A Quantitative Representation of Particle Entanglements via Bohm's Hidden Variable According to Hadronic Mechanics

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In this note, we first recall the 1935 historical view by A. Einstein, B. Podolsky and N. Rosen according to which *"Quantum mechanics is not a complete theory"* (EPR argument), because of the inability by quantum mechanics to provide a quantitative representation of the *interactions* occurring in particle entanglements. We then show, apparently for the first time, that the completion of quantum entanglements into the covering *EPR entanglements* formulated according to hadronic mechanics provides a *quantitative representation of the interactions occurring in particle entanglements* by assuming that their continuous and instantaneous communications at a distance are due to the overlapping of the wave packets of particles, and therefore avoiding superluminal communications. According to this view, entanglement interactions result to be non-linear, non-local and not derivable from a potential, and are represented via Bohm's variable λ hidden in the quantum mechanical *associative* product of Hermitean operators $AB = A \times B$ via explicit and concrete, axiom-preserving realizations $A \hat{\times} B = A \lambda B$, with ensuing *non-unitary* structure, multiplicative unit $U 1 U^{\dagger} = \hat{I} = 1/\lambda$, $\hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A$, *inapplicability'* of Bell's inequalities and consequential validity of Bohm's hidden variables. We finally introduce, also apparently for the first time, the completion of quantum computers into the broader *EPR computers* characterizing a collection of extended electronic components under continuous entanglements, and show their apparent faster computation, better cybersecurity and improved energy efficiency.

According to clear experimental evidence dating back to the early part of the past century, particles that were initially bounded together and then separated, can continuously and instantaneously influence each other at a distance, not only at the particle level (see e.g. [1, 2] and papers quoted therein), but also at the classical level [3].

The above experimental evidence is generally *assumed* to be represented by quantum mechanics and, therefore, particle entanglements are widely called *quantum entanglement* (Figure 1). However, Albert Einstein strongly criticized such an assumption because it would imply superluminal communications that violate special relativity. This occurrence motivated the 1935 historical view by A. Einstein, B. Podolsky and N. Rosen according to which *"Quantum mechanics is not a complete theory"* (EPR argument) [4].

In fact, quantum mechanics can only represent interactions derivable from a potential while *no* quantum mechanical potential is conceivably possible to represent continuous and instantaneous interactions at a distance. More explicitly, the quantum mechanical equation for two interacting particles with coordinates r_k, $k = 1, 2$ on a Hilbert space $\mathcal{H}$ over the field C of complex numbers is given by the familiar Schrödinger equation (for $\hbar = 1$)

$$\left[\Sigma_{k=1,2} \frac{1}{2m_k} p_k p_k + V(r)\right] \psi(r) = E\, \psi(r)\,. \tag{1}$$

When the two particles are entangled, in view of the absence of any possible potential $V(r)$, the above equation becomes

$$\begin{aligned} &\Sigma_{k=1,2} \frac{1}{2m_k} p_k p_k \psi(r_1)\psi(r_2) = \\ &= \left[\Sigma_{k=1,2} \frac{1}{2m_k} \left(-i\frac{\partial}{\partial_{r_k}}\right)\left(-i\frac{\partial}{\partial_{r_k}}\right)\right] \psi(r_1)\psi(r_2) = \\ &= E\, \psi(r_1)\psi(r_2) \end{aligned} \tag{2}$$

and *can only represent two free particles* characterized by the individual wave functions $\psi(r_k)$ without any possible or otherwise known interaction.

At the *2020 International Teleconference on the EPR argument* [5–7], R. M. Santilli proposed the new notion of *Einstein-Podolsky-Rosen entanglement* (Sect. 7.2.3, p. 61 of [6]) which is based on the sole conceivable interaction responsible for particle entanglements, that due to the overlapping of the wave packets of particles (Figure 2), thus being nonlinear as first suggested by W. Heisenberg [8], non-local as first suggested by L. de Broglie and D. Bohm [9] and non derivable from a potential as first suggested by R. M. Santilli at Harvard University under DOE support [13, 14], because of contact, thus continuous and instantaneous character, by therefore voiding the need for superluminal communications.

