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The Mathematization of Physics and the Neo-Thomism of Duhem and Maritain

Stephen M. Barr

Abstract. Pierre Duhem and Jacques Maritain, influenced by positivist philosophies of science that prevailed in the late nineteenth and early twentieth centuries, adopted markedly non-realist views about the mathematical theories of the modern physical sciences. The philosophies of science they developed were a hybrid of Thomism and positivism. This paper argues that the ideas of Duhem and Maritain about the relation of the mathematical theories of modern physics to physical reality are inadequate in light of the insights modern physics has yielded about the physical world.

I. Introduction

Generally speaking, Catholic authors, including philosophers in the Aristotelian-Thomistic (AT) tradition, have defended a "realist" view of science. There are a few, however, who take a less realist view when it comes to the highly mathematical accounts that modern physics gives of the physical world. They admit that mathematical physics "works," in the sense that it fits experimental data (approximately anyway) and allows successful prediction and control of phenomena, but they suggest that it does not give access to essences, natures, and causes—it remains at the level of the merely quantitative aspects of things, seen by them as relatively superficial. They regard the mathematical entities of modern physics as mere "mental constructs": useful indeed, but improperly reified by scientists themselves, contributing to physicalist reductionism. This instrumentalist view of mathematical physics is found pre-eminently in Pierre Duhem, Jacques Maritain, and certain writers influenced by their views on science. It has roots in certain aspects of Aristotelian thought (specifically certain ideas about matter, quantity, and the process of mathematical abstraction), but really owes much more to the positivist account of science that was dominant in the early twentieth century and embraced by both Duhem and Maritain.

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My goal in this article is to argue against the views of Duhem and Maritain on the relation of mathematical physics to physical reality, and to show their inadequacy in the light of what mathematical physics has revealed to us about the physical world. I start by tracing some history in section II, beginning with some Aristotelian-Thomistic ideas and how they were applied in thinking about modern science in the early modern period. I go on to discuss, in much greater detail, how Duhem and Maritain applied a strongly positivistic twist to these ideas to produce a markedly non-realist philosophy of science. In sections III through VI, I critique the Duhem-Maritain account of mathematical physics and rebut many of their claims. Specifically, in section III I argue that mathematical physics (what Maritain called "empiriometric science") does tell one about essences; in section IV that it can tell one about physical causes; in section V that it does not just yield endlessly revised and patched models that are never more than approximate summaries of the results of measurements; and in section VI that it is not limited to explaining the quantitative aspects of things, but explains also qualities.

In section VI, I argue, in a more positive vein, that the mathematics of modern physics does not remain simply at the level of appearances but goes deeply into physical reality, yielding not only an understanding of essences and causes, but also of qualities, concrete features of things, matter (as an individuating principle), and change. I will suggest that Aristotelian ideas about mathematics can be developed in a direction much different than Duhem and Maritain took them, which might harmonize better with the role mathematics plays in modern physics. In particular, the Aristotelian notion of "intelligible matter" (or "mathematical matter") and the related notion of *materia signata quantitate*, may allow for a mathematical description of matter itself (in the sense of an individuating principle).

In section VII, I note some reasons that mathematical physics cannot give a full account even of physical reality, despite some of the points made in sections III through VI.

II. From Aristotle to Maritain: Some Ideas on how Mathematics Relates to Nature

Aristotelian-Thomistic (AT) thought distinguishes three orders of abstraction from matter.[1] These give rise to physical, mathematical, and metaphysical concepts, respectively. Physical concepts abstract from particular things, but are

[1]William A. Wallace, *The Modeling of Nature: Philosophy of Science and Philosophy of Nature in Synthesis* (Washington, DC: Catholic University of America Press, 1996), 135–4; Yves Simon, "Maritain's Philosophy of Science," *The Thomist* 5 (1943): 87–8; Anthony Rizzi, *The Science Before Science: A Guide to Thinking in the 21st Century* (Baton Rouge: AIP Press, 2004), 137–42.

still tied to matter. The physical concepts "horse" and "red" grasp horseness and redness apart from this or that horse or red thing. But both presuppose matter, since a non-material horse or red thing makes no sense. Mathematical concepts, by contrast, though abstracted from material things, can be considered without any reference to matter, as happens in pure mathematics. Still further removed from matter are metaphysical concepts.

AT philosophers call concepts, whether physical, mathematical, or metaphysical, that refer to something real outside the mind "real concepts." The intellect, however, can form concepts that have no direct extra-mental reference, such as those of pure logic, and these are considered by AT philosophers to be "beings of reason" (*entia rationis*).[2]

The mathematical concepts that were known to Aristotle were primarily those of arithmetic and Euclidean geometry in two and three dimensions, many or all of which can be understood as arising directly through abstraction from material objects. For example, the concept "sphere" can arise by abstraction from (approximately) spherical objects, such as the earth or a bronze ball; and the concept "five" by abstraction from groups of five perceptible objects. Therefore, such mathematical concepts would clearly fall under the category of "real concepts," as referring to extramental realities. Modern mathematics, on the other hand, deals with many concepts that are not obtained directly by abstraction from material things, and probably have nothing corresponding to them in the physical world. Examples would be such concepts as the Mandelbrot set, fifteen-dimensional spheres, or *p*-adic numbers. These would fall under the category of *entia rationis*. There are other concepts of modern mathematics whose status is more ambiguous. These concepts did not arise in the first place by abstraction from material things, and yet have been found to be of crucial importance in modern physics. A simple example would be negative numbers. These should be considered *entia rationis*, according to one author,[3] because there cannot be (say) −2 objects; and yet the theories of modern physics cannot be formulated without negative numbers. The same considerations would apply to many of the key mathematical concepts used in fundamental physics, such as matrices, complex numbers, tensors, spinor representations of Lie groups, Hilbert spaces, and so on. In this paper, I take no position (for in fact I hold none) on whether these should be regarded as *entia rationis*. Obviously, we cannot know how Aristotle himself or St. Thomas would have regarded such concepts and their relation to physical reality. Indeed, there is some controversy among interpreters of Aristotle even about the status he accorded such mathematical entities as triangles and

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[2]Wallace, *The Modeling of Nature*, 137, 141; Rizzi, *The Science Before Science*, 366.
[3]Rizzi, *The Science Before Science*, 78.

spheres. Presumably, these and other questions in the philosophy of mathematics could be answered in a variety of ways within a broadly AT framework.

