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Modalities in Medieval Logic

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Credo ut intelligam

—Saint Anselm of Canterbury

Contents

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Chapter 1

The changing scope of logic

The title of this dissertation, *Modalities in Medieval Logic*, combines two temporally distinct and seemingly disparate fields of logic. The use of term 'modalities' in the plural, as opposed to 'modality' in the singular, is one of the hall-marks of recent developments in logic, which recognizes that there is no single 'correct' choice of modality, but that rather a range of modalities can be fruitfully studied with the tools of formal modal logic. This modern view can be contrasted with the distinctly non-modern view of logic indicated by the other part of the title, 'medieval', which brings to mind the narrow and rigid formal system of syllogistics and Scholasticism.

This apparent discrepancy between the two parts of the title immediately raises the question of what can possibly be gained from combining modern logic and medieval logic in the same research programme. Another way to state this question is to split it into two, and to ask what benefit a modern logician might have from looking at medieval logic, and what benefit a historian of medieval logic might have from looking at modern logic.

At a very general level, there are two reasons why the study of medieval logic is of interest to the modern logician. The first reason is to see how closely medieval logical theories in different branches (modal logic, temporal logic, quantifier logic, etc.) resemble modern logical theories in these same branches. The second is to see how much they differ. If the medieval theory is similar to the modern theory, one can ask to what extent we can shed new light on the medieval theory by modeling it with modern formal tools. If the medieval theory differs from the modern theory, one can ask what the causes of these differences are, whether they are purely historical, accidental, or whether they reflect conscious differences in goals and aims, and, if the latter, what we can learn from these differences.

On the other side of the question, a similar answer can be given. Many medieval logical theories often leave something to be desired in terms of clarity. This can be the result of at least two different factors. The first is that the medieval theories were developed within natural language, and even when this

natural language is used in a semi-formal fashion, the possibility for ambiguity still remains. The second is that because medieval logical theories were developed essentially as tools for modeling specific philosophical and theological problems, they often carry extra, non-logical, baggage. Abstracting away from this baggage, often metaphysical in nature, allows for a clearer understanding of the underlying logical theory.

Given this natural synthesis and synergy between modern and medieval logic, one might wonder why one needs to justify the study of medieval logic with modern tools. That this combination seems natural is a relatively recent development; two generations ago, modern logicians had a very different view of the utility of both medieval logic and of using modern techniques to study medieval logic. We discuss this in the next section. In §1.2 we introduce an operator 'history of', and show how changes in the scope of logic over time affect what counts as the 'history of logic'. The consequences of this change in the scope of logic, and hence in the history of logic, especially with respect to the topics of the current dissertation, are discussed in §1.3, where we also give an outline of the contents of the rest of the dissertation.

1.1 Two views on the scope of logic

In their classic 1962 work [KnKn84], Kneale and Kneale note that since there is no *a priori* definition of logic, such a definition "can be settled only by linguistic legislation", and this legislation can be "well- or ill-advised" [p. 741]. The definition of logic that they settle on is this:

> [Logic] is best defined as the pure theory of involution, that is to say, the theory of the general form of principles of involution without regard to the special natures of the propositions contained in the classes between which the relation holds [p. 742].

One consequence of this definition of logic is that

> If we think that the logic of tradition has been concerned primarily with principles of inference valid for all possible subject-matters, we must reject as unprofitable an extension of usage which allows such phrases as 'the logic of "God" '... For the word 'logic' is connected traditionally with discussion of rules of inference; and while it is strange to apply it to any axiomatic system such as that of Frege, it is even more strange to apply it to a system in which the consequences of the axioms are not all accessible by inference from the axioms... We therefore conclude that the theory of identity may be conveniently excluded from the scope of logic, and that our science is best defined as the pure theory of involution, that is to say, the theory of the general

> form of principles of involution without regard to the special natures of the propositions contained in the classes between which the relation holds [KnKn84, pp. 741–742].[1]

Restricting the proper application of the term 'logic' to "principles of inference valid for all possible subject-matters" excludes from the scope of the term 'logic' things such as the theory of identity, set theory, and logic of the trinity, because the principles of inference involved in these areas are not applicable to all fields. However, the Kneales do admit that

> No doubt in practice logic as we define it will always be studied together with other subjects which are relevant to the organization of knowledge, and in particular with those with which it has been associated by Aristotle, Chrysippus, Leibniz, Bolzano, and Frege. For we have seen that logic *in our narrow sense* is not even coextensive with [first-order logic] [KnKn84, p. 742, emphasis added].

This narrow view of logic advocated by Kneale and Kneale can be contrasted with the modern view of logic, which we will call the 'wide scope' view, where logic is considered to be the formal study of reasoning and information in general.[2] This view of logic has come more and more into the forefront in the last few decades, and is neatly captured in the definition of 'applied logic" in the *Encyclopædia Britannica*, which defines 'applied logic' as:

> The study of the practical art of right reasoning. The formalism and theoretical results of pure logic can be clothed with meanings derived from a variety of sources within philosophy as well as from other sciences. This formal machinery also can be used to guide the design of computers and computer programs [EB08, s.v. applied logic].

Under this view, the definition of logic has an essentially pragmatic flavor; we can change our definition of logic to meet the specific application that we have in mind,

[1] Moritz Pasch is often held to hold a similar view (e.g., according to Shapiro, Pasch "developed the idea that logical inference should be topic-neutral" [Shap00, p. 151], in reference to [Pas26]; Schlimm (private correspondence) notes that Shapiro probably took his information from [Na39]). Pasch's position as presented in [Schl–] is considerably more subtle than Shapiro's characterization, and it is less clear how similar Pasch's view is to Kneale and Kneale's.

A dissenting view can be found in Boolos, who says that "Indispensable to cross reference, lacking distinctive content, and pervading thought and discourse, *identity* is without question a logical concept" [Boo84, p. 430].

[2] This is not to say that this view of logic *only* developed in recent times, for clearly this is not true: Dürr quotes C.I. Lewis's *Symbolic logic*, where he says "'exact logic can be taken in two ways: (1) as a vehicle and canon of deductive interference [sic], or (2) as that subject which comprises all principles the statement of which is tautological' (cf. L.a.L., p. 235)" [Dür51, p. 3]. However, it is only in the last couple of decades that the wide view of logic has become a relatively wide-spread and generally accepted dominant paradigm.

whether this is the development of so-called 'ethical' robots implementing mechanized deontic logic [ArBrBe05], logic-based bidding languages for combinatorial auctions [Uc$_1$ChEnLa09], the formalization of the concepts of 'public announcement' and 'common knowledge' [vDvdHK07], or analyzing beliefs of characters in television dramas [LöwPa08].

If we take seriously the narrow view of logic advocated by Kneale and Kneale and others, then there is only one system which can be truly called 'logic' (what this system is Kneale and Kneale do not say, though plausible candidates are classical propositional and predicate logic), and any other system which we call 'logic' is usurping the term. On such a view, we are forced to consider much of the work done by medieval logicians, who did not focus on broad, abstract logical principles but rather on logic as it could be applied in concrete situations, in one of two ways: Either the medievals were not doing logic at all (though in some cases it may have resembled logic), or they were, but they often failed to do it correctly, where "correctness" is judged with respect to one definitive system of logic. This latter view is expressed by many people working on medieval logic in the '50s and '60s, of which Dürr can be taken as representative. When Dürr researches Boëthius on hypothetical syllogisms, which he takes to be a theory of conditionals, his procedure is to render Boëthius's Latin language-statements into the language of *Principia Mathematica*, and then evaluate the result to see if it is a thesis of propositional logic, either with material implication or with strict implication. This leads him to say things such as:

> We will now show that the eight inference schemes of the first group are correct but the inference schemes of the second group are incorrect [Dür51, p. 38].

and

> We may now say that Boethius failed to see the possibility of correlating inference schemes with the propositional forms of the second subclass; modern logic shows that there is such a possibility [Dür51, pp. 52–53].

and

> Finally, we will show that the theorems stated by Boethius in this connection and presented by us above are represented in the modern presentation of the logic of modalities, i.e., they are correct... We have thus shown that there is in fact considerable agreement between those theorems of Boethius represented by us and the results of the modern logic of modalities [Dür51, pp. 63, 65].

Similar views are expressed by Moody in [Mood75], whose research in medieval logic often focuses on how close the medieval logicians got to modern logic. He

notes that "It appears to be impossible to give a consistent interpretation of the medieval doctrine of supposition that corresponds to any branch of modern logic" [Mood75, p. 384], and, when considering the "the important question of whether modal propositions are to be construed as object-language statements (*de re*) or as metalinguistic statements (*de dicto*)", he says that it was "resolved in favor of the second (and correct) interpretation; this involved recognition that modal logic belongs to the logic of propositions and not to the logic of terms" [Mood75, p. 386].[3]

Quite clearly one must disagree with the assessment of medieval logic that either it is not logic or that it is wrong logic for the rest of this dissertation to have any value whatsoever, and in order to disagree with this assessment we must reject the narrow view of logic. In the next section we discuss the consequences that rejecting this narrow view of logic and embracing the wide scope view have on the study of the history of logic.

1.2 'History of' as an operator

The recent decades which have witnessed the change in how the scope of logic is viewed (by logicians at least) are contemporaneous with another development, which superficially may seem unrelated. In the last fifty years or so new areas of academic study have developed that are in one way or another derivative of other, well-established areas. An example of an area of study which is derived from another is 'history of science', derivative of the field 'science'. For pretty much any established field of academic study X, it is now possible to find the derivative areas of 'history of X', 'philosophy of X', 'sociology of X', 'didactics of X' or 'X education', etc.

The connection between these two observations (the broadening of the definition of 'logic' and the development of derivative fields of study such as 'history of X') is that the bounds of these derivative fields are essentially tied to the bounds of the fields from which they are derived. That is, if the boundaries of X change, the meaning of 'Y of X' also changes. Of the particular Ys that we mentioned, the operator 'history of' is special among these because it, by definition, covers not only what is X now but what has counted as X in the past; in fact, in many disciplines many things fall under 'history of X' which are not considered to be X today. For instance, because optics was seen as part of mathematics in the Renaissance, it can be considered a subject for 'history of mathematics' when looking, say, at 16th-century mathematics, but it wouldn't be if you read 'history of mathematics' strictly, as 'history of what is currently called math'. As the scope of mathematics changes through time, so changes the scope of 'history of mathematics'.

[3] As we will show in Chapter 4, this is not a correct characterization of the medieval views of modality.

This phenomenon is especially clear when we take X to be logic. The material from the previous section shows just one example from the history of logic where the scope of logic has broadened. What is interesting to notice is that this changing approach to logic from a very narrow definition to a much wider one is not new to the late 20th and early 21st centuries. Over time, we can see cycles of narrowing and broadening. On the extreme narrow end, logic is seen solely as the study of correct inferences from a fixed set of correct rules, such as Aristotelian syllogistic logic (this is the view espoused by Kneale and Kneale as cited above). On the extreme wide end, logic encompasses all aspects of the formal study of reasoning and information in general; on this view, logic, especially applied logic, as practiced in the 21st century has much in common with the medieval view of logic as *ars sermocinalis*. We have seen above how this change is manifest in recent developments in logic.

We can see a pattern of similar changes in the scope of the field of logic throughout history, where one group of people take a very narrow and strict view of logic, and another group of people who rebel against this narrow view, with the 'rebellion' resulting in a period of concentrated and active research in wildly new and different branches of logic (in the broad sense). We briefly highlight two such examples: The development of the *via antiqua* and the *via nova* in the 12th to 14th centuries, and the Ramistic revolution against scholastic logic in the 16th century and its connections to the development of Humanist logic in the 17th.

1.2.1 The *via antiqua* and *via nova*

At the beginning of the 12th century, Latin translations of only two Aristotelian texts on logic were available to Western Europe: Boëthius's translations of *Categoriae* and *De interpretatione*, made in the early 6th century. These two texts, along with Porphyry's *Isagoge*, formed the basis for the transmission of classical logical thought. The three books served as the standard textbooks for the teaching of logic in the *trivium*.[4]

Boëthius had also translated the *Analytica Priora*, the *Topica*, and the *Sophistici Elenchi*, but these translations were lost to Western Europe and not rediscovered there until the 1120s [Dod82, p. 46]. In addition to the rediscovery of the lost translations of Boëthius in the 1120s, in the first half of the 12th century new Latin translations of otherwise unknown works by Aristotle were made and disseminated throughout the Latin west, along with works by Arabic and other Greek writers. Earliest on the scene of translation was James of Venice, who between 1125 and 1150 completed the Aristotelian logical corpus by translating the *Analytica Posteriora*. He also produced a new translation of

[4] A manuscript written between 1230 and 1240 containing a manual for students of the Arts Faculty in Paris gives the following text books for the *trivium*: Priscian and Donatus (for grammar); Cicero's *De inventione* (for rhetoric); and Aristotle's *Organon*, Porphyry's *Isagoge*, and Boëthius's logical treatises (for dialectic, that is, logic) [Loh82, p. 85].

the *Sophistici Elenchi*, as well as translations of the non-logical *De anima* and *De morte* [Dod82, pp. 74–79]. After the introduction of these new Aristotelian texts into the Latin west, the first three books became collectively known as the *logica vetus*, and the new translations as the *logica nova*.[5] The second half of the twelfth century saw continued and sustained translation activity, with the result that, by the beginning of the thirteenth century, a wealth of new secular material, both original texts and commentaries on those texts, was available to Latin scholars. This translation work culminated in William of Moerbeke, who in the third quarter of the thirteenth century translated anew and revised the complete Aristotelian corpus, including two texts which had not previously been available, the *Politica* and the *Poetica* [Dod82, p. 49].

The sudden influx of new treatises on logic engendered a corresponding growth in the range and application of logic—logic as presented in these treatises wasn't just new applications of old methods, but rather wholly new logic, ripe for application to previously unconsidered problems. The new books were very quickly disseminated; for example, Dod notes that "in 1159 John of Salisbury in his *Metalogicon* shows a familiarity with all these works" [Dod82, p. 46]. Not only were the *logica nova* texts much more accessible in style compared to the relatively compressed and difficult books of the *logica vetus* (with the exception of the *Posterior Analytics*, which, Dod notes, "was regarded as difficult" [Dod82, p. 69]), the logic contained in the *logica nova* texts is radically different from the syllogistic reasoning found in *De interpretatione*, and the introduction and later assimilation of these new texts into the standard logical corpus was the cause of significant and innovative developments. Over the course of the next two centuries, the scope of logic widened from mere syllogistic and topical inferences to include reasoning about *insolubilia*, the study of *syncategoremata* and the birth of terminist logic, *obligationes*, and theories of *significatio* and *suppositio*. All of these new branches of logic can be seen as part of a move from rote syllogistic reasoning towards applied logic.

In the second half of the 12th century, the *Sophistici Elenchi* was particularly studied [Dod82, pp. 48, 69]. This text gave birth to the medieval fields of study of *insolubilia* and *fallaciae*, which, of the new developments in logic, can be seen as being the most applied. A vocal proponent of logic being applied to real-life situations, and not merely an *inutilis* tool for syllogistic wrangling[6], was John of Salisbury. John was born between 1115 and 1120 in Salisbury, England.[7] Between 1154 and 1161, he wrote the *Metalogicon*, "a defense of logic in its broad sense" [JoS55, p. xvi], "[c]omposed to defend the arts of verbal expression and reasoning

[5]For further discussion of this, see [Dod82, especially p. 46].

[6]*Siquidem cum opera logicorum uehementius tanquam inutilis rideretur* [JoS29, p. 2], "Since, however, the labors of the latter [logicians] were being lampooned as a waste of time" [JoS55, p. 5].

[7]For more details on John's life and works, see the introduction of [JoS55].

comprised in the Trivium" [JoS55, p. xix].[8] John begins his defense of logic with a definition of the term, noting that 'logic' can be used in either a broad or a strict sense:

> *Congrediamuer, ergo, et quid censeatur nomine logice proferatur in medium. Est itaque logica, (ut nominis significatio latissime pateat) loquendi uel disserendi ratio. Contrahitur enim interdum, et dumtaxat circa disserendi rationes, uis nominis coartatur* [JoS29, p. 27].[9]

But whichever way we interpret 'logic', *profecto desipiunt qui eam dicunt esse inutilem.*[10] John makes it clear (e.g., in Book I, Ch. 13) that he subscribes to the broad view, but also allows that the question of the full extension of logic (e.g., whether 'logic' also covers 'grammar' or not) need not be settled definitively for his argument for its utility to succeed [Book II, prologue]. What is interesting is that even in the narrow sense, where logic is defined as *ratio disserendi* ('the science of argumentative reasoning'), John's definition of logic is much broader than that of, e.g., Kneale and Kneale cited above.

The 13th and 14th centuries see the continuing broadening of logic occur in two ways. First, there is the further development of the non-demonstrative parts of Aristotelian logic, through the study of *insolubilia* and *fallaciae*. Second, there is the birth of wholly new branches of logic such as terminist logic and theories of signification and supposition. Moody notes that:

> Characteristic of this *logica moderna* were its metalinguistic method of presentation, its extensional approach to language analysis, and its formal treatment of both the semantical and the syntactical structure of language [Mood75, p. 375]... It is in these areas [semantical problems of meaning, reference, and truth], rather than in that of pure formal logic, that the work of medieval logicians not only anticipated, but in some respects surpassed, that of twentieth century logicians concerned with these problems [Mood75, p. 387].

They stressed the connection between and inseparability of logic and language; such a view can be contrasted with that of Anselm of Canterbury's strict division between the *usus proprie* and *usus loquendi* of terms, which we discuss in Chapter 3.

The medieval logicians still wanted to recognize the authority and supremacy of Aristotle as a logician (even if his preëminence as a natural philosopher had

[8] John notes in his prologue *Et quia logice suscepi patrocinium,* METALOGICON *inscriptus est liber* [JoS29, p. 3], "This treatise... is entitled THE METALOGICON. For, in it, I undertake to defend logic" [JoS55, p. 5].

[9] "First, bear with me while we define what 'logic' is. 'Logic' (in its broadest sense) is 'the science of verbal expression and [argumentative] reasoning'. Sometimes [the term] 'logic' is used with more restricted extension, and limited to rules of [argumentative] reasoning [JoS55, p. 32].

[10] "surely those who claim that it is useless are deluded" [JoS55, p. 32].

to be given up in the wake of the contest between his philosophy and Christian theology), and quite often they cite various Aristotelian texts in an attempt to provide a grounding for their new developments (see, e.g., [Uc$_2$MaRy09, §2.2]). However, close inspection of the developments in the 13th and 14th centuries shows "how far beyond Aristotelian logic medieval logic eventually developed in various directions, but the non-Aristotelian character of later medieval logic is most striking in its semantic theories" [Kre82, p. 5].

1.2.2 The Humanist revolution

The new advances in logic that started in the 12th century flourished for about two centuries, culminating in the works of logicians such as Ockham, Buridan, and Bradwardine in the first half of the 14th century. However, the second half of the 14th century saw a sharp decrease in the amount of novel logic being produced; instead, we begin to see systematic and thorough compilations of the state of the art, e.g., Paul of Venice's *Logica magna* at the beginning of the 15th century. A century later, new developments had all but disappeared from the logical landscape.[11] Most 16th-century logical texts can be classified into one of four categories: Humanist logic, Ramistic logic, Aristotelian textbooks, and commentaries on Aristotle [As82, p. 791].[12]

One might wonder why it is that the late scholastics did not continue the work of their predecessors. Ashworth asks:

> Why did these interesting and varied treatments of medieval logical themes cease so abruptly after 1530? Humanism alone cannot be the answer, since it apparently triumphed only by default... The judgment of a contemporary logician might be that medieval logic came to an end because no further progress was possible without the concept of a formal system and without the development of a logic of relations. This view is borne out by the desperate, complicated attempts to analyse such propositions as 'Every man has a head' that are to be found in the writings of the Parisian logicians [As82, pp. 795–96].

The fact that the scholastics of the 15th and 16th century did not follow in the footsteps of their late medieval predecessors, and did not continue to develop the innovations in logic which we saw in the previous section, but rather concentrated their energies in "desperate, complicated" analyses, combined with the

[11] "The creative period of medieval logic commenced in the early twelfth century and reached its completion before the end of the fourteenth century... Although logic was intensively cultivated for two centuries after this period, it does not appear to have made any significant advances over fourteenth century logic, but to have been primarily a reworking and further systematization of the latter" [Mood75, p. 374].

[12] Though many texts on Aristotelian logic were produced in the 16th century, this period also saw a strong anti-Aristotelian movement; see [Sei78].

anti-Aristotelian sentiment, especially among the Humanists, lead to many people taking a very dim view of the utility and scope of 'logic' in the 17th century. In the eyes of many in the 17th century, scholastic logic as a discipline was unfruitful and dead, focusing on traditional subjects with no new innovations and potentially many hazards. Bacon, Descartes, and Locke are all indicative of this narrow and pessimistic view of logic in this time.[13] We can contrast these views with the views of philosophers such as Leibniz and Wallis[14], and with those of the Humanists, who viewed logic primarily as a rhetorical tool to be applied in every

[13]Bacon says, *Siquidem dialectica quae recepta est, licet ad civilia et artes quae in sermone et opinione positae sunt rectissime adhibeatur, naturae tamen subtilitatem longo intervallo non attingit; et prensando quod non capit, ad errores potius stabiliendos et euasi figendos quam ad viam veritati aperiendam valuit* [Bac$_1$90, p. 24] ("For the logic now in use, though very properly applied to civil questions and the arts which consist of discussion and opinion, still falls a long way short of the subtlety of nature; and in grasping at what it cannot hold, has succeeded in establishing and fixing errors rather than in opening up the way to truth" [Bac$_1$00, p. 10]). Descartes says, *[O]mittamus omnia Dialecticorum praecepta, quibus rationem humanam regere se putant, dum quasdam formas disserendi praescribunt, quae tam necessario concludunt, ut illis confisa ratio, etiamsi quodammodo ferietur ab ipsius illationis evidenti & attenta consideratione, possit tamen interim aliquid certum ex vi formae concludere... [R]ejicimus istas formas ut adversantes nostro instituto... [P]atet, illos ipsos ex tali forma nihil novi percipere, ideoque vulgarem Dilaecticam omnino esse inutilum rerum veritatem investigare cupientibus* [De66, pp. 36–37] ("[W]e omit all the rules by which the logicians think they regulate human reason. These prescribe certain forms of argument which involve such necessary implications that the mind which relies upon this method, even though it neglects to give clear and attentive consideration to the reasoning, can nevertheless reach certain conclusions on the strength of the form of the argument alone... [W]e reject this formal logic as opposed to our teaching... [I]t is clear that the logicians themselves learn nothing new from such formal procedures, and that ordinary logic is completely useless to those who seek to investigate the truth of things [De77, p. 184]"). And Locke says, "To this abuse, and the mischiefs of confounding the Signification of Words, Logick, and the Liberal Sciences, as they have been handled in the Schools, have given Reputation; and the admired Art of Disputing, hath added much to the natural imperfection of Languages, whilst it has been made use of, and fitted, to perplex the signification of Words, more than to discover the Knowledge and Truth of Things" [Loc75, p. 493–94].

[14]Leibniz says, "Or comme la *logique* est l'art qui enseigne l'ordre et la liaison des pensées, je ne vois point de sujet de la blâmer. Au contraire c'est faute de logique que les hommes se trompent" [Lei66, p. 299] ("As for *logic*: since it is the art which teaches us how to order and connect our thoughts, I see no grounds for laying blame upon it. On the contrary, men's errors are due rather to their lack of logic" [Lei96, p. 343]). Wallis says, *Non enim ideo tantum tradit censendum est praecepta Logica (quae Juvenum praesertim magna pars sensisse videatur) ut per Annum unum aut alternum (aut tertium etiam quartumve) in Sophistarum Scholis vixandi aut altercandi, de praeceptis ipsis, materiam subministrent; atque tum denum, recedentibus, cum Toga Academica deponatur; quasi in reliquum in vitae inutilia: Sed ut formandis Academicorum animis inserviant; quo possint, per universam vitam, rationes rite instituere, atque dilucide justoque ordine conceptus suos apud se formare, & apud alios proferre* [Wall87, pp. 2–3] ("The precepts of logic are not taught (as many of the young seem to have thought) to supply the means for quarreling and wrangling over sophistical theses for a couple of years..., being useless in the rest of their lives after they have taken off the academic gown, but to be able, for their whole lives, to set up reasonings well, to form clear concepts for themselves, and to put them forward to others in the right order", trans. by Jaap Maat).

discourse. Trentman explains that the Humanists reacted against the scholastic tradition as encapsulated by the contemporary Aristotelian commentary because these late scholastic authors "were not much interested in formal logic as such. They were far more interested in the philosophy of logic and language than in formal logic" [Tre82, p. 819].

Two prominent critics of Aristotelian logic in the late 15th century were *Valla, Lorenzo* and Rudolph Agricola. For both Valla and Agricola, logic was to be developed within the confines of rhetoric [ØhHa95, p. 110].[15] Agricola's textbook, *De inventione dialectica*, completed in 1479, returns to the classical tradition of rhetoric. Agricola speaks of "the corruption of modern logic" [Mac93, p. 136, fn. 20] and little traditional Aristotelian logic can be found in his textbook. Agricola's works had a strong influence on Petrus Ramus [As82, p. 791][Gau89, p. 35], working in the 16th century, though Ramus's followers often reinstated Aristotelian material into their works.

The Humanists reacted against the scholastic view of Aristotelian logic primarily by shunning it. This approached can be contrasted with the Port Royalists, who retained some of the traditional Aristotelian discussions but did not limit themselves to that. The foremost Port-Royalists were Antoine Arnauld and Pierre Nicole, authors of the *La logique, ou l'art de penser*, better known as the *Logique de Port-Royal* or the *Port-Royal Logic*, published anonymously in 1662. The *Port-Royal Logic* has been called variously the "most influential logic from Aristotle to the end of the nineteenth century", "never been superseded" [Fino97, p. 393], and a "most important contribution to the development of modern logic" [Mi69, p. 262].[16] Carr describes the book as follows:

> Postulating truth as a prerequisite of beauty (III.20.b.2), the *Logic* attacks the highly figured baroque prose that was gradually losing the popularity that it had enjoyed earlier in the century, describing it as 'an artificial style typical of rhetoric classes, composed of false and hyperbolic thoughts and exaggerated figures' (Second Discourse) [Car96, p. 547].

Finocchiaro argues that the *Port-Royal Logic* should be seen as "a precursor of the contemporary field of informal logic and/or argumentation theory" [Fino97,

[15] Øhrstrøm and Hasle say that "[t]his considerable change in the conception of logic was to a large extent a reaction against the perceived maltreatment of the Latin language by Scholastic logicians" [ØhHa95, p. 110].

[16] It is interesting to note that Locke read the English translation of the *Port-Royal Logic* before it was published, and his reaction to it was quite a bit more positive than his general view towards logic: "In fact, it is certain that Locke had read the *Port Royal Logic* before its first publication in English in 1685" [Wo70, p. 257] and "In any case there is no doubt that Locke had at least *read* the *P.R.L.* before 1685. Writing in his Journal on 7 March 1678... he suggests that 'apres avoir bien conceu la maniere de philosopher dans sa methode on peut lire sur le sujet de la Logique celle que nous ont donnée M^{rs} de Port Royal qui est un ouvrage le plus accompli qui ait encore paru en ce genre' (Aaron and Gibb, p. 107)" [Wo70, p. 259].

p. 394], though if this characterization is correct, "almost any 17th century logic textbook can be seen as such".[17]

From this we can see how in both the 12th–14th centuries, and then again in the Renaissance, we have periods where the scope of logic was construed very narrowly, and that there were reactions against this narrowing resulting in a dramatic widening, usually over the course of a relatively short period of time (50 to 100 years). Thus the pattern that we opened with in §1.1 is just another repetition in a well-established cycle.

1.3 A modern view of medieval logic

These changes in the scope of logic have two important implications for modern-day logicians, one which looks backwards and one forwards:

History of logic: A modern logician working within a wide scope of logic has a much better chance of fully appreciating the historical logicians who worked within a similarly wide scope.

Practice of logic: With the broad view of logic, and hence a broad view of the history of logic, we open up the possibility that we can take historical logical theories which fall outside the narrow scope and re-implement them within modern contexts.

The different approaches to what counts as logic (the wide view and the narrow view) have consequences for how modern logicians approach medieval logic. As we noted above, in the view of the narrow-scope logician, there are two conclusions that can be drawn about the work of the ancient and medieval authors:

- Either they were mistaken in their logical theories (Boëthius's theory of hypothetical syllogism is wrong because it is not propositional logic, supposition theory is wrong because it is not quantifier logic),
- or they simply weren't discussing logic at all.

That is, on this narrow view, most of the topics in this dissertation are not 'logic', but they are on the wide view of logic.

A 21st-century logician who is interested in the societal factors which affect the development of various branches of logic cannot help but notice that one of the biggest differences between logic as it is currently practiced and logic as it was practiced in the Middle Ages is the role of the Church. Problems in theology have very little influence on modern logic, whereas medieval logicians were often trained theologians and those who were not were still connected to the Church via the Church's role in everyday life and its connections to the university and

[17] Personal communication with Jaap Maat.

academic life. As a result, it is natural to ask whether there were specific actions of the Church which altered the development of logic in the Middle Ages. This is the topic of the most historically-oriented chapter of this dissertation, Chapter 2, where we investigate the consequences that two ecclesiastical actions in Paris and Oxford in 1277 had on the development of modal and temporal logic in the succeeding decades and century.

The remaining chapters of the dissertation cover various topics in medieval modal logic and are ordered in roughly historical order.

Most discussions of modality in the Middle Ages seem to leap directly from Boëthius to Peter Abelard, without any discussion of modal theories in the intervening period.[18] This is because 'modality' is usually taken in a very narrow sense of the term, referring only to the modalities of possibility, contingency, necessity, and impossibility. But when we expand our view to include modalities such as agency and action, then a very important figure arises in the intervening period: Saint Anselm of Canterbury. With Chapter 3 we begin our tour of modalities in medieval logic by looking at Saint Anselm's theory of agency and deontic logic in fragmentary writings written towards the end of his life and compiled after his death in 1190.

Chapter 4 deals with the traditional subject matter of modal logic, namely the logic of the necessity and possibility modalities. We compare three different 13th-century works on necessity and possibility, both with each other and with modern theories of modal logic, showing that the medieval conception of modality differs from the modern one in some fundamental ways.

In Chapter 5 we consider the other 'standard' modal logic, namely temporal logic. We show how analyses of tensed natural language sentences in the context of a theory of supposition provide the theoretical grounding for quantified temporal logic. Our analysis highlights aspects of medieval logic which are essentially pragmatic in nature, developed to be tools for the analysis of properties of natural language (even if that natural language was sometimes the semi-formalized Latin of the late scholastics).

The logics considered in the final two chapters, Chapter 6 and Chapter 7, diverge from the traditional definition of modal logic. Chapter 6 deals with a relatively new modality, that of 'public announcement' developed within dynamic epistemic logic, which we compare to the "modality" used in Roger Swyneshed's characterization of self-falsifying sentences. Chapter 7 deals with trinitarian reasoning, a topic which was not uncommon in medieval logic and philosophy but vanishingly rare in modern logical discussions. In both cases we can show how tools developed by modern logic can help us to understand and explain the medieval theories, even though these medieval theories in some cases have little application in modern context.

[18] See, e.g., [Knu80], though this is partially redressed in [Knu93].

Appendix A contains details about basic logical knowledge that we assume throughout the dissertation. In Appendices B and C we provide various translations of supporting material. Throughout this dissertation, translations of Latin quotes which do not have citation references are my own.

Chapter 2

Logic and the condemnations of 1277

2.1 Introduction

When looking for clear examples of theological issues affecting medieval logic, one might think he need look no further than condemnations and prohibitions by Church authorities of the teaching and dissemination of certain logical doctrines, for surely the charge of heresy and the threat of excommunication would have had some effect on logicians' choice of theories. In this chapter we look at various 13th-century condemnations propounded at the universities of Paris and Oxford, focusing on the two most important, the condemnation of 219 propositions by Stephen Tempier, bishop of Paris, and the prohibition to teach of 30 propositions by Robert Kilwardby, archbishop of Canterbury, also, both in March 1277. Both the condemnation and the prohibition were reactions against the growing influence of Aristotelian philosophy, both because of the rediscovery of new Aristotelian texts in the 12th century and through the commentaries on Aristotle by the 12th-century Muslim philosopher Averroes. Tempier's condemnation, in particular, has "gained great symbolic meaning in the minds of modern intellectual historians" [Thi03] because of the influence that it had in the teaching and propagation of Aristotelian philosophy at the University of Paris from the end of the 13th century onwards.

Since 1977, their 700th anniversary, these condemnations have received extensive attention from historians and scholars of medieval philosophy[1], but with a few exceptions, these historians have focused primarily on the parts of the condemnations that connect to natural philosophy and to science.[2] We are interested

[1]The seminal work on the Paris condemnation, [His77], was written to commemorate this anniversary. Hissette's work was the first modern scholarly consideration of the condemnation, and remains one of the most important introductions to the topic.

[2]For example, Murdoch in says explicitly that he addresses the effects of the condemnations only with regard to natural philosophy, and not theology or philosophy in general [Mu98, p. 111]. Perhaps the most well-known modern comment on the condemnations is the claim by Pierre Duhem that Tempier's censure gave birth to modern science. See [Duh06–13] and [Duh13–59];

here in the condemnations with respect to their connections to logic and dialectic. We begin in §§2.1.1, 2.1.2 by locating the historical facts of these condemnations, and provide some overview of previous literature discussing the condemnations. Next in §2.2 we turn to a discussion of events leading up to the condemnations, as well as of the general intellectual and academic features which played a role in these events. We then look at the condemned propositions themselves in §2.3, focusing on those which treat with specifically logical matters, the role of logic or philosophy in theological reasoning, or the nature and structure of time. After discussing these propositions of logical interest, we briefly survey the documented effects of the condemnations in §2.4, and finally, draw some conclusions in §2.5 about how these condemnations affected and guided the development of logic, with special attention to the development of modal and temporal logic, in the end of the thirteenth century and into the fourteenth century. It turns out that these condemnations did not have as large an impact as one might think, for reasons that derive from the medieval conception of both modality, time, and logic.