The non-linear, non-local and non-potential character of the assumed interactions render them ideally suited for their representation via the *isotopic (i.e. axiom-preserving branch of hadronic mechanics,* [15–17]), comprising *iso-mathematics and iso-mechanics* (see [18] for an outline, [19–21] for a

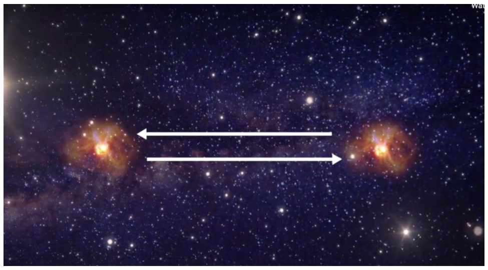

Fig. 1: In this figure, we illustrate the entanglement of particles with continuous and instantaneous interactions at a distance, and recall the argument by A. Einstein, B. Podolsky and N. Rosen on the lack of completeness of quantum mechanics due to its inability to represent said entanglement in a way compatible with special relativity [4].

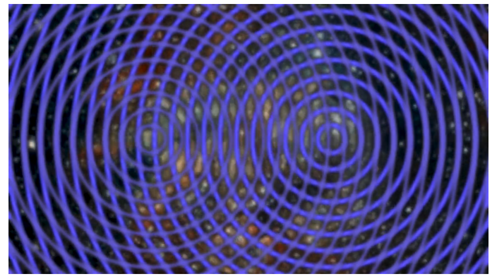

Fig. 2: In this figure, we illustrate the new Einstein-Podolsky-Rosen entanglement of particles introduced by R. M. Santilli in the 2020 overview [6] which is characterized by contact, therefore continuous and instantaneous interactions due to the overlapping of the wave packets of particles represented via Bohm's hidden variable (9), by therefore avoiding the need for superluminal interactions.

review and [22–30] for independent studies) which are characterized by the isotopy $\hat{\xi} : \{\hat{A}, \hat{B}, ..., A\hat{\times}B\}$ of the universal enveloping associative algebra $\xi : \{A, B, ..., AB = A \times B\}$ of quantum mechanical Hermitian operators $A, B, ...$ with *iso-product* (first introduced in Eq. (5), p. 71 of [14])

$$\hat{A}\hat{\times}\hat{B} = \hat{A}\hat{T}\hat{B}, \quad \hat{T} > 0\,, \tag{3}$$

where $\hat{T}$, called the *isotopic element,* is positive-definite but possesses otherwise an unrestricted functional dependence on coordinates, momenta, wave function and any other needed local variable, with related *iso-unit*

$$\begin{aligned} &\hat{I} = 1/\hat{T} > 0\,, \\ &\hat{I}\hat{\times}A = \hat{A}\hat{\times}\hat{I} = \hat{A}, \ \forall\, \hat{A} \in \hat{\xi}\,, \end{aligned} \tag{4}$$

completion of Lie's theory into the *Lie-Santilli iso-theory* [14] (see [23, 28] for independent studies) with iso-brackets for an N-dimensional iso-algebra

$$[X_i \hat{,} X_j] = X_i\hat{\times}X_j - X_j\hat{\times}X_i = X_i\hat{T}X_j - X_j\hat{T}X_i = C^k_{ij}X_k\,, \tag{5}$$

iso-Heisenberg's equation (first proposed in Eq. (18), p. 163 of [14])

$$i\,\frac{dA}{dt} = [A\hat{,}H] = A\hat{\times}H - H\hat{\times}A\,, \tag{6}$$

and related *iso-Schrödinger's equation*

$$H\hat{\times}|\hat{\psi}\rangle = H\hat{T}|\hat{\psi}\rangle = E|\hat{\psi}\rangle\,. \tag{7}$$