Given their account of mathematical concepts as arising at the second level of abstraction from matter, some AT philosophers have seen them as more remote from and giving a more impoverished view of the material world than physical concepts. They say that mathematical abstraction "leaves matter behind," or more of it behind than does physical abstraction. On the other hand, according to AT philosophy, matter itself can be "signed with quantity" ("*signata quantitate*") in that it can have extension and be divisible into parts. This suggests an intimate connection between quantity and matter, rather than remoteness. However, this signing of matter by quantity has often been seen as having more to do with the individuation of things than with their essences. There is a consequent tendency to see mathematical form as relating primarily to adventitious accidental features of things rather than to their natures and causes. Whether a cat is here or there, is moving or at rest, is large or small, is in this or that spatial configuration (such as sitting or standing), or whether it is one of a large or small number of cats, makes no difference to what a cat is or to its essential properties.

This is one reason that Aristotelian philosophers of the early modern period saw astronomy as a branch of mathematics rather than of natural philosophy or what we would now call physical science. The techniques of geometry could be used to predict accurately where heavenly bodies would appear in the sky at particular times, much as a modern train schedule is useful for predicting when trains will arrive at various stations. But just as a train schedule does not tell you what a train is or what makes it move, the view of many seventeenth-century Aristotelians was that mathematics gives no real insight into the underlying physical causes of celestial phenomena; that was the job of a type of natural philosophy in which mathematics had a peripheral role.[4]

This way of thinking played a part in the dispute over heliocentrism. Galileo's opponents were aware that *apparent motion* was a relative question, and that for the purposes of mathematical calculation of appearances one could equally well assume that the Earth or the Sun was at rest, and that it might even be advantageous to assume the latter. But actual physical motion they took to be an absolute question, which mathematics is powerless to resolve. Motions have physical causes, the investigation of which is the province of natural philosophy proceeding in a largely non-mathematical way.

The idea that quantity only scratches the surface of reality, without giving access to the causes and natures of things, is understandable if we think of the social sciences. An opinion survey can quantify how many people consider

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[4]Marcus Hellyer, *Catholic Physics: Jesuit Natural Philosophy in Early Modern Germany* (Notre Dame, IN: University of Notre Dame Press, 2005), 117.

themselves "liberal" or "conservative." But the political ideas of any actual person cannot be captured by a statistic. Nor can a few statistics, such as GDP and trade balance, capture the vast complexity of nation's economic life. In a similar way, it seemed obvious to some AT philosophers in the early modern period that quantities cannot take one very deeply into the reality of the physical world.

The gradually increasing mathematization of the physical sciences after Newton contributed to a widening separation from Aristotelian-Thomistic thought, to the point where they became almost disjoint universes of discourse. Many AT philosophers have regarded this as a tragic development and have tried to build bridges across the divide. This includes thinkers as diverse in their approaches and views as Charles De Koninck, Bernard J. F. Lonergan, Benedict M. Ashley, William A. Wallace, and Jude P. Dougherty.

Pierre Duhem (1861–1916), however, had a very different view. Although his metaphysical views were very much in the AT tradition, he saw the divide as a healthy development and proper division of intellectual labor. Duhem is very important to the present topic, because, as an eminent Catholic scientist and important historian and philosopher of science, he had a great impact on Catholic thinking about science that continues to this day. And his views on theoretical physics, which were thoroughly and avowedly positivistic,[5] helped shape those of Jacques Maritain, and through him a certain strain of neo-Thomistic thought. (Duhem and Maritain's positivist views of modern science have been criticized by other AT philosophers.[6])

Duhem wholeheartedly welcomed the mathematization of physical science. In *The Aim and Structure of Physical Theory*, he wrote,

> There is one science in which logic attains a degree of perfection which makes it easy to avoid error and to recognize it when it has been committed, namely . . . arithmetic, with its extension to algebra. It owes this perfection to a . . . symbolic language in which each idea is represented by an unambiguously defined sign, and in which each sentence of the deductive reasoning is replaced by an operation combining the signs in accord with strictly fixed rules whose accuracy is always easy to test. . . .
>
> [The] geniuses . . . [of] the sixteenth and seventeenth centuries . . . [recognized] that physics would not become a clear and precise science, exempt from perpetual and sterile disputes . . . and would not be capable of demanding universal assent to its doctrines as long as it would not speak

[5]Pierre Duhem, *The Aim and Structure of Physical Theory* (reprint, New York: Athenaeum, 1981; original copyright, New Jersey: Princeton University Press, 1954), 275–82. Page numbers in the text are to the 1981 version.

[6]For Wallace's critique, see Wallace, *The Modeling of Nature*, 207–9, 224–7; for De Koninck's critique, see William A. Wallace, *From a Realist Point of View: Essays on the Philosophy of Science*, 2nd ed. (Lanham, MD: University Press of America, 1983), 1–21.

> the language of geometers. They created a true theoretical physics by their understanding that it had to be mathematical physics. (Duhem, 107)

For Duhem, what made the application of mathematics to physics possible was that some physical attributes (those in the category of quantity) could be subdivided and added in a way that paralleled arithmetic operations. For instance, two rods of equal length could be laid end to end to make something of twice the length. Thus, the physical operation of laying rods end to end could be "represented by" arithmetical operations on numbers, which in turn could be "symbolized by" various signs. Therefore, in Duhem's words, "Theoretical physics does not grasp the reality of things; it is limited to representing observable appearances by signs and symbols" (Duhem, 115). Moreover, since it is by measurement that *physical* quantity gets related to *mathematical* quantity, it is only those observable appearances that are measurable which can be represented by signs and symbols.

In Duhem's view, theoretical physics *should* deal only with quantities, as only those can be symbolized by mathematics. But the "real properties of bodies underneath the observable appearances," which theoretical physics "does not have the power to grasp" (Duhem, 115), include qualities. Although qualities can have magnitude, they do not have the kind of magnitude that can be added together in a way that parallels mathematical operations. Duhem illustrated this with the quality of mathematical talent (Duhem, 112). Although such talent can be present to a greater or lesser degree in different people, one cannot add together many mediocre mathematicians to make a brilliant one. He also used the example of heat. In his view, there is a quality of heat that is greater, say, in an oven than in a snowball; but he observed that it would be absurd to ask (as Diderot once jokingly did) how many snowballs would have to be added to heat an oven (Duhem, 112).

