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Ann. Rev. Biochem. 40, 237–258 (1971) is a contemporary literature review of periodic chemical phenomena with references to theoretical treatments. (I)

183. **"Oscillation and Chaos in Physiological Control Systems,"** M. C. Mackey and L. Glass, Science 197, 287–289 (1977) discuss two examples of physiological control processes and their abnormalities (fluctuations in white cell count and irregularities in breathing) within a common theoretical framework. (I)

184. **"Neural Circuits for Generating Rhythmic Movements,"** W. O. Friesen and G. S. Stent, Ann. Rev. Biophys. Bioeng. 7, 37–60 (1978) is a literature review of experimental results and of models of rhythmn generators. (E)

185. **"Oscillatory Neural Networks,"** A. I. Selverston and M. Moulins, Ann. Rev. Physiol. 47, 29–48 (1985). (I)

186. **"Intersegmental Coordination of the Leech Swimming Rhythm, I. Roles of Cycle Period Gradient and Coupling Strength,"** R. A. Pierce and W. O. Friesen, J. Neurophysiol. 54, 1444–1459 (1985). (I)

187. **"Chemical Clocks: Experimental and Theoretical Models of Nonlinear Behavior,"** E. V. Mielczarek, J. S. Turner, D. Leitner and L. Davis, Am. J. Phys. 51, 32–42 (1983). (I)

188. **"Chemical Clocks, Feedback, and Nonlinear Behavior,"** A. A. Harms and O. E. Hileman, Am. J. Phys. 53, 578 (1985) comments upon the previous reference. (I)

189. **"The Random Element in Bird Navigation,"** D. H. Wilkinson, J. Exp. Biol. 29, 532–560 (1952). (I)

190. **"Optimization of Brownian Search Strategies,"** G. Hoffmann, Biol. Cybern. 49, 21–31 (1983). (I)

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The Cavendish experiment as Cavendish knew it

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The Cavendish experiment is always interpreted now as a measurement of *G*, the universal gravitational constant, but that is an interpretation that Cavendish did not make. He thought that he had measured the mean density of the Earth, and he was only one of many experimenters measuring the density of the Earth by many methods during the 18th century.

I. INTRODUCTION

In 1798 Henry Cavendish performed an experiment now always described in physics textbooks as a measurement of the universal gravitational constant *G*. Cavendish did not report his work as a measurement of a gravitational constant, however, and in fact that did not become the standard interpretation for over 100 years. The paper in which Cavendish reported his experiment was titled "Experiments to Determine the Density of the Earth,"[1] and it is interesting to understand why he interpreted the work in that way.

The determination of the density of the Earth was an important problem from the time of Newton until approximately 1900, and to understand why we must go back to Newton and the law of universal gravitation.

Newton never stated the law of universal gravitation as we memorize it, and in the *Principia* it appears only as Proposition VII, Theorem VII of Book III: "That there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they obtain" and Corollary II to that theorem: "The force of gravity towards the several particles of any body is inversely as the square of the distances of places from the particles ..."[2] There is no suggestion in the *Principia* that Newton ever wrote the statement of the law of gravitation in the form of an equation with a constant; when he carried out computations using the law, he worked with ratios rather than with an equation. If Newton ever considered introducing a constant which could be measured, he probably gave the idea up because of an error in calculation. In *The System of the World* he said that two spheres, 1 ft in diameter, "of a like nature to the earth," placed 1/4 in. apart, would attract one another with a force so small that even in a region devoid of resistance, they would not move to touch in less than a month's time. In fact, the time for two spheres arranged as described to move to touch because of their mutual gravitational attraction is more nearly 5 min than one month, but the incorrect estimate of the force between two terrestrial objects convinced Newton that "experiments in terrestrial bodies do not count."[3]

At the same time that he did not suggest that a constant be introduced into an equation for universal gravitation, Newton did raise a question about the density of the Earth. In Book III of the *Principia* he gave the relative densities of the Sun, Jupiter, Saturn, and the Earth as 100, $94\frac{1}{2}$, 67, and 400,[4] respectively, so that if the density of the Earth were known the densities of the Sun and the two planets whose satellites were known to Newton could be calculated. On the basis of the known density of rocks at the surface of the

Earth and of rocks from mines, Newton guessed that the average density of the Earth is 5 or 6 times the density of water. His guess was quite accurate, but it was only an estimate. Newton left natural scientists with a motive to measure the density of the Earth similar to the motive to measure the astronomical unit. In the one case the dimensions of the solar system were known in terms of the astronomical unit, and when it was measured in terrestrial units the scale of the solar system was known. Similarly, the densities of the Sun and some of the planets were known in terms of density of the Earth, and when the Earth's density was measured, the other densities were known. Great efforts went into measuring both those basic numbers.