We begin with an introduction to the condemnations themselves. The propositions which were condemned and that they were condemned are perhaps the only clear facts of the situation. Less clear are the motives of their condemners, Stephen Tempier, bishop of Paris, and Robert Kilwardby, archbishop of Canterbury (did he undertake the condemnation on his own? Was he acting on a papal mandate?), who was targeted by the condemnation (was anyone? Was it just scholars in the Arts Faculty at Paris? Was it St. Thomas Aquinas?), and what effect the condemnation had in the following centuries (was there any? Did this signal the strength of the stranglehold of the Church upon science? Was this the birth of modern science?). Many of these questions simply cannot be answered given the available evidence. But in order to discuss the impact that this condemnation had on the development of logic, in the next section we briefly consider some of these questions and the arguments in support of different answers.

2.1.1 The condemnation in Paris

On 7 March 1277, the third anniversary of the death of St. Thomas Aquinas, Stephen Tempier, bishop of Paris, issued the condemnation of 219 errors of theology, natural philosophy, and logic.[3] In addition to the 219 propositions, Tempier

and [Mu91] for further discussion. Others, on the other hand, argue that, e.g. "the effects of the condemnation of 1277 were narrow and largely ignored" [Goo06, p. 43]. See also [Gr79] and [Gr96].

[3]The condemnations are printed in [CUP, I, pp. 543–61]. The propositions were reprinted in a different order in [Man08], and the first serious extensive systematic historical and doctrinal study of the articles is [His77].

also condemned two specific texts, *De amore* or *De deo armoris*[4] and a book of geomancy with the incipit *Existimaverunt Indi*, and any tract dealing with necromancy, invocations of the devil, incantations which may endanger lives, or fortune telling. Anyone teaching, defending, upholding, or even listening to any of these propositions who did not turn themselves in to ecclesiastical authority within seven days faced excommunication and any other punishment as required by the nature of the offense [FoO'N63, p. 337].

We start our discussion of the condemnation by presenting the standard view of the events leading up to Tempier's 7 March condemnation. On 18 January 1277, Pope John XXI[5] wrote to Tempier saying that he had heard rumors of errors circulating within Paris, and charging him with investigating these rumors and reporting on them to him:

> Episcopo Parisiensi. *Relatio nimis implacida nostrum nuper turbavit auditum, amaricavit et animum, quod Parisiis, ubi fons vivus sapientie salutaris abundanter hucusque scaturiit, suos rivos limpidissimos, fidem patefacientes catholicam usque ad terminos orbis terrae diffundens, quidam errores in prejudicium ejusdem fidei de novo pullulasse dicuntur. Volumus itaque tibique auctoritate presentium districte precipiendo mandamus, quatinus diligenter facias inspici, vel inquiri a quibus personis et in quibus locis errores hujusmodi dicti sunt sive scripti, et que didiceris sive inveneris, conscripta fideliter, nobis per tuum nuntium transmittere quam citius non omittas. — Dat. Viterbii, xv kalendas februarii, anno primo* [Cad92, p. 51].[6]

The traditional view is that this letter was the instigation for Tempier's condemnation.[7] Assuming that the letter took a month to travel from Viterbo to Paris[8],

[4] [Bia98, p. 91], [Thi03, §3], and [Wip03, p. 65] identify the author of this text as Andreas Cappellanus or Capellanus.

[5] This pope is the one originally known as *Petrus Hispanus*, and is no longer thought to be identical with the Petrus Hispanus who was the author of a very popular *Summula Logicales*. See [D'Or97] for a further discussion of this issue.

[6] "*To the bishop of Paris.* An exceedingly worrisome relation has recently disturbed our hearing and excited our mind, that in Paris, where hitherto the living font of salutary wisdom has been lavishly spreading its most clear streams showing the Catholic faith all the way to the ends of the earth, certain errors in judgment of the same faith are said to have sprung forth anew. And so by the authority of these presents we wish and strictly enjoin that you should diligently cause to be inspected or inquired by which people and in which places the errors of this kind are spoken or written, and whatever you may hear about or find, you should not omit to faithfully write them down, to be transmitted to us through your messenger as quickly as possible. — dated at Viterbo, 18 January, in the first year."

[7] [Cal55, p. 11]; [Gr96, p. 71]; [Kno42, p. 184]; [Thi97a, p. 92]; [Wip95b, §4].

[8] Thijssen in [Thi97a, p. 93, fn. 29] says that "According to Robert Wielockx in *Aegidii Romani Opera Omnia*, vol. 3, pt. 1, *Apologia*... correspondence from the papal court to Paris took about a month to arrive."

this means that Bishop Tempier created his list of erroneous propositions and suspect texts in just over two weeks, and issued his condemnation without waiting for the list to be ratified by the pope.[9] This short interval, and the impetuosity it implies on Tempier's part, are cited as explanations for the evident lack of systematicity and the seeming haphazardness in the construction of the list, and the facts that certain of them appear to be condemning positions of orthodoxy, that apparently contradictory propositions appear in the list, and that the authors of these erroneous propositions are not named.[10]

In drawing up his list of erroneous propositions, Bishop Tempier sought the advice of sixteen masters of theology, including Henry of Ghent (d. 1293), whom he first met in 1276. The identity of the other fifteen masters is not known, though it is likely that they include John of Alleux, university chancellor; Simon of Brion, papal legate; and Ranulphe of Houblonnière, Tempier's successor as bishop.[11] In the introduction to the list of condemned propositions, Tempier says:

> *Magnarum et gravium personarum crebra zeloque fidei accesna insinuavit relatio, quod nonnulli Parisius studentes in artibus proprie facultatis limites excedentes quosdam manifestos et execrabiles errores... in rotulo seu cedulis presentibus hiis annexo seu annexis contento.*[12]

There are two things of note in this selection of Tempier's introduction. The first is that the perpetrators of the errors are members (not even specifically masters) of the Arts Faculty, and that they are described as "exceeding the boundaries of their own faculty". This is a reference to the voluntary oath of 1272 which we will discuss in §2.2.2, which forbade members of the Arts Faculty from pursuing theological questions. As we'll also see in §2.2.2, the 1277 condemnation was not the only act of censure issued at or applying to the University of Paris in the thirteenth century. In comparison to the other lists of censured propositions, the fact that Tempier's was sponsored by a bishop, directed at the Faculty of Arts, and anonymous (in that only the propagators of the errors, not their originators, were named) makes it unique.[13]

[9] Note that in most cases, this would not be remarkable; Thijssen in [Thi98] indicates that many cases of censure, especially in the 13th century, only reached the pope on appeal. However, if the traditional view is correct, that the condemnation was triggered by the papal letter, then it would be remarkable given the request of Pope John that Tempier send him a list of the problematic theses.

[10] See, e.g., [Wip77].

[11] [EmSp01, p. 3]; [FoO'N63, p. 355]; [Thi97a, p. 103]; [Wip95a, p. 237]. Tempier himself had been a member of the Theology Faculty before he became bishop of Paris.

[12] "We have received frequent reports, inspired by zeal for the faith, on the part of important and serious persons to the effect that some students of the arts in Paris are exceeding the boundaries of their own faculty and are presuming to treat and discuss, as if they were debatable in the schools, certain obvious and loathsome errors... which are contained in the roll joined to this letter" [FoO'N63, p. 337].

[13] [Thi97a, p. 87]; [Thi03, §4].

The second item of note is that there is no mention at all of any papal injunction underpinning this action. One would think that if Tempier was acting specifically on papal command, that rather than referring to "reports" from "important and serious persons", he would refer to the command received from Pope John in the 18 January letter quoted above. As Thijssen notes, if Tempier's actions were triggered by the 18 January papal letter, "it is surprising that he does not mention it in the introduction" [Thi03, §1]. He argues that "the evidence suggests that Tempier acted independently from the pope and that when he received the papal letter of 18 January 1277 he was already in the process of preparing his condemnation" [Thi97a, p. 92]. Further evidence that Tempier's action was not caused by the first papal letter can be found in the existence of a second letter from the Pope to Tempier, dated 28 April 1277, which "gives no indication whatsoever that the pope knew about Tempier's action" and where the pope asks Tempier "to inform him about the names of the propagators" of errors both in the Faculty of Arts and in the Faculty of Philosophy.[14] If this is the case, as seems likely given Thijssen's arguments, then this claim of Wilshire's seems unwarranted:

> in a second letter, dated April 28, 1277, and after the condemnations themselves, Pope John XXI commended the 1277 Paris Condemnations as dealing with the Arts Faculty and asked, upon explicit instructions, that a further inquiry be made into suspect theologians [Wils97, p. 153].

If the cause for Tempier's actions is not to be rooted in papal mandate, where can it be found? Thijssen argues that the roots of the 1277 condemnation can be traced back to 23 November 1276: "On that date Simon du Val, the Inquisitor of France, cited Siger of Brabant together with Bernier of Nivelles and Goswin of Chapelle to appear before this court" [Thi97a, p. 94]. Thijssen argues that the results of the inquisition against Siger are part of the information received from important people (*magnarum et gravium personarum*) which Tempier refers to in his introduction. In sum, "the picture of Tempier as an overzealous bishop is simply untrue. More likely, Tempier was disturbed by the charges that were raised against Siger of Brabant's teaching toward the end of 1276. Probably, the bishop used the dossier collected against Siger and the other two masters in drawing up the censure of 1277" [Thi97a, p. 105].

Even though the sources of the errors are not generally explicitly named in the condemnation, the standard view is that the condemnations were directed against Thomas Aquinas, Siger of Brabant, and Boëthius of Dacia (like Siger, a member of the Faculty of Arts).[15] However, contra, Wippel notes that "efforts to find

[14] [Thi97a, p. 93]. See also [Wip77, pp. 186–87].

[15] See discussions in, e.g., [Eb98], [Wip77], [Wip95a], and [Wip95b, p. 25]. The standard source on Siger's life and works is [Man08]. For information on Boëthius's life and works, see

these explicit doctrines in the writings of Siger or of other members of the Arts Faculty prior to the date of the decree's issuance have not, to our knowledge, met with success" [Wip77, p. 180], Thijssen in [Thi97b] disagrees that Aquinas was a target for Tempier's condemnation, and Normore casts the condemnations as just one more cycle in an on-going struggle between what he terms the Philoponeans and the Simplicians.[16] We do not attempt to offer any further arguments in settling these questions here.

2.1.2 The prohibition in Oxford

Eleven days after Tempier's condemnation in Paris, on 18 March 1277, Robert Kilwardby, archbishop of Canterbury, was on a pastoral visit to the university of Oxford in his capacity as patron of Merton College. During his visit, Kilwardby, who by this time had been a teacher in the Theology Faculty at Oxford for over twenty years[17], issued his own list of erroneous propositions. These propositions were thirty in number: four concerning grammar, ten concerning logic, and sixteen concerning natural philosophy [Lef68, p. 291]. Kilwardby's motives and the relationship between his acts and those of Tempier raise many interesting questions. It is variously argued that Kilwardby's issue was a reaction to Tempier's condemnation; that it was in collusion with Tempier's[18]; that it, like Tempier's, was triggered by a papal letter[19]; and that in any case, Kilwardby was acting,

[Wip77, fn. 63] and [Wils97, p. 158]. [deM06, fn. 2] notes that two MSs of the condemnation, Paris, Bibl. nat. de France, lat. 4391 and lat. 16533, carry marginalia identifying "the heretic Siger and Boethius" in one case and "a clerk named Boethius". However, these MSs are noted as being among the more unreliable ones.

[16] "The leading idea I want to introduce is that the Oxford and Paris Condemnations should be understood as another round in a debate between philosophy and Christian theology which had begun in late antiquity, was carried on vigorously in the Islamic world and was being replayed with several new elements in Latin in the thirteenth century" [No95, p. 95].

[17] [Shar34, p. 306]; [GlLiWa05].

[18] Both of these options are *a priori* possible; in the late 13th and early 14th centuries, an important message could have gotten from Paris to Oxford in five, and perhaps as few as four, days (cf. [Hil42, pp. 25, 27]; my thanks to Wendel Bordelon for pointing me to this reference), thus leaving Kilwardby six or seven days to create his list.

[19] "[I]t is likely that events in Paris precipitated Kilwardby's action" [Cat84, p. 499]; "In this light, Archbishop Kilwardby's action at Oxford in 1277 could well be seen as a prudential procedure inspired by the recent measures taken at Paris" [Ir01, p. 261]; "Ten days after Tempier's decree, and almost certainly through collusive action—perhaps also in response to a monition from Rome—Archbishop Kilwardby... 'visited' the University of Oxford" [Kno42, p. 184]; "Kilwardby's condemnations were in fact a backwash from the philosophical controversies which had their storm-centre in this period in the faculty of arts at Paris" [Law84, p. 116]; "Kilwardby's condemnations of 1277 at Oxford were largely a reflex action of those ten days earlier at Paris...[Kilwardby's] condemnation of 1277 took place on March 18, ten days after that of Etienne Tempier's at Paris. It can therefore be regarded principally as a response to events outside Oxford" [Lef68, pp. 272, 291]; we should see the "Oxford Condemnations as a hastily drawn addendum to the Paris Condemnations" [Wils97, p. 164].

as Tempier was, to "check the rapid departure from Augustinianism... [and] to ensure that every doctrine taken over from Aristotle and the Arabians should be capable of reconciliation with Augustine and with Christian thought in general" [Shar34, p. 306]. We have seen above that there is reason to doubt that Tempier's actions were the result of the papal letter of January 1277. If, as Thijssen suggests, Tempier's condemnation was a natural outgrowth of his investigations into Siger of Brabant in late 1276, then it seems less plausible that Kilwardby was acting in collusion with Tempier, since there is no reason that Oxford would be connected to the investigation of Siger's views. Furthermore, if we accept that the papal bull was not the cause of the 7 March condemnation, then given that "no document has ever been found instructing Kilwardby to inquire into errors at Oxford" [Wils97, p. 164][20], postulating a papal bull received by Kilwardby seems unjustified. This, combined with the fact that "there is little in the Oxford theses to match those condemned at Paris" [Lew81, p. 235], makes it unlikely that Kilwardby's actions were either done in concert with Tempier or that they were stimulated by papal decree.

It is also often argued that the Oxford condemnation, like the Paris one, was directed specifically against Thomas Aquinas.[21] Knowles argues that the activities of one Richard Knapwell, a vocal supporter of Thomistic ideas, "had very possibly provided the immediate material for Kilwardby's interference in 1277; it is almost certain that his somewhat noisy propaganda on behalf of Thomism brought about Pecham's action in 1284, for in that year Knapwell incepted as regent master" [Kno42, p. 186].[22] We discuss Pecham's actions further in §2.4.

The questions of whether the Oxford condemnation was in reaction to the Paris one, and whether Aquinas was a specific focus in either condemnation do not interest us here, because their answers have little bearing on the effect these condemnations had on logic. However, because they are so widely discussed, especially the latter, it is important that we at least mention them.

Kilwardby's list of erroneous propositions differs from Tempier's in two important respects. It is manifest from Tempier's introduction that his intent was to publish a list of heretical propositions, adherence to which could result in excommunication (see §2.1.1). On the other hand, not too long after *his* prohibition, Kilwardby wrote to Peter of Conflans, Dominican archbishop of Corinth then acting as papal envoy to the French court in Paris[23], explaining that the intent behind his list of propositions was not "to stigmatize them as heretical, but simply

[20] See also [Wils93, p. 114].

[21] See, e.g., [Cal55], [Cr50], [Wip77]; but contra, [Shar34], [Wils74], [Wils93, p. 113].

[22] Knapwell, an Oxford member of the Dominican Order, was a vocal proponent of Aquinas's views; his "attempts to bring reconciliation between the pluralist and unitarian standpoints backfired, thus provoking an unfavourable reaction on the part of the authorities" [Ir01, p. 290], and in 1286 he was excommunicated. For a detailed discussion of Knapwell, see [Ir01], specifically §§4.2, 4.3. Sharp in [Shar34, p. 306] calls Knapwell "Clapwell".

[23] [Kno42, p. 184]; [Wils93, p. 114].

to prohibit them from being taught, upheld and defined publicly in the schools" [Cal55, p. 13].[24] As further evidence that he did not intend his prohibition to be taken as a condemnation of heresy, he cites the penalties for propagating the erroneous theses. The penalty in Paris was excommunication, whereas the penalties in Oxford were merely temporal: Masters were to be deprived of their chairs and bachelors were not to be promoted to the mastership and were to be expelled.[25]

The second point of departure is that Kilwardby was acting with the full faith and agreement of all the masters at Oxford, both regent and non-regent, both theologians and philosophers.[26] This is in contrast to Tempier, whose condemnation was made with only the advice of doctors of theology (see §2.1.1).

2.2 Historical background

Before we turn to a discussion of the propositions themselves, we will first sketch, from a primarily logical point of view, the historical events which culminated in the 1277 condemnations. We do so in order to have a framework for discussing the effect of the condemnations on subsequent work.

2.2.1 The structure of a university

In §1.2.1 of Chapter 1, we discussed the introduction of new Aristotelian material into the Latin west. This new material was not embraced with universal enthusiasm. Different people with different agenda quite naturally reacted in different ways. The organization of and the relationships between the faculties at the universities of Paris and Oxford deeply affected the reception of the new Aristotelian material in the thirteenth century. In order to understand these effects, we must first understand the structure, and some of the history, of the universities. We

[24] See also [GlLiWa05]. Kilwardby was writing in response to a letter of Conflans's denouncing his actions as unduly harsh. Conflans's letter is no longer extant, but Kilwardby's response, which is a point by point reply to issues raised by Conflans, has been edited in [Eh89] and in [Bir22]. This point is reiterated by Wilshire when he notes that "we have seen in his letter to Peter of Conflans how Kilwardby stated that he had framed his statements as propositions rather than as condemnations and had repudiated categorically that he had condemned any thesis. He had instead issued a mere prohibition to teach, or to maintain certain tenets with 'pertinacity' in the schools" [Wils97, p. 174].

[25] [Cal55, p. 13]; Bianchi in [Bia98, pp. 93–94] argues that since Kilwardby's intent was to discourage the teaching of said propositions, and not to condemn them as heretical, that it's "highly likely" (p. 93) that this was Tempier's motive too—after all, "Tempier did not condemn the articles themselves, but their teaching" [p. 94]. He says that this "explains (although it does not justify) the prohibition of perfectly orthodox articles, or of articles completely unrelated to the Christian faith—a fact that has perplexed historians and which indeed, would be hardly understandable in a true doctrinal condemnation" [Bia98, p. 95]. Given Tempier's introduction to his list, Bianchi's conclusions do not seem reasonable.

[26] [Cr50, p. 248]; [Cal55, p. 13]; [Cou87, p. 179]; [Wils97, p. 154].

give here but a sketch. For a more complete picture, see, e.g., [Cou87], [Lef68], and [Ra36, vols. 1, 3].

The universities of both Paris and Oxford have their roots in the early thirteenth century.[27] This is not to say that there were no centers for learning in either place previous to the thirteenth century; this is manifestly not the case. The university of Paris grew out of the cathedral schools of St. Geneviève, St. Victor, and Notre Dame. As such, it fell directly under the auspices of the bishop of Paris. As a result, the university of Paris was more closely entangled with ecclesiastical matters than Oxford.

Oxford had its connections with the Church[28], but they were in many cases more subdued. While Oxford was the home of the monastery of St. Frideswide, founded in 1121 [Lef68, p. 77], there is no evidence that the university was connected with the monastery in any fashion[29], or even that it was connected with any monastic or cathedral school. And while it also fell under the supervision of a bishop, this bishop was the bishop of Lincoln [Lef68, p. 15], and Lincoln is nearly 225km from Oxford. By virtue of distance, the bishop of Lincoln simply couldn't be as involved in Oxford's university doings as the bishop of Paris could be in Paris's. Once the episcopal cathedral for the diocese was moved from Dorchester to Lincoln in the late 11th century by Bishop Remigius, the bishop's primary residence was in Lincoln but he "moved perpetually through his diocese, from one to another of his estates, or to London to his residence in the Old Temple" [Ow71, p. 20]. But this is not to say that the bishop wasn't involved with the university: In the early part of the 13th century the university was "for a time... entirely under the control of the Bishop of Lincoln", but "so long as the see of Lincoln was filled by Robert Grosseteste... almost unbroken harmony prevailed between the university and the diocesan" [Ra36, pp. 114, 115]. (Grosseteste was bishop of Lincoln from 1235 to 1253.)

Both universities were organized into four different Faculties: Arts, Theology, Law, and Medicine.[30] In theory, all students started off under the auspices of the Arts Faculty, where they studied the *trivium* (see §1.2.1), which formed the basis of a liberal arts education. Once a student has become a bachelor in the Arts

[27]Paris is generally accepted to be the earlier of the two universities, but an interesting claim that Oxford is older than Paris is found in a letter in Richard de Bury's Letter Book ("This *Liber Epistolaris*... is a collection of upwards of 1500 documents, comprising mostly copies of letters and other public documents from various sovereigns, popes, and other important persons. They range in date from the twelfth century to 1324", according to [Has41, p. 284, fn. 5].). The letter is addressed to the pope and "urges by way of argument that since England was the *fons et origo* of the French universities, it is fitting that Oxford be given the same privileges as those enjoyed by the French universities" [Has41, p. 285].

[28]As Lyte says, "The University of Oxford [was] scarcely less ecclesiastical in character than that of Paris" [Ly86, p. 8].

[29]That is, until 1525 when the archbishop of York suppressed the abbey and founded Cardinal College, the predecessor of Christchurch, on its grounds. See [Kin$_1$62].

[30][Ly86, p. 54]; [Ra36, pp. 321–3].

Faculty, he could continue towards a doctorate in any of the four faculties. This means that many members of the Theology Faculty, both students and teachers, had close connections with the Arts Faculty. On the other hand, not all of the teachers in the theology went first through a university course in arts: Theologians trained at one of the local cathedral schools (which remained in place even after the university was well established) could be appointed to teaching positions within the university's Theology Faculty.[31] Naturally, such appointments were not wholly welcomed by the Arts Faculty, because ecclesiastical funding for teaching chairs in the Theology Faculty put the Arts Faculty at a disadvantage when it came to attracting qualified teachers, but also that the role that liberal arts and the Arts Faculty played in the academic career of both students and teachers was diminished. It is somewhat ironic that one result of this emphasis on the liberal arts was the eventual lowering of their status. As Lyte points out, "the [Arts] Masters of 1253, in their very anxiety to do honour to the liberal arts, unwittingly caused them to be regarded as mere preliminary studies for men aiming at higher knowledge. The Faculties of Theology, Law, and Medicine, soon took rank above the Faculty of Arts, and the teachers of these superior Faculties came to be styled Doctors, in contradistinction to the Masters of Arts" [Ly86, p. 54].

There were two main tensions affecting the Arts and Theology Faculties at both universities.[32] The first was within the Theology Faculty. The thirteenth century saw the rise of the mendicant orders (the Dominicans and Franciscans[33]), of which many masters and doctors in the Theology Faculty were members, and consequently the rise of controversies between the orders.[34] The second tension was between the faculties. On the one hand, members of both faculties were at least nominally working within the constraints of Catholic orthodoxy. On the other hand, solving and clarifying doctrinal problems was not the purpose of the Arts Faculty. Because their subject matter was temporal, rather than eternal, members of the Arts Faculty often didn't concern themselves with the eternal consequences of their temporal subjects that the Theology Faculty was quick to see. This was seen most clearly in the introduction of the Aristotelian metaphysics.

[31] See [Ve92, p. 160]; [Ly86, p. 52] discusses the case of Thomas of York, who in 1253 "came forward to claim a theological degree" which would allow him to teach in the Theology Faculty (a position he was well qualified for), even though he had never taken an arts degree. In light of the fact that a similar situation at Paris had resulted in the Dominicans having a perpetual right to one of the public chairs in the Theological Faculty, the decision of the Chancellor, Masters, and chief Bachelors (reached after three meetings), was that an exception would be granted for Thomas of York, but that after that, no one would be allowed to teach in one of the higher faculties unless he had already received a master of arts from some university.

[32] That is, tensions within the university; we do not consider here tensions between the university and its surrounding town.

[33] See [Ben37] and [Moorm68].

[34] One such controversy, both within and between the orders, was the question of poverty. This question was still being debated in the middle of the 14th century. See, e.g., [Kil90, pp. xxxi–xxxiv].

The Arts Faculty were interested in this new knowledge for its own sake, with blithe unconcern for the implications it may have for theological issues. Naturally, the same could not be said of the Theology Faculty. This tension between the theologian's approach to doctrinal issues and the philosopher's approach played a significant role in the various restrictions put on the two faculties in the thirteenth century, which we discuss further in the next section.

The influx of the new translations into the Latin west over the course of the 12th century upset the tenuous balance between philosophy and theology which had been in place ever since the early church fathers had "grudgingly [come] to tolerate [pagan philosophy and science] as handmaidens to theology" [Gr79, p. 211]. When Pope Gregory IX in 1228 required that members of the Faculty of Theology abstain from Aristotle's natural philosophy, he was seeking "to preserve the traditional relationship between theology and philosophy, with the latter serving as handmaiden to the former" [Gr96, p. 73]. It was difficult to relegate the new Aristotelian logic and philosophy to the position of mere handmaiden, "as abortive attempts to ban and then expurgate the texts of Aristotle in the first half of the thirteenth century at Paris bear witness" [Gr79, p. 211]. We discuss these bans and attempted expurgations in the next section.

2.2.2 Previous condemnations and strictures

The time from the introduction of the new Aristotelian material into the university curricula to the first condemnation of Aristotelian theses by the Church was brief. All of *De generatione et corruptione*, *De sensu*, *De caelo*, *Physica*, and books I–IV of the *Meteorologica* were translated by the end of the 12th century [Dod82, p. 47], and in 1210 the provincial synod of Sens, headed by Archbishop Peter of Corbeil and which included the bishop of Paris, proclaimed that "*nec libri Aristotelis de naturali philosophia nec commenta legantur Parisius publice vel secreto, et hoc sub penae xcommunicationis* [sic] *inhibemus*" [CUP, I, p. 70].[35] This prohibition applied only in Paris and only to the Arts Faculty [Wip95a, p. 233]. According to Wippel, Grabmann "suggested that the Faculty of Theology may have been consulted and had some influence in the measures taken in 1210 and 1215" [Wip77, p. 173]. Whether this is the case or not, the fact that the prohibition did not extend to the Faculty of Theology meant that the theologians were free to continue reading and teaching Aristotle (though while the former happened, the latter never occurred much in the Theology Faculty); this changed in 1228, when Pope Gregory IX "ordered the theological masters at Paris to exclude natural philosophy from their theology" [Gr96, p. 73].

The 1210 prohibition of Sens was reaffirmed, by the papal legate Cardinal Robert of Courçon, in 1215 specifically for the University of Paris, and in 1245

[35] "Neither the books of Aristotle on natural philosophy nor their commentaries are to be read at Paris in public or in secret, and this we forbid under penalty of excommunication" [Thor44, p. 26].

Pope Innocent IV extended the ban to cover the University of Toulouse.[36] However, while the prohibition of Sens persisted for another forty years [Gr96, p. 71], it grew steadily weaker, such that already by the 1230s it must hardly have had much affect. It was during this time that the English scholars Robert Grosseteste and Roger Bacon were studying at Paris; were the prohibition against Aristotle still strongly in force, they would likely have remained in Oxford, where no such prohibition existed.[37] It is clear that a blanket prohibition of Aristotle would not have been followed. But from an early date, it was also understood that such a prohibition might not be necessary. On 23 April 1231, Pope Gregory IX wrote to Master W. (that is, William of Auxerre), archdeacon of Beauvais; Simon of Alteis, canon of Amiens; and Stephen of Provins, canon of Reims, saying that as he had learned that

> *libri naturalium, qui Parisius in Concilio provinciali fuere prohibiti, quedam utilia et inutilia continere dicantur, ne utile per inutile vitietur, discretioni vestre, de qua plenam in Domino fiduciam obtinemus, per apostolica scripta sub obtestatione divini judicii firmiter precipiendo mandamus, quatinus libros ipsos examinantes sicut convenit subtiliter et prudenter, que ibi erronea seu scandali vel offendiculi legentibus inveneritis illativa, penitus resecetis ut que sunt suspecta remotis incunctanter ac inoffense in reliquis studeatur* [CUP, I, pp. 143–144].[38]

Ultimately, nothing came of this charge; the proceedings of the three masters were presumably halted by the death of William of Auxerre in 1231, and never taken back up [Gr74, p. 43]. But even without the help of an expurgation, the corpus of Aristotelian natural philosophy continued to infiltrate the university of Paris; we see evidence of this in the Statutes of 1255, where the *Physics*, *Metaphysics*, *De animalibus*, *De caelo* (referred to by the title *De celo et mundo*), *De anima*, the *Meteorology*, and others were all included in a teaching schedule distributed throughout the Arts Faculty, indicating by which feast days lectures on which books should be concluded; this mean that all the known works of Aristotle were listed as required reading for members of the Arts Faculty.[39]

Signs that the academic tensions in Paris were coming closer to a breaking point can be seen clearly from 1270 on. In that year, Giles of Lessines, a young

[36] [Gr96, p. 70]; [Wip03, p. 66].

[37] [Gr74, p. 42]; [Gr96, p. 71]; [Wip77, p. 173]; [Wip95a, p. 234].

[38] "The books on nature which were prohibited at Paris in provincial council are said to contain both useful and useless matter, lest the useful be vitiated by the useless, we command to your discretion, in which we have full faith in the Lord, firmly bidding by apostolic writings under solemn adjuration of divine judgment, that, examining the same books as is convenient subtly and prudently, you entirely exclude what you shall find there erroneous or likely to give scandal or offense to readers, so that, what are suspect being removed, the rest may be studied without delay and without offense" [Thor44, pp. 39–40].

[39] [Gr74, pp. 43–44]; [CUP, I, pp. 486–87, pp. 277–278].

Dominican, sent to Albert the Great a list of fifteen propositions that "were being taught in the schools of Paris by men of reputation in philosophy" [Cou94, p. 191].[40] Tempier wielded his episcopal power, in a shadow of what was to come nearly twenty-fold just seven years later, by condemning thirteen of these theses and excommunicating anyone who "shall have taught or asserted them knowingly."[41] The content of these theses will be discussed briefly in §2.3.

The next event of interest, as we trace the history leading up to the 1277 condemnations, was not itself a condemnation. On 1 April 1272, the "masters of logical science or professors of natural science at Paris, each and all spontaneously agreed" [Thor44, p. 85] that

> *nullus magister vel bachellarius nostre facultatis aliquam questionem pure theologicam, utpote de Trinitate et Incarnatione sicque de consimilibus omnibus, determinare seu etiam disputare presumat, tanquam sibi determinatos limites transgrediens... Superaddentes iterum quod si magister vel bachellarius aliquis nostre facultatis passus aliquos difficiles vel aliquas questiones legat vel disputeet, que fidem videantur dissolvere, aliquatenus videatur; rationes autem seu textum, si que contra fidem, dissolvat vel etiam falsas simpliciter et erroneas totaliter esse concedat* [CUP, I, p. 499].[42]

Though this oath remained in effect until the fifteenth century [Gr96, p. 76], one cannot help but wonder, when reading these strong, definitive terms, just how spontaneous the agreement was. Were the threat of retaliation from Church authorities not already hanging over the Arts Faculty, even if such a threat was unspoken and not formal, no formal commitment to a restriction of their enterprise should have been needed.

This oath was followed up on 2 September 1276 by a decree applying to the entire university "which prohibited teaching in secret or in private places, with the exception of logic and grammar", and that all other lessons needed to be in public [Wip77, pp. 185–86].[43] Such oaths were not restricted to the Arts Faculty, however. During this period, teachers in the Theology Faculty were also required

[40] For more on Albert the Great's role in the university of Paris in the second half of the 13th century, see [Wip95b, §2].

[41] The proclamation was issued "on the Wednesday following the feast of blessed Nicholas"; if blessed Nicholas is the popular Saint Nicholas of the 6 December feastday, that puts the following Wednesday on 10 December [WipWo69, p. 366]. Many of the propositions condemned in 1270 had already been denounced by Bonaventure in 1267 and 1268 [Wip77, p. 180].

[42] "No master or bachelor of our faculty should presume to determine or even to dispute any theological question, as concerning the Trinity and incarnation and similar matters, since this would be transgressing the limits assigned him... adding further that, if any master or bachelor of our faculty reads or disputes any difficult passages or any questions which seem to undermine the faith, he shall refute the arguments or text as far as they are against the faith or concede that they are absolutely false and entirely erroneous" [Thor44, pp. 85–86]

[43] See [CUP, I, 539].

to swear "not to teach anything in favor of articles that had been condemned at the Roman Curia or in Paris" [Thi97b, p. 97].

These restrictions imposed by the Church were not specific to Aristotelian natural philosophy. Throughout the 13th and 14th centuries the Church was heavily involved with academic life in Paris, through the condemnation of theses and the trials of academics on the charge of heresy. As Thijssen notes, "Tempier's [1277] condemnation is only one of the approximately sixteen lists of censured theses that were issued at the University of Paris during the thirteenth and fourteenth centuries" [Thi97a, p. 85]. One curious point to note is that "with few exceptions, all cases of academic condemnations between 1200 and 1378 concerned masters or bachelors of theology" [Thi95, p. 218]. The focus on the members of the Arts Faculty in the various ecclesiastical prohibitions and restrictions on Aristotle's books on natural philosophy is a symptom of how dangerous the church felt this new material was: No longer were errors in theology only being found among those studying theology as their primary function, but also among those whose focus was secular.

Such was the state of affairs in Paris at the time of Tempier's condemnation. In Oxford in the early part of the thirteenth century, there were no similar bans against the dissemination of the Arabic and newly discovered Greek philosophies. As a result, by the 1240s the study of these texts was firmly in place in Oxford, with such scholars as Roger Bacon taking this information over to Paris [Lef68, p. 272]. However, the proximity of the two universities, both geographically and academically, and the exchange of students and teachers between them meant that Oxford was by no means wholly isolated from the trials faced by Paris.