Even if the "real properties of bodies underneath the observable appearances" may be qualities unquantifiable in the manner required by theoretical physics, Duhem noted that they can have physical *effects* that are quantifiable. For example, heat can cause the mercury in a thermometer to expand, and the height of a column of mercury is measurable. Thus, a science of thermodynamics is possible, even though it cannot tell us what the quality of heat really is in itself.

For Duhem the statements of theoretical physics were only about appearances, but even about appearances could not be considered "true." This judgement was based on several points, which I will explain in different language than he used.

First, for Duhem, the laws of theoretical physics are mathematical formulas that express relationships among measured quantities. But measurements are always approximate; the data points have error bars. Consequently, there are always an infinite number of different equations that fit the data to that approximation. Therefore, said Duhem, "to the physicist all these laws are equally acceptable, for all determine [the values of the measured quantities] with a closer approximation than can be obtained with our instruments" (Duhem, 171). And that would remain so no matter how much one's instruments were improved.

Second, any physical situation to which one applies the laws of physics is too complex to treat exactly, and so an "idealization" or "model" of the situation must be made. A theorist may keep refining his model, but always something is left out.

Third, theories based on quite different principles can yield the same equation, or equations whose solutions are extremely close numerically. For example, Newton's and Einstein's theories of gravity are conceptually different but give nearly the same numerical answers for masses moving slowly in weak gravitational fields. Therefore, even the fact that a theory fits the data with extreme precision does not guarantee that its conceptual basis is correct. The theories of theoretical physics are inherently and always provisional, whereas, notes Duhem, true propositions are true for all times.

Duhem concluded from all this that the laws of theoretical physics "are always symbolic . . . [and] a symbol is not, properly speaking, either true or false; it is rather something more or less well suited to stand for the reality it represents, and pictures that reality in a more or less precise . . . [and] detailed manner" (Duhem, 168). "Physics makes progress because experiment constantly causes new disagreements to break out between laws and facts, and because physicists constantly touch up and modify laws in order that they may more faithfully represent facts" (Duhem, 177). In an article entitled "Physics of a Believer," he wrote,

> [A] physical theory is neither a metaphysical explanation nor a set of general laws whose truth is established by experiment and induction; . . . it is an artificial construction manufactured with the aid of mathematical magnitudes; [and] the relation of those magnitudes to the abstract notion emergent from experiment is that . . . of signs. . . . [Thus] theory constitutes a kind of synoptic painting or schematic sketch suited to summarize and classify the laws of observation. (Duhem, 277)

Now, if theories merely summarize the results of observations without grasping the underlying reality, it raises the question how some theories are able to predict accurately new and utterly surprising effects. This remarkable fact led Duhem to concede that such theories must in some way be "natural classifications" whose "principles express profound and real relations among things" (Duhem, 28, 30). While these theories cannot grasp real causal or metaphysical relationships, they may in some way reflect them.

Duhem did not try to bridge the widening gap between AT philosophy and modern physical science, but on the contrary tried to construct what William A. Wallace called a "wall of separation" between them. In Wallace's words, "In effect, he placed the 'perennial philosophy' of the Church beyond question, while according only a conjectural status to the advances made by modern science."[7]

A similar divide existed in the thought of Jacques Maritain, who, like Duhem, accepted a positivistic account of modern science. In the words of Michael Heller, "Maritain and others applied [Aristotle's and St. Thomas's] theory of knowledge to the sciences and supplemented it with . . . elements of the [then] current philosophy of science . . . [which] was mainly of a positivistic and neo-positivistic origin."[8] According to Wallace, Maritain is "the Thomist philosopher most often quoted in the U.S. on topics relating to the philosophy of science."[9] In what follows, I am relying on an article in *The Thomist* entitled "Maritain's Philosophy of Science," written in 1943 by his disciple and colleague Yves Simon.[10]

Maritain distinguished between natural philosophy and modern science, which he termed "empiriological science." They study the same objects, but use different modes of analysis. For example, says Simon, an AT natural philosopher would define man as a "rational animal," an "animal" as a "living body endowed with sense knowledge," and a "living body" as one that "moves itself" and is "the active origin of its own development" (Simon, 90). To proceed further in this direction would require metaphysical categories such as act, potency, and cause. Thus, the AT natural philosopher's analysis "ascends" towards metaphysics. Maritain called these ascending analyses "dianoetic" because they penetrate *through* sensible appearances to attain knowledge of the underlying essences. Yves Simon admitted, however, that natural philosophy has rarely been able to achieve this goal (Simon, 94).

By contrast, says Simon, an empiriological scientist (in this case, zoologist) would define man as a "mammal of the order of Primates," a "mammal" as a "vertebrate characterized by the presence of special glands secreting a liquid called milk," and "milk" in terms of "color taste, average density, biological function, chemical components, etc." (Simon, 91). One sees that this analysis "descends" towards sensory experience. Maritain called the descending analyses of empiriological science "perinoetic," because they do not go beyond the sensible phenomena, but stay on the periphery, never attaining a knowledge of essences.

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[7]Wallace, *The Modeling of Nature*, 209.

[8]Michael Heller, *Creative Tension: Essays on Science and Religion* (Philadelphia: Templeton Foundation Press, 2003), 72.

[9]Wallace, *The Modeling of Nature*, 224.

[10]Yves Simon, "Maritain's Philosophy of Science," 85–102.

Maritain called empiriological sciences that use mathematics "empiriometric." These remain at the level of phenomena that are not only sensible, but measurable.

Maritain's views echo Duhem's, but Maritain was willing to concede that empiriological science can yield propositions that are true and give genuine knowledge of the physical world, even if only at the level of appearances rather than the "ontological level." It can do this because "the matter remains physical" even if the "form is mathematical" (Simon, 101). Nevertheless, the "attraction of mathematical form" tends to lead empiriometric scientists to mistake mere beings of reason for real beings.