Since the isotopic element $\hat{T}$ is *hidden* in the abstract axiom of associativity and becomes visible only in the isotopic realization (3)

$$A\hat{\times}(B\hat{\times}C) = (A\hat{\times}B)\hat{\times}C\,, \tag{8}$$

R. M. Santilli proposed in Sect. 6.8, p. 150 on, Eq. (5.8.19) in particular, of [16], as part of the isotopy of the SU(2) spin algebra and Pauli's matrices in particular, the *identification of the isotopic element $\hat{T}$ with Bohm's hidden variable λ* [31]

$$\lambda = \hat{T}(r, p, \psi, ...)\,, \tag{9}$$

and consequential realization of the iso-unit $\hat{I} = 1/\lambda$, with a variety of explicit and concrete realizations that, for the entanglement of two particles, are of the type [21]

$$\begin{aligned} &\lambda = \hat{T} = 1/\hat{I} = \Pi_{\alpha=1,2}\,\text{Diag.}\left(\frac{1}{n^2_{1,\alpha}}, \frac{1}{n^2_{2,\alpha}}, \frac{1}{n^2_{3,\alpha}}, \frac{1}{n^2_{4,\alpha}}\right)e^{-\Gamma}, \\ &\quad n_{\mu,\alpha} > 0, \quad \Gamma > 0, \ \ \mu = 1, 2, 3, 4, \ \ \alpha = 1, 2\,, \end{aligned} \tag{10}$$

providing:

1) A representation of the dimension and shape of particles via semi-axes $n^2_{k,\alpha}$, $k = 1, 2, 3$ normalized to $n_{k,\alpha} = 1$, $k = 1, 2, 3$, $\alpha = 1, 2$ for the vacuum.

2) A representation of the density of particles via $n^2_{4,\alpha}$ normalized to the value $n^2_{\mu,\alpha} = 1$ for the vacuum.

3) A representation of the non-potential character of the interactions due to the mutual penetration of particles via the exponential term e^{Γ}, where $\Gamma(\hat{r}, \hat{p}, \hat{\psi}, ...)$ is a positive-definite quantity with an unrestricted functional dependence on iso-coordinates $\hat{r} = r\hat{I} = r/\lambda$, iso-momenta $\hat{p}$, iso-wave-functions $\hat{\psi}(\hat{r})$, and other local variables.

By recalling the basic expression of the *iso-linear iso-momentum* characterized by the completion of the local Newton-Leibnitz differential calculus into the non-local *iso-differential calculus* [32] (see [29, 30] for independent studies)

$$\begin{aligned} &\hat{p}\hat{\times}\hat{\psi}(\hat{r}) = \hat{p}\hat{T}(\hat{r}, ...)\psi(\hat{r}) = -i\,\frac{\hat{\partial}}{\hat{\partial}_{\hat{r}}}\hat{\psi}(\hat{r}) = \\ &= -i\hat{I}(\hat{r}, ...)\,\frac{\partial}{\partial_{\hat{r}}}\hat{\psi}(\hat{r}) = -i\,\frac{1}{\lambda}\,\frac{\partial}{\partial_{r/\lambda}}\hat{\psi}(r/\lambda)\,, \end{aligned} \tag{11}$$

the non-relativistic version of the EPR entanglement is characterized by the iso-Schrödinger equation (see [16] for the relativistic extension)

$$\Sigma_{k=1,2}\,\frac{1}{2m_k}\,\hat{p}_k\hat{\times}\hat{p}_k\hat{\times}\hat{\psi}(\hat{r}) =$$

$$= \left[\Sigma_{k=1,2}\,\frac{1}{2m_k}\,\left(-i\hat{I}\frac{\partial}{\partial\hat{r}_k}\right)\left(-i\,\hat{I}\frac{\partial}{\partial\hat{r}_k}\right)\right]\hat{\psi}(\hat{r}) =$$

$$= \left\{\Sigma_{k=1,2}\left[-\frac{\hat{I}^2}{2m_k}\left(\frac{\partial}{\partial\hat{r}_k}\right)\left(\frac{\partial}{\partial\hat{r}_k}\right) - \frac{\hat{I}}{2m_k}\left(\frac{\partial\hat{I}}{\partial\hat{r}_k}\right)\left(\frac{\partial}{\partial\hat{r}_k}\right)\right]\right\}\hat{\psi}(\hat{r}) = \quad (12)$$

$$= \left\{\Sigma_{k=1,2}\left[-\frac{\hat{I}^2}{2m_k}\left(\frac{\partial}{\partial\hat{r}_k}\right)\left(\frac{\partial}{\partial\hat{r}_k}\right) - \frac{\Gamma}{2m_k}\left(\frac{\partial\Gamma}{\partial\hat{r}_k}\right)\left(\frac{\partial}{\partial\hat{r}_k}\right)\right]\right\}\hat{\psi}(\hat{r}) =$$

$$= \hat{E}\hat{\times}\hat{\psi}(\hat{r}) = E\hat{\psi}(\hat{r}) = E\hat{\psi}(\hat{r}_1)\hat{\times}\hat{\psi}(\hat{r}_2)\,,$$

with the following primary characteristics:

1. Iso-equation (9) characterizes a *new entanglement interaction* represented by Bohm's hidden variable $\lambda = \hat{T}$ which is absent in quantum mechanical equation (2);

2. The new entanglement interaction is manifestly *non-linear* (in the wave-function), yet the theory is *iso-linear* [15, 16], namely, it is linear on the Hilbert-Myung-Santilli iso-space $\hat{\mathcal{H}}$ [33] with iso-states $|\psi\rangle$ and iso-normalization [12]

$$\langle\psi|\hat{\times}|\hat{\psi}\rangle = \langle\psi|\lambda|\hat{\psi}\rangle = \lambda\,, \quad (12)$$

over Santtilli iso-field $\hat{C}$ of *iso-real, iso-complex or iso-quaternionic iso-numbers* [34]

$$\hat{n} = n\hat{I} = \frac{n}{\lambda} \quad (13)$$

with iso-superposition principle $\psi(\hat{r}) = \hat{\psi}(\hat{r}_1\hat{\times}\hat{\psi}(\hat{r}_2)$;

3. The new entanglement interaction is manifestly *non-local* in the sense of occurring in *volumes* represented by the iso-unit $\hat{I} = 1/\lambda$ and characterized by the overlapping of two volumes $V_k = (1/n^2_{1,k}, 1/n^2_{2,k}, 1/n^2_{3,k}),\ k = 1, 2$, each being on ontological grounds as big as experimental measurements can allow;

4. The new entanglement interaction is manifestly of contact, zero-range character, thus not being derivable from a potential, and therefore avoiding the need for superluminal interactions required by quantum entanglements [4];

5. The new entanglement interaction verifies, by conception and construction, the abstract axioms of relativistic quantum mechanics although realized via the indicated universal iso-associative envelope [35–37].

It should be indicated that (12) can be equally derived via a *non-unitary transformation* of quantum mechanical equation (2)

$$U \times 1 \times U^\dagger = \frac{1}{\lambda} = \hat{I}\,,$$
$$U \times (A \times B) \times U^\dagger = A'\lambda B',\ \ \lambda = (U \times U^\dagger)^{-1}\,, \quad (14)$$
$$A' = U \times A \times U^\dagger,\ \ B' = U \times B \times U^\dagger\,.$$

The *invariance* of the numeric value of Bohm's hidden variable is then assured by the *Lorentz-Poincaré-Santilli iso-symmetry* [20] with structure [38]

$$U = \hat{U} \times \hat{T}^{1/2}$$
$$\hat{U}\hat{\times}\hat{U}^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \frac{1}{\lambda}\,. \quad (15)$$

It may be of some interest to indicate the expected EPR completion of other branches of physics, such as the completion of quantum computers into new computers, here suggested under the name of *EPR computers,* for the description of extended electronic constituents in global, continuous and instantaneous communications, by therefore approaching the new notion of living organisms attempted in [39], with the following expected advances:

1) Faster computations, since all values of Bohm hidden variable λ are very small according to all available fits of experimental data [6], with ensuing rapid convergence of iso-perturbative series (see also Corollary 3.7.1, p. 128 of [20]). As a confirmation of this expectation, we recall the achievement via iso-mathematics and iso-chemistry of the first known *attractive* force between the *identical* electrons of valence coupling (see Chapter 4 of [40]), resulting in a *strong valence bond* that allowed the first known numerically exact representation of the experimental data for the hydrogen [41] and water [42] molecules with iso-perturbative calculations at least one thousand times faster than their conventional chemical counterparts.

2) Better cybersecurity, due to the formulation via iso-mathematics, with the consequential availability of iso-cryptograms equipped with an algorithm changing the numeric value of the iso-unit with such a frequency to prevent a solution within a finite period of time (Appendix 2C, p. 84 of [15] and [43]).