2.3 The propositions

The propositions in Tempier's list are not arranged in any systematic fashion, a fact which many people take as evidence for his quick action upon receipt of the papal letter (See §2.1.1). Mandonnet in [Man08] reordered the propositions by subject, and renumbered them accordingly; the first division is between errors in philosophy (179) and errors in theology (40). The errors in theology are divided into those on the Christian law (five), on the dogmas of the Church (15), on the Christian virtues (13), and on the last ends (7). Those in philosophy are divided into those on the nature of philosophy (7), on the knowability of God (3), on the nature of God (2), on divine science or knowledge (3), on divine will and power (11), on the causation of the world (6), on the nature of intelligences (23), on the function of the intelligences (8), on the heavens and on the generation of lower beings (19), on the eternity of the world (10), on the necessity and contingency of things (15), on the principles of material things (5), on man and the agent intellect (27), on the operations of the human intellect (10), on the human will (20), and on ethics or moral matters (10). Of course, such divisions are subjective,

and there are propositions which Mandonnet places in one category which could easily have gone into another: No great significance should be read into these categories. We use the numbering system found in the *Chartularium*, not Mandonnet's renumbering; the English translations in [FoO'N63] use Mandonnet's renumbering.

Not one of these 219 propositions refers to what modern logicians, strictly speaking, would call logic. There is no mention of validity, truth conditions, or even more generally about reason or argumentation. However some of the propositions are connected to logic in a more broad sense, in that they concern the nature and the scope of philosophy and philosophical method:

37 *Quod nichil est credendum, nisi per se notum, vel ex per se notis possit declarari.*[44]

40 *Quod non est excellentior status, quam vacare philosophie.*[45]

145 *Quod nulla questio est disputabilis per rationem, quam philosophus non debeat, disputare et determinare, quia rationes accipiuntur a rebus. Philosophia autem omnes res habet considerare secundum diversas sui partes.*[46]

151 *Quod ad hoc, quod homo habeat aliquam certitudinem alicujus conclusionis oportet, quod sit fundatus super principia per se nota. — Error, quia generaliter tam de certitudine. apprehensionis quam adhesionis loquitur.*[47]

154 *Quod sapientes mundi sunt philosophi tantum.*[48]

These should be seen not so much as attacks on the application of logical methods to theological problems but as condemning positions which do not allow for the acquiring of knowledge through supernatural revelation or in which the members of the Arts Faculty might be appearing to overstep the bounds that they had voluntarily agreed to six years earlier. So long as these two issues are respected, one could continue to apply philosophical methods such as logic and dialectic to theological problem without falling afoul of Tempier's condemnation. Such practice throughout the 14th century shows that whatever restriction on the application of philosophical method the condemnation had, it was minor and short-lived.

[44] "That one should not hold anything unless it is self-evident or can be manifested from self-evident principles" [FoO'N63, no. 4].

[45] "That there is no more excellent state than to study philosophy" [FoO'N63, no. 1].

[46] "That there is no rationally disputable question that the philosopher ought not to dispute and determine, because reasons are derived from things. It belongs to philosophy under one or another of its parts to consider all things" [FoO'N63, no. 6].

[47] "That in order to have some certitude about any conclusion, man must base himself on self-evident principles.—The statement is erroneous because it refers in a general way both to the certitude of apprehension and to that of adherence" [FoO'N63, no. 3].

[48] "that the only wise men in the world are the philosophers" [FoO'N63, no. 2].

On the other hand, there are a large number of propositions concerning the nature of time and necessity, the condemnation of which affects the types of modal or temporal logic which can be adopted without being heretical. A number of the propositions require the denial that the world is eternal. But one proposition says more than that—one must deny not only that the world is eternal, but also that time is eternal in both directions:

4 *Quod nichil est eternum a parte finis, quod non sit eternum a parte principii.*[49]

87 *Quod mundus est eternus, quantum ad omnes species in eo contentas; et, quod tempus est eternum, et motus, et materia, et agens, et suscipines; et quia est a potentia Dei infinita, et impossibile est innovationem esse in effectu sine innovatione in causa.*[50]

89 *Quod impossibile est solvere rationes philosophi de eternitate mundi, nisi dicamus, quod voluntas primi implicat incompossibilia.*[51]

99 *Quod mundus, licet sit factus de nichilo, non tamen est factus de novo; et quamvis de non esse exierit in esse, tamen non esse non precissit esse duratione, sed natura tantum.*[52]

101 *Quod infinite precesserunt revolutiones celi, quas non fuit impossibile comprehendi a prima causa, sed ab intellectu creato.*[53]

200 *Quod evum et tempus nichil sunt in re, sed solum in apprehensione.*[54]

Proposition 87 is closely connected with the one proposition of possible logical interest in Tempier's 1270 condemnation, namely Proposition 5 which condemns the view "that the world is eternal" [WipWo69, p. 366].

[49] "That nothing is eternal from the standpoint of its end that is not eternal from the standpoint of its beginning" [FoO'N63, no. 87].

[50] "That the world is eternal as regards all the species contained in it, and that time, motion, matter, agent, and receiver are eternal, because the world comes from the infinite power of God and it is impossible that there be something new in the effect without there being something new in the cause" [FoO'N63, no. 85].

[51] "that is is impossible to refute the arguments of the Philosopher concerning the eternity of the world unless we say that the will of the first being embraces incompatibles" [FoO'N63, no. 89].

[52] "That the world, although it was made from nothing, was not newly-made, and, although it passed from nonbeing to being, the non-being did not precede being in duration but only in nature" [FoO'N63, no. 83].

[53] "That there has already been an infinite number of revolutions of the heaven, which it is impossible for the created intellect but not for the first cause to comprehend" [FoO'N63, no. 91].

[54] "That eternity and time have no existence in reality but only in the mind" [FoO'N63, no. 86].

Grant in [Gr96, p. 73] picks out three major controversies which are illustrated in the Paris condemnations. Two of these controversies could bear on the nature of modal or temporal logic: (1) the eternity of the world and (2) God's absolute power.[55] The first affects the nature of time, and the second is connected to the nature of possibility and necessity. Concerning the first, the idea of an eternal world was "regarded as potentially dangerous" says Grant, noting that "27 of the 219 articles condemned in 1277 (more than ten percent) were devoted to its denunciation" [Gr96, p. 74]. Concerning the second, "theological authorities wanted everyone to concede that God could do anything whatever short of a logical contradiction" [Gr96, pp. 78–79], thus stressing logical possibility, rather than temporal or physical possibility, as the preeminent type of possibility.

As we discussed earlier, in many cases it is extremely difficult to find examples of texts where any of these 219 propositions are explicitly endorsed.[56] With respect to the absolute eternity of the world, denying the creation of the world in time by God, "no one has yet been identified who held this heretical opinion without qualification" [Gr96, p. 75].[57] What we find, instead, is this opinion held with qualification. One person who held such a qualified view was Boëthius of Dacia. Boëthius argued not that the world did not come into being, but that its coming into being could not be demonstrated by philosophical methods.[58]

The denial of the eternality of the world raises many questions, such as whether this forces time also to be not eternal? If time is not eternal, was it created when the world was created? The view that time is not eternal, but was created simultaneously with the world is that of William of Conches, who wrote in the second quarter of the 12th century that "Aristotle held that the world did not begin 'ever', that is, in time, but rather with time, as Augustine had taught" [Dal84, p. 170].[59] Dales notes that this view was extremely influential in the 13th century, and shows how the Aristotelian view of time could be brought into harmony with the more traditional Augustinian view. We discuss these issues further in §2.5.

[55] For medieval views on the eternity of the world, see [DalAr91].

[56] Donati provides an example of an author who appears to be explicitly endorsing opinions condemned by Tempier. In [Don98] she discusses an anonymous text which appears to date from around the time of the condemnations. She says that "our author maintains that the first cause cannot produce temporal effects immediately" and that "this doctrine contradicts the Christian doctrine of the possibility of immediate divine causality even in the realm of temporal effects and was condemned in 1277" [Don98, p. 374]; the relevant articles are 43, 54, 61, and 93. But it is not clear to what extent these opinions have a bearing on the nature of time or necessity.

[57] It is tempting to draw the conclusion that this fact is an explanation why Tempier did not name anyone explicitly in his condemnation. However, because of the number of manuscript sources which have not yet been seriously investigated or which have been lost in the intervening time, such a conclusion is unwarranted.

[58] "The eternity of the world, however, is no more demonstrable than is its creation" [Gr96, p. 75].

[59] See also [Dal84, fn. 7].

1	*Quod contraria simul possunt esse vera in aliqua materia.*
2	*Item quod sillogismus peccans in materia non est sillogismus.*
3	*Item quod non est suppositio in propositione magis pro supposito quam pro significato, et ideo idem est diecere, cujuslibet hominis asinus currit, et asinus cujuslibet hominis currit.*
4	*Item quod animal est omnis homo.*
5	*Item quod signum no distribuit subjectum in comparatione ad predicatum.*
6	*Item quod veritas cum necessitate tantum est cum constancia subjecti.*
7	*Item quod non est ponere demonstracionem sine rebus entibus.*
8	*Item quod omnis propositio de futuro vera est necessaria.*
9	*Item quod terminus cum verbo de presenti distribuitur pro omnibus differentiis temporum.*
10	*Item quod ex negativa de predicato finito sequitur affirmativa de predicato infinito sine constancia subjecti.*

Table 2.1: Propositions concerning logic in the Oxford prohibition [Wils93].

In terms of logical matters, the Oxford prohibition is significantly more interesting.[60] Kilwardby's list contained ten logical propositions whose teaching was prohibited [CUP, I, 558–60]; these ten propositions are listed in Table 2.1. Three are connected with issues in time and modality:

6 *Item quod veritas cum necessitate tantum est cum constancia subjecti.*[61]

8 *Item quod omnis propositio de futuro vera est necessaria.*[62]

9 *Item quod terminus cum verbo de presenti distribuitur pro omnibus differentiis temporum.*[63]

Proposition 8 is the problem of the truth value of future contingent statements, a problem which would have been familiar to 13th-century readers of Aristotelian philosophy because of Aristotle's discussion of whether there will be a sea battle tomorrow in *De interpretatione* 9. The problems that assigning truth values to contingent sentences about the future raises in connection with free will and determinism of action are of crucial importance to the Christian philosopher as a

[60] Curiously, no form of the Thomistic proposition about the possibility of an eternal world (nos. 87, 89 in the Paris condemnation) appears in Kilwardby's prohibition. See [Wils74, p. 126].

[61] "That necessary truth depends on persistence of the subject" [Lew81, p. 245].

[62] "That every true proposition concerning the future is necessary."

[63] "That a term with a verb in the present is distributed for all differences of time" [Lew81, p. 250].

deterministic position on action may be impossible to reconcile with responsibility for one's actions.[64]

Now that we are familiar with the propositions in both the Paris and the Oxford actions which could be relevant to the development of modal and temporal logic in the next centuries, we can turn to a discussion of the aftermath of the condemnation and prohibition.

2.4 After the condemnation

In §2.1.1 we raised some questions about the longer-term effects that the condemnations had: Was there any long-term effect, or did the furor die down by the middle of the 14th century? Was this a signal of the growing strength of the stranglehold the Church had upon science and scientific learning? Or was this the birth of modern science, as Pierre Duhem has claimed? There is no doubt that one of the effects of Tempier's condemnation is that the already eroding relationship between theology and philosophy, specifically natural philosophy, was further damaged.[65] What is less clear is how much the condemnation affected *logic* specifically, and that is our concern for the rest of the chapter. Before investigating in §2.5 the extent of the effects the two actions had on logic, we first sketch some of the historical facts in the centuries following 1277.

2.4.1 Oxford

The prohibition in Oxford drew immediate protest from the Dominicans. In the Dominican chapter of 1278, two lectors from Montpellier, Raymond Mevouillon and John Vigoroux, were appointed to make an investigation of Kilwardby's prohibition and were given the power to remove from office anyone who cast aspersive light on Aquinas's works [Wei84, p. 468]. However, they were not able to touch the instigator of the prohibition. On 12 March 1278, probably at the instigation of Peter of Conflans, Pope Nicholas III promoted Kilwardby as cardinal bishop of Porto, and as a result he had to resign as archbishop of Canterbury, which he did on 5 June 1278.[66] On 28 January 1279, Nicholas appointed Franciscan John Pecham in Kilwardby's place, and Pecham was consecrated on 19 February 1279.[67]

Five years later, on 29 October 1284, Pecham summoned the masters of Oxford to the abbey of Osney, where he both renewed and strengthened Kilwardby's

[64] A discussion, from a formal point of view, of the problem of future contingents as they connect to free will, determinism, and the foreknowledge of God can be found in [Fo08].

[65] This is discussed by Grant in, e.g., [Gr74, Gr79, Gr96]. Also, Thijssen notes that "[i]n the historiography of science, Tempier's condemnation has generally been perceived as a symptom of the conflict between science and theology" [Thi97a, p. 85].

[66] [Cr50, p. 248]; [Kno42, p. 184], [Wils74, p. 128].

[67] [Cal55, p. 16]; [Cr50, p. 249]; [Kno42, p. 178].

prohibition of 1277.[68] The thirty propositions were no longer just prohibited, but also condemned [Ir01, p. 256]. However, he tempered this in a letter dated 10 November 1284, where he asked the chancellor of the university to investigate the condemned propositions and those propagating them, with a view towards reexamining them.[69] Pecham said further that he would consider whether all thirty propositions still needed to be condemned, or whether perhaps they could start being taught again in the schools. Until such time as he made a decision, however, the penalties for propagating these theses were to remain in place as before.[70]

However, 1284 appears to be the last moment of real controversy on these issues in Oxford. Just two years later, in 1286, the Dominican Chapter accepted as orthodox the full works and views of Aquinas, one of their most preeminent members.[71] After this, "the [Oxford] prohibitions were generally ignored" [Law84, p. 117]. That whatever effect the prohibition may have had was mitigated with the passage of time is evidenced by the fact that in Lyte's 19th-century history of the university of Oxford, Tempier's actions in Paris and Kilwardby's and Pecham's in Oxford are mentioned only obliquely, in a passing reference to the condemnation of two of the views of Thomas Aquinas [Ly86, p. 114].

2.4.2 Paris

Though the sources of the propositions in the Oxford and Paris actions were never definitively identified, the people the condemnations were generally thought to be directed against (Siger of Brabant, Boëthius of Dacia, Richard Knapwell, etc.) all were strongly affected by the condemnation and prohibition.[72] As we noted earlier (fn. 25), some historians today believe that Tempier's condemnation was like Kilwardby's, that is, a prohibition to teach and not really a condemnation. Regardless of whether it was, Tempier's action "was certainly interpreted as [a condemnation] by late-medieval philosophers and theologians... In fact, authors such as Richard of Middleton, Scotus, Bradwardine, Gerson, and many others referred to the theses proscribed in 1277 as articuli *damnati*, *condamnati*, or even

[68] [Law84, p. 118]; [Wils97, p. 174]. For a full discussion of Pecham's visitation, see [Dou52].

[69] [Wils93, pp. 115–6]. This letter was partly motivated by a complaint made by the Dominicans at Oxford against Pecham's action at Osney [Wils97, p. 174].

[70] [Cal55, p. 16]. On one proposition Pecham stood firm: "one particular thesis should be most strictly proscribed, namely, the execrable error of those who posited one single form in man". A week earlier when Pecham met with William of Hothum, the Dominican Provincial, Pecham told William that he intended to "renew Kilwardby's condemnation of Thomist teaching on unity of form" in his visitation a week later [Cr50, p. 251].

[71] See [Ben37, p. 70]. Even so, Kilwardby's condemnation of the unity of the substantial form "was to be remembered, among others, by Richard FitzRalph and Adam of Woodham in their debates in the early 1330s" [Lef68, p. 292].

[72] "Siger of Brabant, threatened by the inquisition, never returned to teaching. Boëthius of Dacia disappeared from the academic scene. Richard Knapwell lost his appeal at the papal curia" [Wils97, p. 178].

excommunicati" [Bia98, pp. 95–96]. Though the legal force of the condemnation was only in Paris, "its influence occasionally spread to England where eminent English scholastics found occasion to cite one or more of the articles as if relevant to, and authoritative in, England" [Gr79, p. 214]; two examples of these eminent scholastics that he gives are John Duns Scotus and William of Ockham, and Murdoch mentions *pro forma* references to condemned articles by Ockham and Peter Ceffons [Mu98, p. 114]. Another example is the English theologian Walter Catton. In his commentary on the sentences (c. 1322–23), he mentions the Paris condemnation in the prologue of his second question:

> *Secundo sic: Si visio requiretur ad illam notitiam abstractivam causandam, hoc esset in aliquo genere causæ; non materialis, formalis, finalis certum est; ergo in genere causæ efficientis. Sed omnem efficientiam alterius potest Deus supplere. Oppositum istius est articulus condemnatus in Parisius. Ergo istam notitiam potest Deus causare visione ista circumscripta* [O'C55, p. 235].[73]

But even though the members of the Arts Faculty probably respected the Parisian condemnations and did not "willfully repudiate its separate articles" [Gr79, p. 213, fn. 5], there were efforts on the part of members of the Theology Faculty to reduce the scope and effect of Tempier's condemnation. Only twenty years later, Godfrey of Fontaines, a member of the Theology Faculty at Paris, noted in his *Quodlibet* XII of 1297/98 that "at the University of Paris, the Paris Condemnations of 1277 were ignored completely or interpreted in a way entirely contrary to the intentions of their framer" [Wils97, p. 178]. He went so far as to publicly defend the tenets of Thomas Aquinas, and to "conclude that the then Bishop of Paris should at least suspend the condemnation of those propositions which appeared to have been taught by Thomas" [Wip95b, p. 19].

This was done in 1325, when Tempier's successor as bishop of Paris, Stephen de Bourret, proclaimed that the 1277 condemnation "had no canonical value" with respect to any censured Thomistic proposition. This revocation of censure on the Thomistic thesis of the unity of the form was reaffirmed by Pope Pius X, who in 1914 said that "the thesis, proscribed at Oxford in 1277 and 1284–6, is one of the most fundamental tenets in the Thomist synthesis" [Cal55, p. 34].[74] Despite this revocation, "the condemnation was generally effective at Paris throughout the fourteenth century" [Gr79, pp. 213–214], and over a century later, the Dominican John of Naples "found it necessary to write an apology to the effect that Aquinas's views were not affected by Tempier's condemnation, and that, consequently, it was

[73] "The second thus: If vision is required for that abstract appearance to be caused, that [cause] is in some genus of cause: it is certainly not material, formal, [or] final; therefore it is in the genus of efficient cause. But God is able to supply [a cause] different from all efficient [causes]. The opposite of this is an article condemned at Paris. Therefore that God is able to cause this appearance by vision is ruled out."

[74] See also [Thi97b, p. 88]; [Wip95a, p. 239]; [Wils97, p. 177].

legitimate to teach Aquinas's works at Paris without danger of excommunication" [Thi97b, p. 88].

From this, we can to a large extent agree with Knowles when he says that

> [t]he Paris and Oxford condemnations of 1277, like similar events in the history of ideas before and since, had had precisely the opposite effect to that intended by the two prelates concerned. Instead of strangling the infant Thomism in the cradle, they had brought about a new solidarity among all its adherents [Kno42, p. 186].

Nevertheless, as Grant shows in [Gr79], natural philosophers throughout the 14th century frequently cited or make implicit reference to propositions condemned by Tempier (he argues that these references "should convince us that it was taken seriously throughout the fourteenth century and that it encouraged innumerable invocations of God's absolute power in a variety of hypothetical physical situations" [p. 239]). The question still remains, however, what effects, if any, that these condemnations had on the narrower fields of modal and temporal logic in the 14th century. We attempt to answer this in the next section.

2.5 Modal and temporal logic in the 14th century

The two topics covered in Tempier's condemnation and Kilwardby's prohibition which are most relevant to modal and temporal logic are, as we noted in §2.3, the questions of the nature of necessity and of the eternality of the world and of time. There is evidence that William of Ockham knew of the Condemnations, but probably did not know the articles themselves[75], and we know that two of the most influential 14th-century logicians were familiar with Tempier's condemnation: the French logician and member of the Arts Faculty John Buridan, and English logician and master at Oxford Thomas Bradwardine. In his treatises on motion written in the 1340s and 50s, Buridan mentions the 1277 condemnation, and Grant says that he "not only upheld them, albeit reluctantly, but occasionally used them to advantage" [Gr79, p. 232; p. 213, fn. 5]. Buridan also referenced condemned articles in his questions on *De caelo*, on *Physica*, and on *Ethica* [Mah01, p. 921]. Bradwardine referred to proposition 52, *quod id, quod de se determinatur ut Deus, vel semper agit, vel numquam; et, quod multa sunt eterna*, when discussing the eternality of the world [Gr79, p. 237], and there are many references to various articles in his *De causa Dei* [Mah01, p. 923]. In the

[75] Mahoney notes that "[w]hether or not Ockham's thought was influenced by the Condemnation, it remains that he did indeed allude to it in his 'Dialogus'. However, in the text... he does not reveal knowledge of much of its content nor does he appear to be wholly accurate as to what he says regarding Thomas and the Condemnation" [Mah01, p. 920].

following century, the issue of the bounds of the Arts Faculty arises again, with references to the 1277 condemnation; Mahoney says that "[i]n his 'Contra curiositatem studentium', John Gerson (†1429) deplores the fact that philosophers have not stayed within proper limits, since they have raised unanswerable questions about the beginning of the world" [Mah01, p. 923].

But though we have clear evidence that some of the most prominent logicians in the 14th century were familiar with the condemned articles, there is little evidence that they ever referred to them in the context of pure logic. Perhaps the most telling evidence is Mahoney's article, the goal of which is simply to chronicle references to the Condemnation found in later philosophers, that is, from the late thirteenth to the late sixteenth century" [Mah01, p. 902]. Not a single reference that he discusses comes from a logical treatise. Why is this?

2.5.1 Modal logic

Grant argues that one major consequence of the Paris condemnation was to "manifest and emphasize the absolute power of God" [Gr79, p. 214]. The only necessity by which God can be bound is logical or conceptual necessity. Contemporary philosophers of logic who are interested in modal logic often focus on the question of "what is the [correct] nature/conception of necessity and how can we model it?" From this point of view, one could look at the 1277 condemnation in Paris as an attempt to answer this question, namely that the correct conception of necessity, at least insofar as God is concerned, is logical necessity, and so any logic which expresses a more restricted notion of necessity may be a useful tool, but will not, ultimately, be a guide to truth. If a contemporary philosopher was presented with Tempier's condemnation, then a natural reaction would be to say[76]

> The most important type of necessity is logical necessity, and as this is modeled in the modal logic S5, this means that S5 is the correct logic of necessity. Any other modal logic may be a useful heuristic, but it is not the *real* logic of necessity.

But this is a very modern reaction. Medieval logicians working in modal logic generally took as a starting point the relationships of implication between modal sentences and moved next to the validity of modal syllogisms, following Aristotle. Questions about the nature of necessity or even the truth conditions for individual modal sentences were often completely disregarded (we discuss this in more detail in §4.3 of Chapter 4). The task of finding modal axioms which correspond to different conceptions of necessity and possibility was simply not one in which medieval logicians were involved.

[76]Such a reaction would presumably be espoused by, e.g., R. Kane, who says that "a case can be made for saying that $S5$ expresses our intuitive idea of logical possibility in the broadest, unconditional sense" [Kan84, p. 342].

There is one 14th-century logician who did take seriously the idea of possibility as logical non-contradictoriness: John Duns Scotus. His theory of modality is rooted in the intellect of God: Anything which can be understood or conceived receives its being as intelligible or understandable in the intellect of God. These intelligible beings make up the various possibilities. Some intelligible beings are not compossible with others; compossibility, understood as logical compossibility, partitions the conceptual space into sets of beings all of which are compossible. One of these sets God actualized, and the others though unactualized are possible. (In this, Duns Scotus is rejecting the view often attributed to Aristotle that something is possible only if it is actualized at some point.) These unactualized sets of possible intelligibles and their actions are seen by many to be conceptual correspondents with the modern idea of using possible worlds to evaluate modal propositions.[77] However, while Duns Scotus was undoubtedly familiar with the condemned propositions, I have found no indication that he ever cited them as grounds for developing the modal theory that he did.

2.5.2 Temporal logic

We noted in §2.3 that a number of the propositions condemned by Tempier dealt with the nature of time, e.g., whether the world is eternal, whether time is created, etc. The situation of modern philosophers working in modal logic that we presented in the previous subsection is also true for those working in temporal logic. For a modern-day temporal logician, restrictions on the nature of time have an immediate effect on the logic. This is most clearly seen in the early literature in temporal logic, when correspondence results were the primary focus and a number of different axioms were proved to correspond to certain properties of time, such as density, linearity, backwards linearity, discreteness, etc.[78] In this modern perspective, with its emphasis on representing properties of time by logical axioms, an ecclesiastical statement to the effect that time had a beginning would mean that the only theologically sound temporal logics are those which have no infinitely descending chains of temporal points.

But, as with modal logic, this was not the mindset of the medieval logician. For the most part, there was no separate branch of medieval logic which could be called temporal logic, strictly speaking. Instead, as we discuss in further detail in Chapter 5, questions of tense were dealt with by theories of supposition. In a present-tensed proposition, the supposition of a term would be a subset of presently existing objects, but in a past- or future-tensed proposition, the supposition of a term was said to be *ampliated* by the verb, that is, to have its range of application extend to both presently existing objects and past or future objects. I argue in Chapter 5 that the way that the tense of a verb affects the supposition

[77] For further information on Duns Scotus's modal theory, see §4 and the bibliography of [Knu03].

[78] See, e.g., any work on temporal logic by Arthur Prior, such as [Pr67].

of a term by ampliation is best understood, from the point of view of modern logic, as a type of quantified temporal logic. This means that the proposition with the most relevance to temporal logic is not that in Kilwardby's prohibition about the necessity of future contingents, but rather Proposition 9, *quod terminus cum verbo de presenti distribuitur pro omnibus differentiis temporum.* Lewry provides a short survey of early 14th-century Oxford texts which address this proposition:

> The author of the questions on the *Prior Analytics* preserved by John Aston maintained that a verb, such as *currit*, had from its imposition a form such that whenever and by whomsoever it was uttered it would refer to the present of the utterance. [John] Stykborn said, at the end of his questions on the *Praedicamenta*, that if the reference were to the present instant, then when that became past, the time-reference would be to the past, so there must be a reference to a common time, which could be realized in many instants successively rather than confusedly for many at once. For these authors the admission of a common time, rather than a confused time, was a way of respecting the contextually determined reference of tensed verbs [Lew84, p. 424].[79]

The question which is raised by this proposition is related to the idea, found in Kilwardby's commentary on the *Perihermeneias* and based on views of Augustine, that every tensed statement can be rephrased as a present tense statement [Lew81, p. 250]. For example, the statement *Caesar fuit* is equivalent to *Caeser est praeteritus*, and *Antichristus erit* is equivalent to *Antichristus est futurus* (and in this way the connection with Proposition 8, on the truth value of future contingents, becomes more clear).

But while this proposition and its adherents and opponents are of great logical interest, unfortunately, as Lewry notes, in this case "there is little to suggest that theology had imposed an orthodoxy upon logic" [Lew84, p. 425]. So while we have an example of a logical proposition whose teaching was prohibited on the basis that it could lead to error, which is clearly connected to issues found in medieval temporal logic, and which was discussed by logicians after 1277, it is not clear to what extent this can be seen as a case where the course of logic was *changed* because of an ecclesiastical action.

[79]The anonymous text is found in Worcester Cath. MS Q.13; Stykborn's commentary is found in Caius MS 344/540. [Lew81, fn. 141, 143] provides the relevant quotes: *Dico quod habet ab impositore quod quandocumque et a quocumque proferatur, consignificet praesens, non hoc tamen praesens vel illud, sed illud quod est vere praesens respectu prolationios. Quod dico de praesent, dico etiam de praeterito et futuro...* and *Ad quaestionem dicendum est quod hoc verbum, 'est', copulat pro tempore communi et non confuso. Ratio est ista, quia si significaret instans quod unc instat, cum illud fiet praeteritum, aliquando esset consignificatum suum praeteritum, cum tamen semper copulat pro praesenti.*

2.6 Concluding remarks

If we now return to our original question concerning the effects of the condemnations of 1277 on the development of modal and temporal logic, the perhaps puzzling answer appears to be negative. Since the goal of studying the 1277 condemnations to determine how they affected logic in the succeeding centuries seemed so natural, one is left wondering why the results are so seemingly unsatisfactory. Our discussion in §2.5 gives an explanation: For a contemporary logician working in modal and temporal logic, the nature of necessity and of time are of crucial importance, because one of the primary goals is to develop axioms which model these properties and to prove correspondence results concerning those axioms. But these were simply not the priorities of medieval logicians. Instead, their interests lay in the inferential relations between modal propositions and in the reference of terms in quantified tensed statements. We are surprised that the 1277 condemnation of Paris and the 1277 prohibition of Oxford did not have a greater affect on logic than they appear to have had because we have projected the priorities of a 21st-century logician onto the 14th-century logicians. Once we understand this difference in priorities and goals, then it is no longer so surprising that the effect the actions had on the development of natural philosophy and natural science did not spill over into logic.

Chapter 3

St. Anselm on agency and obligation

3.1 Agency as a modal notion

In Chapter 1 we said that we reject the narrow view of modality, which treats only necessity and possibility as *real* modality, and instead we should embrace the expanded interpretation 'modality', covering notions such as agency, knowledge, belief, obligation, time, modes of being, and so forth. When we consider this extended list of modalities, the period between Boëthius and Abelard is not so devoid of developments as some may believe (cf. p. 13), but instead becomes a rich and fruitful period of study because of the works of Saint Anselm, archbishop of Canterbury.

Anselm was born in Aosta, in the kingdom of Burgundy, in 1033. At the age of 27, he joined the Abbey of Bec, where he served as abbot from 1078 to 1093. In 1093, he was made archbishop of Canterbury. Anselm spent much of his monastic life teaching and writing. Part of the attraction of the Abbey of Bec was the school opened there by Lanfranc, where logic, rhetoric, and theology were taught to all comers, not just those who intended to join the church. Anselm taught in this school for many years, before beginning to put his teachings into writing, with his first work, the *Monologion*, written in 1076. Anselm continued to write until the end of his life.[1]

From a logical point of view, the most interesting material can be found in fragmentary notes which were compiled and organized by Eadmer, Anselm's friend and later biographer, shortly after Anselm's death in 1109. These notes along with letters of Anselm's, collectively called the Lambeth fragments as the primary manuscript is preserved in the Lambeth Palace, in London, were first edited in [Schm36] and then partially again in [SoSc69]. The first edition rearranges the fragments into a more conceptually coherent organization; the second retains the original arrangement made by Eadmer. The text is partially translated with de-

[1]For more information on Anselm's life and works, see [DavLe04], specifically [DavLe04a] and [Ev04].

tailed commentary in [He67] and completely translated, with little commentary, in [Hop72]. Among other topics, the Lambeth fragments contain modal analyses of certain Latin verbs, including *facere* 'to do', *velle* 'to will', and *posse* 'to be able'. It is believed that Anselm composed these parts of the Lambeth fragments while he was archbishop of Canterbury [Kin_2–, p. 1]. In these fragments, Anselm's primary focus is on *facere*, with his analyses of *velle* and *posse* being modeled on the analysis of *facere* for the most part. In this chapter, we focus on the writings which deal with concepts of action, agency, and obligation, placing them in the context of the history of logics of agency.

In his discussion of the meaning and function of the Latin verb *facere* 'to do', Anselm identifies four types of doing and further subdivides each type into six different modes. The relationships between the four types can be placed neatly into a square of opposition. According to [BelPeXu01], it is this square of opposition which "clearly indicates that he [Anselm] had in mind a modal logic of agency" [p. 19]; they note that Anselm appears to be the first person to consider the modal interpretation of agency in a rigorous fashion. This modal interpretation of agency found in Anselm shows that the idea of treating agency as a modal concept is far older than many action theorists might have thought.[2] In Chapter 1, we noted that there are two reasons why a modern logician might be interested in a historical logical theory. With respect to Anselm's theory of agency, we can ask the purely formal question of what the modal logic of Anselm's theory of agency actually is, whether it is identical with any of the standard modern agentive logics or whether Anselm's constraints result in something new. We can also ask the more philosophical question whether this historical theory has any insights to offer to modern problems and questions of agency. Our focus in this chapter is primarily the formal questions. However, in order to answer the formal questions we must consider the philosophical and theological motivations of Anselm which underpin various aspects of this theory. We hence start by giving a brief introduction to the theory, followed by consideration of the non-logical motivations in §3.2.1. After that, we turn to the details of the theory, the four types and six modes of agency, and the square of opposition in which they can be placed, in §3.2.2. §3.3 and §3.4 are devoted to considering how the theory might be formalized using modern techniques. In particular, we look at a syntax for Anselm's theory proposed by Walton in [Walt76a] and [Walt76b]. We show that because Walton did not have an adequate semantics, his syntax introduced features which are not found in Anselm's original theory, and that given an adequate semantics, namely that of neighborhood models, we can give formalizations for different interpretations of Anselm's theory. Additionally, we comment briefly on stit-theory, for which Anselm's views are often cited as inspiration. Lastly, in §3.5, we then see how certain unsystematic comments that Anselm makes connecting actions

[2]The first modern author to recognize Anselm's theory as developing a modal interpretation of agency is Henry, in [He53] and [He67].

to goodness could be used to develop interesting deontic notions which take into account both natural language usage of terms and various theologically-specific problems.

3.2 Anselm on *facere*

Anselm's discussion of *facere* consists of a dialogue between a teacher and his student. The opening statement of the teacher contains all the details of the theory in a nutshell:

> *Mag. Verbum hoc, quod est, "facere", solet poni pro omni verbo cuiuslibet significationis, finito vel infinito*[3]*, etiam pro "non facere". Cum enim quaeritur de aliquo: "quid facit?": si diligenter consideretur, ponitur ibi "facere" pro omni verbo, quod responderi potest, et quodcumque verbum respondetur, ponitur pro "facere". Non enim recte redditur ullum verbum interroganti: "quid facit?", in quo non intelligitur facere, de quo interrogatur. Nam cum respondetur: "legit aut scribit", valet idem ac si dicatur: hoc facit, scilicet legit aut scribit"* [Schm36, p. 25].[4]

[3]The terminology of 'infinite' vs. 'finite' verbs Anselm has taken from Boëthius, and should not be confused with modern linguistic use of these terms. For Anselm, an infinite verb is one which is the complement of a finite verb, i.e., one which indicates a finite action. For example, 'run' is a finite verb, and 'not run' is an infinite verb.