III. Ontological versus Empiriometric: Is the Divide Real?

Let us look at Maritain's and Simon's distinction between the "ontological" and "empiriometric," using some specific examples taken from Simon's article. The first example concerns the simultaneity of physical events. Simon says that whether two events are simultaneous is an absolute question "ontologically considered," but a relative one "if referred to definite possibilities of accurate measurements." He goes on to say that "relative simultaneity is a physico-mathematical *ens rationis* founded in the real and imposed on the mind of the physicist by the very nature of his scientific point of view" (Simon, 100).[11]

To evaluate these statements properly, it is helpful to consider first an issue far less technical than simultaneity, but analogous to it (as I will explain later): are the concepts "above" and "below" absolute or relative? At first glance, they seem absolute. Any two people would agree that a cat up a tree is above the ground and that the first floor of a building is below the second. But upon considering that the Earth is round and a center of gravitational attraction, they would realize that the subject is more involved. To a person in the US, Australia is below, while to a person in Australia the US is below. To a person on Earth looking at Mars, Mars is above, while to a person on Mars looking at Earth, Earth would be above.

This has nothing to do with absurdities such as truth being relative, but simply with the fact that the terms "above" and "below" at any point on the Earth's surface are defined with respect to the direction of the Earth's gravity at that point. For two points a few feet or a few miles apart, it would indeed require very "accurate measurements" to tell that gravity is pointing in slightly

[11]Simon does not seem to be contrasting two different notions of time, but two different approaches (ontological and the empiriometric, or dianoetic and perinoetic) to analyzing it, just as in his earlier example these ascending and descending analyses both study "man." When Simon says that simultaneity is "ontologically absolute," he is certainly speaking of physical time, since for him the ontological perspective is closer to physical reality than the empiriometric. The empiriometric scientist also claims to be studying physical time.

different directions at those points, and the import of those measurements might be unclear and seem like technical quibbles to anyone but a scientist or surveyor. But why "above" and "below" are different in the US and Australia would be intuitively obvious to an astronaut looking at the Earth from afar, or even to a school child looking at a globe in his classroom. No one would then say that "above" and "below" being relative to one's location on the Earth is a "physico-mathematical *ens rationis* imposed on the mind of the physicist by his scientific point of view," or that people pointing down in Australia and the US (or on Mars and on the Earth) are actually pointing in the same direction "ontologically speaking."

What is the analogy with simultaneity? "Above" and "below" are defined with respect to the direction of gravitational acceleration, which depends on an object's location. "Simultaneity" of physical events is defined with respect to a "time direction," and according to Einstein's theory of relativity the time directions defined by objects moving with respect to each other are different.[12] If those objects are moving slowly (compared to the speed of light) with respect to each other, it would take extremely accurate measurements to detect that difference. But there are situations where such differences are large and obvious without calculations, precise measurements, or empiriometric theory.

In the famous "twin paradox," for instance, one considers a situation in which one twin remains on Earth, while the other takes a round trip to a distant star in a rocket ship that travels at near light speed. The traveling twin would have experienced a much shorter lapse of time during the trip. It could be that years have passed for the stay-at-home twin, while only a few days have passed for the traveler. The time that differed would not be just some precisely measured "physico-mathematical" time, but time as we all intuitively understand and experience it. The stay-at-home twin would have had years' more of life experiences, thoughts, deeds, and bodily aging. While this scenario has not yet been carried out, it could be, and there is not the slightest doubt among scientists what the result would be. The predictions of Einstein's theory, including time dilation and related effects are seen in the laboratory every day and have been confirmed

[12]In contemporary physics, physical time is ultimately defined in terms of changes undergone by physical things, whether the swaying of a branch, the weathering of a rock, the growth of a plant, or the vibrations of an atom. One never measures "time itself," but always the alteration of some features of changing physical things. Any statement about physical time intervals or durations implicitly refers to the changes undergone by actual physical entities, and each such entity thereby defines a time direction. (In essence, to a physicist all physical things are "clocks.") According to the theory of relativity (in contrast to Newtonian physics) the time directions defined by things that are in motion with respect to each other are not aligned. This is the basis of the analogy made in the text.

on distance scales from the subatomic to the astronomical.[13] We see, then, that the "relativity of simultaneity" is not a matter of the requirements of "accurate measurements," or the putative limitations of empiriometric science; it has to do with physical realities that can be experienced quite directly.

Someone might still object that as the "twin paradox" scenario has not yet been carried out it remains at the level of empiriometric theory rather than demonstrated fact. Yves Simon, however, does not question the ability of empiriometric theories to predict sensible and measurable phenomena correctly; on the contrary, he presumes it (as did Maritain and Duhem). Nor does Simon claim that the absoluteness of simultaneity "ontologically considered" would lead to experimental consequences different from those of the theory of relativity. Were it to do so, the ontological analysis would be making statements on the same empiriometric level as the theory of relativity. That is, it would be a rival empiriometric theory, and the claimed gap between the ontological and empiriometric levels would close.

The second example in Simon's article is his statement that "no one knows what the essence of silver is" (Simon, 94). One can only circle around it perinoetically, he says, by cataloguing its various properties, such as melting at 961.8 degrees Centigrade. Not all Aristotelians would agree, however. The essence of something in Aristotelian philosophy is that which makes it the type of thing that it is rather than some other type, which grounds all of its properties, and which is the basis of an adequate definition of it. Physicists and chemists would define silver as that substance whose atoms have forty-seven protons in their nuclei. (All silver satisfies that definition, and nothing that is not silver satisfies it.) They would say that this is what makes silver what it is rather than something else. And they would say that from the fact that silver is made of atoms with forty-seven protons in their nuclei, and the laws of physics, one can in principle derive all of silver's properties.

Finally, let us examine an assertion that is made in two recent books by different authors who are influenced by Maritain.[14] They say that "real space" is "ontologically Euclidean," while non-Euclidean geometry is an *ens rationis*, despite the fact that physicists have found non-Euclidean geometry necessary to account for spatial and temporal relationships in our universe.

Let us leave aside the concept of "space-time," as that would embroil us in discussions about the assimilation of time to space. Consider just a three-dimensional *space* gotten by taking a "snap-shot," so to speak, of the universe at

[13]The special relativistic "time dilation" effect and related effects have nothing to do with the size of objects, but only with their speed, and have been confirmed in macroscopic systems, including GPS satellites and the Hulse-Taylor binary pulsar.

