined by a static observer inside the accelerated, charged shell should be the same as that determined by an observer accelerating in the opposite sense inside a static shell. Since the electromagnetic field *tensor* is zero inside the static shell, it should remain zero for the accelerated observer.

The conclusion that we are led to by the above argument is that the strong principle of relativity is not valid for electromagnetic phenomena.

But it *can* be applied to the corresponding gravitational situation. This leads to the following: the gravitational field determined by an inertial observer inside the accelerated shell must be the same as that determined by an observer accelerating in the opposite sense inside an inertial shell. This is perfectly correct. But a statement such as, "If a static observer finds that the electromagnetic field tensor vanishes inside a nonaccelerated shell, then it vanishes for an observer that accelerates through it, too," cannot be applied to the gravitational case. A uniform gravitational field is described not by a field tensor, but by certain Christoffel symbols. These are not invariant with respect to a transformation between an inertial and a non-inertial reference frame.

Like other laws of nature the strong principle of relativity has a restricted range of validity. An example of a similar kind is the nonvalidity of the principle of parity-invariance for the weak interaction. An interesting possibility should be mentioned, however. According to the Kaluza–Klein theory electromagnetism may be given a geometrical interpretation in a five-dimensional world, free of charges and electromagnetic fields. We conjecture that there exists a five-dimensional version of the strong principle of relativity with a region of validity encompassing electromagnetic phenomena.

5.7. The Translational Bucket Experiment

We now consider the variant of Newton's bucket experiment in which a bucket with water at the surface of the Earth is accelerated horizontally by, for example, a rocket [22]. Since the Earth moves freely, the observed acceleration coincides with the horizontal component of the bucket's cosmic acceleration, which is due to a nongravitational force F^{μ}.

Because of the force from the accelerated bucket upon the water, the surface of the water will not be horizontal.

The frame-independent law governing the shape of the water surface in the bucket is the equation of motion of the liquid particles, which may be stated as follows: the rate of change of four-momentum of a particle equals the sum of the four-forces acting on it. For a particle with constant rest mass m_0 this gives

$$F^{\mu} = m_0 U^{\mu}{}_{;\nu} U^{\nu} \tag{18}$$

where U^{μ} are the components of the four-velocity of the particle. To first order in v/c and Φ/c^2 the spatial components of Eq. (18), as referred to an arbitrary frame of reference, reduce to

$$\mathbf{F} = m_0[\mathbf{a} - \tfrac{1}{2}c^2 \nabla h_{00} + \partial \mathbf{h}/\partial t] \tag{19}$$

where $\mathbf{a}$ is the observed acceleration of the test particle.

According to our assumption above, concerning the mass and radius of the universe, the translational inertial dragging due to the cosmic mass makes an inertial reference frame coincide with a cosmic reference frame as far as horizontal motion is concerned.

Referred to a member of this class of reference frames, the equation reduces to the ordinary form of Newton's second law. The water accelerates due to the force $\mathbf{F}$, and one finds that the surface of the water is inclined at an angle α to the horizontal, given by $\tan \alpha = a/g_0$, where g_0 is the acceleration of gravity.

Observed in the rest frame R^1 of the bucket, the force $\mathbf{F}$ keeps the bucket at rest. According to this observer the gravitational field at the position of the bucket has two components: one due to the Earth and one due to the accelerated cosmic mass. Thus the equipotential surfaces are not parallel to the surface of the Earth. Using the expression for $-\frac{1}{2}c^2 \nabla h_{00} + \partial \mathbf{h}/\partial t$ given in Eq. (15), we find that the water is inclined at an angle β to the horizontal, given by $\tan \beta = (11GM/3c^2R)(a/g_0)$. If $11GM/3c^2R = 1$, this will be consistent with the analysis as referred to R.

5.8. The Strong Principle of Relativity, Mach's Principle, and Translational Inertial Dragging

The induced gravitational acceleration observed in R^1 has a Machian character. In a homogeneous universe it tends to minimize the relative acceleration between a free test particle and the mass of the universe. This acceleration field gives an explanation of why it is necessary to apply a nongravitational force to accelerate a particle relative to the average motion of the matter in the universe. Also, the existence of this field is necessary in order for the strong principle of relativity to be valid for gravitational phenomena.

All four-forces F^{μ} may, in principle, be reduced to the known interactions of nongravitational nature (electroweak and color interaction). They are determined by the distribution of matter surrounding the test particle. If *one* such force acts on the test particle as referred to R, then *one* such force acts on the particle as observed in R^1 also. Then it would be impossible to explain how the particle can stay at rest in R^1 if there were no induced gravitational forces acting in R^1. Thus the kinematical and dynamical consequences of a transformation between R and R^1 would be inconsistent with each other. This would constitute a violation of the strong principle of relativity.