[4] *Teacher*: We commonly use the verb 'to do' in place of all other verbs, regardless of the signification of these other verbs and regardless of whether they are finite or infinite. In fact, 'to do' may even stand for 'not to do'. If you think about it carefully, you will see that when we ask about someone 'What (how) is he doing?' here 'doing' stands for any verb that can be given in answer. And so too, these other verbs stand for the verb "to do". For in a correct reply to one who asks "What (how) is he doing?" any verb at all will indicate a doing on the part of the person asked about. If someone were to respond, "He is reading" or "He is writing", it is the same as if he were saying, "He is doing this, namely, reading", or "He is doing that, namely, writing" [Hop72, p. 218]. Anselm goes on to write: *Potest autem omne verbum reddi sic interroganti. Et in pluribus quidem palam est, ut: cantat, dictat; in aliquibus vero fositan dubitatur, ut sunt ista, scilicet: est, vivit, potest, debet, nominatur, vocatur. Sed nemo reprehendit, si interroganti: 'quid facit?', respondetur, quia est in ecclesia, aut: vivit sicut bonus vir, aut: potest super totam civitatem, in qua habitat, aut: magnam debet pecuniam, aut nominatur super omnes vicinos suos, aut: vocatur ante omnes alios, ubicumque sit* [Schm36, p. 25] ("So then, any verb can be used in the answer. In many cases this is obvious, as for example when we reply, 'He is singing' or 'He is composing'. In other cases, however, the substitution may seem somewhat problematical, as for example when we reply, 'He is', or 'He lives', 'He is powerful', 'He owes', 'He is named', 'He is summoned'. But no one would reproach us if we were to answer someone who asked 'What (how) is so-and-so doing?' by saying, 'He is in church' or 'He is living as a good man should live', 'He is powerful (ruler) over the whole domain in which he lives', 'He owes much money', 'He is named above his neighbors', 'Wherever he is, he is summoned before all others'" [Hop72, p. 218]). It is worth noting here that both transitive and intransitive verbs are used; this casts some doubt on Dazeley's claim that "the

Perhaps the most striking feature of Anselm's theory is its breadth, namely that for Anselm, an analysis of *facere* will encompass an analysis of *all* verbs.[5] Some may object that it is *too* broad, and that either it cannot be used in particular situations or that too many things end up counting as examples of agency. The objection is essentially this: How can we expect to find a reasonable explication of what *facere* means and how it functions, if we will not be distinguishing it from any other verb? In the succeeding sections, where we present the philosophical and theological foundations which motivate Anselm's theory, we will show how Anselm is able to handle this issue, and show that any more restricted conception of agency would be untenable for him. While his scope of agency is wider than many modern theories, this very breadth of his account gives it more flexibility and allows it to be applied to cases of agency beyond the rather narrow setting of human agency.

3.2.1 Philosophical and theological motivations

We can isolate two non-logical motivations underlying Anselm's development of his theory. The first motivation can be classified as methodological. Much of Anselm's discussions of logical matters involves separating questions of logic and logical usage from questions of grammar and everyday usage, separating the *usus proprie* from the *usus non proprie* (also called *usus loquendi* and *usus communis locutionis*). This distinction is the main topic of his *De grammatico*, many themes of which are echoed in the Lambeth fragments.[6] In *De grammatico*, Anselm points out that everyday usage (*usus loquendi*) of words is often sloppy, and what we say doesn't always accurately represent what we mean. The aim of the grammarian is to explain the *usus loquendi* of terms; his goal is *descriptive*. The logician, on the other hand, has two options. He can either ignore the *usus loquendi* altogether, and make his aim strictly *prescriptive*, by focusing on the proper, logical uses of the terms involved, even when this explication seems at odds with our everyday uses of the terms. Alternatively, he can allow his logical explication to be broad enough to cover and hence to *explain* to some extent, the *usus loquendi*.

Parts of Anselm's logical works take the former route, but in his discussions on agency he always allows for taking into account the latter route. In discussing

sorts of verbs which can most readily be dealt with in Anselm's system are transitive" and intransitive verbs are "the sorts of verbs that seem to be causing the trouble" [DaGo79, p. 77]. It is also interesting to contrast this with [An70, p. 232], where Anderson indicates that a patient of some sort is a necessary condition for agency.

[5]This broad interpretation of 'to do' has also been noticed by modern philosophers, for "J.L. Austin told us that 'The beginning of sense, not to say wisdom, is to realize that "doing an action", as used in philosophy, is a highly abstract expression—it is a stand-in used in place of any (or almost any) verb with a personal subject...'" [BelPe88, p. 175]. The quote is from [Au56–57, p. 178].

[6]*De grammatico* is edited in [AoC38–61, vol. 1]; a comprehensive discussion of the text and its relation to Anselm's other works appears in [Bosc06]. See also [He60b].

facere he notes that *est et alia consideratio de verbo eodem, scilicet, quot modis usus loquendi dicat "facere"* [Schm36, pp. 28–29][7]; as Henry says, these "are to be codified so that the deviations of these uses from the proper sense become evident" [He67, p. 123]. We cannot fully understand the proper usage of a term until we understand how ordinary usage differs from proper usage. Henry says that Anselm's discussion of *facere* "is intended as a means of analyzing the senses of verbs as they occur in customary utterance (*usus loquendi*), in non-strict oblique uses as measured against the standard of their precisive or strict signification, the latter being shown by exemplifying the simplest overt meaning of the verb in question" [He60a, p. 377]. The goal, then, is to produce a logical explanation for the result of the grammarian's study of the word.

This explanatory motivation is connected to the other motivation guiding Anselm's account of agency. As Lagerlund notes, "Anselm's thinking and writing is always motivated by his interest in religion and theological problems" [Lag08, p. 318]. To ignore the *usus non proprie* is a mistake on the part of the logician: scriptural usage of terms is often improper. Since it is everyone's responsibility to seek further understanding of the scriptures, it follows that logicians should be interested in providing logical explanations for improper usage of terms. Thus, a medieval logician should be interested in providing a grounding for the improper or non-logical usage of terms, and any theory of agency which Anselm proposes needs to be able to explain why *facere* is used the way that it is in scripture. An explication of agency which does not make sense of scriptural usages of *facere* will not be adequate for Anselm, because just as *usus loquendi* is very broad, so too is scriptural use. Anselm specifies this in the Lambeth fragments: *Siquidem et dominus in evangelio ponit "facere" vel "agere"—quod idem est—pro omni verbo* [Schm36, p. 28][8], and again in *De veritate*:

> *Facere autem non solum pro eo quod proprie dicitur facere, sed pro omni verbo dominus voluit intelligere... Usus quoque communis locutionis hoc habet, ut et pati et multa alia dicat facere, quae non sunt facere* [AoC38–61, vol. 1, V, p. 182].[9]

If our logical theory of agency can provide an explanation of the *usus loquendi*, then we will also have an explanation of the theological usage of the word, because the two combine.

The desire to give an adequate account of the scriptural usage of *facere* is not

[7] "We must consider another thing about the verb 'to do', namely, the different modes in which it has a use in our language" [Hop72, p. 221].

[8] "Indeed, the Lord Himself in the Gospel uses *facere* and *agere*—which are the same—in place of every other verb" [Hop72, p. 220].

[9] "The Lord wishes to convey that 'to do' may be used not only in respect of that which is properly asserted to constitute 'doing' but also in respect of all verbs ... The ordinary use of language also has this feature, namely, it treats as 'doing' both *undergoing* and many other things which are not really cases of *doing*" [He67, pp. 182–183].

merely an idle exercise in logic and grammar. Because the concept of agency is closely connected to issues in responsibility for actions and hence culpability and sin, an explanation of the proper conditions under which agency can be ascribed will have implications for ethics as well as logic. These issues can be seen in Matthew 25:31–46, where on the day of judgment God will separate the sheep from the goats on the basis of what they did and didn't do[10], and a similar sentiment is found in *De casu diaboli*:

> *Cum enim iste dicitur quia fecit esse nudum aut non esse indutum, non aliud intelligitur nisi qui cum posset non fecit, ut non esset nudus aut ut manteret indutus* [AoC38–61, vol. 1, I, p. 234].[11]

Knowledge of correct ascriptions of agency, both in proper usage and in common usage, is hence important because it gives knowledge concerning eternal culpability.

It is important to point out that in these cases, we are dealing with active, *human* agency. This point hardly seems worth noting; modern agency theorists focus on explications of 'x does' where x is an efficient agent, and indeed many of Anselm's examples are of this type as well. But he says that *omnis tamen causa, sicut dixi, facere dicitur et omne, quod facere perhibetur, causa nominatur* [Schm36, p. 29].[12] This means that his concept of agency covers more than just human agency. For example, on this view, if I trip over a tree which has fallen across the path, then it is perfectly plausible to ask "What did the tree do?" and respond "It tripped me." Even though he generally uses cases of human agency as examples of the logical properties of the theory, this is done only for pragmatic reasons:

> *Nota 2. Haec quidem exempla, quae posui de "facere esse" et de "facere non esse", de causis efficientibus assumpsi, quoniam in his clarius apparet, quod volui ostendere. Sicut autem in efficientibus causis praedicti sex modi cognoscuntur, ita etiam in non efficientibus, si quis eos diligenter investigare voluerit, inveniuntur* [Schm36, p. 32].[13]

[10] Matthew 25:40 "The King will reply, 'I tell you the truth, whatever you did for one of the least of these brothers of mine, you did for me'," and 25:45 "He will reply, 'I tell you the truth, whatever you did not do for one of the least of these, you did not do for me'".

[11] "For when in the latter instance, someone is said to bring it about that the victim is naked, or that the victim is not clothed, the exact import is that although the person was capable of doing so, he did not bring it about that the victim was not naked, or that the victim remained clothed" [He67, p. 184].

[12] "Nonetheless, every cause (as I mentioned) is said to do something, and everything which is said to do something is called a cause" [Hop72, p. 221].

[13] "Note 2. These examples which I've given about 'causing to be' and 'causing not to be' all concerned efficient causation. I adopted these examples of efficient causation since what I wished to point out can be seen more clearly in them. but the same six modes are also found in the case of nonefficient causation, as one may discover if he cares to pursue the matter intently" [Hop72, p. 225].

This point needs to be stressed. We must be careful to separate the logical aspects of the theory from those aspects that can be called, broadly speaking, the applied aspects. For the present purposes, we are interested solely in the logical aspects of the theory, without regard to their application to discussions of human agency; as a result, any logic which is developed to address Anselm's discussion of *facere* must not turn on any specifically human (or even sentient) property. These facts only have a use when we are trying to give a full account of *human* agency, in which case an evaluation of *velle* ('to will') will also be necessary, as Serene makes clear:

> Because the analysis of *facere* is meant to apply to all instances of agency, whether or not the subject is human, rational, conscious or even an efficient cause of the outcome, it does not constitute a complete or a specific account of human action. His full theory of human agency also includes some explanation of the nature of willing [Ser83, p. 144].

We return briefly to this point at the end of §3.5.

3.2.2 The types and modes of doing

Now that we have seen some of the motivations underlying the informal statement of the theory, we can turn to the details of the account. Anselm says:

> *Quidquid autem facere dicitur, aut facit ut sit aliquid, aut facit ut non sit aliquid. Omne igitur facere dici potest aut 'facere esse' aut 'facere non esse'; quae duo sunt affirmationes contrariae. Quarum negationes sunt: 'non facere esse' et 'non facere non esse'* [Schm36, p. 29].[14]

Henry paraphrases this as: "For all x, if 'x does' is true, then x does so that something either is so or is not so. Hence the analysis of 'doing' will in fact be an analysis of x's doing so that p, and of x's doing so that not-p, [where 'p' is a clause describing a state of affairs, and 'not-p' is short for 'it is not the case that p']" [He67, p. 124]. Hence, *doing* will always result in something being or not being the case. Something can either be or not be the case because it is either caused or not caused. This gives us four types of agency:

facere esse	'to cause to be'	(A)
facere non esse	'to cause not to be'	(B)
non facere non esse	'not to cause not to be'	(C)
non facere esse	'not to cause to be'	(D)

[14] "Now, whatever is said to do (or to cause) either brings about something's being or else brings about something's not-being. Therefore, every doing can be called either (A) causing something to be or (B) causing something not to be. These are contrary affirmations, whose negations are, (C) not causing something to be, and (D) not causing something not to be" [Hop72, p. 221].

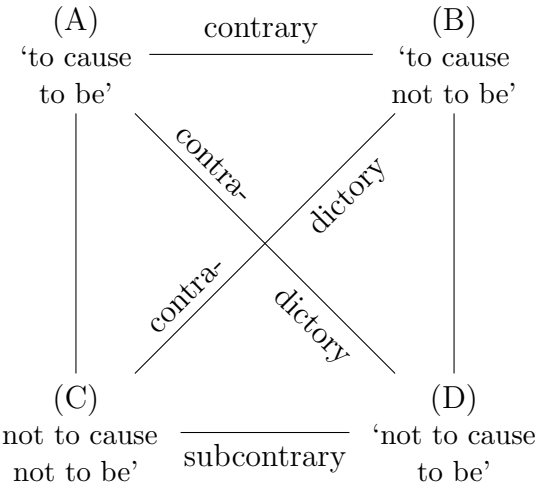

Figure 3.1: Agentive Square of Opposition

In these glosses, 'to cause [p] to be' should be understood as a shorthand for 'to do such that p is the case'; if an agent x does such that p is the case, then he causes p to be, and vice versa, so this shorthand is licit. Types (A) and (B) are called *affirmative*, and they are contraries. Types (C) and (D) are called *negative*; though he does not say so explicitly, they are also contraries. The implication relationships between these four types of agency form a square of opposition (see Figure 3.1). The graphical square itself is not present in Anselm's work, but the verbal descriptions of the relations fix the graphical square uniquely. Each of the four types of action can be further divided into six modes, each of which picks out a different way that the main type of action can be brought about. For example, of type (A) *facere esse*:

> *Sex ergo modis 'facere' pronuntiamus: duobus videlicet, cum facit idipsum esse aut non facit idipsum non esse causa, quod facere dicitur; quattuor vero, cum aut facit aut non facit aliud esse vel non esse. Dicimus namque rem quamlibet facere aliquid esse, aut quia facit idipsum esse, quod facere dicitur, aut quia non facit idipsum non esse; aut quia facit aliud esse, aut quia non facit aliud esse, aut quia facit aliud non esse, aut quia non facit aliud non esse* [Schm36, p. 29].[15]

that is,

> *Quidquid enim dicitur facere non esse aliquid, aut ideo dicitur, quia facit hoc ipsum non esse, aut quia non facit hoc ipsum esse aut quia*

[15] "We speak in six modes of 'causing to be': We say that x causes y when x causes y itself to be; or when x does not cause y itself not to be; or when x causes y to be by causing z to be, by not causing z to be, by causing z not to be, or by not causing z not to be" [Hop72, pp. 221–222].

> *facit aliud esse aut quia non facit aliud esse aut quia facit aliud non esse aut quia non facit aliud non esse* [Schm36, p. 30].[16]

Let us illustrate these six different modes with an example, 'to cause to be dead' (this is the example that Anselm uses):

Killing directly	(*facere idipsum esse*)	(A1)
Not making not dead	(*non facere idipsum non esse*)	(A2)
Making the killer have arms	(*facere aliud esse*)	(A3)
Not arming the victim	(*non facere aliud esse*)	(A4)
Making the victim not armed	(*facere aliud non esse*)	(A5)
Not making the killer not armed	(*non facere aliud non esse*)	(A6)

This list distinguishes between positive agency (where the agent does something) and negative agency (where the agent does not do something), as well as direct *per se* agency (where the agent brings about the effect himself) and indirect *per aliud* agency (where he causes some other being to bring the effect about). There is a further distinction that can be made in cases of *per aliud* agency. They divide into cases where the agent indirectly performs an action and where the agent indirectly does not perform an action (we can call this 'proximal' and 'distal', respectively). Thus, the six types listed above can be classified as follows:

Positive *per se*	(1)
Negative *per se*	(2)
Positive, proximal, *per aliud*	(3)
Negative, proximal, *per aliud*	(4)
Positive, distal, *per aliud*	(5)
Negative, distal, *per aliud*	(6)

Each of the four types of agency can be expressed in each of the six modes, which means that we have potentially twenty-four types of agency. Within each type, the six modes are all independent; they can neither be defined by each other nor do they imply each other. The relationships between the four types do, in a sense, 'trickle down' to the modes within each type. For example, (A1) is the contrary of (B1), (D3) is a contradictory of (A3), and so on. As a result, it turns out that types (A1) and (C2) are identical, and likewise (C1) and (A2), and the same for (B1) and (D1), and (B2) and (D2). The other sixteen combinations of modes and types are all logically independent, hence the result is twenty distinct ways that agentive statements can be expressed.

[16] "We say that a thing causes something else not to be either because (1) it directly causes this other thing not to be, or (2) it does not directly cause it to be, or (3) it causes an intervening thing to be, or (4) it does not cause an intervening thing to be, or (5) it causes an intervening thing not to be or (6) it does not cause an intervening thing not to be" [Hop72, p. 223].

Anselm's thesis is that any ascription of doing will be one of these twenty-four forms, but that in ordinary usage, the twenty logically distinct forms are often used interchangeably, as if they were not distinct but equivalent. He notes that in *usus loquendi*, we often use affirmative claims as a short hand when what we really mean is the negation of the contrary:

> *Sed affirmatio "facere esse" ponitur aliquando pro negatione, quae est "non facere non esse"; et conversim "non facere non esse" pro invicem ponuntur. Dicitur enim facere male esse aliquando aliquis idcirco, quia non facit ea non esse; et mala non facere non esse, quia facit ea esse; et facere bona non esse, quia non facit ea esse; et non facere bona esse, quia facit ea non esse* [Schm36, p. 29].[17]

In a more detailed explanation, Anselm says:

> *Et notandum, quia in modis negandi primus simpliciter negat nihil aliud insinuans; quinque vero sequentes habent negationem pro contrario suae affirmationis. Qui enim resuscitat aliquem, dicitur in secundo modo "non facere illum esse mortuum" pro "facere non esse mortuum", et "non facere non esse vivientem" pro "facere esse viventem"* [Schm36, p. 33].[18]

That is, forms (C2)–(C6) and (D2)–(D6) are often used equivalently with forms (B2)–(B6) and (A2)–(A6), respectively, even though, strictly speaking, forms (C3)–(C6), (D3)–(D6), (B3)–(B6) and (A3)–(A6) are all nonequivalent. (As noted earlier, (C2) is equivalent to (A1), and so on for the first and second modes of each type.) This is an example of Anselm demonstrating how ordinary usage of terms can be explained in part by their logical definitions and relations. The same phenomenon shows up later in the same philosophical fragments, when Anselm uses his explication of *facere* as a model for his discussion of *esse* 'to be', *habere* 'to have', and *debere* 'to be obliged, ought'. He says:

[17] "But the affirmation (A) 'causing something to be' is sometimes used in place of the negation (D) 'not causing something not to be', and vice versa. Likewise (B) 'causing something not to be' and (C) 'not causing something to be' are sometimes used in place of each other. thus, someone may on occasion be said to cause evil to be because he does not cause it not to be; or he may be said not to cause evil not to be, because he causes it to be. In the same way, he may be said to cause good not to be, because he does not cause it to be; and he may be said not to cause good to be, because he causes it not to be" [Hop72, p. 221].

[18] "It must be noted that while the first mode of the negative tables [modes (C) and (D)] simply negates, without implying anything else, each of the five subsequent modes in the negative tables contains statements which can be substituted for those statements which appear in that table which is the contrary of their corresponding affirmative table. For example, whoever revives someone may be said 'not to cause him to be dead' in the place of 'to cause him not to be dead'; and we may also substitute 'not to cause him not to be living' for 'to cause him to be living'..." [Hop72, p. 227].

> *dicimus etiam nos "non debere peccare" pro "debere non peccare". Non enim omnis, qui facit, quod non debet, peccat, si proprie consideretur... Sed si memores eorum, quae supra dicta sunt, sicut dicimus "non facere esse" pro "facere non esse": ita dicimus "non debere facere" pro "debere non facere"; et ideo, ubi est "debere non peccare", dicitur pro eo "non debere peccare". Quod in tantum obtinuit usus, ut non aliud intelligatur, quam "debere non peccare'* [Schm36, p. 36].[19]

Nevertheless, we need to remember that though we may use the locutions interchangeably, *tamen differeunt* [Schm36, p. 32], and only the first mode of each type represents *usus proprie*.[20]

Note that this is partially contrary to Serene's assertion that "only ascriptions made in mode one are 'proper', since this is the only mode in which the agent's action directly causes the outcome ascribed to him. Ascriptions in mode two are 'improper' because the directly relevant factor is the agent's failure to act rather than his directly doing what is ascribed to him." [Ser80, p. 123]. Ascriptions in the second mode of the negative types ((C2) and (D2)) must count as proper if the first mode of the two positive types are to count as proper, since they are identical. And likewise, if the first mode of the negative types are to be considered proper ascriptions of agency, then the second mode of the positive types must also be considered proper, for the same reason.

3.3 Semantics for non-normal modal logics

In this section we introduce a semantics for modal logic which we use in the next section to examine the syntax proposed by Walton in [Walt76a] and [Walt76b]. It will turn out that the type of modal logic which best expresses the features of Anselm's account of agency is a non-normal modal logic. Traditional semantics for normal modal logics are not adequate for modeling non-normal modal logics, so we will use instead neighborhood semantics.

Definition 3.3.1. A *normal* modal logic is any extension of propositional logic with at least one modal operator $\Box$ which contains axioms K, M, C, and N of

[19] "We also say that we are not 'obliged to sin' (*non debere peccare*) as a substitute for saying that we are 'obliged not to sin' (*debere non peccare*). But properly speaking not everyone who does what he is not obliged to do sins... Now as you remember, we said earlier that 'not to cause to be' may be used in place of 'to cause not to be'. In the same way, we say 'is not obliged to' for 'is obliged not to', and 'is not obliged to sin' for 'is obliged not to sin'. But our [Latin] usage is such that 'is not obliged to sin' we really mean 'is obliged not to sin'" [Hop72, pp. 231–232]. In modern linguistics, this phenomenon is called "negation raising" or "neg raising". For general information on negation raising, see [Horn89]. My thanks to Laurence Horn for drawing to my attention this parallel occurrence concerning *debere*.

[20] "[T]hey are different from each other" [Hop72, p. 225]; *siquidem ille proprie facit esse, qui facit, ut sit, quod non erat* [Schm36, p. 32]; "Thus, properly speaking, he causes to be who causes there to be what previously was not" [Hop72, p. 225].

C: $(\Box\varphi \wedge \Box\psi) \rightarrow \Box(\varphi \wedge \psi)$
E: $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
K: $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M: $\Box(\varphi \wedge \psi) \rightarrow (\Box\varphi \wedge \Box\psi)$
N: $\Box\top$
T: $\Box\varphi \rightarrow \varphi$

Figure 3.2: Common modal axioms and their standard names [Che80, ch. 1]

Table 3.2, and is closed under *modus ponens*, uniform substitution, and the rule of necessitation RN (from $\vdash \varphi$ infer $\vdash \Box\varphi$).

The minimal modal logic which contains all of these axioms and satisfies these rules of inferences is called K. The standard semantics for a normal modal logic are Kripke relational semantics: a structure is a *frame* $\mathfrak{F} = \langle W, R\rangle$ where W is a non-empty set and R is a binary relation on W, and a model is a frame plus a valuation function, e.g., $\mathfrak{M} = \langle \mathfrak{F}, V\rangle$, where V is a map from atomic sentence letters to $\mathcal{P}(W)$ (for a more detailed presentation of Kripke semantics, see Appendix A.2.)

As we will see in more detail in §3.4, some of the rules of inference and axioms of normal modal logics are problematic when we try to apply them to agency. We therefore look at axiom systems which are weaker than K, namely ones that do not have the necessitation rule and which omit one or more of the axioms listed above. Since K is characterized by the class of all Kripke frames, these sub-K, non-normal logics cannot have Kripke frames as their semantics. Instead, non-normal modal logics are usually modeled with neighborhood semantics (called 'minimal models' in [Che80]), developed by Montague and Scott in [Mon68] and [Sco80], respectively.[21]

Definition 3.3.2. A *neighborhood model* is a structure $\mathfrak{M} = \langle W, N, V\rangle$ where

- W is a set of points, called worlds.
- N is a function from W to $\mathcal{P}(\mathcal{P}(W))$, such that $N(w)$ is called *the neighborhood of* w.
- V is a function from atomic sentence letters to $\mathcal{P}(W)$. If $w \in V(p)$, we say that p is true at w, or $V(p, w) = 1$.

Here and elsewhere in this dissertation, we will often abuse notation and identify V with the valuation function V' extended from V in the natural way to cover the boolean operators.

[21] For the formal properties of logics which correspond to different classes of neighborhood semantics, including questions of expressivity and completeness, see [Che80]. Because these formal properties do not affect our analysis of Walton's syntax in the next section, we will say little about them here.

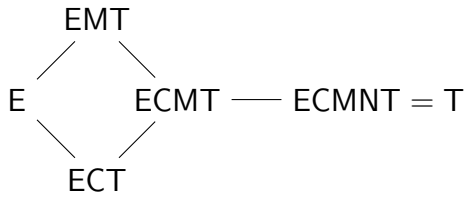

Figure 3.3: Lattice of logics between E and K containing T

In these models, each formula is associated with a *truth set*.

Definition 3.3.3. Let $\mathfrak{M}$ be a neighborhood model and φ a formula. The *truth set for φ in $\mathfrak{M}$* is $\|\varphi\|^{\mathfrak{M}} = \{w \in W : w \in V(\varphi)\}$

The clauses in the truth definition for the propositional connectives are as expected. We add the following for the modal operator:

Definition 3.3.4. Let $\mathfrak{M}$ be a neighborhood model, w a world in $\mathfrak{M}$, and φ a formula. Then

$$\mathfrak{M}, w \vDash \Box\varphi \quad \text{iff} \quad \|\varphi\|^{\mathfrak{M}} \in N(w)$$

The modal logic characterized by the class of all neighborhood models is the logic E (so called in [Che80]). E has one axiom, E, and one rule of inference RE (from $\vdash \varphi \leftrightarrow \psi$ infer $\vdash \Box\varphi \leftrightarrow \Box\psi$).

We define EM to be the smallest logic containing both E and M (cf. Table 3.2) and closed under *modus ponens* and uniform substitution, and similarly for EN, EC, ET, ECM, ECMT, etc. The logics between E and K extended by M, N, and C form a boolean lattice where each combination is distinct and ECMN = K. Each of these with T as a further axiom are also all distinct logics (see Figure 3.3). We prove a few of the cases. The other proofs are either straightforward or found in [Che80, pp. 214–217].

Theorem 3.3.5. EMT $\nvdash$ C

Proof. Let $\mathfrak{M} = \langle W, N, V\rangle$ where

$$\begin{aligned} W &= \{0,1,2\} \\ N(0) &= \big\{\{0,1\},\{0,2\},\{0,1,2\}\big\} \\ N(1) &= \emptyset \\ N(2) &= \big\{\{0,1,2\}\big\} \\ V(p) &= \{0,1\} \\ V(q) &= \{0,2\} \end{aligned}$$

Then C is falsified at 0, because $\|p\|^{\mathfrak{M}} = \{0,1\} \in N(0)$ and $\|q\|^{\mathfrak{M}} = \{0,2\} \in N(0)$, but $\|p\|^{\mathfrak{M}} \cap \|q\|^{\mathfrak{M}} = \{0\} \notin N(0)$. Further, E, M, and T are true everywhere. (In addition, note that this model also does not satisfy N, because of 1.) □

Theorem 3.3.6. $\mathsf{ECT} \nvDash \mathrm{M}$

Proof. Let $\mathfrak{M} = \langle W, N, V \rangle$ where

$$\begin{aligned} W &= \{0, 1\} \\ N(0) &= \big\{\{0\}\big\} \\ N(1) &= \big\{\{1\}\big\} \\ V(p) &= \{0, 1\} \\ V(q) &= \{0\} \end{aligned}$$

Then, since $\Box p$ is true nowhere, both T and C are satisfied everywhere, but $0 \vDash \Box(p \wedge q)$, and hence M is falsified. (It should also be clear that this model also does not satisfy $\Box\top$, either.) □

As we've introduced them, these neighborhood models are mono-modal. If we wish to introduce a 'does' modality for each agent, then we need to work within a multi-modal setting, which requires us to modify slightly the definition of neighborhood model given above, with a corresponding modification to the truth conditions for agentive formulas.

Definition 3.3.7. A *multi-modal neighborhood model* is a structure

$$\mathfrak{M} = \langle W, A, N_a \text{ for } a \in A, V \rangle$$

where

- W is a set of points, called worlds.
- A is a set of agents.
- Each N_a is a function from W to $\mathcal{P}(\mathcal{P}(W))$. We call $N_a(w)$ "the neighborhood of w for a".
- V is a function from atomic sentence letters to $\mathcal{P}(W)$. If $w \in V(p)$, we say that p is true at w, or $V(p, w) = 1$.

Definition 3.3.8. Let $\mathfrak{M}$ be a multi-modal neighborhood model, w a world in $\mathfrak{M}$ and φ a formula. Then

$$\mathfrak{M}, w \vDash \delta_a \varphi \quad \text{iff} \quad \|\varphi\|^{\mathfrak{M}} \in N_a(w)$$

where we read $\delta_a p$ as 'a does so that p'.

In the next section we apply these different classes of models to the syntax developed in [Walt76a] and [Walt76b] to see whether certain questions which he leaves open can be settled.

3.4 The syntax of agency

In this section we look at modern syntactical representations of Anselm's theory, focusing on the one based on standard modal logic given by Walton in [Walt76a] and [Walt76b].[22] Walton introduces his syntax without providing any semantics, and he leaves open some questions about which axioms can be legitimately introduced, because he has no semantic theory to decide the question. We answer these questions with the help of the semantics presented in the previous section. We start with a classical propositional language with the following signature (we will throughout this dissertation identify languages with their signatures, always giving the syntax explicitly):

$$\mathcal{L}_A := \{P, A, \wedge, \vee, \neg, \rightarrow, \hookrightarrow, \rightsquigarrow, \delta_a\}$$

where P is an infinite set of propositions and A is a non-empty (and possibly but not necessarily infinite) set of agents. We let $G \in A$ stand for the agent 'God'. We define the well-formed formulas of $\mathcal{L}_A$ as:

$$\varphi := p \text{ for } p \in P \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \rightarrow \psi \mid \varphi \hookrightarrow \psi \mid \varphi \rightsquigarrow \psi \mid \delta_a\varphi \text{ for } a \in A$$

where $\hookrightarrow$ is causal implication, $\rightsquigarrow$ is *per aliud* causal implication, and the operators δ_a, one for each agent in A, are our 'does' modalities.[23] When the specification of the agent is not necessary, we drop the subscript; δp is read 'someone does such that p'. Note that in this syntax, we have just one type of modal operator; because there is no good natural language expression that corresponds to the dual notion of agency, we do not introduce a separate operator for the dual $\neg\delta\neg$.

[22] We omit from discussion the syntax introduced by Danto in [Dan73], which is both cursory and unfortunate. Danto says very little about Anselm; there is a brief mention and then a footnote. He uses Anselm as a justification for introducing the expression mDa, to be read "m makes happen the event a by doing a". He says that

> the locution mDa covers the stiltedness of the expression '... makes ... happen by ...-ing' and permits us to treat actions in a generalized manner by treating 'does' for the moment as an auxiliary of action verbs, much as 'knows' may be an auxiliary of cognitive verbs. In doing so, I follow the illustrious precedent of Anselm of Canterbury who in discussing the Latin verb *facere* treats it in similar auxiliary fashion [Dan73, p. 7].

Danto then quotes Anselm, and footnotes this with a reference to the Lambeth fragment, and notes that a translation of the fragment by Ernst Van Haagen was "scheduled for publication in the *American Philosophical Quarterly*" [Dan73, p. 199], but I have unfortunately not been able to find any further record of this publication.

This is an unfortunate case where symbolic notation is introduced as a method of clarifying the underlying structure of the sentences being discussed but where in fact the notation ends up merely hiding the relevant issues without explaining them.

[23] We make no assumption about any of the properties of these agents, other than that they are agents, in as weak a sense as possible. This is in line with what we discussed at the end of §3.2.

(A1)	δp
(A2)	$\neg\delta\neg p$
(A3)	$\delta q \wedge (q \rightsquigarrow p)$
(A4)	$\neg\delta q \wedge (q \rightsquigarrow \neg p)$
(A5)	$\delta\neg q \wedge (q \rightsquigarrow \neg p)$
(A6)	$\neg\delta\neg q \wedge (q \rightsquigarrow p)$

Figure 3.4: The six modes of type (A)

facere esse	δp
facere non esse	$\delta\neg p$
non facere esse	$\neg\delta p$
non facere non esse	$\neg\delta\neg p$

Figure 3.5: The four proper modes of agency

With this syntax, we are able to represent all six modes of the four different types of agency introduced in §3.2.2. As an example, we give the six modes of type (A) (*facere esse*) in Table 3.4. Recall that, for Anselm, only the first two types of each mode count as proper, from a logical point of view. This means that, for discussing just the logical aspects of the theory, we need not say anything about how the relationship expressed by $q \rightsquigarrow p$ is to be interpreted, since this only shows up in the four improper forms. The four proper forms are listed in Table 3.5.

There is an important respect in which using a language like the one we've outlined, and like the one Walton uses in his reconstruction, is best described as Anselm*ian*, and *not* Anselm's actual ideas (beyond the surface difference that Anselm never gave this type of formalism). Anselm explicitly allows as answers to the question "What is he doing?" only atomic actions and negations of atomic actions. Because our language allows *any* type of formula to be substituted in for p in $\delta_a p$, this system cannot be taken as being a reconstruction of Anselm's actual ideas.[24] However, because Anselm himself says that the answer to "What is he doing?" can be *any* verb, this extension of our syntax is not unreasonable, because it makes just as much sense to say "He is reading and sitting" and "He makes it the case that if he reads he is sitting" as it does to say "He is reading" or "He is sitting" (see footnote 4).

After Walton introduces his syntax, he considers different possible candidate theorems for a logic of Anselmian agency. The first he proposes is both necessary and obvious:

[24]Walton is aware of this: "St. Anselm did not, to my knowledge, take the next step that would be of interest to a student of modern sentence logic, namely extension to conjunctive, disjunctive, and materially conditional states of affairs" [Walt76b, p. 301].