[14]Rizzi, *The Science Before Science*, 220–2; Jude P. Dougherty, *The Nature of Scientific Explanation* (Washington, DC: Catholic University of America Press, 2013), 14–5.

one time. Because simultaneity is not absolute, some specification of what one means by "at one time" is required. No matter how this is done, however, the resulting three-dimensional space will be found to have "intrinsic curvature." Therefore, the geometry of that space would be non-Euclidean. For example, the angles of a triangle in that space would not add up to 180 degrees. Why is this not an "ontological" statement about "real space"?

Let us consider an example. Let us take the snapshot in the most natural way for cosmology, i.e., at a particular "cosmic time." (Cosmic time is defined as the time since the Big Bang as measured in frames of reference "comoving" with the cosmic background radiation.) Let us suppose that the universe is approximately homogeneous on large scales (which all evidence to date shows), and is of finite volume, i.e., "spatially closed" (which is one of two possibilities that are equally probable given everything that is known at present). Then the three-dimensional space of our universe at one cosmic time would be simply what mathematicians call a "three-sphere." (Not a perfect three-sphere, but a slightly bumpy one, because galaxies, etc., warp the space in their vicinity.)

No Aristotelian would deny that the surface of the earth or a bronze ball being a (slightly bumpy) "two-sphere" is a statement about physical reality. The concept of two-sphere is regarded as a "real concept" arising at the second level of abstraction. It might be objected that speaking of the shape of a material object, like the earth or a bronze ball, is different from speaking of the shape of space itself. But rather than speaking of the shape of space itself, we can speak of the shape assumed by the gas of cosmic radiation that fills all of space. That gas of course conforms to the contours of the space it fills, which is a three-sphere. (Just as paint on the surface of a bronze ball conforms to the shape of a two-sphere.) Of course, we only know about the geometry of the universe by indirect means. But the spherical shape of the earth was also first discovered by very indirect means.

IV. The Mathematization of Physics and Causal Explanations

Already with the discoveries of Newton, scientists began to see that mathematical calculations could tell them about physical causes and not just appearances, as had been widely assumed before that. In particular, in Newton's famous second law, $F = ma$, one can think of the force F as a physical cause and the acceleration a as its effect. Historically, it was primarily this that allowed physicists to resolve the question of whether the Sun is orbiting the Earth or vice versa.

To see how this works, consider the following simple situation: a man swinging a little rubber ball around himself at the end of an elastic string. Here Galileo and his opponents would have agreed that the ball is going in circles around the man, not the man around the ball. Of course, one can describe it mathematically either way. That is, one can choose systems of coordinates (i.e., "frames of reference") in which either the man or the ball is at rest. Indeed, even for some physics purposes the latter frame may be more convenient. Nevertheless, a modern physicist would say that the description in which the ball is moving is more "physical." The reason has to do with measurable forces. An object that moves in a circle of radius r at constant speed v has an acceleration towards the center of the circle that is given by v^2/r, which is measurable. (This follows from the mathematical definition of acceleration.) Newton's second law says that there must be a force acting on the object to cause that acceleration, a so-called centripetal force.

In the frame in which the ball is circling, it is obvious that the centripetal force is the tension of the stretched elastic string. One can determine the magnitude of this force by measuring the string's elastic properties and how much it is stretched when the ball is circling. The force of the string thus determined would be found equal to the ball's acceleration times its mass, satisfying Newton's law,

In the frame where the man is circling the ball, however, the string's force is far too tiny to keep the man going in a circle, because his mass is so large. To satisfy Newton's law, therefore, one must posit a huge so-called "fictitious force" acting on the man, a force that does not correspond to any physical cause. It is the need to posit such fictitious forces that reveals a frame of reference to be "non-inertial" and hence less "physical." Here we see very vividly how a *mathematical* analysis in terms of measurable *quantities* can tell us what is really going on *physically*. A similar analysis tells us why, and in what sense, heliocentrism is correct and geocentrism wrong.

Since Newton's time, the role played by mathematics in theoretical physics has grown and deepened greatly. And it is precisely this mathematization that has enabled physicists to find the physical causes of a vast range of phenomena. Returning to Simon's example of silver, its properties are understood well, both in qualitative and quantitative terms. Few if any scientists would doubt that all its properties follow from its atomic and subatomic structure and the laws of physics, and that both this structure and those laws are mathematical.

V. Fundamental Laws versus Models of Situations

Let us now consider the notion, advanced by Duhem 100 years ago but still enjoying wide currency, that the "laws of physics" are nothing but an endless succession of approximate models continually in need of "touching up," like an old cloak being patched. It is understandable that Duhem could have thought so, but things look far different today.

As physical theory has advanced over the last century, it has become increasingly unified and coherent. Rather than more seams and patches appearing in its fabric, it has become more seamless. This continued a trend that started with Newton. Newton's theories unified terrestrial and celestial physics. Maxwell's theory unified electricity, magnetism, and optics. Atomic theory unified all of chemistry and much of physics. Einstein's theories unified space and time, as well as mass and energy, and also revealed an even more profound unity of electric and magnetic fields. Quantum field theory unified particles and forces. Gauge theory gave a more unified picture of the three non-gravitational forces. For decades quantum theory and general relativity (i.e., Einstein's theory of gravity) resisted unification, but superstring theory has shown that they too can be unified in a stunningly beautiful way that also unifies gravity with the other forces. (Though it remains to be seen whether this is the right way.)

While, as Duhem emphasized, a particular set of numerical data describing a particular range of phenomena might be fit equally well (within the experimental "error bars") by many different equations, that does not make those equations "equally acceptable" to the physicist, as Duhem averred (Duhem, 171). Such data-fitting is not the only criterion by which theories are judged. It is important that the theoretical assumptions and the equations used to account for one set of phenomena cohere with those used to account for others. This requirement radically narrows the range of plausible theories beyond what mere data-fitting could ever yield, which is just an assortment of *ad hoc* and patched up approximate models. As theoretical physics has become more unified and coherent, however, it increasingly gives the appearance of merging into a single mathematical edifice that is profoundly unified and tightly constrained by a small number of deep principles.