II. MEASUREMENTS TO DETERMINE THE DENSITY OF THE EARTH

Between 1687, when Newton published the *Principia*, and 1892, when C. V. Boys read a paper titled "On the Newtonian Constant of Gravitation,"[5] at least 38 papers had been published dealing with measurements of gravitation,[6] and only one of those, published in 1884, had mentioned the gravitational constant in its title. That 1884 paper was "Eine Neue Methods zur Bestimmung der Gravitations-Constante."[7] All the other papers were entirely directed at the density of the Earth.

Most of the methods used to measure the density of the Earth during that period of two centuries could not possibly have yielded a gravitational constant. At least five different methods were tried.

The first attempt to determine the density of the Earth used the deflection of a pendulum hung near a mountain. It was first tried in France in 1749, but the results were so poor that the experimenter realized that the experiment was quantitatively useless. In 1775 the same method was used at the mountain Schelhallien in Scotland; in 1814 the same method was used at Mimet, a few miles north of Marseilles; and in 1856 the method was used again at Arthur's Seat, just outside Edinburgh. In the application of the method, the line of a pendulum hanging near the mountain was compared to the line expected if the mountain were not there, with a sighting of a star providing the reference system. After the deviation from normal was found, the density of the rocks of the mountain was estimated, the shape of the mountain was determined by surveying, and the relative attraction of the mountain and the Earth on the pendulum was determined. The density of the Earth could then be computed in terms of the measured and estimated density of the material of the mountain.

A second method also used a mountain, but the value of g was measured at the top of the mountain and then that was compared to the value of g computed for the same altitude assuming the Earth's surface to be smooth. The difference was attributed to the attraction of the mass of the mountain, and of course, the accuracy of the computed density of the Earth was limited by uncertainty in the knowledge of the mass of the mountain just as in the first method. That method was tried at Mount Cenis in Italy in 1824 and again at the same place in 1841, and in 1880 T. C. Mendenhall[8] used the same method at Mount Fujiyama, which is particularly well suited to application of the method because of the symmetry of the cone.

A third method involved measuring g, using a seconds pendulum, at the surface of the Earth and then at the bottom of a mine shaft. Between 1826 and 1885 the method was used at least five times in England, Bohemia, and in Germany.

The fourth method measured the attraction between a small mass in the pan of a balance and a large sphere of lead placed below the pan. In order to reduce the attraction of the lead sphere on the second pan, the two pans were hung at different distances from the balance beam; in one case one pan was hung 5 m lower than the other, and in a repetition of the experiment one pan was 21 m below the other.

The fifth method was use of the torsion pendulum, the method first used by Cavendish. Between the time of his experimental work and 1885, at least six other experimenters used the same method to determine the density of the Earth.

From this recital of experimental attempts to measure the density of the Earth, one may infer that the problem was of great importance to the experimenters of the 19th century. We may conclude, however, that that period of interest in the density of the Earth had ended by 1892, when C. V. Boys read his paper "On the Newtonian Constant of Gravitation" at the Royal Institution. The paper was published in Nature in 1894.[5] In the paper Boys explained his understanding of his work thus:

> Owing to the universal character of the constant G, it seems to me to be descending from the sublime to the ridiculous to describe the object of this experiment as finding the mass of the earth or the mean density of the earth, or less accurately the weight of the earth. I could not lecture here under the title that has always been chosen in connection with this investigation. In spite of the courteously expressed desire of your distinguished and energetic secretary, that I should indicate in the title that, to put it vulgarly, I had been weighing the earth, I could not introduce as the object of my work anything so casual as an accidental property of an insignificant planet. To the physicist this would be equivalent to leaving some great international conference to attend to the affairs of a county council, I might even say of a parish council. That is the business of the geologist. The object of these investigations is to find the value of G. The earth has no more to do with the investigation than the table has upon which the apparatus is supported. It does interfere and occasionally, by its attraction breaks even the quartz fibres that I have used. The investigation could be carried on far more precisely and accurately on the moon, or on a minor planet, such as Juno; but as yet no means are available for getting there.