Axiom 3.4.1 (Success)**.** $\delta_a p \to p$

This is the agentive parallel to the axiom T introduced in the previous section. Its intuitive plausibility follows from the fact that after agent a does so that p, then p must be the case; to say otherwise would be to that a succeeded in bringing about p, even though p is still false, which makes no sense. Beyond its intuitive plausibility, there is second reason to adopt this axiom. This axiom implies $\neg\delta_a p \vee \neg\delta_a\neg p$, which in turn is equivalent to $\neg(\delta_a p \wedge \delta_a\neg p)$, the truth of which is required for the relations in the square of opposition to hold (cf. [Seg92, p. 349]).

Next Walton considers the following pair of potential axioms:

Axiom 3.4.2 (Conjunction Elimination)**.** $\delta_a(p \wedge q) \to (\delta_a p \wedge \delta_a q)$

Axiom 3.4.3 (Conjunction Introduction)**.** $(\delta_a p \wedge \delta_a q) \to \delta_a(p \wedge q)$

These are converses of each other. Walton argues that we cannot accept both of these as axioms or theorems. He claims that adding

$$\delta_a(p \wedge q) \leftrightarrow (\delta_a p \wedge \delta_a q)$$

is too strong, because this equivalence plus the T axiom is provably equivalent to the standard normal modal logic T [Walt76b, p. 303, fn. 17]. He says that this is unacceptable because T, being a normal modal logic and hence an extension of K, both proves versions of the paradoxes of strict implication and also validates the rule of necessitation RN. From an agentive point of view, RN violates intuitions that we have about agency and tautologies. It should not be the case that any agent can *cause* it to be the case that a tautologous state of affairs is obtained. Such states of affairs will obtain vacuously, whether or not we ever do anything, and even in spite of our actions. The problems with this rule also apply to adopting either $\delta_a p \leftrightarrow p$ (material equivalence) or $\delta_a p \Leftrightarrow p$ (strict or causal equivalence) as theorems.

Walton is wrong in rejecting the acceptance of both Axiom 3.4.2 and Axiom 3.4.3 out of hand, for two reasons. The first is that

$$\delta_a(p \wedge q) \leftrightarrow (\delta_a p \wedge \delta_a q) \;+\; \delta_a p \to p$$

is equivalent to

$$\delta_a(p \to q) \to (\delta_a p \to \delta_a q) \;+\; \delta_a p \to p$$

only in the presence of the further axiom $\delta_a\top$. Without $\delta_a\top$, RN is not sound. If we wanted to take both Axiom 3.4.3 and Axiom 3.4.2 as axioms, we can without sacrificing our intuitions about doing: The resulting logic is ECMT.

The second reason is that his objection to RN relies on a certain narrow conception of agency. Under such a narrow conception, agency is always active and causal. But insisting that we interpreted Latin *facere* as 'to cause' is too

restrictive. If we remember that the analysis of *facere* is an analysis of doing, not of causation, then it wouldn't seem that unreasonable if someone said 'agent a does such that $p \vee \neg p$'. In fact, I myself am doing such that an infinite number of tautologies are true. Here is a case where the ordinary usage (*usus loquendi*) of terms contradicts some intuitions about their potentially more narrow logical functions.[25] For insofar as tautologies are necessary, $\neg\delta_a\neg p$ ('it is not the case that a brings it about that not p') will always be true when p is a tautology; and then, as mentioned earlier, it *does* follow that $\delta_a p$ holds whenever p is a theorem. If we are interested in the logical properties of *facere* at the possible expense of ordinary usage, then the necessitation rule is unacceptable and we must look elsewhere for axioms and rules. If, however, we are interested in explaining in logical terms our ordinary usage of *facere*, as Anselm appears to be doing, then T presents itself as a most plausible choice.

That being said, we will continue to focus on the more strictly logical, rather than common usage, analysis of doing. Walton concludes, incorrectly, that one of Axiom 3.4.2 and Axiom 3.4.3 must be given up. He gives up the latter, because this is the route taken in [Fitc63][26], but his argument for accepting Axiom 3.4.2 is simply to state what it says, and note that adopting it plus axiom T "would give us the rudiments of a seemingly not very contentious, if rather minimal, system of agency" [Walt76b, p. 302]. But the same could be said if we took Axiom 3.4.3 instead of Axiom 3.4.2.

After accepting Axioms 3.4.1 and 3.4.2, Walton next proposes, and quickly rejects, the following:

Axiom 3.4.4. $(\delta_a p \wedge (p \to q)) \to \delta_a q$

His reason for rejecting this is that this axiom is even stronger than Axioms 3.4.3 and 3.4.2 combined. In this he is correct, both in his rejection of the principle and his reason for doing so. Axiom 3.4.4 is stronger than the axiom K (and in fact

[25] This case is similar to one presented by Anderson, when he notes the two possible answers to the question of "Who (wrongly) left the door open?":

> Devotees of quantification theory might immediately point out that if the door was left open, then *everyone* left the door open, on the grounds that no-one closed it. But it ought to be clear that the questioner does not want to hear "everyone" in response to his question.
>
> Just *who* left the door open may depend on lots of moot questions, and certainly it depends on the rules governing the situation [An70, p. 240].

Putting Anderson's answers in Anselmian terms, the answer "everyone" is the correct answer according to *usus proprie*, and the more palatable answer, say, "Bob", is correct according to *usus communis* or *loquendi*.

[26] Fitch gives no argument for why we should take this over Axiom 3.4.3. He claims outright that he's assuming it's true: "We assume that the following concepts, viewed as classes of propositions, are closed with respect to conjunction elimination: striving (for), doing, believing, knowing, proving" [Fitc63, p. 137]. He makes no argument for the truth of this assumption.

its alethic counterpart $\Box p \wedge (p \rightarrow q)) \rightarrow \Box q$ is invalid in many standard modal logics), as it implies $(\delta_a p \wedge q) \rightarrow \delta_a q$. This is clearly too strong, so Axiom 3.4.4 should be rejected.

Instead, Walton offers a version of the K axiom as an alternative to Axiom 3.4.2:

Axiom 3.4.5. $(\delta_a p \wedge \delta_a(p \rightarrow q)) \rightarrow \delta_a q$

He says that the system combining Axiom 3.4.1 with Axiom 3.4.5 is stronger than that containing just Axioms 3.4.1 and 3.4.2, because Axiom 3.4.5 implies Axiom 3.4.2 but that "the converse implication does not seem to hold. [The claim] is inconclusive, in the absence of a δ_a-semantics" [Walt76b, p. 304]. As we noted earlier, he is wrong in saying that Axiom 3.4.5 implies Axiom 3.4.2; it does so only in the presence of the further axiom $\delta\top$, which we have reason to reject when modeling the proper, logical usage of *facere*. However, now that we have provided a type of δ_a-semantics, we can confirm that his second claim is correct; Axiom 3.4.2 does not imply Axiom 3.4.5.

Finally, Walton puts forward one further possible axiom or theorem:

Axiom 3.4.6 (Causal implication)**.** $(\delta_a p \wedge (p \hookrightarrow q)) \rightarrow \delta_a q$

The reason that this axiom is formulated as causal implication instead of just standard implication is because Walton wishes to block $(\delta_a p \wedge (p \rightarrow q)) \rightarrow \delta_a q$ as a theorem, as this implies $(\delta_a p \wedge q) \rightarrow \delta_a q$, which has as an unfortunate instance the following: "If Socrates scratches his head and Plato dies, then Socrates brings it about that Plato dies" [Walt76b, p. 304]. Walton discusses this theorem in the context of agency *per aliud*. Agency of this type only becomes relevant when we are trying to give an analysis of the *usus loquendi* of the term *facere*; it plays no role in the analysis of the strict logical usage of the term. A full analysis of the improper usage of the term is much more difficult, and as it is one best left to the grammarian or linguist, we do not pursue it further here.

There is a relevant sense in which Walton's approach, in developing the syntax and leaving any questions of semantics behind, more adequately captures what is found in the Anselmian texts, and in which our semantical proposal is irredeemably anachronistic. As Serene notes, Anselm in the texts discussed above "presents the modes as a disjunctive *necessary condition* for ascriptions of agency, but he does not to my knowledge assert that any relationship, no matter how remote, between a subject and a state of affairs provides a sufficient condition for agency" [Ser83, p. 146] (emphasis added). This is a crucial feature of his theory. If there were such a sufficient condition for ascriptions of agency, then given how encompassing his theory of action is, it would be possible to make practically every person (or indeed, every object) an agent for every action, because failure to act counts, in his theory, as action. With the ensuing consequences such a

move would have for personal culpability and sin, this is clearly a move we do not want to take.[27]

Walton's syntax can be seen as an extension of Anselm's necessary conditions, in which any ascription of agency will have one of twenty distinct possible syntactic constructions. When we add semantics, we are essentially adding sufficient conditions; we can say that when such-and-such conditions hold, we can then make a true statement about agency.

In no way, then, should the discussions in the preceding two sections be taken as a formalization of *what Anselm said*, as it is not. The formalizations should be viewed as inspired by, and hopefully capturing, the brilliant insights of his theory of agency. We have seen that the breadth of Anselm's conception of agency is a point in its favor, and not a reason for discard. Further, Walton's extension of Anselm's discussion of agency to non-atomic actions seems thoroughly plausible, and we can provide both syntax and semantics to accommodate this extension. If we are content to divorce the logical theory from any ethical theory, there is no problem with the addition of semantics from the formal point of view.

With that caveat expressed, we draw the following conclusions about Walton's syntax of Anselm's agentive logic. Depending on specific ideas about agency, there are a number of different choices for logics:

T The normal multi-modal logic T, which has as axioms both $\delta_a(p \wedge q) \leftrightarrow (\delta_a p \wedge \delta_a q)$ and $\delta_a \top$, corresponds to at least some aspects of our ordinary usage of the word *facere.*

EMT, ECT These both block the unwanted inference of $\delta_a \top$, which is desirable from the standpoint of the logical usage of *facere*, as well as barring the equivalence found in T, thus satisfying the syntax provided by Walton.

ECMT This blocks the unwanted inference of $\delta_a \top$ but allows for the equivalence noted above, for which the only argument against was the incorrect claim that it caused the logic to collapse into T, and for which arguments in favor can be provided.

With this we have answered one of the questions mentioned at the beginning of this chapter. We have shown that each of these logics is characterized by a class of models, and hence that each system is sound, and that they are all distinct. Which class of models should be preferred depends on the context of usage. From the point of view of proper logic, ECMT is the most expressive logic capturing Anselm's views. We show in §3.5 that if we wish to consider the *usus loquendi*, especially as exemplified by theological usage, then T becomes a natural candidate.

[27] We would also be faced with a variant of the problem of evil, namely that God, because he does not do so that it is the case that people always do good, thereby *does* such that evil exists.

Before we move to adding moral concepts to the logical theory, we comment briefly on the second question raised at the beginning of §3.2, namely whether Anselm's logical theory is identical with any standard modern logics. One natural candidate would be a member of the most prevalent class of modern agentive logics, namely variants on stit-theory. Belnap and Perloff introduced stit-theory in [BelPe88] in an attempt to "augment the language with a class of sentences whose fundamental syntactic and semantic structures are so well designed and easily understood that they illuminate not only their own operations but the nature and structure of the linguistic settings in which they function" so that we can "progress toward a deeper understanding of an agent doing an action" [BelPe88, p. 175]. However, despite the fact that Anselm's modal conception of agency is regularly referred to in literature on stit-theory (e.g., [HortBe95], [Xu95], [Mü05], and [TroTryVi06]), often in the context of offering a justification for certain aspects of the theory, stit-theory is actually a remarkably poor choice for modeling Anselm's logic. In the presentation of Anselm's logic given above, we have that $\delta_a(p \vee q) \leftrightarrow (\delta_a p) \vee (\delta_a q)$, but this equivalence does not hold in standard stit-theory.[28]

3.5 Human agency, obligation, and goodness

We have just seen that when we consider the purely logical aspects of Anselm's agentive theory, these do not pick out a unique logic, but rather a class of logics, each of which has different features and applications. In §3.2.1, we noted two features of Anselm's purely logical theory of agency, namely that the *usus proprie* of *facere* is insufficient to explain the theological use of the term, and that the theory was not a theory of human agency. In this section we comment briefly on both points, in the context of developing some of the deontic notions found in the Lambeth fragments. We show that by combining comments about the theological usage of the word *facere* with certain statements Anselm makes about obligations we can give a formal analysis of the *usus commune* of the terms via their *usus proprie*. Such an analysis gives a formal justification for Anselm's informal arguments that improper uses of agentive and deontic terms in scripture can be grounded in proper, logical usage.[29]

[28] [BelPeXu01, pp. 84–85] notes that $[\alpha \operatorname{stit} p] \vee [\alpha \operatorname{stit} q]$ follows from $[\alpha \operatorname{stit} p]$ but that $[\alpha \operatorname{stit}(p \vee q)]$ does not, so $[\alpha \operatorname{stit} p] \vee [\alpha \operatorname{stit} q]$ and $[\alpha \operatorname{stit}(p \vee q)]$ cannot be equivalent. However, variants of stit-theory which do not have the usual clause in the truth condition of the operator could allow for this equivalence. Such a variant can be found in [Hort01]. My thanks to Thomas Müller for pointing this out to me.

[29] The formal analysis could also be used to provide a framework for Anselm's views of free will as discussed in *De libertate arbitrii*. According to King, in this work Anselm "defends a unilateral normative conception of freedom according to which an agent is free when two conditions are jointly satisfied: (a) she has the ability to perform a given action; and (b) that action is the one she ought to perform", and one of the purposes of the material in the Lambeth

The result is of twofold interest, both logically and historically. From the logical point of view, the system is essentially different from standard contemporary deontic logics because we must be able to model uniquely theological aspects, such as the role God plays in forming obligations. From the historical point of view, the present investigations are of interest because they allow us to extend the history of deontic logic by a couple of centuries. Modern interest in deontic logic can be traced back to von Wright's article [vW51], though his paper was by no means the first on the subject. Knuuttila in [Knu81] argues that Dagfinn Føllesdal and Risto Hilpinen's identification of Ernst Mally as the first person to study normative language from a logical point of view, in his monograph [Mal26], is mistaken and that in fact we can find discussions of deontic concepts from a logical standpoint in some of the 14th-century logicians, such as Robert Holcot and William of Ockham. Knuuttila even goes so far as to say that "discussion of the logic of norms comparable with the modern deontic logic started only in the fourteenth century" [p. 226] and not earlier. In the following we will show this is not wholly the case.

Anselm cites scriptural usage of *facere* as an example of the *usus loquendi* or *usus improprie* of the term. We already saw a comment on this from *De veritate* (cf. p. 45). When he makes the same point early in his discussion of *facere* in the Lambeth fragments, he does so in the context of discussing the obligations of (human) agents:

> *Siquidem et dominus in evangelio ponit "facere" vel "agere"—quod idem est—pro omni verbo, cum dicit: "Omnis qui male agit, odit lucem", et "qui facit veritatem, venit ad lucem". Male quidem agit, qui facit, quod non debet, aut non facit, quod debet; quod similiter intelligitur de omni verbo... Veritatem autem facit, qui facit, quod debet, et qui non facit, quod non debet... Hoc modo redigit dominus omne verbum positivum vel negativum in "facere"* [Schm36, p. 28].[30]

While Anselm's purpose in this quote was to provide an argument for discussing the non-logical usage of *facere* as a part of his logical account of agency, the passage is independently interesting because it is talking about *human* agency

fragments was to track "connections among ascriptions of ability, responsibility, and the cause of an action" [Kin$_2$–, p. 6]. However, we do not explore the use of agent ability here.

[30] "Indeed, the Lord Himself in the Gospel uses *facere* and *agere*—which are the same—in place of every other verb when He says, 'Whoever does evil hates the light' and 'Whoever does the truth comes to the light' (John 3:20–21). For he who does what he ought not or does not do what he ought does evil. And any other verb can be substituted for "does" in this sentence... But he does the truth who does what he ought and who does not do what he ought not do... In this way the Lord reduces every verb, whether positive or negative, to a form of 'to do'" [Hop72, p. 220]. In these verses, and in the verses from Matthew cited earlier, the original Greek has a form of ποιεω; this word has the same generality that *facere* has in Latin and 'to do' in English, in that it can be used to stand in for any type of doing or making action.

specifically. Here, Anselm's account of human agency differs radically from traditional modern accounts. Serene notes that:

> Many modern theorists assume that an important part of the problem of human action is to explicate the contrast between what an agent does and what merely happens to him. One line of response is to explain human agency in terms of intentionality, or in terms of the characteristic causation of action by our desires and beliefs [Ser83, p. 149].

This is not the solution that Anselm takes. Instead, he disagrees that the problem even exists. The argument is roughly this:

- The theological usage of terms such as *facere*, *agere*, and *debere* is often improper.
- This means that not doing such that not p is the same as doing p ($\neg\delta\neg p \leftrightarrow \delta p$); and that not doing such that p is the same as doing such that not p ($\neg\delta p \leftrightarrow \delta\neg p$).
- This means that we cannot separate what an agent does from what he fails to do. Since what happens to him is what he fails to prevent, we cannot separate what an agent does from what happens to him.

While this may be grossly in contrast with our (modern) intuitions, when there is a conflict between what scripture says and what our intuitions say, Anselm is happy to sacrifice intuition. For him, it is part of the business of the logician to give rational foundation to theological problems, and to do so we must take what scripture says seriously. That is, we must be prepared to take

$$\delta_a p \leftrightarrow \neg\delta_a\neg p$$

as an axiom. Adding this as an axiom automatically implies that the N axiom $\delta_a\top$ is valid, which means that the correct logic of the *usus commune* of human agency will be an extension of T. It also causes the square of opposition to collapse, since types (A) and (C), which properly are distinct, are made equivalent, as are types (B) and (D).

The passage is also interesting because of the connections that are drawn between between doing, obligation, and good and evil. When we consider these issues, the first point of note is Anselm's contrasting of "doing evil" not with "doing good" as one would expect, but rather with "doing the truth". For Anselm, "all truth either is God or somehow reflects God" [ViWi04, p. 205], and God is both truth and goodness. Anselm explicitly equates the two in Chapter V of *De veritate*, where the student and the teacher discuss what it means for an action to be true or false. The teacher again quotes the passage from John, saying:

> *Nam si male agere et veritatem facere opposita sunt, sicut ostendit dominus cum dicit: "qui male agit, odit lucem"; et: "qui facit veritatem, venit ad lucem": idem est veritatem facere quod est bene facere. Bene namque facere ad male facere contrarium est. Quapropter si veritatem facere et bene facere idem sunt in oppositione, non sunt diversa in significatione. Sed sententia est omnium quia qui facit quod debet, bene facit et rectitudinem facit. Unde sequitur quia rectitudinem facere est facere veritatem. Constat namque facere veritatem esse bene facere, et bene facere esse rectitudinem facere. Quare nihil apertius quam veritatem actionis esse rectitudinem* [AoC38–61, vol. 1, IV, p. 181].[31]

Pragmatically, equating the two is also the easiest option, since it makes more sense to speak of agents doing good and evil than it does to speak of them doing the truth or doing the falsity. Thus we are justified, both pragmatically and theoretically, for substituting "good" for "the truth".[32] If we want to add "doing good" and "doing evil" to our model, we must address the question of what the nature of "good" and "evil" is. Are good and evil states of affairs or properties? If properties, are they properties of actions, states of affairs, or agents? If we want to make good and evil properties of actions, then we have to represent actions explicitly in our model. The central tenet of Anselm's theory is that we do not have to specify the action being done, we can simply use the general term "does". If we introduced explicit actions into our model, we would be removing this rather distinctive feature of Anselm's agentive theory. If actions are not specified explicitly (an agent does such that p results or does not result, without any further specification of what he does to do this), then it is most natural to interpret "evil" and "good" as designated propositions. Thus, when we construct our model, we will speak of agents doing such that there is good and such that there is evil, rather than doing actions which are good or bad.

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[31] "Since to do evil and to do the truth are opposites—as the Lord declares when He says, 'He who does evil hates the light' and 'He who does the truth comes to the light'—doing the truth is the same thing as doing good. For doing good and doing evil are opposites. Therefore, if doing the truth and doing good have the same opposite term, namely, doing evil, then they are not different in their signification. But everyone maintains that whoever does what he is supposed to do does what is good and what is right, or correct. So it follows that to do what is right, or correct, is to do the truth. For it is evident that to do the truth is to do what is good, and to do what is good is to do what is right. Therefore, nothing is clearer than that the truth of an action is its rightness" [AoC67, p. 98].

[32] Of course this is a very simplified version of the issue; we do not go into further detail here because most of the details are theological or ethical, and not logical, in nature. One point is worth commenting on briefly, and that is what Anselm sees is the relationship between "doing good" or "doing evil" and *being* good or *being* evil. Following Augustine, Anselm believed in salvation *sola gratia*; without the grace of God, no amount of doing good would make a person *be* good, at least, not good enough to merit salvation. Anselm's primary discussion of these issues is in *Cur deus homo* [AoC38–61, vol. 2, II].

We represent this formally by adding two designated propositions to our language, e and g. We also add a new propositional operator O 'it is obligatory that'. The resulting language is $\mathcal{L}_A^{\mathrm{D}} := \mathcal{L}_A \cup \{e, g, \mathrm{O}\}$ (with $e, g \notin P$), and we extend our syntax. The set of well-formed propositional $\mathcal{L}_A^{\mathrm{D}}$ formulas is defined as expected, and the set of well-formed modal $\mathcal{L}_A^{\mathrm{D}}$ formulas is defined as:

$$\varphi := \delta_a\psi \mid \mathrm{O}\,\psi \text{ for } a \in A \text{ and } \psi \text{ propositional}$$

The set of well-formed $\mathcal{L}_A^{\mathrm{D}}$ formulas is:

$$\varphi := \varphi \text{ propositional} \mid \varphi \text{ modal}$$

Definition 3.5.1. Let $\mathfrak{M} = \langle W, A, \{N_a : a \in A\}, V\rangle$ be an agentive model as defined in §3.3. We make an *agentive-deontic model* $\mathfrak{M}^{\mathrm{D}}$ by augmenting $\mathfrak{M}$ with an *obligation function* $\mathbf{O} : W \to \mathcal{P}(P \cup \neg P)$, where $\neg P = \{\neg p : p \in P\}$, such that $\mathbf{O}(w)$ is consistent.

Intuitively, the function picks out for each world w a state of affairs which is obligatory at that point. Theoretically, $\mathbf{O}$ could be any function such that $\mathbf{O}(w)$ is consistent. However, there is a natural way to define $\mathbf{O}$ given that God is the root of both goodness and obligation, and that is to let $\mathbf{O} = N_G$ the neighborhood function of God.[33] Then $w \vDash \mathrm{O}\,p$ 'p is obligatory' iff $V(p) \subseteq \bigcap N_G(w)$. We will use this definition of $\mathbf{O}$ in what follows.

We add to our truth definition the following clause. If $\mathfrak{M}^{\mathrm{D}}$ is an agentive-deontic model, w a world in $\mathfrak{M}^{\mathrm{D}}$, and $p, q \in P$, then

$$\mathfrak{M}^{\mathrm{D}}, w \vDash \mathrm{O}\,p \quad \text{iff} \quad p \in \mathbf{O}(w)$$

The truth conditions for $\mathrm{O}\,\varphi$ where φ is complex are defined recursively in the expected fashion.

We can now formalize the material in the passage quoted at the beginning of this section. It is clear that we are given four axioms connecting obligation, action, failure to act, and moral consequences. A naïve paraphrase straightforwardly results in the following four principles:

- If you ought not do p, and you do p, then you do evil.
- If you ought to do p and you don't do p, then you do evil.
- If you ought to do p and you do p, then you do good.
- If you ought not do p and you don't do p, then you do good.

[33] This definition is supported by *Monologion* 9, where God's being supreme goodness is given as the cause of created beings' obligations. I'm indebted to Prof. Marilyn McCord Adams for this reference.

In English, 'ought not do p' is ambiguous between 'ought not do p' and 'ought do $\neg p$', in precisely the same way that *facere* is in Latin. However, because we have adopted $\delta_a p \leftrightarrow \neg\delta_a\neg p$ as an axiom, this ambiguity is not problematic. We can formalize the axioms as:

1. $\mathrm{O}\,\neg p \wedge \delta_a p \rightarrow \delta_a e$
2. $\mathrm{O}\,p \wedge \delta_a\neg p \rightarrow \delta_a e$
3. $\mathrm{O}\,p \wedge \delta_a p \rightarrow \delta_a g$
4. $\mathrm{O}\,\neg p \wedge \delta_a\neg p \rightarrow \delta_a g$

Note that if we have uniform substitution, we need only the first and third axioms.

The axioms above only give sufficient conditions for doing good and doing evil. However, given that we've defined the obligation function in terms of God's neighborhood, and that God is the source of all goodness, it makes sense to extend these principles to also be necessary conditions, and hence offer a definition of doing good and doing evil. We can thus offer the following truth conditions for e and g:

$$\begin{array}{lll} \mathfrak{M}^{\mathrm{D}}, w \vDash g & \text{iff} & \exists a \in A, p \in \mathbf{O}(w),\ \mathfrak{M}^{\mathrm{D}}, w \vDash \delta_a p \\ \mathfrak{M}^{\mathrm{D}}, w \vDash e & \text{iff} & \exists a \in A, \neg p \in \mathbf{O}(w),\ \mathfrak{M}^{\mathrm{D}}, w \vDash \delta_a p \end{array}$$

If we define goodness and evil solely in terms of obligation, then there should be some actions which result in states of affairs which are neither good nor evil: That is, we want to allow that some states of affairs are obliged, some are obligated not, and some are neither. By defining $\mathbf{O}$ as N_G, we automatically ensure that this is the case, for not every state of affairs or its negation will be obligated by God. We are also able, in this framework, to express things like that if the state of affairs resulting from some action implies evil, then the agent was not obliged to bring that state of affairs about, e.g., $(\delta_a\varphi \wedge (\varphi \rightarrow e)) \rightarrow \neg\,\mathrm{O}\,\varphi$, and other natural statements connecting obligation to goodness and agency.

While we have not developed the deontic extension of the models in full detail and generality, the sketch above shows various ways how this could be done. This is sufficient to show that the roots of "deontic logic" can in fact be traced at least as far back as the late 11th century.

Chapter 4

Three 13th-century views on quantified modal logic

In this chapter we compare contemporary philosophical modal logic with three 13th-century views of modal logic. The comparison demonstrates that there is a fundamental difference between how these 13th-century logicians approached and used modal logic and how philosophical logicians of the 21st century approach and use modal logic. This gives us cause to be careful that we do not discount medieval modal logic as being narrow or unfruitful: Because its aims were different from ours, we should not expect it to be applicable in the same circumstances.

The three 13th-century texts that we consider in this chapter are William of Sherwood's *Introductiones in logicam* [WoS95] (translated into English with commentary in [WoS66]), the short text *De propositionibus modalibus* [Aq06], and Pseudo-Aquinas's *Summa totius logicae Aristotelis* [Pse06]. Of the three, the provenance of the *Introductiones* is best known; the author can be ascribed with confidence, and while a definitive date of the text is not known, it is quite likely that the text was compiled between 1240 and 1248, a period in which Sherwood was a master in the Arts Faculty at the University of Paris [WoS66, p. 8]. The other two texts are both connected to St. Thomas Aquinas. Aquinas was long considered to be the author of the *Summa*, though current thought is that this is highly unlikely. Conversely, the authorship of the *De modalibus* text was considered doubtful until the early 20th century when Grabmann attributed it to Aquinas; if he is the author of the *De modalibus*, it is a juvenile and early work [O'G97, p. 13]. We shall follow Grabmann in attributing *De modalibus* to Aquinas, but reflecting our uncertainty about the authorship of the *Summa*, we will refer to the author of that text as Pseudo-Aquinas. Despite questions about the authorship of the two texts, it is clear from their content that they date from the same period as Sherwood's *Introductiones* or slightly later [Es56].

Before we can discuss the views of these three authors, and how they compare to modern approaches to modal logic, we must first specify what we mean by the phrase "modal logic". In this chapter we do not take the broad approach, but

rather focus on just the alethic modalities. The term "modal" comes from Latin *modus* 'mode, mood', and when medieval authors speak of adding a *modus* to a sentence, they generally specify that it is one of the following six modes: *verum* 'true', *falsum* 'false', *necessarium* 'necessary', *impossibile* 'impossible', *possibile* 'possible', *contingens* 'contingent'. We follow this customary usage in this chapter, and restrict our attention to just the modalities of necessity and possibility.

The course of this chapter follows the normal course of developing logic in medieval treatises: first we discuss modal propositions, in §4.1, discussing their construction, quantity, and quality. Next in §4.2 we discuss the inferential relations that hold between modal propositions, including conversions of modal propositions, implications between sets of modal propositions, and general modal syllogistic reasoning. We then compare this information with modern approaches to modal propositions and their inferential relations in §4.3. A note about references: Citations from William of Sherwood refer to page numbers unless a section number is explicitly indicated. The Aquinas text is referenced by sentence number, and Pseudo-Aquinas by tract, chapter, and sentence number. Latin texts which are not translated in footnotes in this chapter can be found translated in Appendix B.

4.1 Modes and modal propositions

All three of the 13th-century authors define modal propositions as being constructed from categorical propositions (recall that a categorical proposition or statement is, à la Sherwood, *cuius substantia consistit ex subiecto et praedicato* [WoS95, p. 12][1]). The class of modal propositions is defined in a jointly semantic-syntactic fashion. First, on the syntactic side, a modal proposition is a categorical proposition to which a mode has been added. The three authors all give slightly different definitions of *modus* 'mode'. Aquinas says that a mode is a *determinatio adiacens rei, quae quidem fit per adiectionem nominis adiectivi, quod determinat substantivum... vel per adverbium, quod determinat verbum* [Aq06, 2], that is, both adverbs and adjectives are modes. Pseudo-Aquinas says a mode is an *adjacens rei determinatio; idest, determinatio facta per adjectivum* [Pse06, tract. 6, cap. 11, 2], that is, modes are adjectives. And Sherwood takes the other route; his definition of mode includes only adverbs: *Modus igitur dicitur communiter et proprie. Communiter sic: Modus est determinatio alicuius actus, et secundum hoc convenit omni adverbio* [WoS95, p. 32].[2,3]

[1] "one whose substance consists of a subject and a predicate" [WoS66, p. 27].

[2] "The word 'mode' is used both broadly and strictly. Broadly speaking, a mode is the determination of an act, and in this respect it goes together with every adverb" [WoS66, p. 40].

[3] As de Rijk argues in [deR76, pp. 39, 41–42], these definitions show that not only did Aquinas *not* "cop[y] Sherwood almost *verbatim*", as Prantl, argues but rather that "there is not any reason to think that Aquinas here 'copies' (as Prantl says) Sherwood or is dependent

But not all categorical sentences to which adverbs or adjectives have been added are, strictly speaking, modal. The second part of the definition, which the three authors all include, is the semantic side: It is only those categorical statements where the adverb determines or modifies the composition of the subject and the predicate that are correctly called modal.[4] This *determinatio* is a semantic concept, as it modifies the *significatio* ('signification', roughly, the meaning) of the sentence. The six modes which can determine the inherence expressed in a categorical sentence are *verum*, *falsum*, *necessarium*, *impossibile*, *possibile*, and *contingens*. However, because the addition of "true" and "false" to a categorical proposition does not change its signification (because *nihil addunt supra significationes propositionum de inesse* [Aq06, 9]) these two modes will be omitted from consideration and the focus will be on the four modes *necessarium*, *impossibile*, *possibile*, and *contingens*.

At this point in his presentation of modality, Sherwood makes a distinction which the other two authors do not. He notes that there are two ways that *impossibile* and *necessarium* can be used. Both ways can be expressed in terms of temporal notions:

> *uno modo, quod non potest nec poterit nec potuit esse verum, et est impossible per se... alio modo, quod non potest nec poterit esse verum, potuit tamen ... et est impossibile per accidens. Et similiter dicitur necessarium per se, quod non potest nec potuit nec poterit esse falsum... Necessarium autem per accidens est, quod non potest nec poterit esse falsum, potuit tamen* [WoS95, p. 34].[5]

Essentially, Sherwood is defining the necessity operators as follows, translated into the familiar notation of temporal logic:

$$\begin{aligned} \Box_{ps}\varphi &:= \varphi \wedge G\varphi \wedge H\varphi \\ \Box_{pa}\varphi &:= \varphi \wedge G\varphi \wedge \Diamond\neg H\varphi \end{aligned}$$

(We discuss the correct interpretation of the $\Diamond$ in the definition of necessity *per accidens* in §4.3.) As we'll see in §4.2, we can define the impossibility operators from the necessity operators by negation, so we do not need to list them separately.

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upon him".

[4] *Quidam determinat compositionem ipsam praedicati ad subiectum... et ab hoc solo modo dicitur propositio modalis* [Aq06, 6]; *modalis vero in qua inhaerentia praedicati ad subjectum modificatur* [Pse06, tract. 6, cap. 7, 4]; *proprie sic: modus est determinatio praedicati in subiecto* [WoS95, p. 32], "strictly speaking, a mode is the determination of [the inherence of] the predicate in the subject" [WoS66, p. 40].

[5] "[impossible] is used in one way of whatever cannot be true now or in the future or in the past; and this is 'impossible *per se*'... It is used in the other way of whatever cannot be true now or in the future although it could have been true in the past... and this is 'impossible *per accidens*'. Similarly, in the case something cannot be false now or in the future or in the past it is said to be 'necessary *per se*'... But it is 'necessary *per accidens*' in case something cannot be false now or in the future although it could have been [false] in the past" [WoS66, p. 41].

According to Sherwood, *possibile* and *contingens* also have twofold usage. On the one hand, they can be used of statements which can both be true and be false, and so are neither impossible or necessary; this is the sense which is generally ascribed to "contingent" in modern usage. On the other hand, they can be used of statements which can be true, even if they cannot be false; this is the sense which is generally ascribed to "possibility" in modern usage, under the assumption that the axiom $\Box\varphi \rightarrow \Diamond\varphi$ is valid. While some medieval authors follow this distinction, using *possibile* for things which can be true, even if they cannot be false, and *contingens* in the stricter fashion for things which can be true or false, the two terms were regularly conflated, and as they were in the three texts we're considering, we'll follow their lead.