That is why theoretical physicists are increasingly confident that underlying everything is a set of fundamental laws that form a unified mathematical structure and that are *exactly* satisfied by all natural physical processes. Of course, no proof of this would ever be possible. But to fundamental physicists, the trend lines are unmistakable and the case compelling.

This brings us to a crucial distinction that is often overlooked: namely, that between the "fundamental laws of physics" and "physics models." Even if the fundamental laws were known to physicists completely and held exactly, they would still have to use approximate models of any physical *situations* to which they wanted to apply those laws. For, however mathematically elegant the fundamental laws may be, physical *situations* in the real world are vastly complicated and messy. So the fact that *models* of actual situations must always be approximate in no way implies that there do not exist exact physical *laws.*

This raises the question whether an exact mathematical description of the physical world exists and, if so, what aspects of physical reality, if any, it would leave out. It would certainly leave out important things, as I will briefly argue at the end of this paper. Before that, however, in the next section, I am going to argue that far less must be left out by a mathematical description of physical reality than it is often contended, and that it isn't obvious that anything about physical entities *as such* must be left out.

VI. What Does Mathematical Physics Leave Out?

I am going to discuss several aspects of physical reality that, it is often argued, are inevitably left out by a mathematical account, and present some reasons to doubt that they are.

VI.A. Qualities

There exist many attributes that cannot be described mathematically or explained by mathematical physics, such as justice, courage, love, and intelligence. However, I am speaking about *physical* reality and thus restricting my attention to the realm of non-sentient physical entities, or the "non-sentient realm" for short, where attributes such as those just listed do not exist. By qualities, then, I here mean *physical* qualities, such as redness, coldness, wetness, brittleness, hardness, and the like, as well as such qualities as being alive and healthy, which non-sentient organisms can also have.

When it comes to sensible qualities one must distinguish between the quality as it inheres in the object as an accident or property, and as it is perceived by some sentient being. Since I am speaking of the non-sentient realm, I am concerned only with the former: that is, with the qualities of physical objects as they would exist even had no organisms ever evolved that could sense those objects or their qualities.

Let us start with color. When a human being sees a red rose, two stages are involved. First, the rose interacts with the electromagnetic field in such a way that light in the red part of the visible spectrum is reflected toward the percipient. Second, the light enters the eyes of the percipient and sets off a chain of electrochemical processes that lead (in a very mysterious way) to perception. The redness of the rose as something inherent in it, a property of it, is that feature of the rose that causes it preferentially to reflect red light. It pertains to the first stage. The rose would have the same properties if the second stage never occurred.

There are, it turns out, distinct and quite different properties that are commonly referred to as redness. The redness of the rose is that which causes it to preferentially *reflect* red light, whereas the redness of a flame is that which causes it to preferentially *emit* red light. Similarly, for other colors. The "blueness" of a Blue Jay's feathers is that which causes them to produce mostly blue light by *refraction*, whereas the "blueness" of blue eyes is that which causes them to *scatter* blue light preferentially. Some things that appear green are reflecting light in the green part of the spectrum, while others are reflecting light in the blue and yellow parts of the spectrum.

In every case, colors (as inherent properties of physical things) are physical features that determine how they interact with light, i.e., the electromagnetic field. The same is true of all other optical properties, such as those referred to as transparency, translucence, iridescence, luminescence, coruscation, shininess, and so forth. Physicists generally agree that those interactions of physical objects with the electromagnetic field that would make any difference to the perceptions of a human or animal are satisfactorily described by the currently known facts and laws of physics, and that physicists either understand them or have in hand adequate theoretical tools to do so.[15]

A well-known argument imagines the existence of a person (often called "Mary") who knows all the physics and physiology of sight, but who was raised in a black-and-white environment (or perhaps was blind from birth) and has therefore never seen color. Upon first seeing color, Mary would acquire objective knowledge of reality that she had not possessed before. This implies that there are knowable realities not explicable by physical science. This argument, which I regard as absolutely compelling, is usually advanced to support the thesis that consciousness, qualia, and other mental realities are not describable by physics. Could one not get more from this thought experiment, however, and draw the conclusion that Mary is also learning something new about the inherent physical properties of, say, a red rose? If the inherent physical properties of the rose that were newly known to Mary were ones that do not affect in any way how the rose interacts with light, then it is hard to see how Mary could learn anything about them via the light that has come from the rose. If, on the other hand, the these newly known physical properties do affect light, then their effects on light should also lead to effects in experimental devices that interact with light. And all the effects found so far in such devices have been explicable by mathematical physics. But even without such devices, since the effects of these properties are discernable by Mary's now normally functioning senses, they should also be discernable by the ordinary senses of scientists. And there is no evidence so far

[15]There are aspects of electromagnetism that are not yet understood. In particular, it is known that our present theory of electromagnetism must break down at sufficiently short wavelengths; but those wavelengths cannot affect the perceptions of any organism, for reasons that are also not understood. Could there be as-yet-unthought-of physical features of objects that would affect the visible light emanating from them in a way perceptible to humans, but that were beyond the ken of mathematical physics as such? This is imaginable. All one can say is that after many decades of exhaustive and extremely careful studies no evidence of any such lacuna in our theories of the interactions of light and matter has emerged.

of physical properties of roses or anything else that affect light in a discernable way but are not explicable by mathematical physics.

In a certain sense, of course, Mary has learned something "about the rose" that she hadn't known before: how it looks to a human being whose senses are not impaired. That may be an inevitable consequence of the rose's properties and the nature of our senses, but it is not itself a property of the rose in the sense defined in the third paragraph of this subsection.

A similar analysis can be applied to other sensible qualities, such as warmth and coldness. The thermal properties of things are believed by contemporary physicists to be very well explained by Statistical Mechanics and Condensed Matter Physics. Nothing we know at this point in scientific history gives us any reason to posit the existence of a Duhemian "quality of heat" that inheres in physical objects and produces physical effects but is beyond the ken of mathematical physics. It is not illogical to suppose that such properties exist; it is just that exhaustive physical investigations have revealed no evidence for them as yet.