3) Increased energy efficiency, due to the fact that EPR entanglements are caused by *interactions without potential energy,* thus being more energy efficient than quantum computers.

The following comments are now in order:

I. Verifications of the EPR argument. In 1964, J. S. Bell [44] released a theorem according to which, under the assumption of quantum mechanics, the representation of the spin 1/2 of particles via Pauli's matrices, statistical independence and other assumptions, *a system of point-like particles with spin* 1/2 *does not admit a classical counterpart,* by therefore preventing any possibility of recovering Einstein determinism [4] under the indicated assumptions. The theorem was proved by showing that a certain expression D^{Bell} (whose explicit value depends on the relative conditions of the particles) is always *smaller* than the corresponding classical value D^{Clas},

$$D^{Bell} < D^{Clas}\,, \quad (16)$$

for all possible values of D^{Bell}.

By assuming that nuclear forces have a non-linear, non-local and non-potential component represented via the isotopic element $\hat{T}$, R. M. Santilli initiated in 1981 [10] the studies on the inevitable *completion of Heisenberg's uncertainties and Bell's inequalities for strong interactions.*

Following the achievement of maturity for iso-mathematics and iso-mechanics, by representing the *extended* character of nuclear constituents via the *isotopic completion of Pauli's matrices with Bell's hidden variables* [16], today called the *Pauli-Santilli iso-matrices* formulated on a Hilbert-Myung-Sanilli iso-space [33] over Santilli iso-fields C [34] (see Eqs. (6.8.20), p. 254 of [16])

$$\hat{\sigma}_k = U\sigma_k U^\dagger, \;\; UU^\dagger = \hat{I} = 1/\hat{T} = \text{Diag.}(\lambda^{-1}, \lambda),$$
$$\hat{\sigma}_1 = \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, \;\; \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \tag{17}$$
$$\hat{\sigma}_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix},$$

with *iso-commutation rules*

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = \hat{\sigma}_i \hat{\times} \hat{\sigma}_j - \hat{\sigma}_j \hat{\times} \hat{\sigma}_i = i2\epsilon_{ijk}\hat{\sigma}_k\,, \tag{18}$$

and *conventional spin* $1/2$ *iso-eigenvalues*

$$\hat{\sigma}_3 \hat{\times} |\hat{b}\rangle = \hat{\sigma}_3 \hat{T} |\hat{b}\rangle = \pm |\hat{b}\rangle\,,$$
$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{b}\rangle = (\hat{\sigma}_1 \hat{T} \hat{\sigma}_1 + \hat{\sigma}_2 \hat{T} \hat{\sigma}_2 + \hat{\sigma}_3 \hat{T} \hat{\sigma}_3) \hat{T} |\hat{b}\rangle = 3|\hat{b}\rangle\,. \tag{19}$$

R. M. Santilli proved in 1998 the following *completion of Bell's inequalities for strong interactions* (Eq. (5.8), p. 189 of [11])

$$D^{HM} = \frac{1}{2}(\lambda_1 \lambda_2^{-1} + \lambda_1^{-1}\lambda_2)\, D^{Bell}\,, \tag{20}$$

where λ_1 and λ_2 are the hidden variables of the two particles. Additionally, Santilli proved that D^{HM} can indeed be equal to the corresponding D^{Clas} with specific examples, by therefore confirming Einstein's view on the possible recovering of classical determinism.

Finally, by combining the results of [10] and [11], in 2019 R. M. Santilli [12] (see the review in [20]) proved the following *completion of Heisenberg's uncertainties for strong interactions* (Eq. (35), p. 14 of [12])

$$\Delta r\, \Delta p \approx \frac{1}{2} \left| \langle \hat{\psi}(\hat{r}) | \hat{\times} [\hat{r} \hat{,} \hat{p}] \hat{\times} | \hat{\psi}(\hat{r}) \rangle \right| \ll \frac{1}{2} \hat{T} = \frac{\lambda}{2} \tag{21}$$

establishing that *the standard deviations* Δr *and* Δp*, individually as well as their product, progressively approach Einstein's classical determinism with the increase of the density in the interior of hadrons, nuclei, and stars, and achieve a full classical determinism at the limit of Schwarzschild's horizon for which* $\lambda = \hat{T} = 0$.