Thus we find, more than 100 years after Cavendish published the description of his experiment, a forceful rejection by Boys of the concern with the density or mass of the Earth and the interpretation of the experiment familiar to most of us.[9]

III. THE CAVENDISH EXPERIMENT

If Cavendish did not use a constant in his calculations and did not think in the terms we use, how did he analyze his results? How can one use observations made with the torsion pendulum to calculate the density of the Earth without introducing the gravitational constant? The answer is that Cavendish related the deflection of his torsion

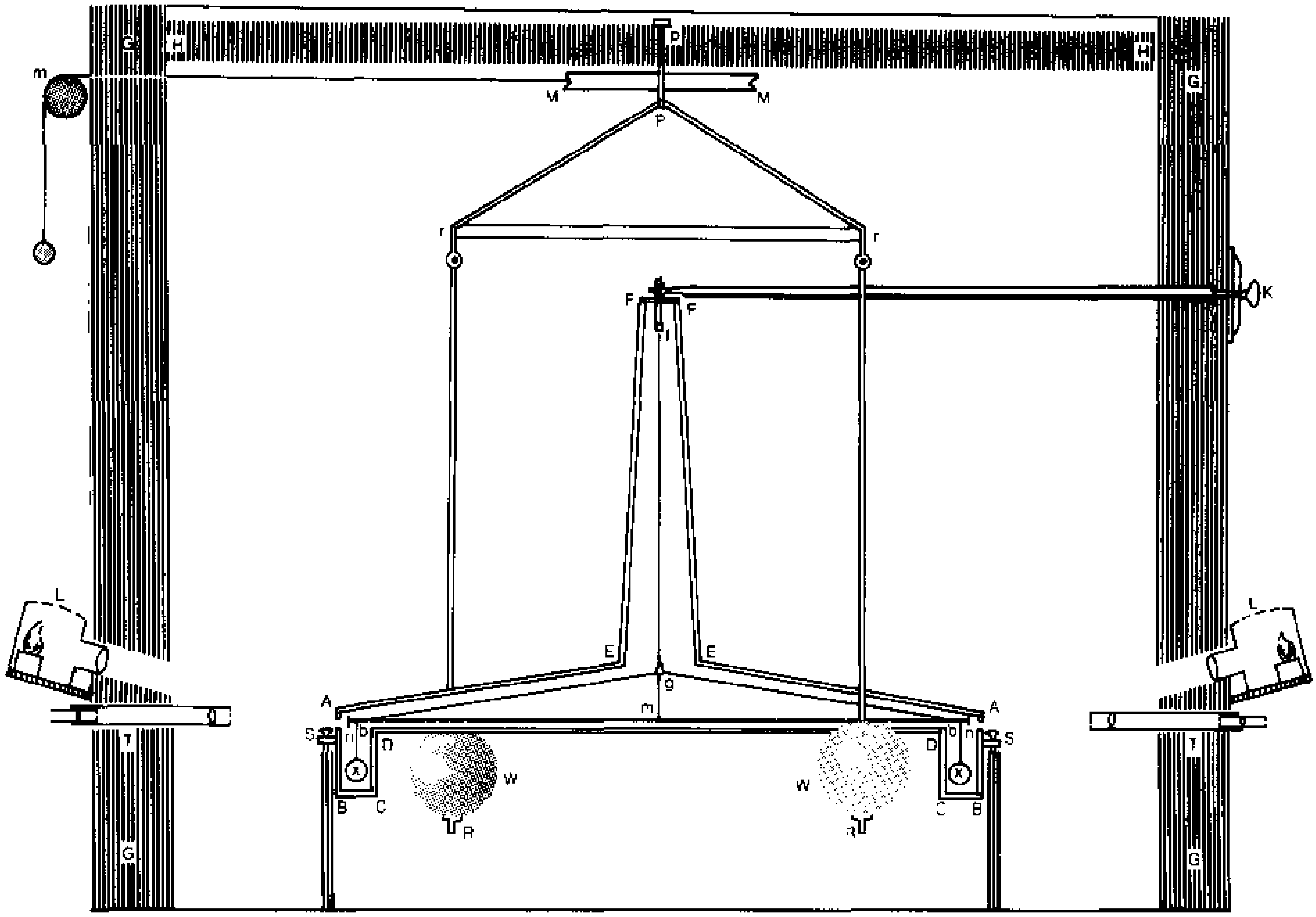

Fig. 1. A longitudinal vertical section through Cavendish's instrument and the building in which it was placed. His description: "ABCDDCBAEFFE" is the case; x and x are the two balls, which are suspended by the wires bx from the arm gbmb, which is itself suspended by the slender wire gl. This arm consists of a slender deal rod bmb, strengthened by a silver wire bgb; by which means it is made strong enough to support the balls, though very light. "The case is supported, and set horizontal, by four screws, resting on posts fixed firmly into the ground: two of them are represented in the figure, by S and S; the two others are not represented, to avoid confusion. GG and GG are the end walls of the building. W and W are the leaden weights; which are suspended by the copper rods RrPrR, and the wooden bar rr, from the center pin Pp. This pin passes through a hole in the beam HH, perpendicularly over the center of the instrument, and turns round in it, being prevented from falling by the plate p. MM is a pulley, fastened to this pin; and Mm, a cord wound round the pulley, and passing through the end wall; by which the observer may turn it round and thereby move the weights from one situation to the other." Observations were made from outside the room by the telescopes shown.

pendulum to the density of the Earth by first comparing it to a simple pendulum.

Cavendish used a torsion pendulum comprising a beam 73.3 in. long suspended by a fiber 39.25 in. long. From each end of the beam hung lead spheres about 2 in. in diameter. The suspension fiber first used was of such a stiffness that the pendulum had a period of about 15 min when it was disturbed. That fiber did not offer enough resistance to twisting and was replaced by one which produced a period of about 7 min. Outside the box in which the pendulum was suspended (to control air currents) were two lead spheres, each weighing about 348 lbs. Observations were made of the natural period of the pendulum and of the displacement of the pendulum from its rest position by the attraction of the external lead spheres, twisting the pendulum first in one direction and then in the other direction. (See Fig. 1.)