It should be noted that Duhem, though a significant thermodynamicist, was handicapped in thinking about the ontological status of thermal qualities by his skepticism about atoms and therefore about the microscopic and statistical basis of thermodynamic concepts such as heat, temperature, entropy, pressure, free energy, and so on. Duhem and Ernst Mach (both of whom died in 1916) were the last well-known physicists to doubt the existence of atoms, a rejection that was part of their thoroughgoing positivism.

I have not been arguing that mathematical physics must *a priori* be able to account for all qualities (as inherent physical properties) of physical objects, or that it has been demonstrated that it can or will. I note only that no counterexample has yet been discovered by physicists either through measurement or through ordinary perception. But even were there counterexamples, it would remain the case that mathematical physics explains an enormous amount about a vast range of physical qualities.

VI.B. The Concretely Physical

Some authors have emphasized that the same equation can describe systems that are physically very different. This is an example of what Yves Simon called the indifference of mathematics to the reality of its object (Simon, 101). A good illustration is the so-called "harmonic oscillator equation." It can describe the oscillations of electrical current in an "LC circuit," the swinging of a pendulum, the vibrations of a tuning fork, and many other things. Obviously, electrical circuits, pendulums, and tuning forks are very different, physically speaking; so if the same mathematics can be used to describe them, it must be leaving out a lot of the physical reality.

That is correct: the harmonic oscillator equation, when applied to such systems, leaves out a tremendous amount. The important point, however, is that what is left out is just as mathematically describable as what is included. Each of the systems I mentioned—circuit, pendulum, tuning fork—contains a vast number of elementary constituents, including typically around 10^{26} atoms. A full mathematical description and explanation of such a system would require a vast number of variables, together with the equations that govern them. If one looked only at the harmonic oscillator equation with its one variable, one wouldn't be able to tell what kind of system it was describing. But from the *full* equations of a system, including all the variables, it would be possible in principle to tell that it was a circuit made of copper wires sheathed in red plastic with electrical current flowing through it, etc., and not a pendulum or tuning fork.

Mathematics does not only pertain to things in abstraction. Much of the concreteness of physical things lies in their mathematically describable details.

VI.C. Matter

Maritain said that in modern physics the "form is mathematical" but "the matter remains physical." Certainly, mathematics deals with forms and structures. And a form or structure, at least in the physical world, requires something that *gets* formed and structured, which one might suppose to be *not* a form and therefore *not* mathematical.

If the form of a statue is its shape (which is something geometrical, and hence mathematical), there must be some "stuff" that is shaped, say bronze or marble, which would appear to be non-mathematical. But what is bronze? It is itself a mathematically describable structure composed of copper and tin atoms. Are those atoms a kind of stuff? It turns out that copper and tin atoms are themselves structures, composed of electrons, protons and neutrons, while protons and neutrons, in turn, are structures composed of quarks and gluons. In every case, the structures are mathematically describable. But at some point, when we get to the *deepest* level of structure, mustn't we be faced with some kinds of basic stuff that are not analyzable in terms of mathematical structure and form?

We haven't reached that deepest level, but for the purposes of discussion, let's assume we have. In our present theory, electrons, quarks, and gluons are treated as fundamental entities. If this theory is correct, would an electron and quark be different because one is a little clump of "electron stuff" and the other a little clump of "quark stuff"?

Such an idea is completely superfluous. The reason was well explained by the great physicist Paul Dirac. He said,

> When you ask what are electrons . . . [i.e., what they are made of], I ought to answer that this question is not a profitable one to ask and does not

really have a meaning. The important thing about electrons is . . . how they behave—how they move. I can describe the situation by comparing it to the game of chess. In chess, we have various chessmen, kings, knights, pawns and so on. If you ask what a chessman is, the answer would be [that] it is a piece of wood, or a piece of ivory, or perhaps just a sign written on paper, [or anything whatever]. It does not matter. Each chessman has a characteristic way of moving and this is all that matters about it. The whole game of chess follows from this way of moving the various chessmen.[16]

Before proceeding, I should note that both in chess and in physics the mathematical rules do not just govern local movement, but also other kinds of change. For example, a pawn reaching the last rank can turn into a queen. When a piece lands on the square of an enemy piece, the enemy piece is annihilated. Similarly, in physics, the rules say that some types of particles can turn into other types, and that when certain types of particles land at a point occupied by certain other types they are annihilated and something else appears. In physics, then, the rules are much richer and more full of potentialities than the common but misleading billiard-ball analogy would suggest.

One might reply to Dirac by saying that even if it doesn't matter what a chess piece is made of, it is certainly made of *something*. And so, perhaps, an electron also is made of something. This would be to miss Dirac's point, however. He would have agreed that if electrons were found to be made of more elementary constituents, then it would matter greatly what those constituents were and what mathematical rules governed *their* behavior. For, from those more fundamental rules, it would be possible to derive the rules obeyed by electrons. Dirac was tacitly assuming, as we are, for the sake of discussion, that electrons are truly fundamental. In that case, the mathematical rules governing electrons' behavior are all that needs to be said, or meaningfully can be said, about what electrons are. Positing some kind of stuff out of which electrons are composed would be superfluous to any physical explanation.

To return to Dirac's analogy, consider some famous game of chess, say Lasker's win over Capablanca in 1914. It doesn't matter whether they were using wooden or ivory pieces: it would be the same game of chess in either case. The figurines used by chess players are merely mnemonic devices and aids to imagination that help them grasp what are essentially purely mathematical relationships. To think of the mathematical structure of the chess game as abstracting from the movements of the wooden or ivory pieces is in a way to get things exactly

[16] P. A. M. Dirac, "Address to the Indian Science Congress in Baroda," 1955, quoted in Alex Montwill and Ann Breslin, *The Quantum Adventure: Does God Play Dice?* (London: Imperial College Press, 2011), 188–9.

backwards. In chess, the mathematical structure is the fundamental thing, and the wooden and ivory pieces are added as mere aids to thought, in the same way that the structure of non-Euclidean geometry is what it is, whatever crude diagrams a mathematician might draw on the blackboard or paper.

To the modern theoretical physicist, the physical world is mathematically describable structure *all the way down*. In a certain sense, physical "matter" is itself mathematically describable. This may sound strange to some Aristotelian-Thomistic ears, but it may already be adumbrated in the traditional AT notion, already mentioned, of "matter signed with quantity."