4.1.1 Construction

Once the relevant modes have been identified, the syntactic ways that they can be added to a categorical proposition must be distinguished. There are two ways that a mode can determine the composition of a categorical proposition. The three authors each make the distinction, but in slightly different ways and with different labels.

Aquinas's text divides modal propositions into those which are modal *de dicto* and those which are modal *de re*. This text is generally credited as being the source of the use of this distinction in modern philosophy and modal logic.[6] He makes the distinction this way:

> *Modalis de dicto est, in qua totum dictum subiicitur et modus praedicatur, ut Socrates currere est possibile; modalis de re est, in qua modus interponitur dicto, ut Socratem possibile est currere* [Aq06, 16].

The *dictum* of a sentence is what the sentence expresses; a categorical proposition's *dictum* can be formed, as Aquinas tells us, by substituting the infinitive form for the indicative verb, and the accusative case for the nominative subject.[7] This same distinction is found in Pseudo-Aquinas but in a more elaborate fashion:

> *[N]otandum quod quaedam sunt propositiones modales de dicto, ut, Socratem currere est necesse; in quibus scilicet dictum subjicitur, et*

[6] See [vW51, p. 1], where the terms are first introduced in modern contexts. Von Wright credits Aquinas with this distinction, probably in reference to *De modalibus*, as this was attributed to Aquinas by the 1950s. Dutilh Novaes in [Dut04, fn. 9] notes that von Wright was introduced to the distinction by Peter Geach.

Though the terminology appears to be new, the distinction is not. When discussing modal statements in Chapter 12 of *De interpretatione*, Aristotle asked whether 'the modal word modif[ies] the predicate of a sentence or the whole sentence?' According to [KnKn84, p. 83], Aristotle's answer is that "the modal words must be taken as modifying the whole sentence in which they occur and not any single word or phrase in it".

[7] *[Q]uod quidem fit si pro verbo indicativo propositionis sumatur infinitivus, et pro nominativo accusativus* [Aq06, 12].

> *modus praedicatur: et istae sunt vere modales, quia modus hic determinat verbum ratione compositionis, ut supra dictum est. Quaedam autem sunt modales de re, in quibus videlicet modus interponitur dicto, ut, Socratem necesse est currere: non enim modo est sensus, quod hoc dictum sit necessarium, scilicet Socratem currere; sed hujus sensus est, quod in Socrate sit necessitas ad currendum* [Pse06, tract. 6, cap. 11, 14–15].

Sherwood makes the same distinction but does not use the *de dicto*/*de re* terminology. Instead he distinguishes between adverbial modes and nominal modes; categorical propositions with adverbial modes correspond to the class of *de re* modal sentences, and those with nominal modes correspond to the class of *de dicto* modal sentences [WoS95, pp. 34–38].

The distinction is best understood with a few examples. If we take the assertoric categorical sentence "Some red thing is black", and the mode "possible", then the *de dicto* composition of the two is "It is possible that some red thing is black", which is not only false but necessarily false (since colors are considered to be exclusive). The *de re* composition is "Some red thing is possibly black", which is true (namely, this red door which at some point in the future I will be painting black).

4.1.2 Quantity

The type of modal sentence (that is, whether it is *de re* (or adverbial) or *de dicto* (or nominal)) must be established before the further properties of the sentence can be determined. Modal propositions, like categorical propositions, have both quantity and quality, and the authors give rules by which the quantity and quality of modal propositions can be recognized. The quantity of a categorical proposition can be one of four types: singular, particular, universal, or indefinite. A categorical proposition is singular when the subject term picks out only one object, e.g., because it is either a proper name or because it is modified by a demonstrative pronoun such as *hoc* or *illud*. It is particular when the subject term picks out more than one object, and is modified by a particular quantifier such as *quoddam* or *aliquid*. It is universal when the subject term picks out more than one object, and is modified by a universal quantifier such as *omnem* or *nullum*. Finally, a categorical is indefinite when the subject term refers to more than one object, but is not modified but either a universal or a particular quantifier.[8]

The division into modal statements *de dicto* and *de re* is motivated partly by the differences in how the quantity of the two types of statements is determined. Modal *de re* statements have the same quantity as their underlying categorical

[8]These are essentially Sherwood's definitions [WoS95, p. 14], which are typical for the period. Note that these definitions are in contrast with the modern use of 'universal', 'particular', and 'singular', which are purely syntactic categories.

sentences. But this is not the case for modal *de dicto* statements. According to both Aquinas and Pseudo-Aquinas, *de dicto* statements always have singular quantity, even though they may contain universal or particular quantifiers within them.[9] This is because the subject of a *de dictum* sentence is a *dictum*, and a *dictum* is essentially a proper name; it has a unique referent. Because Sherwood doesn't use the *de dicto*/*de re* distinction, his identification of the quantity is phrased somewhat differently, but with the same end result: When a categorical statement with a nominal mode is interpreted as if it had an adverbial mode, then the quantity of the sentence is determined by the quantity of the underlying categorical claim. But when it is not interpreted this way, then the *dictum* of the sentence is the subject, and this is singular.

4.1.3 Quality

The quality of a proposition (categorical or otherwise) is determined by the presence or absence of a negation: For categorical sentences, it is the negation of the composition between the subject and the predicate, for modal sentences it is the negation of the mode. If the composition or the mode is affirmed, then the sentence is affirmative, and if it is denied, then it is negative. In this way, a categorical proposition which is negative can become positive when made into a modal proposition, and similarly a positive categorical proposition can become negative when made modal, because, as Aquinas notes, *propositio modalis dicitur affirmativa vel negativa secundum affirmationem vel negationem modi, et non dicti* [Aq06, 19]. As an example, *Socrates non currit* is a negative categorical proposition, but *Socrates non currere est possibile* is an affirmative modal proposition. Note that the quantity of a proposition is a syntactic property, because it depends on the presence or absence of the term *non*, whereas the quality of a categorical proposition is semantic, because it does not depend on the addition of a specific term but rather on the truth conditions of various predications of the subject term on different objects.

The importance of being able to determine the quality and quantity of a modal proposition is grounded in the importance which is ascribed to the inferential relations of modal propositions, as it is the quality and the quantity that determines which propositions can be inferred from which others. We discuss these next.

[9] *Sciendum quod omnes enunciationes modales de dicto sunt singulares, quantumcumque sit in eis signum universale* [Pse06, tract. 6, cap. 11, 21]; *sciendum est autem quod omnes modales de dicto sunt singulares, eo quod modus praedicatur de hoc vel de illo sicut de quodam singulari* [Aq06, 17].

ordo 1	*possibile* *contingens* *non impossible* *non necessarium non*	*non possibile* *non contingens* *impossibile* *necessarium non*	*ordo* 3
ordo 2	*possibile non* *contingens non* *non impossibile non* *non necessarium*	*non possibile non* *non contingens non* *impossibile non* *necessarium*	*ordo* 4

Figure 4.1: The four *ordines*

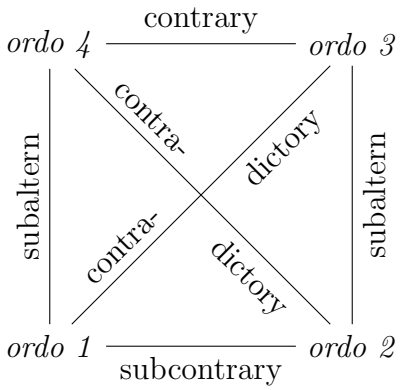

Figure 4.2: Modal square of opposition

4.2 Inferential relations

The inferential relations which are discussed in the three treatises can be divided into two groups: implications and conversions. The implications considered are the relations of contradiction, contrariety, subcontrariety, subalternation, and superalternation (the relations which make up the square of opposition). The conversions considered are the traditional Aristotelian ones, conversion *per accidens* and conversion *simplex*, along with equivalences which can be generated through the square of opposition. These implications and conversions are used to develop a modal syllogistic.

4.2.1 Implications

Sherwood notes that modes can be combined with negation in one of the following four ways [WoS66, p. 48]:

without negation	(A)
with more than one negation[10]	(B)
with one negation, before the mode	(C)
with one negation, after the mode	(D)

Since we have four modes, and four ways that a mode can be combined with negation, this gives us sixteen syntactically different modes; these modes can occur both adverbially and nominally (or, to say the same thing, in *de dicto* and in *de re* statements). The question is whether these sixteen syntactically different modes are all semantically distinct, or whether there are any pairs which are equipollent (to call them by their standard medieval name *equipollens*). The answer is that each of the sixteen can be placed into one of four groups, called *ordines* 'orders' or 'series' (see Figure 4.1). An order is essentially an equivalence class, since *omnes propositiones, quae sunt in eodem ordine, aequipollent* [Aq06, 23].

The four orders make up the corners of a square of opposition illustrating the inferential relationships (see Figure 4.2).[11] This square of opposition can be found in the manuscripts of each of the three treatises. After the square of opposition is presented, Aquinas's treatise makes reference to the mnemonic poem for constructing the square, whereupon the text ends.[12]

[10] I.e., one before the mode and one after.

[11] By treating *contingens* and *possibile* as strictly equivalent, rather than defining *contingens* as *possibile et possibile non*, all three authors avoid the problems faced by Aristotle in Chapter 13 of *De interpretatione* discussed in [KnKn84, pp. 84–86].

[12] The mnemonic poem shows up in various forms in 13th-century texts. Sherwood gives the text as follows:

4.2.2 Conversions

Both Sherwood and Pseudo-Aquinas discuss how modal propositions can be converted from one form to another. By "conversion" both authors mean the two types of conversion which Aristotle presents in giving rules for the proving of syllogisms, conversion *simplex* or *per se* and conversion *per accidens*. Simple conversion of a categorical proposition exchanges the subject and predicate terms, leaving the quality and the quantity of the sentence unchanged; accidental conversion swaps the subject and predicate, but also changes the quantity, from universal to particular or vice versa.[13] Sherwood also mentions a third type of conversion, conversion *per contrapositionem*, where the subject and predicate are swapped and replaced with their infinite counterparts (e.g., 'man' is replaced with 'non-man'; *infinitum* is the standard medieval name for such terms.)

In tract. 7, cap. 3, Pseudo-Aquinas tells us that *propositiones de necessario et impossibili eodem modo convertuntur sicut propositiones de inesse, et per idem principium probantur* [Pse06, 2]. Though he does not say so explicitly, it is clear from all of his examples that he is discussing conversion principles for *de dicto* statements; all of his examples use nominal modes, not adverbial ones. Here Pseudo-Aquinas follows Aristotle, whose treatment of the conversion of modal propositions was "guided almost entirely by the insight of the *De Interpretatione* that the modal word qualifies a whole proposition" [KnKn84, p. 86].

Because it is not obvious that necessary and impossible propositions can be converted in the same way that assertoric (that is, categorical) propositions can be, he gives proofs for various conversions. We give the first, because it exemplifies the techniques used in the rest. It is a proof that

$$\textit{necesse est nullum } b \textit{ esse } a \tag{4.1}$$

Sit tibi linea subcontraria prima secunde.
Tertius est quarto semper contrarius ordo.
Tertius est primo contradictorius ordo.
Pugnat cum quarto contradicendo secundus.
Prima subest quarte vice particularis habens se.
Hac habet ad seriem se lege secunda sequentem [WoS66, fn. 91].

Aquinas rearranges the lines, adds a few of his own, and omits some of Sherwood's: *Tertius est quarto semper contrarius ordo. Pugnat cum quarto contradicendo secundus. Sit subcontraria linea tibi prima secundae. Tertius est primo contradictorius ordo. Prima subest quartae vicem particularis habens. Sed habet ad seriem se lege secunda sequentem. Vel ordo subalternus sit primus sive secundus. Primus amabimus, edentulique secundus. Tertius illiace, purpurea reliquus. Destruit u totum sed a confirmat utrumque, destruit e dictum, destruit i que modum* [Aq06, 35–4]. Pseudo-Aquinas reduces the poem to just the names for each of the corners of the square: *Amabimus, edentuli, illiace, purpurea* [Pse06, tract. 6, cap. 13, 22].

[13] *Dicitur autem conversio simplex, quando de praedicato fit subjectum, et de subjecto praedicatum, manente secunda propositione in eadem qualitate et quantitate cum prima. Per accidens vero dicitur, quando de subjecto fit praedicatum, et e converso, manente eadem qualitate propositionis, sed mutata quantitate* [Pse06, tract. 7, cap. 2, 4–5]; a similar definition can be found in [WoS95, cap. 3, §2].

can be simply converted into

necesse est nullum a esse b (4.2)

First, Pseudo-Aquinas notes that the opposite of (4.2) implies the opposite of (4.1). But the opposite of (4.2),

non necesse est nullum a esse b (4.3)

is equipollent to

possibile est aliquod a esse b (4.4)

The equipollence between (4.3) and (4.4) holds because *impossibile* and *non necessarium non* are equipollent (as we saw in the previous section), and this latter equipollence holds because *non nullus* and *aliquis* are equipollent. Next, he notes that from (4.4) the following can be proved through an expository syllogism (an expository syllogism is one in which one premise is a singular proposition. Pseudo-Aquinas discusses these in tract. 7, cap. 2.):

possibile est aliquod b esse a (4.5)

But (4.5) is the contradictory of (4.1). Since we were able to prove the contradictory of the antecedent from the contradictory of the consequent, we can conclude that (4.1) can be converted into (4.2). That (4.2) can be converted back into (4.1) by similar reasoning is obvious.

The other proofs are similar and so will not be discussed further here.

4.2.3 Modal syllogisms

Sherwood tells the reader, before he even gives the definition of a mode, that the reason it is important to separate modal propositions from assertoric ones is that

> *[c]um intentio sit de enuntiatione propter syllogismum, consideranda est sub differentiis, in quibus differentiam facit in syllogismo. Quales sunt haec: ... modale, de inesse et aliae huiusmodi. Differt enim syllogismus a syllogismo per has differentias* [WoS95, p. 30].[14]

Kretzmann points out that "in spite of this remark, which seems to promise a consideration of the modal/assertoric difference as it relates to the syllogism, there is no treatment of modal syllogisms in any of the works that have been ascribed to Sherwood" [WoS66, fn. 58].

[14] "[s]ince our treatment is oriented toward syllogism, we have to consider them under those differences that make a difference in syllogism. These are such differences as... modal, assertoric; and others of that sort. For one syllogism differs from another as a result of those differences" [WoS66, p. 39].

Since Aquinas's text ends before discussing conversions and modal syllogisms (cf. §4.2.1), this leaves us with the *Summa*. Pseudo-Aquinas discusses modal syllogisms in tract. 7, caps. 13–15. Unfortunately, in many cases, his presentation is less than clear. The three chapters are devoted to the different combinations of necessary, impossible, and contingent premises with assertoric premises in syllogisms. Each combination is considered, and if it is valid, no argument is given, and if it is invalid, a counterexample is given. The result is an unfortunate tangle of case-by-case examples and rules with limited applicability.

Additionally, in giving the various examples of valid and invalid syllogisms, Pseudo-Aquinas moves between *de dicto* and *de re* formulations indiscriminately. For example, when he says that a syllogism in any mood or figure (for the technical details and terminology of Aristotelian syllogisms, see Appendix A.1) which has two necessary premises will have a necessary conclusion, the example that he gives is the following [tract. 7, cap. 13, 7–9]:

Necesse est omnem hominem esse animal.
Necesse est omne risibile esse hominem.
Ergo necesse est omne risibile esse animal.

But when he gives an example to show that a necessary conclusion does not follow from an assertoric major and a necessary minor, he uses *de re* modalities [tract. 7, cap. 13, 21–23]:

Omnis homo est albus.
Omne risibile necessario est homo.
Ergo omne risibile necessario est album.

The unclarity which results from his indiscriminate use of *de dicto* and *de re* statements in his examples is compounded by the fact that very few explicit rules for resolving the validity of classes of syllogisms are given. In assertoric syllogisms, the two rules commonly discussed are the *dici de omni* and the *dici de nullo*:

> *Est autem dici de omni, quando nihil est sumere sub subjecto, de quo non dicatur praedicatum; dici vero de nullo est, quando nihil est sumere sub subjecto, a quo non removeatur praedicatum* [Pse06, tract. 7, cap. 1, 36].

Pseudo-Aquinas often appeals to these two rules when he gives arguments for the invalidity of certain syllogisms with one modal and one assertoric premise. It is only when he considers syllogisms which have one necessary premise and one contingent or possible premise that he formulates a new rule. The rule is:

> *si aliquod subjectum sit essentialiter sub aliquo praedicato, quicquid contingit sub subjecto, contingit sub praedicato* [Pse06, tract. 7, cap. 15, 10].

Clearly this rule is an attempt to make a modal variant of the *dici de omni*.

It is at this point in the treatise that the modern logician could be forgiven for finding himself frustrated. The lack of both precision and perspicuity make one wonder whether there is anything to be gained in further study. If one is interested solely in developing a reliable modal syllogistic, there are other authors where this material is more easily accessible. But if one is interested in understanding the parts of the modal theory which are difficult not just because they are unclear but because they are fundamentally different from modern modal theories, then there are a number of things that can be said; we turn to these in the next section.

4.3 Contrasts with modern views of modal logic

We are now in the position to note two places where the medieval conception of modality and modal reasoning diverge from the modern conception of the same, with interesting consequences for our understanding of medieval modal logic.

4.3.1 The nature of modality

The first is that in modern propositional modal logic, the modality being expressed is the *de dicto* modality. A modal operator is an operator at the level of *formulas*. A formula of the form $\Box\varphi$ is read "it is necessary that φ", where the addition of "that" before "φ" is the syntactic construct in English for forming the *dictum* of a sentence. It isn't even clear that *de re* modality, with its emphasis on the inherence of the subject in the predicate, can be interpreted in a propositional context in a coherent fashion. Because of the subject-predicate nature of the medieval *de re* sentences, it is clear that the medieval logicians were working with some type of first-order logic, not a propositional logic. But in that context, there is some temptation to say either that *de re* statements do not say anything other than *de dicto* statements[15] or that *de re* statements aren't *really* about modality; they're just about a (perhaps special) type of predicates which we could call, e.g., possibly-P. But syntactically, these predicates are just like any other predicate, and semantically, we would be perfectly within our bounds to give the truth conditions to predicates like possibly-P in the same way that we do predicates like P, through an assignment function. Then we could use $\Box$ and $\Diamond$ to express *real* modality, modality applying at the level of entire formulas.

Both of these approaches to modality are in direct contrast with that of William of Sherwood. Sherwood is reluctant to accept categorical statements with nominal modes (that is, *de dicto* modals) as modal statements [WoS95, p. 36].[16] Recall that in §4.1 when we presented the different definitions of 'mode',

[15]The reduction of *de re* formulas to *de dicto* formulas, called "*de re* elimination", is done informally in [Kn62, §§5–6] and formally in [Fine78b], [Kam97], and [Schw97].

[16]A similar view is taken by Abelard according to Kneale, who says of the *de re*/*de dicto*

all three authors agreed that under the most strict interpretation, only those categorical sentences where the mode determines the inherence of the subject and predicate are really modal. Pseudo-Aquinas is willing to allow sentences such as *Socratem currere est necesse* to count as modal [tract. 6, cap. 11, 14], but specifically says that:

> *Sunt autem et aliae enunciationes quae videntur modales, et non sunt: quando videlicet modus subjicitur, et dictum praedicatur: ut, possibile est Socratem currere. Ratio hujus est, quia denominatio debet sumi a forma: formale autem in enunciatione est praedicatum, et ideo a praedicato debet denominari* [Pse06, tract. 6, cap. 11, 18–19].

Aquinas is also willing to let sentences such as *possibile est aliquod a esse b* to count as being determinations of the subject a in the predicate b, but does so without really spelling out how we are to understand this determination, but Sherwood will only call such sentences modal when they are interpreted in the *de re* fashion.[17] Under this interpretation:

> *Si enim dicam 'Socratem currere est contingens', idem est secundum rem ac si dicerem 'Socrates contingenter currit'* [WoS95, p. 38].[18]

Can modifications in the inherence of a subject in a predicate be represented in first-order modal logic? If the underlying categorical statement is universal or particular, then the distinction between the nominal and adverbial modes is easy: It is just the distinction between, e.g., $\Box\forall x F(x)$ and $\forall x \Box F(x)$ [FittMe98, §4.3]. Contemporary philosophical logicians working with the *de re*/*de dicto* distinction in modal logic formally define a formula φ to be *de dicto* if no free variable occurs in the scope of $\Box$, and *de re* otherwise (cf. [Fine78a, p. 135]).[19] But such a definition will not work for singular or indefinite statements; because they have no quantifier, all variables in such statements will be free, and hence on this definition all singular and indefinite statements would be *de re*.

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distinction that "whatever we may in the end come to think of [its] value... we can scarcely deny that there is something unfortunate in Abelard's doctrine that genuine modal statements never involve application of modal adjectives to *dicta*" [Kn62, p. 626].

[17] This contradicts Moody's assertion that one of the medieval contributions to modal logic was that "the important question of whether modal propositions are to be construed as object-language statements (*de re*) or as metalinguistic statements (*de dicto*) was raised, and resolved in favor of the second (and correct) interpretation; this involved recognition that modal logic belongs to the logic of propositions and not to the logic of terms" [Mood75, p. 386], since many medieval accounts of modal logic *rejected* the *de dicto* reading in favor of the *de re* reading. Moody's assertion is at variance with the views of Abelard, but in line with Aristotle, cf. footnotes 16 and 6, respectively.

[18] "if I say 'that Socrates is running is contingent', it is just the same, with respect to what is signified, as if I were to say 'Socrates is contingently running'" [WoS66, p. 45].

[19] For a sketch of an alternative way of handling *de re* modalities in a quantified context, see [GovRo03].

In [Fitt99, p. 108], Fitting gives two different ways that the formula $\Diamond P(c)$ could be read. Let $\mathfrak{M}^{\mathrm{Q}}$ be an extended frame (cf. §A.3); then the two possibilities for $\mathfrak{M}^{\mathrm{Q}}, w \vDash \Diamond P(c)$ can be represented as:

1 There is a world x such that wRx and $\mathfrak{M}^{\mathrm{Q}}, x \vDash_V Py$ where $V(y) = I(c, x)$.

2 There is a world x such that wRx and $\mathfrak{M}^{\mathrm{Q}}, x \vDash_V Py$ where $V(y) = I(c, w)$.

The first reading is interpretable as modality *de dicto*: The most natural reading of "it is possible that c is P" is "there is a possible world where the interpretation of c at that world is in the interpretation of P". The second is a plausible reading of modality *de re*, namely that what c actually is in the current world, that very thing is in the interpretation of P in another possible world.

This means that sentences of the form $\Diamond P(c)$ are essentially ambiguous: Their syntactic structure gives no clues as to whether they should be interpreted in the first or the second way. But from the point of view of the medieval logicians, this is precisely what they want: Natural language sentences such as *Socrates est possibile currere* are ambiguous, and we, as users of natural language, must make a choice in the interpretation of the sentence (perhaps based on context) when we wish to reason about it in a formal setting. The choice of interpretation will, naturally, affect the validity of the syllogisms in which these premises are found.[20]

This distinction is given in terms of simple predications, but its analysis easily extends to more complicated sentences such as *Omnis homo est possibile currere*. If we formalize this as $\forall y(Hy \rightarrow \Diamond_{dr} Cy)$[21] to show that we are interested in the *de re* analysis, then $\mathfrak{M}^{\mathrm{Q}}, w \vDash \forall y(Hy \rightarrow \Diamond_{dr} Cy)$ is true if and only if for arbitrary m,

$$\begin{aligned} &\text{if } I(m) \in I(H, w), \\ &\qquad \text{then there exists } x, wRx \text{ and } \mathfrak{M}^{\mathrm{Q}}, x \vDash_v C(y) \\ &\qquad\qquad \text{where } y \in I(m, w) \end{aligned} \tag{4.6}$$

Note that x can be different for different m; this is exactly what we want, for if we required that it be the same world where all the currently existing men are running, then the sentence would collapse into the *de dicto* reading.

4.3.2 The truth conditions of modal sentences

The second discrepancy between modern modal logic and medieval logic as presented in these three texts comes from the emphasis. In modern modal logic, emphasis is placed on the truth conditions of the modal propositions considered

[20] Since this is not an acceptable solution for many contemporary logicians, Fitting in [Fitt99, §3], Fitting and Mendelsohn in [FittMe98, ch. 9], and Garson in [Gar06, ch. 19] all introduce lambda abstraction to solve the problem.

[21] For present purposes it does no harm to omit consideration of existential import.

in and of themselves; when working with Kripke semantics, this emphasis manifests itself in the choice of the R relation or a restriction on the valuation functions for the propositions. This is in contrast to the three texts that we've seen, where the emphasis is placed on the inferential relations between modal propositions, e.g., the relations which form the square of opposition, conversions and of modal propositions, and classes of valid syllogisms. (Speaking anachronistically, we could say that the medieval logicians were more interested in proof theory than in model theory.) Pseudo-Aquinas does not provide any explicit truth conditions for modal propositions *considered in themselves* (as opposed to *considered with respect to other modal propositions*). This is most surprising when considered in conjunction with the stated goal of the entire treatise. *Omnes homines natura scire desiderant*, the text opens [Pse06, Prologue, 1].[22] But, he goes on to say, knowledge only comes as a result of demonstration, and a demonstration is a valid syllogism with necessarily true premises. Because this is the only route to knowledge (valid syllogisms which have merely, but not necessarily, true premises can only lead to probable knowledge; these syllogisms are subsumed under 'dialectic', which our author says he will not consider in this treatise [Pse06, Prologue, 11]), it is quite surprising that nothing is said about how to determine whether a premise is necessarily true, or (a slightly different question) whether a necessary premise is true.

Aquinas devotes two sentences to the truth conditions of modal propositions, when he draws a conceptual parallel between the four modes and the four combinations of quality and quantity in categorical propositions. He says:

> *Attendendum est autem quod necessarium habet similitudinem cum signo universali affirmativo, quia quod necesse est esse, semper est; impossibile cum signo universali negativo, quia quod est impossibile esse, nunquam est. Contingens vero et possibile similitudinem habent cum signo particulari: quia quod est contingens et possibile, quandoque est, quandoque non est* [Aq06, 20–21].

This interpretation of necessity and impossibility corresponds to Sherwood's definition of necessity and impossibility *per se* that we saw in §4.1. And as we saw in §4.2 that impossibility can be defined from necessity using negation, so too can possibility; so the type of possibility that Aquinas is discussing here is possibility *per se*, meaning that we can also formalize it with temporal notions, as

$$\Diamond_{ps}\varphi := (\varphi \vee F\varphi \vee P\varphi) \wedge (\neg\varphi \vee F\neg\varphi \vee P\neg\varphi) \tag{4.7}$$

But if this temporal formula expresses the truth conditions of sentences of possibility and contingency, and there is little reason to think that Sherwood would reject this definition while accepting the other, then we are left with the question

[22] "Every man by nature desires to know."

of what exactly Sherwood means when he says that a statement which is necessary *per accidens* "could have been false in the past". That is, we must ask what type of possibility is being expressed by the $\Diamond$ in

$$\Box_{pa}\varphi := \varphi \wedge G\varphi \wedge \Diamond\neg H\varphi \tag{4.8}$$

We can prove easily that $\Diamond\varphi$ here cannot be a shorthand for $(\varphi \vee F\varphi \vee P\varphi) \wedge (\neg\varphi \vee F\neg\varphi \vee P\neg\varphi)$. Let w be an arbitrary point where $\Box_{pa}\varphi$ is true. We know then that $w \vDash \varphi$ and $w \vDash G\varphi$, and

$$w \vDash (\neg H\varphi \vee F\neg H\varphi \vee P\neg H\varphi) \wedge (H\varphi \vee FH\varphi \vee PH\varphi) \tag{4.9}$$

The problem is the second conjunct. If $\Box_{pa}\varphi$ is to be distinguished from $\Box_{ps}\varphi$, we know that neither $H\varphi$ nor $FH\varphi$ can be true, for then the two would be equivalent. Thus there is some $t \ll w$ such that $t \vDash H\varphi$; t cannot be an immediate predecessor of w or otherwise $PH\varphi$ would be equivalent with $H\varphi$, if we assume reflexivity. But this with the first conjunct forces there to be some t', $t < t' < w$ where $t' \vDash \neg\varphi$. And in this case, the interpretation of $\Box_{pa}\varphi$ would be that φ is true now and always in the future, but was false at some point in the past, and not that φ is true now and always in the future but *could have been* false in the past (even if it never was). If we take seriously Sherwood's counterfactual truth conditions for necessity *per accidens*, then the possibility involved cannot be temporal possibility.

There is a natural solution to the problem, though it is not one explicitly endorsed by Sherwood. If we remember that φ is not just a simple propositional construct, but a subject-predicate sentence like *Socrates est necessario currere*, then we can solve the question of the interpretation of $\Diamond$ by using the formal distinction between the *de re* and *de dicto* readings that we presented in the previous section. Then if *Socrates est necessario currere* is interpreted with necessity *per accidens*, it can be rewritten as

$$C(s) \wedge \Box_{ps}FC(s) \wedge \Diamond_{dr}\neg C(s) \tag{4.10}$$

that is, Socrates is running now, it is necessary *per se* that he is running in the future, but he is possibly (*de re*) not running. The reason that this explication doesn't collapse the same way that the other one did is that the *de re* possibility here is not defined with respect to past, present, or future times, but to possible worlds; i.e., this type of possibility is in a sense perpendicular to the temporal notion of possibility (see Figure 4.3). And thus we see how Sherwood's insistence that it is the adverbial modal sentences which are the real modal sentences, and not the nominal ones, can be used to explain how, under a temporal notion of modality, the distinction between necessity *per accidens* and necessity *per se* can be maintained in the way that he has defined them.

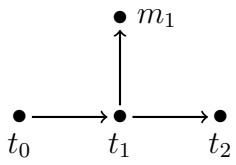

Figure 4.3: $\Box_{ps}$ is evaluated w.r.t. t_n, $\Diamond_{dr}$ w.r.t. m_1

4.3.3 Concluding remarks

In [FittMe98, §4.4], Fitting and Mendelsohn address the question "is quantified modal logic possible?" They note that

> for much of the latter half of the twentieth century, there has been considerable antipathy toward the development of modal logic in certain quarters. Many of the philosophical objectors find their inspiration in the work of W.V.O. Quine, who as early as (Quine, 1943), expressed doubts about the coherence of the project... Quine does not believe that quantified modal logic can be done coherently... [FittMe98, p. 89]

Fine echoes this sentiment when he discusses the philosophical position of *de re* skepticism, which he defines as "the doctrine that quantification into modal contexts does not, as it stands, make sense" [Fine78a, p. 125]. Such philosophical doubts are cited as the cause for the lack of development of quantified modal logic in modern times; Garson in his introduction says

> The problem is that quantified modal logic is not as well developed... Philosophical worries about whether quantification is coherent or advisable in certain modal settings partly explains this lack of attention [Gar06, p. xiii]

This suspicion of quantified modal logic is deep-seated and pervasive among contemporary logicians both philosophical and mathematical (skim through any article which discusses quantified modal logic from a philosophical point of view, and you will find at least one disparaging remark about it; likewise, see the references in footnote 15 to see mathematical logicians' attempts to remove *de re* modality from their systems). In this chapter we have shown that the medieval logicians took a very different approach to modality, believing that quantified modal logic is not a scary, intractable field of study, but in fact can be developed in a systematic fashion from the logic of simple categorical statements, and not only is this development conceptually quite natural, it is in some sense more natural than a modal logic for unanalyzed propositions.

Chapter 5

A quantified predicate logic for ampliation and restriction

5.1 Introduction

Natural language occurs in written, spoken, and mental utterances. For the medieval logicians, it is these utterances, not the propositions expressed by them, which bear the truth values. Since, in Latin (the standard language of the logicians, whatever their vernacular was), every utterance has either a past-, present-, or future-tensed verb, the analysis of the truth conditions for an utterance essentially involves time. Thus, there was no such differentiation of logic into tensed and untensed. As a result, no analysis of the truth conditions of natural language sentences can be done without some reference to the tense of the main verbs. In a sense, for the medieval logicians, all logic was temporal logic.

This medieval approach to temporal logic, which is highly pragmatic in nature, contrasts with the development of the modern discipline of temporal logic, which is treated separately and as distinct from ordinary logic, whether predicate or propositional. Modern propositional logic is essentially timeless, dealing with timeless properties and relations. The addition of a temporal structure involves the addition of a more complex semantics, usually involving possible worlds or states of affairs, and an accessibility relation between those worlds.

The fact that medieval logic is essentially temporal in nature is well illustrated in the theories of supposition that developed in the 12th century and later.[1] While the present-tense fragments of various theories of supposition have been studied from a formal perspective in recent years, the parts of these theories that deal with future- and past-tensed sentences have not been so studied.

Our purpose in this chapter is to use the techniques of modern temporal logic to provide a formal analysis of the future- and past-tensed parts of supposition

[1]For an overview of medieval theories of supposition, see [Re08, §§3ff], and the references cited therein, and also [Bos–].

theory. Such a formal analysis has many benefits: it will show that supposition theory, far from being mere scholastic wrangling with innumerable rules and definitions (as later scholars tended to complain), was an implicit solution to various philosophical problems which was lost to modern logicians when supposition theory was lost, in the post-medieval period. Since many of these problems still arise in philosophical discussions today, we will have yet another reason why it is important for contemporary philosophical logicians to be familiar with the developments of their medieval predecessors, as there is much to be gained from a close investigation of their theories. Further, we will give a way that these medieval solutions can be made accessible to modern practitioners by placing the solutions in a context familiar to people working in philosophical logic.

The techniques that we introduce in this chapter will be general in application, but because we cannot create a formal model without a particular theory to formalize, we have picked one text on which to focus. This text is the final chapter, *De suppositionibus et de significationibus*, of Lambert of Lagny's logic textbook, the *Summa Lamberti* or *Logica*.[2] The chapter starts with definitions of signification and supposition, followed by the standard division of supposition into its types[3], and then concludes with a discussion of *appellatio*, *restrictio*, *ampliatio*, *distributio*, and *relatio*. Lambert's supposition theory is not especially remarkable, and so serves as a good 'generic' theory that we can use to exemplify different parts of our formal model.