The reasoning of the analysis, recast into modern terminology, is this: Compare the torsion pendulum of length 73.3 in. (half-length = 36.65 in.) with a simple pendulum of length 36.65 in. When the simple pendulum is pulled aside from its equilibrium position, a restoring force F_0 acts on it to return it to the equilibrium location. If the weight of the mass on the simple pendulum is W, the restoring force is to the weight W as the length of the arc by which the mass is displaced from equilibrium is to the length of the pendulum, or

$$F_0/W = \text{arc length}/l,$$

from which

$$F_0 = W\,(\text{arc}/l).$$

The simple pendulum has a period T_0. Any other pendulum with the same length but a different restoring force will have a period T, related to T_0 by

$$F/F_0 = T_0^2/T^2$$

because the period of a pendulum is inversely proportional to the square root of the restoring force. (Proved by Newton in the *Principia*.)

What Cavendish wished to find was the restoring force F acting on the torsion pendulum, and from the previous equation that can be expressed as

$$F = F_0(T_0^2/T^2) = W(\text{arc}/l)(T_0^2/T^2)$$

or

$$F/W = (T_0^2/T^2)(\mathrm{arc}/l)\ .$$

The force which is to pull the pendulum aside against the restoring force provided by the suspension is the attraction of the lead sphere, but at this point Cavendish introduces an imaginary sphere of water. The mass of the lead sphere is 2 439 000 grains (348 lbs), which is 10.64 times the weight of a sphere of water 1 ft in diameter. The center of the lead sphere is 8.85 in. from the center of the lead ball on the pendulum when it is at its equilibrium position, and therefore the attraction on that ball of the lead sphere is $10.64\times(6\,\mathrm{in.}/8.85\,\mathrm{in.})^2$ as great as would be the attraction of a sphere of water 1 ft in diameter if the attracted ball were on its surface. Therefore if F is the force exerted by the lead sphere, d_w is the density of water, d_E is the density of the Earth, r_E is the radius of the Earth (20 900 000 ft), and r_w is the radius of the sphere of water, (0.5 ft),

$$F/W = 10.64(6/8.85)^2 d_w r_w / d_E r_E\ .$$

By substitution one gets

$$\frac{d_w 2.45}{d_E r_E} = \frac{T_0^2}{T^2}\frac{\mathrm{arc}}{l}\ .$$

With the lead spheres in place, Cavendish measured the arc by which the pendulum was displaced from equilibrium, and then everything in the equation was known except the density of the Earth. Typical values in Cavendish's experiments were $T_0 = 0.97$ s, $T = 424$ s, and $\mathrm{arc}/l = 3/766$. These numbers do not give precisely the results Cavendish quotes because I have neglected a small correction factor required because of misalignment of the spheres in the apparatus.