This example illustrates an important difference between the four-forces F^{μ} and the induced gravitational acceleration. The existence of a four-force F^{μ} depends upon the distribution of matter but not upon its motion. This is a property of the physical situation that does not depend upon the choice of reference frame. Thus it is not, in general, possible to transform away a four-force F^{μ}. The induced gravitational acceleration, on the other hand, depends upon the *motion* of the surrounding matter. It can be transformed away by locally going into a reference frame in which the surrounding matter is at rest (in the cases that such a rest system exists).

Note that the strong principle of relativity, as stated above, does not imply a *global* equivalence of inertial acceleration fields (in Newtonian sense) and gravitational dragging fields. There may exist inertial acceleration fields with properties that cannot be produced by gravitational induction. The inertial acceleration field in a rotating reference frame, for example, has the property that the acceleration of a free test particle is proportional to its distance from the axis. Presumably no vacuum solution exists where this appears as a property of the gravitational field induced by a distant rotating distribution of mass.

The strong principle of relativity, however, says only that an observer *with arbitrary motion* is allowed to consider himself (his laboratory) as at rest. No gravitational experiment in his laboratory can tell whether the laboratory moves or not. In order for this to be true, gravitational force fields must be induced by moving masses, so that results of *local* experiments may be explained equally well under the assumption that the laboratory is at rest, as under the assumption that it moves in an arbitrary manner.

APPENDIX

We consider the electrical field inside an accelerated, charged shell, as described from the hyperbolically accelerated rest frame K' of the shell. In this references frame there is Rindler space with a horizon at $x = 0$.

The potentials at a point $P(x, y, z)$ due to a charge q at rest at (x_1, y_1, z_1) are [27]

$$\Phi = (q/r)(x_1/x)^{1/2}\,(1 + r^2/4xx_1)^{-1/2}\,(1 + r^2/2xx_1) \tag{A1}$$

$$\mathbf{A} = (-q/x)\,\mathbf{e}_1 \tag{A2}$$

where r is the distance between the charge and the field point.

The fields are

$$\mathbf{B} = \nabla \times \mathbf{A} = 0, \qquad \mathbf{E} = -(1/x)\,\nabla(x\Phi) \tag{A3}$$

Let the center of the shell be at $x = b = c^2/g$. We put $x = b + \zeta$ and $x_1 = b + \zeta_1$ and get, for the potential at P due to dq at the shell,

$$d\Phi = (dq/r)(1 + \xi_1/b)^{1/2}\,(1 + \xi/b)^{-1/2}\,[1 + r^2/4(b + \xi)(b + \xi_1)]^{-1/2} \times [1 + r^2/2(b + \xi)(b + \xi_1)] \tag{A4}$$

The acceleration is assumed to be small, i.e., $g \ll c^2/R$, so that $R \ll b$ and $\zeta \ll b$, $\zeta_1 \ll b$. To first order in $\zeta g/c^2$ this gives

$$d\Phi \approx (dq/r)[1 + (\xi_1 - \xi)/2b] \tag{A5}$$

Integrating over the spherical shell, we find at a point inside the shell

$$\Phi = (Q/R)(1 - g\xi/3c^2) \tag{A6}$$

To lowest order Eqs. (A3) and (A6) give

$$\mathbf{E} = -(2Q/3Rc^2) \tag{A7}$$

in accordance with Eq. (6).

One point seems to require a comment. In the electromagnetic case the calculation, as referred to the rest frame of the shell, is based on the Rindler metric, while the corresponding gravitational calculation is based upon the Minkowski metric. This difference is due to the following circumstance. The electromagnetic case concerns the calculation of an electromagnetic field in a given metric. But in the gravitational case the field is essentially the metric itself. The field is a perturbation of the metric inside a static shell, which is the Minkowski metric.

The difference between the electromagnetic and the gravitational cases is an expression of the fact that the strong principle of relativity is valid as far as gravitation concerns, but it is not generally valid in connection with electromagnetic phenomena. For example, there is no electromagnetic field *tensor* inside an inertial shell, whether the description is referred to an inertial or a noninertial observer. But in a shell with cosmic acceleration there is a nonvanishing electromagnetic field tensor, which cannot be transformed away by going into the accelerated rest frame of the shell.

In the gravitational case the electromagnetic field tensor is replaced by an acceleration field, which is given by certain Christoffel symbols. This is not a tensor field. The Christoffel symbols may be transformed away by a suitable choice of reference frame. This *nontensorial property of the gravitational acceleration field* is compatible with the validity of the strong principle of relativity.

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