We thus begin in §5.2 with a discussion of Lambert's basic notions of signification and supposition, as we need to understand these before we introduce, in §5.3, the parts of Lambert's theory which are connected to analyses in terms of time and modality. These analyses we will formalize in a quantified modal-temporal framework in §5.4. Once we have our formal system, we apply it to Lambert's theory and prove some results about it in §5.5. In §5.6, we make some concluding remarks and point towards future work in this area, namely the question of iterated tenses, which Lambert does not address.

[2]The *Logica* is edited in [LoA71], with a translation of the final chapter in [LoA88]. All references are to these editions. The author of this work was previously identified as Lambert of Auxerre by, e.g., the editor and translator of [LoA71] and [LoA88]. The author is now generally identified as Lambert of Lagny [deL81]. Little is known about the life of Lambert, but we know that he was a Dominican friar living in the middle of the 13th century. His *Summa* was most likely written between 1253–57 at Troyes (or possibly Pamplona), and published in Paris probably around 1260 [deR76, p. 39]. It is similar in content and style to those of his contemporaries William of Sherwood (*Introductiones in logicam*, edited with a translation into German in [WoS95] and translated into English in [WoS66]), Peter of Spain (*Summulae logicales* edited in [PoS72] and translated in [PoS90]), and Roger Bacon (*Sumule dialectices* edited in [Bac$_2$40]), and indeed with most of the mid-13th-century logical compendia.

[3]The fact that supposition is so divided is standard; the division itself differs from author to author.

5.2 Basic notions and definitions

Even though the techniques that we present will have general application to ampliation and restriction in a variety of theories of supposition, we make Lambert's theory our case study, and in order to show how the formal tools we develop can be applied, we must give sufficient detail of Lambert's theory. In this section we cover the basic notions and definitions of his theory.

We begin with the properties of terms. It is important to study the properties of terms because sentences (utterances) are made up out of terms, and the properties of the terms will induce the properties of the sentences in which they occur. The most basic concept is that of signification. Both sentences and terms signify, with the signification of terms being prior to the signification of sentences. The signification of a term is, according to Lambert, the *intellectus rei ad quem intellectum rei vox imponitur ad voluntatem instituentis* (205)[4]. Four things are required for signification:

- A thing
- A concept of the thing
- An utterance
- A union of the utterance and the concept

A thing is any extra-mental thing, such as a substance (e.g., Socrates the man), an accident (e.g., the whiteness which is in Socrates), or an activity (e.g., the running which Socrates is doing right now). These extra-mental things are presented to the soul by means of a concept. A term gains signification when it is used in an utterance and the utterance is connected to a concept by the will of the speaker. The concept imposed upon the term in the speaker's utterance is the signification of the term.

Concepts are concepts of things, and terms signify concepts. Signification hence gives us an indirect way to speak of things, but it does not give us a direct way. In order to be able to speak not just of concepts, such as 'man', but of things which fall under those concepts, such as 'Socrates', we need a more sophisticated mechanism. That mechanism is supposition.

The supposition of a term is the *acceptio termini per se sive pro re sua, vel pro aliquo supposito contempto sub re sua vel pro aliquibus suppositis contemptis sub re sua* (206).[5] For example, the term 'homo' signifies the concept of man, but it can supposit either for the word *homo*, the concept man, or individual or multiple men. Supposition is anterior to signification; a term can have signification

[4] "concept on which an utterance is imposed by the will of the person instituting the term" (104).

[5] "acceptance of a term for itself, for its [signified] thing, for some suppositum contained under its [signified] thing, or for more than one suppositum contained under its [signified] thing" (106).

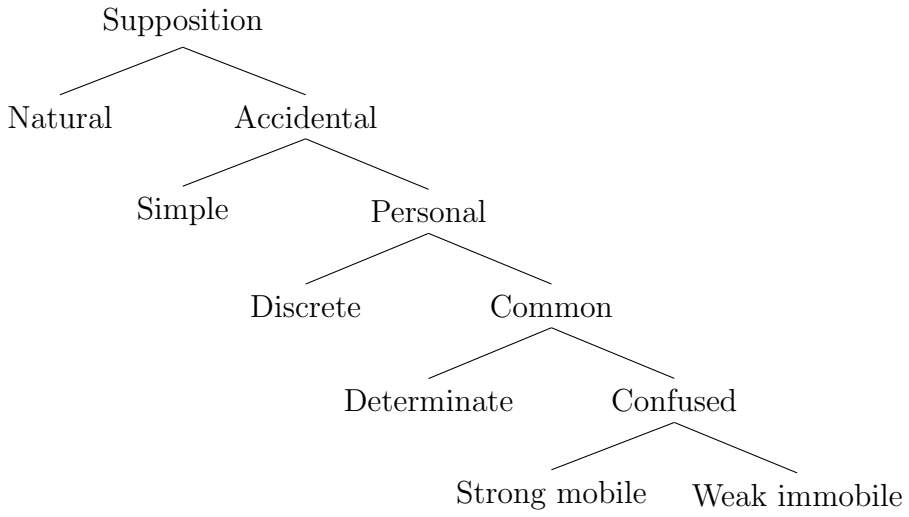

Figure 5.1: Lambert's division of types of *suppositio*.

in isolation, but a term only has supposition within the context of a complete utterance, and it is the context of the sentence that determines what the term supposits for. The different types of supposition can be divided into the tree in Figure 5.1. Lambert's definitions of these types of supposition are by and large orthodox. We present them here briefly. Natural supposition is *quam habet terminus a se et a natura se* (208)[6], e.g., the supposition of 'homo' is men. A term with natural supposition supposits *non solum supponit pro hiis que participant formam suam, immo pro omnibus hiis supponit qui participant formam suam, scilicet pro praesentibus preteritis et futuris* (208)[7]; and here we see how time is brought in to the definition of supposition.

Accidental supposition is *quam habet terminus ab adiuncto* (208).[8] This type of supposition is *secundum exigentiam illius cui adiungitur* (208)[9], and the example given is of a term which is conjoined to a tensed being verb (e.g., *est*, *erit*, *fuit*). Accidental supposition is divided into two types. The first type is simple, which is *illa secundum quam tenetur terminus pro se vel pro re sua, non habito respectu ad supposita sub se contempta* (209).[10] This is contrasted with personal supposition, which is *secundum quam terminus tenetur pro supposito vel pro suppositis* (209).[11] The reason why this latter type is called personal is that

[6] "what a term has on its own and by its nature" (109).

[7] "'not only for the things that share its form, but instead for all the things that share, [have shared, and will share] its form—i.e., for present, past, and future things [of that form]" (109).

[8] "what a term has from what is adjoined to it" (109).

[9] "in keeping with the requirement of that to which it is adjoined" (109).

[10] "the kind according to which a term is interpreted for itself or for its [signified] thing, without relation to the supposita contained under it" (110).

[11] "[the sort] according to which a term is interpreted for a suppositum or for supposita" (110).

suppositum vel individuum in substantia rationali idem est quam persona (209).[12]

Personal supposition is likewise divided into two types, discrete and common. Discrete personal supposition is *quam habet terminus discretus in se... ut quando sumitur terminus communis cum pronomine determinato* (209).[13] In *Socrates currit* and *iste homo currit*, both 'Socrates' and 'homo' have discrete supposition. Common supposition is, as expected, *illa que termino communi convenit* (210).[14]

Common supposition is divided into determinate and confused supposition. Determinate supposition is *illa quam habet terminus communis quando indifferenter potest sumi pro uno vel pro pluribus* (210).[15] The typical example of determinate supposition is *homo* in *homo currit*; the supposition of *homo* here is determinate because it is true *uno homine currente, vel pluribus* (210).[16] Confused supposition is *illa quam habet terminus communis quando de necessitate tenetur pro omnibus suis suppositis vel pro pluribus*; it is called "confused" because *ubi enim est multitudo ibi est confusio* (210).[17]

The last division is of confused supposition into strong mobile and weak immobile. Strong mobile supposition is *illa quam habet terminus communis quando tenetur pro omnibus suis suppositis de necessitate, et potest fieri descensus sub eo* (210).[18] This happens when a term with confused supposition is preceded by a universal affirmative or universal negative quantifier (e.g., *omnis* or *nullus*.) Weak immobile supposition is *illa quam habet terminus communis quando de necessitate tenetur pro pluribus suppositis sub se contemptis, non tamen pro omnibus, nec sub ipso potest fieri descensus* (211).[19] (The terms "mobile" and "immobile" refer to whether or not it is possible to make descents from the term to its *supposita*, cf. [Sp82].)

Our focus in the succeeding sections will be on common terms, terms that can apply to more than one object, because discrete terms *non potest restringi nec ampliari* (213)[20], and as we'll see in the next section, restriction and ampliation of terms are what give Lambert's logic its temporal character. But before we turn to restriction and ampliation, we must make a general point about signification. Signification is a conventional and use-based notion. A term gains signification

[12] "in the case of rational substance a suppositum or individual is the same as a person" (110).

[13] "what a discrete term has in itself... as when a common term is taken together with a determinate pronoun" (111).

[14] "the kind that is appropriate to a common term" (111).

[15] "what a common term has when it can be taken equally well for one or for more than one" (111).

[16] "if one man is running or if more than one are running" (111).

[17] "what a common term has when it is interpreted necessarily for all its supposita or for more than one"; "for where there is plurality there is confusion" (111).

[18] "what a common term has when it is interpreted for all its supposita necessarily and a [logical] descent can be made under it" (112).

[19] "what a common term has when it is interpreted necessarily for more than one suppositum contained under it but not for all, and a descent cannot be made under it" (112).

[20] "are not able to be restricted or ampliated" (116).

when a concept is imposed upon it by the will of a speaker. There are no constraints on what concepts can be imposed on what terms. It is a conventional fact that speakers of English almost always imposed the same concept upon the term 'dog', but that this convention exists is not a logical fact. In order to have a tractable logic, we are forced to gloss over some of the volitional and psychological aspects of signification. We do so by the following stipulation:

> Antecedent to any logical investigation, we set out that we will always impose on a term its standard (that is, dictionary or conventional) signification.

This stipulation is not strictly required, since we could build agents and signification functions for these agents into our logic, but doing so would not add any clarity or indeed any further expressivity for modeling the medieval theory. Lambert, for example, has no discussion what to do about the fact that signification can vary from person to person, or how it is that the hearer knows or understands which concept is imposed upon a term by the speaker. These are interesting questions, but they are not wholly logical ones, so we feel no compunction in leaving them aside here. Instead we take the common sense view that when we use the terms 'dog', 'cat', 'donkey', we intend to pick out the expected concepts.

5.3 Appellation, ampliation, and restriction

After introducing the divisions of types of supposition, Lambert defines five further modifications of supposition: appellation, restriction, ampliation, distribution, and relation. Of these five, we will focus on the first three, since the way each of the three affect the supposition of a term is defined in temporal or modal terms. *Appellatio* is *acceptio termini pro supposito vel pro suppositis actu existentibus* (212).[21] The connection between existence and appellation is stressed a few sections later when Lambert says:

> *Sciendum autem quod proprie loquendo non dicuntur appellata nisi sint acutaliter existentia; appellatur enim proprie quod est, et non quod non est, et ideo bene dicitur quod appellatio est pro existentibus suppositis vel pro supposito* (213).[22]

In the classification of supposition types given in the previous section, appellation is a type of personal supposition.[23] Specifically, it is common terms, not discrete

[21] "the acceptance of a term for a suppositum or for supposita actually existing" (114).

[22] "It is important to know, however, that appellata are not properly so-called unless they are actual existents; for what is, and not what is not, is properly appellated. And so it is right to say that appellation is for existent supposita, or for an [existent] suppositum" (115).

[23] *Appellatio semper est suppositio personalis* (212); "appellation is always personal supposition" (114).

terms, which have appellation. The appellation of a term is, informally speaking, its ordinary reference. If we speak of 'men' without any modifiers, we are speaking of currently existing men. However, there are times when we want to be able to speak of men, but not all currently existing men, or to speak of not just currently existing donkeys, but all donkeys that have ever existed or will ever exist. We can change the supposition of a term by restricting or ampliating it.

Lambert gives the following rule which connects appellation to supposition:

Rule 5.3.1. *Terminus communis substantialis vel accidentalis non restrictus aliunde supponens vel apponens verbo presentis temporis non habenti vim ampliandi a se nec ab alio, restringitur ad supponendum pro presentibus, si appellata habeat; si vero non, recurrit ad non existentiam* (213).[24]

Appellation allows us to speak of currently existing objects of a certain kind. However, we often want to speak of possible objects, or future or past objects, or a subset of objects of a specific kind. This is done through ampliating and restricting the supposition of a term.

Restriction and ampliation are each other's opposites. *Restrictio* is a *minoratio ambitus termini communis, secundum quam pro paucioribus suppositis teneter terminus communis quam exigat sua actualis supposito* (226).[25] One way that a common term can be restricted is through the addition of an adjective. For example, we can restrict the supposition of "man" by adding to it the adjective "white"; "white man" has fewer supposita than "man" unmodified. This type of restriction is called *naturalis*. This is distinguished from *usualis* 'use-governed' restriction, that is, restriction made by convention and not by the addition of a modify word. An example of use-governed restriction is "the queen is coming", where "queen" is not modified explicitly in the statement, but is, by convention, taken to mean the queen of the country in which the sentence is spoken.[26]

It is important to note that not all additions of modifying words or phrases to a common term will result in natural restriction: If the added word or phrase "destroys" the term, then the result is not called restriction, but diminution. An example of this is the modifying clause *qui non est*. Restriction requires that the common term still have appellata, but as defined in Rule 5.3.1, appellata must exist. There is no existing object which falls under the modified common term "man who does not exist".

If the modifying word or phrase does not restrict the supposition of a common term but rather expands it, then we are dealing with the opposite of restriction,

[24] "A substantial or accidental common term that is not restricted by any other means and that serves as the subject for the predicate of a present-tense verb that has no ampliating force of its own or from anything else is restricted to supposition for present things if it has appellata; but if it does not have appellata, it reverts to nonexistents" (116).

[25] "a lessening of the scope of a common term as a consequence of which the common term is interpreted for fewer supposita than its actual supposition requires" (134).

[26] *Cum dicitur: "rex venit"; iste terminus "rex" restringitur ad supponendum pro rege patrie in qua sermo iste dicitur* (277).

which is called ampliation. *Ampliatio* is an *extensio ambitus termini communis secundum quod teneri potest terminus communis pro pluribus suppositis quam exigit sua actualis suppositio* (228).[27] As an example, Lambert offers *homo potest esse Antichristus* (228); in this sentence, *homo* is ampliated to stand not only for current men but for future men.

Ampliation is caused by the addition of names (such as 'possible'), verbs ('can'), adverbs ('potentially'), or participles ('praised'):

> *Quedam enim sunt nomina que habent virtutem ampliandi ut possibile, necessarium; et similiter quedam verba ut potest...; similiter et quedam adverbia ut potentialiter necessario...* (228).[28]

Whichever kind the ampliating term is, ampliation can be divided into two types: *ampliatio...fit ratione suppositorum* and *ratione temporum* (228).[29] Ampliation by reason of *supposita* is caused by *verba quorum actus comparatus ad subiectum de subiecto dicitur, in subiecto tamen non est, ut sunt ista: potest, opinatur, laudatur* (229).[30] A term ampliated in this way stands for both actual and nonexistent *supposita*. Ampliation by reason of times is caused by modifiers which *faciunt terminum extendi ad omnes differentias temporis* (229).[31] Example of this kind of modifier are temporal operators such as "always", modal operators such as "necessarily" and "possibly", and changes in the tense of the verb. We'll call these two types of ampliation '*supposita* ampliation' and 'tense ampliation', respectively.

The interaction of restriction and ampliation with supposition is partly determined by whether the terms in question are accidental. In future or past tensed propositions, the following two rules are relevant:

Rule 5.3.2. *Terminus communis accidentalis non restrictus aliunde supponens verbo preteriti temporis, supponere potest pro presentibus et preteritis; apponens vero terminum supponit pro preteritis; si vero fuerit terminus substantialis supponens vel apponens verbo preteriti temporis, semper pro preteritis supponit* (223).[32]

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[27] "extension of the scope of a common term as a consequence of which the common term can be interpreted for more supposita than its actual supposition requires" (137).

[28] "For there are certain names that have the power of ampliating—e.g., 'possible', 'necessary'—and certain verbs likewise—e.g., 'can'...—similarly also certain adverbs—e.g., 'potentially', 'necessarily'..." (138).

[29] "ampliation...brought about because of supposita" and that brought out "because of times" (138).

[30] "verbs whose corresponding action is related to the subject and said of the subject but is not in the subject—such as 'can', 'is thought', 'is praised'" (138).

[31] "cause a term to be extended to all the differences of time" (138).

[32] "A common term pertaining to accident that is not restricted by any other means and that serves as the subject of a past-tense verb can supposit for present and past things even though the term serving as the predicate supposits [only] for past things; but if a term pertaining to substance serves as the subject or predicate of a past-tense verb, it always supposits for past things" (129).

Rule 5.3.3. *Terminus communis accidentalis non restrictus aliunde supponens verbo futuri temporis, supponere potest pro presentibus et futuris; apponens vero solum tenetur pro futuris; si vero fuerit terminus substantialis supponens vel apponens verbo futuri temporis, semper tenetur pro futuris* (223).[33]

These rules and the definitions of appellation, restriction, and ampliation provide the basis for our formal analysis of the temporal elements of Lambert's theory of supposition, which we give in the next section.

5.4 Constructing a formal model

In this section we adapt the general presentation of Kripke frames and quantified modal logic in Appendix A to our specific purposes. We specify our language and formal model, and give definitions of appellation, ampliation, and restriction.

Our base language is that of predicate logic with temporal operators[34], extended with four copulae for the different types of categorical sentences. That is,

[33] "A common term pertaining to accident that is not restricted by any other means and that serves as the subject of a future-tense verb can supposit for present and future things even though when serving as the predicate it is interpreted only for future things; but if a term pertaining to substance serves as the subject or the predicate of a future-tense verb, it is always interpreted for future things" (129).

[34] In Appendix A, we introduced modal and temporal operators separately. However, the truth conditions for $\Diamond$ and $\Box$ given there do not adequately capture Lambert's view of modality. Lambert follows one of the traditional medieval interpretations of modality, the statistical interpretation (cf. [Knu93, *passim*]), where 'necessarily' and 'possibly' are defined with reference to times, not to possible worlds. As he says:

> *Unde cum dicitur: "home est animal necessario" aliud est ac si diceretur: "id est, in omni tempore convenit homini esse animal", scilicet in presenti preterito et futuro* (229).[35]

Formally, this means the truth conditions of Lambert's modal operators are:

$$\begin{array}{lll} w \models \Diamond p & \text{iff} & \text{there is a } t, t \models p \\ w \models \Box p & \text{iff} & \text{for all } t, t \models p \end{array}$$

That is, we drop any reference to the accessibility relation. If we are dealing with linear time, it is possible to define these modal operators in terms of the temporal operators (note that this only works with linear time; if time branches either forward or backward or both, then these definitions fail):

$$\Diamond p := p \vee Fp \vee Pp$$

$$\Box p := p \wedge Gp \wedge Hp$$

Since we restrict our attention here to linear time, we will omit discussion of the modal operators since they are reducible to the temporal ones.

[35] "Thus when one says 'A man is an animal necessarily', it is the same as if one had said 'That is, in every time being an animal applies to a man: in the present, the past, and the future" (138).

for any predicate S and Q, the following are well-formed formulas:

$$S\mathtt{ua}Q \mid S\mathtt{un}Q \mid S\mathtt{pa}Q \mid S\mathtt{pn}Q$$

We also stipulate that we have a name for every object in our domain; since we are only working with finite domains, this means that our language will still be finite.

Recall that we noted that Lambert's view of modality is statistical, that is, it is defined in terms of temporal instants just as the tense operators are. This simplifies our system somewhat in that we do not need to specify a possibility relation in addition to a temporal ordering on the possible worlds. Formally, our models are 5-tuples $\mathfrak{M} = \langle T, <, O, E, V\rangle$, where:

T is a set of temporal instants. We let variables t with and without subscripts and superscripts, and w, x, y, and z range over T.

$<$ is a transitive, irreflexive, and backward linear relation on T; if $t < t'$ we say that t' is (temporally) later than t and t is (temporally) earlier than t'.

O is a finite set of objects. We let the variables $a, b, c \ldots$ range over O.

E is a function from O to subsets of T; if $t \in E(a)$, then we say that a *exists at* t and write $t \models \hat{a}$. We require that E be such that for every $t \in T$, there is an $a \in O$ such that $t \in E(a)$, that is, at every point in time, at least one object exists, and also that E be such that for every $t \in t$, there is an $a \in O$ such that $t \notin E(a)$, that is, at every point in time, at least one object does not exist.

V is a function from predicate-world pairs to subsets of O; if $a \in V(Q, t)$, then we say that a is Q at t.

As we noted in §A.3, the validity of the Barcan and the Converse Barcan formulas on an extended frame implies that models based on this frame are constant-domain. This is immediately problematic in the context of temporal logic, where we want objects to be able to come into and go out of existence as time passes (for an extensive discussion of the problems, see [Pr57]). As a result, if we want to work in a varying-domain model, we must ensure that neither the Barcan formula nor its converse is valid. Because we wish neither of these to be valid, we require that there exist $t, t', w, w' \in T$ such that $t < t'$ and $E(t) \not\subseteq E(t')$, and $w < w'$ and $E(w') \not\subseteq E(w)$.

We call propositions of the form $\hat{a}$ "existence propositions". We also extend V to a global function V' by setting $V'(Q) := \bigcup_{t \in T} V(Q, t)$. The function V' picks out all the objects in the model that a particular predicate is true of at some point in the timeline; we will use V' in the definition of one type of ampliation.

Finally, if we wish to formalize the medieval concept of existential import, then we can do so as follows:

Definition 5.4.1 (Existential import)**.** A model $\mathfrak{M}$ has *existential import* iff for all predicates Q and for all $t \in T$, $V(Q,t) \neq \emptyset$; that is, V is a function from predicate-world pairs to *non-empty* subsets of O.

From now on we will only work with models which have existential import.

Giving truth conditions for the four types of present-tensed categorical statements is straightforward.

Definition 5.4.2 (Truth conditions for categorical propositions)**.**

$$\begin{array}{lllll} \text{UA:} & t \models S\mathtt{ua}Q & \text{iff} & V(S,t) \subseteq V(Q,t) \\ \text{UN:} & t \models S\mathtt{un}Q & \text{iff} & V(S,t) \cap V(Q,t) = \emptyset \\ \text{PA:} & t \models S\mathtt{pa}Q & \text{iff} & V(S,t) \cap V(Q,t) \neq \emptyset \\ \text{PN:} & t \models S\mathtt{pn}Q & \text{iff} & V(S,t) \not\subseteq V(Q,t) \end{array}$$

It should be immediately clear from these definitions that the truth conditions for the four types of categorical propositions respect the relationships in the traditional square of opposition. We prove this just for the case of the subaltern relation:

Lemma 5.4.3. $t \models S\mathtt{ua}Q$ implies $t \models S\mathtt{pa}Q$.

Proof. Suppose $t \models S\mathtt{ua}Q$ Then by UA of Def. 5.4.2, $V(S,t) \subseteq V(Q,t)$. Since we are working in models with existential import, $V(S,t) \neq \emptyset$; it follows that $V(S,t) \cap V(Q,t) \neq \emptyset$, and hence $t \models S\mathtt{pa}Q$. □

Note that we have not defined the categorical propositions in terms of universally quantified conditionals or existentially quantified conjunctions. This is because we want to reserve the quantifiers for making statements about objects which actually exist at a certain point in time. We will give truth conditions for such statements below, once we have defined the appellation function.

The appellation, ampliation, and restriction functions are defined in terms of V and the existence propositions. These functions, like V itself, will be indexed to worlds.

Definition 5.4.4 (Appellation)**.** $\mathtt{Ap}(Q,t) = V(Q,t) \cap \{a : t \models \hat{a}\}$.

Given a predicate and a world, `Ap` selects all the objects which V assigns to the predicate at the world which actually exist at that world. We use the appellation function to define the truth conditions for simple present-tensed predications:

Definition 5.4.5 (Truth conditions for simple predications)**.**

$$\begin{array}{lll} t \models \exists x Q x & \text{iff} & \mathtt{Ap}(Q,t) \neq \emptyset \\ t \models \forall x Q x & \text{iff} & \mathtt{Ap}(Q,t) = \{a : t \models \hat{a}\} \end{array}$$

Boolean combinations of these simple predications are formed in the expected way, e.g., for any infinite predicate $\neg Q$,

$$t \models \exists x \neg Q x \text{ iff } \mathtt{Ap}(\neg Q, t) = \{a : t \models \hat{a}\} \setminus \mathtt{Ap}(Q, t).$$

Note that these definitions allow for the possibility that $V(Q,t) \neq \emptyset$ but that $t \not\models \exists x Q x$, namely when all of the things which are Q at t do not exist. This allows us to satisfy Lambert's requirement that *nam nullo homine existente hec est falsa: "omnis homo est", ergo sua contradictoria erit vera, hec sciliect: "aliquis homo non est"* (219).[36]

Defining the truth conditions for the quantifiers and the categorical propositions in this fashion means that our models are a sort of hybrid between constant-domain models and varying-domain models, as these were defined in the previous section. Because $\forall x$ and $\exists x$ are defined in terms of objects which actually exist at a given world, we can say that the models have varying-domains (since it is allowed that $E(a) \neq E(b)$ for $a \neq b$). But if we consider just the categorical operators $\mathtt{ua}$, $\mathtt{un}$, $\mathtt{pa}$, and $\mathtt{pn}$, then we have a constant-domain model, since the range of V does not vary.

We are now in a position to define restriction and ampliation. These functions modify the supposition of a term and are defined in terms of the appellation function. The formalization of natural restriction is straightforward:

Definition 5.4.6 (Restriction)**.** If Q is (naturally) restricted by S, then

$$\mathtt{Res}(Q, S, t) = \mathtt{Ap}(Q, t) \cap \mathtt{Ap}(S, t)$$

We omit from consideration here use-governed restriction, as this falls under pragmatics, and not the formal theory itself.

Moving on to ampliation, recall from §5.3 that there are two kinds of ampliation, ampliation because of *supposita* and ampliation because of time. We give separate definitions for each. The definition of tense ampliation is straightforward:

Definition 5.4.7 (Tense ampliation)**.** We have four cases, one for each of the temporal operators:

$$\mathtt{Amp}^F_t(Q, w) = \bigcup_{t>w} \mathtt{Ap}(Q, t) \qquad \mathtt{Amp}^G_t(Q, w) = \bigcap_{t>w} \mathtt{Ap}(Q, t)$$
$$\mathtt{Amp}^P_t(Q, w) = \bigcup_{t<w} \mathtt{Ap}(Q, t) \qquad \mathtt{Amp}^H_t(Q, w) = \bigcap_{t<w} \mathtt{Ap}(Q, t)$$

[36] "when no man is in existence 'Every man exists' is false, and so its contradictory 'Some man does not exist' will be true" (123).

We will see in the next section how these definitions can be used to give formal analyses of Rules 5.3.2 and 5.3.3.

Giving a definition of *supposita* ampliation is much more difficult. Tense ampliation was straight forward because the different cases have clear syntactic definitions. On the other hand, *supposita* ampliation is caused when a term is "related to the subject and said of the subject but is not in the subject", and this is something which does not immediately lend itself to a nice syntactic characterization. To address this difficulty, note that, conceptually, tense ampliation is a subset of *supposita* ampliation, since *supposita* ampliation happens when *terminum [tenet] pro suppositis actu et non existentibus* (229).[37] Formally, we shall restrict the phrase "*supposita* ampliation" so that it only applies to those cases of ampliation which do not fall under tense ampliation, that is, when the nonexistents in question are not past or future existents. This means that we must only address those predicates which ampliate to past, present, and future nonexistents.

Additionally, we stipulate that we antecedently know which predicates are the ones that ampliate in this way. These are not defined by any syntactic property, but can only be collected by ostension. Lambert says that

> *Ad cognoscendum autem que verba ampliant et que non, sciendum quod ad substantiam actus potest comparari duplicitier: uno modo quantum ad illud in quo est et de quo enunciatur... alio modo tamquam ad id de quo enunciatur non tamen in ipso est* (214).[38]

We let $\mathcal{S}$ be the set of all predicates which ampliate by means of *supposita*.

Definition 5.4.8 (*Supposita* ampliation)**.** If $R \in \mathcal{S}$, then $\mathtt{Amp}_s(Q,t) = V'(Q)$.

When we say "a chimaera is thought of" or "a man is praised", this is true if there is *any* chimaera or *any* man, past, present, or future, existing or not, which is thought of or is praised.

This concludes the presentation of our formal model.

5.5 Applying the formal model

In this section we investigate the formal properties of the model presented in the previous section, with particular attention to showing that it satisfies the rules put down in §5.3.

[37] "a term [is] interpreted for both actual and nonexistent supposita" (138).

[38] "But in order to recognize which verbs ampliate and which ones do not, it is important to know that an action can be related to a substance in two different ways: in one way as regards that in which it is and of which it is stated... — in the other way as regards that of which it is stated although it is not in it" (117).

We consider Rule 5.3.1 first, which says that the supposition of a term in a present-tensed, non-ampliated, non-restricted proposition will be the term's appellata if it has appellata, and nonexistents otherwise. The truth conditions that we gave for categorical propositions may seem on first consideration not to respect this rule, since no mention is made of appellata, but it is straightforward to prove lemmas outlining the interaction of categorical propositions with propositions about objects actually existing:

Lemma 5.5.1. If $\mathtt{Ap}(Q,t) \neq \emptyset$ and $t \models Q\mathtt{ua}S$, then $t \models \forall x(Qx \rightarrow Sx)$.

Proof. Let $a \in \mathtt{Ap}(Q,t)$ be arbitrary. Since $a \in \mathtt{Ap}(Q,t)$, $t \models \hat{a}$. From $t \models Q\mathtt{ua}S$, we know that $V(Q,t) \subseteq V(S,t)$. It follows then that $a \in \mathtt{Ap}(S,t)$. Since a was arbitrary, this means that $\mathtt{Ap}(Q,t) \subseteq \mathtt{Ap}(S,t)$, which is sufficient to show our conclusion. □

Similar lemmas for the other three types of categorical statements are easily proven. We can also prove the converses for particular categoricals:

Lemma 5.5.2. If $t \models \exists x(Sx \wedge Qx)$ then $t \models S\mathtt{pa}Q$.

Proof. Assume that $t \models \exists x(Sx \wedge Qx)$. It follows that $\mathtt{Ap}(S,t) \cap \mathtt{Ap}(Q,t) \neq \emptyset$. Since $\mathtt{Ap}(Q,t) \subset V(Q,t)$ and $\mathtt{Ap}(S,t) \subset V(Q,t)$, it follows that $V(Q,t) \cap V(S,t) \neq \emptyset$, and so $t \models S\mathtt{pa}Q$. □

Note that from this result, the soundness of the conversion rule for particular affirmative statements can be derived; if $t \models S\mathtt{pa}Q$, then $t \models Q\mathtt{pa}S$.

Lemma 5.5.3. If $t \models \exists x(Sx \wedge \neg Qx)$, then $t \models S\mathtt{pn}Q$.

Proof. Assume that $t \models \exists x(Sx \wedge \neg Qx)$. Then $\mathtt{Ap}(S,t) \cap \mathtt{Ap}(\neg Q,t) \neq \emptyset$. By the definition of the appellation of infinite predicates, we know that $\mathtt{Ap}(S,t) \cap \{a : t \models \hat{a}\} \setminus \mathtt{Ap}(Q,t) \neq \emptyset$. This implies that there is some a such that $a \in \mathtt{Ap}(S,t)$ and $a \notin \mathtt{Ap}(Q,t)$, so $\mathtt{Ap}(S,t) \not\subseteq \mathtt{Ap}(Q,t)$. It then follows that $V(S,t) \not\subseteq V(Q,t)$ since $\mathtt{Ap}(S,t) \subseteq V(S,t)$, and $b \in \mathtt{Ap}(S,t)$ and $b \in V(Q,t)$ implies $b \in \mathtt{Ap}(Q,t)$. □

That the converses for universal categoricals are not provable is easily demonstrable.

Lemma 5.5.4. $t \models \forall x(Sx \rightarrow Qx)$ does not imply $t \models S\mathtt{ua}Q$.

Proof. Let $\mathfrak{M}$ be such that $O = \{a, b\}$ and there is a $t \in T$ such that $t \in E(a)$ but $t \notin E(b)$. Let $d \in V(S,t)$ for all $d \in O$ and $a \in V(Q,t)$. Then $t \models \forall x(Qx)$ and hence $t \models \forall x(Sx \rightarrow Qx)$. However, $V(S,t) \not\subseteq V(Q,t)$, so $t \not\models S\mathtt{ua}Q$. □

A similar proof can be given for the universal negative categorical propositions.

In order to discuss Rules 5.3.2 and 5.3.3, we need to introduce a few more definitions:

Definition 5.5.5 (Substantial term)**.** Q is a *substantial term* iff $\forall a \in O$ if $\exists t \in E(a)$ and $t \models Qa$, then $\forall t' \in E(a)$, $t' \models Qa$.

Definition 5.5.6 (Accidental term)**.** Q is an *accidental term* iff $\exists a \in O$ such that $\exists t, t' \in E(a)$ where $t \models Qa$ and $t' \not\models Qa$.

Next we need to formalize the different ways in which propositions with tensed verbs, such as 'will', or modal verbs, such as 'is able', can be interpreted. Lambert says:

> *Dicendum quod: "album erit Socrates" habet duas acceptiones: potest enim accipi sub hoc sensu: id quod erit album erit Sortes; vel sub isto: quod est album erit Sortes* (225).[39]

The distinction is between "there is something that exists now and which will be white in the future and will be Socrates in the future" and "there is something which exists now and is white now, and will be Socrates in the future". Since modal terms such as 'can', 'is able', and 'possibly' are all analysed in temporal terms, this means that the same things hold for modal sentences such as "A white thing can be Socrates".

Lambert's distinction here is similar to, but not quite the same as, the distinction that other authors make between the divided and composite interpretations of modal and temporal statements.[40] The divided interpretation of the modal statement "A white thing can be black" is "there is something which is now white and which will be black in the future", which is distinguished from the composite interpretation, which is "it will be the case that there is something which is both white and black". The statement is true under the divided interpretation but false under the composite interpretation.