This method of analysis in terms of a simple pendulum, using the ratio of the restoring force to the weight of the pendulum bob, obviates the need to make a clear distinction between mass and weight and also to introduce a proportionality constant into the gravitation expressions. Cavendish did not make the distinction between mass and weight, and nothing in his analysis could have suggested that he should write the law of gravitation with a constant as we write it.

IV. CONCLUSION

I have suggested thus far that Cavendish described his experiment as a measurement of the density of the Earth because that was an important problem, engaging the attention of numerous investigators, and because that problem had been posed, whereas a measurement of a constant to go into an equation expressing universal gravitation was not a recognized problem. We may think about the matter in another way, however, which may be illuminating. If Cavendish, or one of his contemporaries, had wished to calculate a gravitational constant, how could it have been expressed? The system of units in use did not include a unit for force; no unit of force was proposed until 1873, when the dyne was introduced. Cavendish expressed distances in feet and inches and weights or masses in grains. One might express a gravitational constant without using a unit for force, but surely the idea of measuring such a constant is less likely to occur to an experimenter when no unit for force is available.

We may conclude that Cavendish did precisely what his paper says—he measured the density of the Earth. His report of the experiment gives no hint that he thought in terms of gravitational constant, and his analysis neither needs nor suggests a constant in the equation expressing universal gravitation. It is, of course, not difficult to take the data Cavendish gave and derive from them a value of G, but he did not do that and he did not suggest that he knew that it might be desirable.

[1] H. Cavendish, Philos. Trans. **17**, 469 (1798).
[2] Sir Isaac Newton, *Principia* (University of California, Berkeley, CA, 1973), Vol. 2, p. 414.
[3] Reference 2, pp. 569–570.
[4] Reference 2, p. 417.
[5] C. V. Boys, Nature **50**, 330 (1894).
[6] These papers are listed in the bibliography of J. H. Poynting, *The Mean Density of the Earth* (Charles Griffin & Co., London, 1894), pp. ix–xix.
[7] A. König and F. Richarz, Sitzungsberichte der Berl. Akad. 1884, p. 1203; Wied. Ann. **24**, 664; Nature **31**, 484.
[8] T. C. Mendenhall, Am. J. Sci. **21**, 99 (1881).
[9] Boys perhaps was not the first to reinterpret the Cavendish experiment in terms of the universal gravitational constant, but the fact that he was asked to speak on "Weighing the Earth" or some similar topic indicates that the new interpretation was not common when he delivered his paper. The force of his argument must have been convincing, however, for soon after that time the original statement of Cavendish's work seems to have been forgotten. A physics textbook published in 1896 [E. L. Nichols and W. S. Franklin, *The Elements of Physics* (Macmillan, New York, 1896)] writes the equation for the attraction of the Earth for a body of mass m on its surface in terms of the universal gravitation constant k and then explains that the mass of the earth can be computed if k, g, and r (radius of the Earth) are known. Then this statement follows: "This determination was first made by Cavendish, the difficult experimental part of his work being the observation of the extremely small force F of attraction of two lead balls of known mass at a known distance, in order to determine the quantity, k."

Twenty years after the publication of the paper by Boys the original form of the Cavendish experiment was completely misrepresented in a textbook published in Philadelphia [A. W. Duff, *A Text-Book of Physics* (Blakiston's Son & Co., 1912)]. In it the gravitation equation is written with the constant represented by G as in present notation. Then "To find the magnitude of G it is necessary to measure F in some case where m, m', and r are all known. This was first done by Henry Cavendish..." Later in the discussion this sentence occurs: "The determination of G made it possible to calculate the mass of the earth (hence Cavendish is sometimes said to have been the first to 'weigh the earth')."