In essence, verifications [10–12] of the EPR argument establish that Bell's inequalities are *valid* for the electromagnetic interactions of point-like particles, including electrons and photons, with ensuing lack of hidden variables.

By contrast, following half a century of mathematical, theoretical and experimental studies in the field, this author believes that Bell's inequalities are *inapplicable,* in favor of their completions via hadronic mechanics [10–12], for composite systems of particles at short mutual distances, including hadrons [20] and leptons [47], because the exact representation of their experimental data requires non-unitary transforms of quantum models, under which none of Bell's assumptions can be formulated, with ensuing validity of Bohm's hidden variables.

It then follows that any experiment proving the *violation* of Bell's inequalities, as defined by the above equations, is a direct experimental verification of hadronic mechanics.

II. Conditions of validity of hadronic mechanics. Recall that the wave packet of one electron is identically null only at infinity and, consequently, the universe is a single integrated structure much similar to the total EPR entanglement of living organisms [39]. Recall also that the universe will never admit one single final theory for the representation of all its complexities. Under these recollections, this author believes that the isotopic branch of hadronic mechanics can indeed provide a first axiomatically consistent representation of *stable,* thus time reversible systems, while the genotopic branch of hadronic mechanics can provide a first axiomatically consistent representation of energy-releasing processes, including nuclear fusions and fossil fuel combustion [20], and the hyperstructural branch of hadronic mechanics can at least initiate the search for an axiomatically consistent representation of life [39], here referred to a quantitative representation of the difference between organic and inorganic molecules.

III. Conditions of validity of quantum mechanics. By recalling that "point-like wave packets" do not exist in nature, and that quantum mechanics is identically and uniquely recovered by iso-mathematics and iso-mechanics for $\lambda = \hat{T} = 1$, this author believes that the Copenhagen interpretation of quantum mechanics provides an excellent representation of *stable systems of particles at mutual distances allowing their effective approximation as being point-like,* by therefore solely admitting action-at-a-distance potential interactions, with ensuing validity for atomic structures, particles in accelerators, crystals and numerous other systems. Despite these justly historical achievements, following half a century of studies in the field, this author believes that *quantum mechanics cannot be exactly valid for particle entanglements, as shown in this note, as well as for all composite systems of particles at short mutual distances, thus including leptons, hadrons, nuclei and stars* [17].

It is appropriate in the latter respect to note that the widespread assumption that a single theory, quantum mechanics, can represent all possible complexities of the universe, has been kept for about one century in oblivion of:

A) Clear experimental evidence in various fields of *deviations* of physical reality from quantum predictions in favor of *exact* representations via hadronic mechanics, including deviations in: nuclear physics [45]; electrodynamics [46–48]; nuclear physics [45]; condensed matter physics [49]; heavy ion physics [50]; time dilation for composite particles [51]; Bose-Einstein correlation [52, 53]; cosmology [54, 55]; and other fields.

B) The insufficiencies of quantum mechanics in nuclear physics due to its inability over one century under large public funds to achieve [19–21]: a quantitative representation of the synthesis of the neutron from the hydrogen in the core of stars; an exact representation of nuclear magnetic moments; an exact representation of the spin of nuclei in their true ground state (that without the usual orbital excitations); a representation of the stability of nuclei despite the huge Coulomb repulsion between nuclear protons; a representation of the stability of neutrons when members of a nuclear structure; and other insufficiencies.

C) The inability by quantum mechanics to allow a consistent treatment of energy-releasing processes, including nuclear fusions, due to their time irreversibility compared to the known time-reversibility of quantum mechanics (e.g. because of the invariance of Heisenberg's equation under anti-Hermiticity and for other reasons). Under these conditions, the same Schrödinger equation has to be applied for both, the forward and backward time evolutions, with ensuing violation of causality due to unavoidable solutions in which the effect precedes the cause. This violation of causality may explain the lack of achievement to date of controlled nuclear fusion [56].

Consequently, the continuation of the century-old use and support of quantum mechanics for all possible conditions existing in the universe in oblivion of the teaching by Einstein, Podolsky and Rosen, in oblivion of vast opposing experimental evidence, and in oblivion of fundamental unresolved nuclear problems, may continue to have widely negative implications for our rapidly deteriorating environment.

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