The predominant view among AT philosophers has been that corporeal substances are individuated by matter, and in particular matter signed by quantity. One argument for this is that corporeal entities cannot be individuated by form, because then two entities having the same form would be the same entity.[17] Let us apply the same logic to an entity of pure mathematics. A perfect cube has six faces that are squares. The six squares all have exactly the same form, so why aren't they all the *same* square—why six squares rather than one? One could answer that they are composed of different points, with the points being a kind of fundamental stuff. And note: these points are themselves mathematical entities. One could think of the points of which the lines, squares and cubes of pure mathematics are composed as being a "mathematical matter." This idea may sound like a contradiction in terms from an Aristotelian point of view, since Aristotle held that pure mathematics involved an abstraction *from* matter. And yet, interestingly, the notion of "mathematical matter" is found in Aristotle's own writings.[18]

It is a pervasive feature of modern mathematics, including that used to formulate the theories of mathematical physics, that what is structured *by* a mathematical form is *itself* a mathematical entity, which may be either something primitive, such as a point, or *itself* a structure. A cube is a structure of squares, which are structures of line segments, which are structures of points. A "symmetry group" is a structure whose elements are symmetries, which can be symmetries of a geometrical figures, of equations—or even of symmetries (as in "inner automorphisms").

What I have been suggesting, then, should not be taken as denying that there must be something correlative to form, something that form is the form

[17]Aquinas, *ST* I, q. 11, a. 2, c.

[18]Henry Mendell, "Aristotle and Mathematics," in *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/aristotle-mathematics/ sec. 7.5: "For solving the plurality problem, Aristotle needs to have many triangles with the same form. Since perceptible matter is not parr of the object considered (in abstraction or removal), he needs to have a notion of matter which is the matter of the object: bronze sphere *minus* bronze (perceptible matter). Since this object must be a composite individual to distinguish it from other individuals with the same form, it will have matter. He calls such matter intelligible or mathematical matter."

of. Rather, I am suggesting that in the physical realm (as in the realm of pure mathematics) both the form and what it is the form *of* (the "matter") may be mathematically describable.

VI.D. Change and Potency

There seems to be a contrast between the unchanging forms and truths of mathematics and the real world of flux and change. A circular cloud can change its shape, but a mathematician's circle cannot change. We do not say that two plus two was four, or will be four, but that it is four in a timeless way. So how can the timeless realm of mathematics adequately describe the physical world of time, change, and potentiality?[19]

The answer is that not only can static things have intelligible mathematical structure, but processes also can. The laws of physics are "dynamical laws," which govern how processes unfold in time. That is why modern mathematical physics is able not only to discuss and describe change, but also to explain and even predict it. These dynamical laws do not govern only motion through space, but also changes of one type of thing into another: e.g., of elementary particles of one type into other types, of one chemical element into another, of one form of energy into another.

The mathematical rules therefore govern and explain not only how fast or how much things change, but also what kinds of changes can take place and under what circumstances. Mathematical physics thus describes the "potencies" of things, what "actuates" them, and how forms are "educed" from matter. The mathematics of modern physics need not be seen, therefore, as an alternative to the metaphysical categories of AT philosophy, residing in an "empiriometric" realm separated from the "ontological" realm (as they *were* seen by Duhem, Maritain, and Simon), but rather as what puts flesh and blood on the metaphysical bones.

It is sometimes said that mathematical physics, by its essentially geometric view of time, necessarily assimilates time to space, and thus ends up with a static "block universe" view of time, in which past, present, and future coexist in a timeless 4-dimensional geometry. To the extent this is true, however, it is not because the physics is mathematical. To understand any change and any process that unfolds in time is to grasp a relationship among events that happen at different times and thereby hold those events together in one insight. Both past and future are thus present to the mind in one intellectual act. That "standing outside of time," given by the coexistence in the mind of events that are past and future with respect to each other, is a feature of *any* understanding

[19]W. Norris Clarke, *The One and the Many: Contemporary Thomistic Metaphysics* (Notre Dame, IN: Univ. of Notre Dame Press, 2001), 170–1.

of change, mathematical or not, to the extent that it is an act of the intellect: i.e., to the extent that it is *rational* knowledge as opposed to an act that remains at the level of sensation alone.

VII. But Perhaps Nothing is Purely Physical

I have been making the case that for purely physical entities—what I called the non-sentient realm—both form and matter are ultimately mathematically describable. However, for at least two reasons, I think it can be questioned whether one can really speak about anything in the universe as purely physical.

The first reason has to do with consciousness. It seems most reasonable to suppose that many kinds of animals, such as dogs, have consciousness in the sense of having subjective experiences. It also seems reasonable to suppose that such animals are material beings and arose from inanimate matter by natural material processes. And yet there is no way in principle to derive mathematically or deduce logically from a mathematical account of their physical structure any conclusion about whether such animals have consciousness. There is something about such creatures, therefore, that lies beyond mathematical physics. Moreover, since the matter in an animal's body interacts with matter in the non-sentient realm, the non-sentient realm cannot be isolated from the sentient.

The second reason has to do with "the measurement problem" in quantum mechanics. There are strong arguments that within the logical framework of quantum mechanics one cannot make sense of what happens when measurements are made unless one assumes that the minds of "observers" (those who make measurements) are not fully describable by mathematical physics. In the words of Hans Halvorson, philosopher of physics at Princeton University, "In the case of quantum mechanics, if one presupposes physicalism, then one quickly lands in the measurement problem."[20] Back in 1961, the eminent physicist Eugene Wigner put it this way: "The very study of the physical world led to the conclusion that the content of the consciousness is an ultimate reality."[21]

My conclusion is that mathematics goes very deeply into the nature of physical things, much more deeply than envisioned by the strain of AT thought initiated by Duhem and Maritain, but still leaves something out. What is left out includes *at the very least* consciousness and those things which presuppose it.

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[20]Hans Halvorson, "The Measure of All Things: Quantum Mechanics and the Soul," in *The Soul Hypothesis*, ed. M. Baker and S. Goetz (New York: Continuum Press, 2010), 138–68.

[21]Eugene P. Wigner, *Symmetries and Reflections: Scientific Essays* (Woodbridge, CT: Oxbow Press, 1979; original copyright, Bloomington: Indiana University Press, 1961), 172.