This gives us three ways that past and future tensed statements can be interpreted. Let t indicate the present moment and a be some object:

$$t \models \hat{a} \quad \text{and} \quad \exists t < t', \exists t < t'', t' \models Wa \text{ and } t'' \models a = \texttt{Soc} \tag{5.1}$$

$$t \models \hat{a} \text{ and } t \models Wa \quad \text{and} \quad \exists t < t', t' \models a = \texttt{Soc} \tag{5.2}$$

$$\exists t < t', t' \models Wa \wedge a = \texttt{Soc} \tag{5.3}$$

(To obtain past tense analogues, just change the direction of the $<$.) The first of these corresponds to the divided sense and to Lambert's first interpretation; the second corresponds to Lambert's second interpretation; and the third to the composite sense. It is the composite sense which corresponds to the interpretation

[39] "We have to say that 'A white thing will be Socrates' has two interpretations; for it can be interpreted in this sense: That which will be white will be Socrates; or in this sense: What is white will be Socrates" (133). (It is standard in medieval Latin texts to refer to Socrates by the abbreviated form of his name, *Sortes*.)

[40] See, e.g., [Knu82, pp. 347–48, 354–57]

of the temporal operators given in §5.4. However, this sense is too narrow to capture what we intend to express with tensed quantificational sentences. In general, we want to be able to make statements of types one and two.

It turns out that for accidental terms, the Rules 5.3.2 and 5.3.3 cause the first two distinctions to collapse. This will be clear below when we give formal expressions of the truth conditions given informally in those rules. Before we do so, we first distinguish types 1 and 2 from type 3 by formalizing the latter as $P(Q \odot S)$ where $\odot$ is any of the four categorical connectives, and abusing notation to formalize the former as $QP \odot S$ (we trust that this will not be confusing since we never use P as a predicate variable); we call tenses in sentences of this second type "embedded tenses" or "embedded modalities". To take an example, we read formulas of the form $P(Q\mathtt{pa}S)$ as "it was the case that some Q is S" and ones of the form $QP\mathtt{pa}S$ as "it is the case that some Q was S", and similarly for the other connectives. Now we can give the formal versions of the rules:

Definition 5.5.7 (Rule 5.3.2)**.**
If Q is an unrestricted accidental term and $S \notin \mathcal{S}$, then

$$\begin{aligned}
t \models QP\mathtt{ua}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \subseteq \mathtt{Amp}_t^P(S,t) \\
t \models QP\mathtt{pa}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \cap \mathtt{Amp}_t^P(S,t) \neq \emptyset \\
t \models QP\mathtt{un}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \cap \mathtt{Amp}_t^P(S,t) = \emptyset \\
t \models QP\mathtt{pn}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \not\subseteq \mathtt{Amp}_t^P(S,t)
\end{aligned}$$

and

$$\begin{aligned}
t \models QH\mathtt{ua}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \subseteq \mathtt{Amp}_t^H(S,t) \\
t \models QH\mathtt{pa}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \cap \mathtt{Amp}_t^H(S,t) \neq \emptyset \\
t \models QH\mathtt{un}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \cap \mathtt{Amp}_t^H(S,t) = \emptyset \\
t \models QH\mathtt{pn}S \quad &\text{iff} \quad \{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \not\subseteq \mathtt{Amp}_t^H(S,t)
\end{aligned}$$

If Q is an unrestricted substantial term and $S \notin \mathcal{S}$, then

$$\begin{aligned}
t \models QP\mathtt{ua}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \subseteq \mathtt{Amp}_t^P(S,t) \\
t \models QP\mathtt{pa}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \cap \mathtt{Amp}_t^P(S,t) \neq \emptyset \\
t \models QP\mathtt{un}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \cap \mathtt{Amp}_t^P(S,t) = \emptyset \\
t \models QP\mathtt{pn}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \not\subseteq \mathtt{Amp}_t^P(S,t)
\end{aligned}$$

and

$$\begin{aligned}
t \models QH\mathtt{ua}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \subseteq \mathtt{Amp}_t^H(S,t) \\
t \models QH\mathtt{pa}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \cap \mathtt{Amp}_t^H(S,t) \neq \emptyset \\
t \models QH\mathtt{un}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \cap \mathtt{Amp}_t^H(S,t) = \emptyset \\
t \models QH\mathtt{pn}S \quad &\text{iff} \quad \mathtt{Amp}_t^P(Q,t) \not\subseteq \mathtt{Amp}_t^H(S,t)
\end{aligned}$$

Definition 5.5.8 (Rule 5.3.3)**.** The formalization of Rule 5.3.3 can be obtained by replacing P with F and H with G throughout.

Note that it follows from these rules that sentences with substantial terms as their subject terms can only be interpreted in the first of the two interpretations that Lambert gives.

We make just one more remark before concluding our application of the formal model to Lambert's theory of supposition. Because the temporal operators H and G can be defined as $\neg P \neg$ and $\neg F \neg$, respectively, we focused only on P and F throughout the current section and the preceding one. While it is clear that this interdefinability holds for sentences interpreted in the third way (the composite sense), it is by no means obvious that the same is true when we use P and F as in the two rules. In fact, as we have defined the truth conditions for sentences of the form $QP \odot S$ and $QF \odot S$, the following holds:

Lemma 5.5.9. $t \models QP \odot S$ iff $t \models Q \neg P \neg \odot S$ and $t \models QF \odot S$ iff $t \models Q \neg F \neg \odot S$

Proof. We prove just the case of $t \models QP\mathtt{pa}S$ iff $t \models Q \neg P \neg \mathtt{pa}S$, where Q is accidental and unrestricted, and leave the other cases as exercises for the reader.

($\Leftarrow$) Suppose $t \models Q \neg P \neg \mathtt{pa}S$. Since the categorical propositions respect the relationships in the square of opposition, $\neg\mathtt{pa}$ can be replaced with $\mathtt{un}$. If $\{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \cap \mathtt{Amp}_t^P(S,t) = \emptyset$, then $t \models QP\mathtt{un}S$. Since $t \models Q \neg P \mathtt{un}S$, it follows by modus tollens that $\{\mathtt{Ap}(Q,t) \cup \mathtt{Amp}_t^P(Q,t)\} \cap \mathtt{Amp}_t^P(S,t) \neq \emptyset$, and hence $t \models QP\mathtt{pa}S$.

($\Rightarrow$) As all of the implications involved in the proof of the other direction are equivalences, this case is symmetric. □

5.6 Conclusions and future work

There is one interesting issue which is not generally addressed in medieval theories of supposition and which, because of its potential applicability to modern philosophical problems, warrants further investigation, and that is the issue of iterated tenses. Syntactically, nothing prevents us from nesting temporal operators, e.g. $PFP(Q\mathtt{ua}S)$, $QGHF\mathtt{ua}S$, etc. Three questions immediately arise from this: First, what sense can we give to the interpretation of these strings of temporal operators? Second, how must we modify the definitions in order to allow for iterated temporal operators? Third, what strings of temporal operators result in the same semantic outcome, that is, when can iterated temporal operators be reduced to a single temporal operator?

We briefly comment on the first question. When the iterated tenses are being used in their usual fashion, e.g., $PFP\varphi$, then the answer to the first question is straight-forward: the formula is read from left to right as normally: 'it was the case that it will be the case that it was the case that φ'. When used in the special

way that we introduced above, e.g., $QHF\texttt{ua}S$, the most natural reading of the sentence is to attach the first tense to the copula and the remaining tenses to the predicate, e.g. 'All Q's were-always will-be S'. When there are more than two temporal operators, this natural reading becomes more stilted, but even so we can still make sense of things like $QGHF\texttt{ua}S$ 'All Q's will-always-be were-always will-be S'. Tenses iterated in the first sense are well understood in both linear and branching time structures. This means that future investigation should focus on nested temporal operators used in the second fashion, since they are the ones that make use of the truth conditions based on ampliation and appellation.

The brief excursus into modern logic in the previous section shows the depth and breadth of Lambert's theory of supposition, and opens up the possibility of applying this theory to modern philosophical problems, such as questions about reference to nonexistent entities and issues with combining quantification and modality. We have shown how from a relatively basic theory of supposition a very interesting and distinctive modal and temporal logic can be extracted. What this points to is that that the rise of supposition theory over the course of the 12th to 14th centuries was not just the rise of supposition theory, but the rise of well-defined and widely applicable modal and temporal logic with potential applications to problems in contemporary philosophy.

Chapter 6

Roger Swyneshed's notion of self-falsification and dynamic epistemic logic

6.1 Introduction

Explanations and solution strategies for paradoxes and insolubles have been fruitful catalysts for the development of logic throughout its history. Two paradoxes which are often mentioned in current literature are the *Liar Paradox*, which concerns truth, and *Fitch's Paradox*, which concerns knowledge and knowability. Both paradoxes have given rise to an enormous body of literature; our contribution to this literature is to take a meta-stance with respect to both paradoxes, to compare two solution concepts for these paradoxes and their methodology.

One type of solution strategy that has been proposed for both the Liar and Fitch's Paradox, with fruitful results, is what is called the *restriction strategy*. This strategy *restricts* the scope of applicability of the formula involved in the paradox to a class I^* of allowed instantiations.[1] A trivial choice for I^* is the class of all instantiations that do not give rise to a paradoxical conclusion anymore. Often, restriction strategies have been criticized for being *ad hoc*, in particular, if the choice of I^* depends crucially on the paradox at hand. In order to counter this criticism, more detailed restriction strategies have been devised in order to give independent and non-*ad hoc* descriptions of I^*.

In this chapter, we deal with two of these more elaborate restriction strategies: a medieval solution to the Liar Paradox by the 14th-century logician Roger Swyneshed using the notion of self-falsification, and a recent solution to Fitch's Paradox by Johan van Benthem using dynamic epistemic logic. We point out how these two solutions are structurally similar and how Swyneshed's solution can be seen as a truth-analogue of van Benthem's solution.[2] We begin in §6.2

[1] We give a formal definition of this in §6.2.

[2] Swyneshed's solution is not all that well understood. Spade reports that "the notions of 'relevance', 'self-falsification' and 'signifying as is the case' ... are mysterious ones in Swyneshed's theory and not yet well understood by scholars" [Sp05], and we hasten to add that we do

with a general discussion of paradoxes, where we introduce the Liar and Fitch's Paradoxes in a uniform framework. In §6.3.1, we give an overview of the basics of dynamic epistemic logic (more precisely, of its fragment *public announcement logic*), which we use in our account of van Benthem's solution to Fitch's Paradox (§6.3.2). We then give a brief historical introduction to Swyneshed and his theory in §6.4. In §6.5 we define a dynamic announcement pointer semantics that we use to formalize both Swyneshed's solution and van Benthem's in a uniform framework, and in §6.6, we draw conclusions, and point towards future work.

6.2 Paradoxes

In this section, we begin by informally discussing the general evolution of a paradox and then give a more formal definition of what we call a paradox and what constitutes a solution to a paradox.

It is possible to identify three stages in the evolution of a paradox. The first stage is that the paradox must be be identified as a paradox. This may seem a trivial point, in that prior to this identification there is no paradox of which it can be seen as the first stage. In some situations, it is certainly the case that there is no discussion of the paradoxical statement until it is seen to be a paradox. An example of this type is Frege's basic law V, which only really generated interest and discussion upon Russell's discovery of the antinomy following from it. On the other hand, there are cases where there is a body of literature concerning a paradoxical statement before that statement is recognized as a paradox. As we'll see in greater detail in §6.4, the medieval treatment of the Liar statement falls into this category: Early discussions of the sentence seem not to recognize its paradoxical properties (cf. fn. 14).

Once a statement has been recognized as a paradox, the next step is to identify the cause of the paradoxicality and try to block it. There are two places where the problem can be located, the axioms and the range of application of the axioms. In both cases, the problem can be addressed with a restriction strategy, which we mentioned in §6.1. When the cause is located in the axioms, then the solution, as in the Frege-Russell case, is to reject the offending axiom. When the cause is in the range of application of the axioms, then we adopt a restriction of the range so that the problematic cases fall outside the range. The naïve way to do this is to say that all terms fall under the range of the axioms except those terms whose use leads to a contradiction. However, this naïve way is unsatisfying because it is *ad hoc*. It is not adequate to disregard the problematic cases because they are problematic. What we want is an explanation of why they are problematic so

not claim to have a better grasp of Swyneshed's notions than Spade. However, the dynamic viewpoint offered by the parallelism with van Benthem's ideas might offer a new approach to understanding some of the distinctions in Swyneshed's text. We discuss this in more detail in §6.6.

that we can give a principled characterization of the class of potential paradoxes.

This principled characterization of the problem cases is the third step in the evolution of a paradox. When the cause of the paradox is located not in the axioms but in the range of applicability of the axioms, the goal is to present a non-trivial, non-*ad hoc*, and non-circular definition of those cases which must be excluded from the range, *without* any reference to the fact that they give rise to paradox.

While these three steps cannot be isolated and identified in all paradoxes, they can be in both the Liar and Fitch's Paradoxes, the two paradoxes that will occupy us for the rest of the chapter; we will use this fact to structure our discussion of both in the succeeding sections. First, we give a more formal treatment of the informal ideas just used. In conceptual modeling, we investigate informal concepts (e.g., truth, knowledge, knowability, etc.) for which we, as natural language users, have some intuitions allowing us (in many cases) to intuitively decide the truth value of natural language sentences involving these concepts.

In order to discuss paradoxes in a formal setting, we fix a set I whose elements we call *instances* and a formal language $\mathcal{L}_I$. We say that a family $\langle \langle \chi_\iota, \vdash_\iota \rangle \,;\, \iota \in I \rangle$ is a *paradox setup* if χ_ι is a well-formed formula of $\mathcal{L}_I$ and $\vdash_\iota \subseteq \mathrm{Fml}_\iota \times \mathrm{Fml}_\iota$ is a derivability predicate, typically given by a set of axioms and a set of derivation rules.[3] We now say that $\iota \in I$ is a *weak paradox* if $\chi_\iota \vdash_\iota \bot$, and we say that it is a *strong paradox* if $\top \vdash_\iota \bot$.

6.2.1 Fitch's paradox of knowability

Fitch's Paradox arises from the assumption of certain natural properties about knowledge, including factivity of knowledge and what is called the *Verification Thesis* (VT), $\varphi \to \Diamond \mathrm{K}\, \varphi$, which states that what is true can be known. If the Verification Thesis is instantiated with the statement "q is true but not known", this leads to a contradiction.[4]

We give a formal definition of Fitch's Paradox by fixing a set of propositional letters P and defining the language $\mathcal{L}_{\mathrm{F}} := \langle P, \wedge, \neg, \Diamond, \mathrm{K} \rangle$. The set of well-formed formulas of $\mathcal{L}_{\mathrm{F}}$ is denoted $\mathrm{Fml}_{\mathrm{F}}$. If $\varphi \in \mathrm{Fml}_{\mathrm{F}}$, we write $\mathrm{VT}(\varphi)$ for $\varphi \to \Diamond \mathrm{K}\, \varphi$ (an instance of the Verification Thesis). Let $I = \mathrm{Fml}_{\mathrm{F}}$, then fix $\iota \in I$. We define a paradox setup by $\chi_\iota := \iota$, and let $\vdash_\iota$ be a reasonably tame deduction calculus for $\Diamond$ and K including $\mathrm{VT}(\iota)$ as an axiom.[5]

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[3] I.e., $\varphi \vdash_\iota \psi$ if and only if there is a finite sequence $\varphi_0, ..., \varphi_n$ such that $\varphi_0 = \varphi$, $\varphi_n = \psi$ and for each $0 < i \leq n$, the formula φ_i is either an axiom or the result of applying one of the rules to finitely many formulas φ_j with index $j < i$.

[4] As [vB04, pp. 95–96] notes, Fitch's Paradox is an example of the same type of paradox expressed by Moore in [Moore62] in doxastic terms, 'p, but I don't believe it'. A remarkably similar paradox shows up as Albert of Saxony's *insolubile* no. 17: 'It is possible that Socrates knows that he is mistaken'. See [KreSt88, pp. 363–364] for a discussion of this *insolubile*.

[5] We need the rules $\mathrm{K}(\varphi \wedge \psi) \vdash \mathrm{K}\, \varphi \wedge \mathrm{K}\, \psi$, $\mathrm{K}\, \varphi \vdash \varphi$, $\Box\neg\varphi \vdash \neg\Diamond\varphi$, and the necessitation rule "$\vdash \varphi$ implies $\vdash \Box\varphi$" for this. For more details, see [Will$_2$93, Wan02].

It is easy to show using the axioms mentioned in footnote 5 that for any ι, we get $\top \vdash_\iota \neg\Diamond \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$:

Proof. Suppose for *reductio* that $\top \vdash_\iota \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$. Then:

1. $\top \vdash_\iota \mathrm{K}\, q \wedge \mathrm{K} \neg \mathrm{K}\, q$ by the distributivity of K.
2. $\top \vdash_\iota \mathrm{K}\, q \wedge \neg \mathrm{K}\, q$ by the factivity of K.
3. $\top \vdash_\iota \neg \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$, by *reductio*, discharging our assumption.
4. $\top \vdash_\iota \Box\neg \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$ by the rule of necessitation.
5. $\top \vdash_\iota \neg\Diamond \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$ by the interchange of $\Box\neg$ and $\neg\Diamond$.

Then, Fitch's Paradox is the instance $\iota_{\text{Fitch}} := q \wedge \neg \mathrm{K}\, q$. □

Proposition 6.2.1. The instance ι_{Fitch} is a weak paradox.

Proof. Since $\mathrm{VT}(\iota_{\text{Fitch}}) = (q \wedge \neg \mathrm{K}\, q) \to \Diamond \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$ is an axiom of $\vdash_{\iota_{\text{Fitch}}}$, we get (by *modus ponens*) that $\iota_{\text{Fitch}} \vdash_{\iota_{\text{Fitch}}} \Diamond \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$. Combining this with $\top \vdash_{\iota_{\text{Fitch}}} \neg\Diamond \mathrm{K}(q \wedge \neg \mathrm{K}\, q)$ (see above), we get the desired contradiction. □

6.2.2 The Liar

The Liar Paradox, which is probably the best-known of all paradoxes, is the paradox that arises when the following sentence, and no others, is uttered:

This sentence is false.

This sentence is called the Liar Sentence. In order to formalize the Liar Sentence, we need a language that allows cross-references between sentences. There are many ways to do this; for the purpose of this chapter, we will use Gaifman's pointer semantics [Gai88, Gai92].

We fix a set of propositional letters P and a predicate symbol T, and define the language $\mathcal{L}_{\mathrm{L}} := \langle P, T, \wedge, \neg \rangle$. The set $\mathrm{Fml}_{\mathrm{L}}$ of well-formed formulas of $\mathcal{L}_{\mathrm{L}}$ is defined as:

$$\varphi := P \mid T(p) \text{ for } p \in P \mid \neg\varphi \mid \varphi \wedge \psi$$

We call functions $\Sigma\colon P \to \mathrm{Fml}_{\mathrm{L}}$ *systems* and think of $\Sigma(p)$ as the 'meaning' of p. We let I be the set of all pairs $\langle \Sigma, p \rangle$ such that Σ is a system and $p \in P$. For a given instance $\iota = \langle \Sigma, p \rangle$, we let $\chi_\iota := p$ and let $\vdash_\iota$ be classical propositional logic together with the axioms $T(p) \leftrightarrow p$ and $T(p) \leftrightarrow \Sigma(p)$. Since our logic is a classical two-valued logic, the principle of bivalence holds for formulas; in particular, $T(q) \vee \neg T(q)$ holds.

The Liar Paradox is the instance $\lambda := \langle \Lambda, p \rangle$ with $\Lambda(p) := \neg p$.

Proposition 6.2.2. The instance λ is a strong paradox.

Proof. By bivalence, we have that $\top \vdash_\lambda T(p) \vee \neg T(p)$ and thus (using idempotence) $\top \vdash_\lambda (T(p) \wedge T(p)) \vee (\neg T(p) \wedge \neg T(p))$. From the axioms $T(p) \leftrightarrow p$ and $T(p) \leftrightarrow \Lambda(p) = \neg p$, we immediately get $\top \vdash_\lambda (p \wedge \neg p) \vee (p \wedge \neg p)$, i.e., $\top \vdash_\lambda \bot$. □

6.2.3 Solutions to paradoxes

With these definitions introduced in the previous subsections, we can now give a formal characterizations of two types of solutions to paradoxes. As we noted in the beginning of this section, there are a number of natural reactions to paradoxes of these kinds. Of the two canonical replies, to question either the axiomatization of our intuitions, i.e., the deduction systems $\vdash_\iota$, and replace them with some weaker systems, or the range of formalization, i.e., restrict the set of possible instances I to a smaller set, excluding the problematic case that produced the paradox[6], our focus is the second.

Given a paradoxical situation, what is the right class of instances that will allow us to avoid paradox? The trivial answer to this question is: Just consider the class $I_{\text{safe}} \subseteq I$ of instances defined by $I_{\text{safe}} := \{\iota \in I \colon \iota \text{ is no paradox}\}$. Obviously, such an answer leaves a sense of dissatisfaction. It would be preferable to have a class I^* of instances defined independently of a given paradox ι and of the notion of paradoxicality together with a proof that $\iota \notin I^*$. In addition, one needs to be careful not to exclude too many instances: our intuitions of the informal concept that is being modeled tell us that there are certain unproblematic instances that should not be excluded.

A particular class of restriction solutions are semantic solutions. Let $\mathcal{L}^*$ be a language extending $\mathcal{L}$ and $\Phi(x)$ be an $\mathcal{L}^*$-formula with a parameter x for an $\mathcal{L}$-formula. Let $\mathcal{M}$ be a class of models for $\mathcal{L}^*$. We say that ι is *excluded by* Φ if every model $\mathfrak{M} \in \mathcal{M}$ satisfies

$$\mathfrak{M} \models \Phi(\chi_\iota).$$

We define $I^\Phi := \{\iota \colon \iota \text{ is not excluded by } \Phi\}$. In §§6.3.2, 6.4, we will see two examples of semantic solutions.

6.3 Modern responses to Fitch's Paradox

The conclusion of Fitch's Paradox, that there are unknowable truths, is Theorem 5 of [Fitc63]:

> If there is some true proposition which nobody knows (or has known or will know) to be true, then there is a true proposition which nobody can know to be true [p. 139].

[6] These two strategies are explicitly mentioned in [vB04, p. 95] where they are likened to "turning down the volume on your radio so as not to hear the bad news" and "censoring the news", respectively.

Fitch himself saw nothing problematic with this conclusion. It wasn't until more than ten years after the publication of Fitch's paper that the paradoxicality of Theorem 5 was first discussed, in [HarMc76]. As Kvanvig notes, "the difference between what is paradoxical and what is merely surprising is, perhaps, only a difference in degree and not in kind. Even so, there is a distinction to be drawn here between the unanticipated and the seemingly contradictory, and Fitch's proof engenders the latter experience and not simply the former" [Kv–]. Once its paradoxicality was recognized, Fitch's Paradox became a starting point of a huge discussion in philosophical logic and epistemology. We do not intend to give a detailed overview here, but refer the reader to the excellent synopsis in [BroSa04].

As indicated in §6.2.3, recent literature contains a variety of responses to Fitch's Paradox. These are categorized in [BroSa04, §§2–4] as "logical revisions", "semantic restrictions", and "syntactic restrictions". We are concerned here with "syntactic restrictions", i.e., restrictions of the class of instances that the Verification Thesis can be applied to. Two of the most important attempts at syntactic restriction are those of Dummett [Dum01] and Tennant [Te97, Te01, Te02].

Dummett says that the mistake found in the paradox is found in giving "a blanket characterization of truth, rather than a inductive one" [Dum01, p. 1], and he proposes to define inductively a class of *basic statements*. This solution can be objected to on two grounds. First, as Brogaard and Salerno note, the restriction is unprincipled as "the only reason we are given for restricting the knowability principle to basic statements is that it blocks Fitch's results" [BroSa02, p. 144]. Second, Dummett does not give a full specification of the set of basic statements, but what he says is sufficient for Tennant to notice in [Te02, p. 139] that Dummett's basic statements are reminiscent of Kripke's proposal of the inductive definition of a truth predicate as a least fixed point, going back to [Kri75]. Not surprisingly, Dummett's proposal suffers from similar problems: the least fixed point does not cover all intuitively unproblematic cases, and thus the class of instances allowed by Dummett is too small.[7]

Tennant diagnoses Dummett's problem not in his choice of using a restriction strategy in general, but in choosing the wrong restriction. He says that "the restriction strategy is the strategy that I favour, too" [Te02, p. 136], but he gives a different restriction. Tennant calls a proposition φ *Cartesian* if $\mathrm{K}\,\varphi$ is not provably inconsistent. Tennant's approach has been called both "desperately *ad hoc*" [Will$_2$93, p. 109] and also "unprincipled" [HanKv99, pp. 423, 425]. In [Te–], Tennant attempts to resuscitate his Cartesian restriction against these objections, but for our purposes we are more interested in van Benthem's diagnosis of the problem, which is more refined than that of Williamson and of Hand and Kvanvig. Van Benthem shows in [vB04, p. 96] that Tennant's class is too liberal [vB04, p. 96] to cover the context of what he calls the "natural learning scenario", and proposes a different solution in terms of dynamic epistemic logic. It is this

[7]For details, see [Te02, pp. 139–141].

solution that we want to compare to Swyneshed's medieval solution to the Liar Paradox, so we must first introduce dynamic epistemic logic (DEL).

6.3.1 Dynamic epistemic logic

In this section, we introduce a fragment of dynamic epistemic logic (DEL). In the context of this chapter, we are interested in a fragment called *public announcement logic*. This fragment goes back to [Pl89] (without common knowledge) and [BaltMoSo98] (with common knowledge); for a detailed discussion of public announcement logic and more general forms of dynamic epistemic logic, see [vDvdHK07, §§4, 6].[8]

We fix a set of agents A and a set of atomic statements P and define the language of *public announcement logic* $\mathcal{L}_{\mathrm{PA}} := \langle P, A, \wedge, \neg, \mathrm{K}, [\,] \rangle$. The set $\mathrm{Fml}_{\mathrm{PA}}$ is defined by:

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid \mathrm{K}_a\,\varphi \text{ for } a \in A \mid [\varphi]\psi$$

where $\mathrm{K}_a\,\varphi$ is read 'agent a knows that φ', and $[\varphi]\psi$ is read 'after the (truthful) announcement of φ, ψ is true'.[9,10]

Definition 6.3.1. A structure $\mathfrak{M} = \langle W, \sim, V \rangle$ is called an *epistemic model* if W is a set, $\sim = \langle \sim_a \colon a \in A \rangle$ is a family of equivalence relations on W, and V is a valuation function from P to subsets of W. The relation $w \sim_a w'$ will be interpreted as "states w and w' are epistemically equivalent for agent a", or, in other words, "agent a cannot distinguish between states w and w'".

We define by simultaneous induction the semantics of the language $\mathcal{L}_{\mathrm{K}[\,]}(A, P)$ and the operation of restricting a model by a formula. If we start with a model

[8] For the history of public announcement logic, see in particular [vDvdHK07, §4.13] and the survey [vD07].

[9] For dynamic epistemic logic in general, one would define a class of actions α for modalized sentences $[\alpha]\varphi$, representing 'after action α, φ is true'. Restricting the range of possible actions to truthful public announcements yields public announcement logic. For more details, see [vDvdHK07, §5.2].

[10] Traditional presentations of public announcement logic also include the operator C, where $C\varphi$ is read "it is common knowledge that φ"; this operator can also be indexed with a subset $B \subseteq A$ such that $C_B\varphi$ is read "it is common knowledge among the group B that φ". Common knowledge refers to φ being known by all members of the group of agents, and the infinitary iteration of knowledge statements of this fact (i.e., $\mathrm{K}_a\,\varphi$, $\mathrm{K}_b\,\mathrm{K}_a\,\varphi$, $\mathrm{K}_c\,\mathrm{K}_b\,\mathrm{K}_a\,\varphi$, etc.) Common knowledge is not finitely expressible in public announcement logic without a common knowledge operator [vDvdHK07, Theorems 8.44, 8.48]. However, as van Benthem notes, "In scenarios with just a single agent 1, common knowledge $C_{\{1\}}\varphi$ is just the same as knowledge $\mathrm{K}_1\,\varphi$" [vB09, p. 6]. Because in our presentation, as in van Benthem's, "[o]ne can read the following discussion ... either way, as being about knowledge of a single agent, or about common knowledge in a group" [vB09, p. 6], we chose to focus on the le-agent case and omit common knowledge, to simplify the presentation.

$\mathfrak{M} = \langle W, \sim, V\rangle$, we will define a new model $\mathfrak{M}{\restriction}\varphi := \langle W^{\mathfrak{M},\varphi}, \sim, V\rangle$ where $W^{\mathfrak{M},\varphi} := \{w \in W : \mathfrak{M}, w \models \varphi\} \subseteq W$, and $\sim$ and V are just the restrictions of the original relation and function.

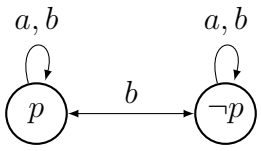
The truth conditions for the propositional formulas as are expected. We give just the semantics for the two modal operators.

Definition 6.3.2. The semantics of the modal operators are as follows:

$$\begin{array}{lll} \mathfrak{M}, w \models \mathrm{K}_a\,\varphi & \text{iff} & \text{for all } w' \text{ such that } w \sim_a w',\ \mathfrak{M}, w' \models \varphi \\ \mathfrak{M}, w \models [\varphi]\psi & \text{iff} & \mathfrak{M}, w \models \neg\varphi \text{ or } \mathfrak{M}{\restriction}\varphi, w \models \psi \end{array}$$

This semantics corresponds to the syntactic modal logic PAC defined in [BaltMoSo98], which is sound and complete with respect to the above semantics [vDvdHK07, Theorem 4.59].

We now give an example to show the semantics of $\mathcal{L}_{\mathrm{PA}}$ at work: suppose that we have two agents, the Papal Legate Antonius, denoted by a, and a Parisian scholar Bertrandus, denoted by b. Let p be the statement "The pope is in Rome", and $A := \{a, b\}$ and $P := \{p\}$. Suppose that the Pope is actually in Rome, and that Antonius knows this but Bertrandus doesn't. This situation is represented by the following model $\mathfrak{M} := \langle W, \sim, V\rangle$:[11]



Here, Bertrandus is unable to distinguish epistemically between the left vertex (where the Pope is in Rome) and the right vertex (where the Pope is not in Rome). In other words, he does not know whether the Pope is in Rome. If now Antonius announces (truthfully) that the Pope is indeed in Rome, we move from the above model $\mathfrak{M}$ to the model $\mathfrak{M}{\restriction}p$ in which the vertices where p is false are removed (together with all corresponding edges). Consequently, we only retain the left vertex and get the following model which represents that both agents know that the Pope is in Rome:

[11] As customary in modal logic, the vertices in the graph correspond to the elements of W and the valuation function V is given by writing the value p or $\neg p$ into the vertices. The indexed edges between vertices represent the equivalence relations $\sim_a$ and $\sim_b$. If there is a edge between w and w' indexed by a, then $w \sim_a w'$, and similarly for b.

6.3.2 Van Benthem's solution

Using the semantics of public announcement logic just defined, we can define a notion of self-refutation for epistemic statements. For simplicity's sake, we follow van Benthem in giving the details only for the one-agent case $A = \{a\}$ and we write $\mathrm{K} := \mathrm{K}_a$.[12]

Definition 6.3.3. If $\varphi \in \mathcal{L}_{\mathrm{PA}}$, we say that φ is *self-refuting* if for all epistemic models $\mathfrak{M}$, we have that $\mathfrak{M} \models [\varphi]\,\mathrm{K}\,\neg\varphi$ [vB04, p. 101].

Note that the language $\mathcal{L}_{\mathrm{PA}}$ lacks a $\Diamond$-modality, so we cannot directly express Fitch's Paradox in this language. Van Benthem's suggestion in [vB09, p. 1] is to read "$\Diamond\varphi$" as "there is some announcement that makes φ known"; this notion of "possible knowledge" is made precise in [Balb et al.08]. By this stipulation, we can consider the language $\mathcal{L}_{\mathrm{PA}}$ as an extension of the language $\mathcal{L}_{\mathrm{F}}$ used in Fitch's Paradox. Definition 6.3.3 gives us a semantic solution in the sense of §6.2.3 via the $\mathcal{L}_{\mathrm{PA}}$-formula schema

$$\Phi(x) := [x]\,\mathrm{K}\,\neg x.$$

Proposition 6.3.4 (van Benthem)**.** The Fitch instance $\varphi := q \wedge \neg\,\mathrm{K}\,q$ is self-refuting.

Proof. Suppose $\mathfrak{M} = \langle W, \sim, V\rangle$ is any epistemic model. We need to show that for all epistemic models $\mathfrak{M}$, we have $\mathfrak{M} \models [\varphi]\,\mathrm{K}\,\neg\varphi$. Consider $M{\restriction}\varphi$; if $w \in W^{\varphi,\mathfrak{M}}$, then $\mathfrak{M}, w \models q$. Therefore, q is true at all worlds in $\mathfrak{M}{\restriction}\varphi$, and therefore trivially $\mathfrak{M}{\restriction}\varphi \models \mathrm{K}\,q$, and thus $\mathfrak{M}{\restriction}\varphi \models \neg\varphi$. Again, this is true at all vertices, whence $\mathfrak{M}{\restriction}\varphi \models \mathrm{K}\,\neg\varphi$. □

Note that the definition of self-refutation does not involve any reference to paradoxicality; it is a purely logical property of a formula of $\mathcal{L}_{\mathrm{PA}}$. We therefore can, in a non-*ad hoc* fashion, define the class $I^{\Phi} := \{\varphi \in \mathcal{L}_{\mathrm{PA}} : \varphi \text{ is not self-refuting}\}$, and thus by Proposition 6.3.4 avoid Fitch's Paradox.

6.4 Medieval responses to the Liar

From the middle of the twelfth century onwards, there was much interest among medieval logicians in such paradoxical sentences, which they called *insolubilia*. The medieval logicians recognized that the task of providing a non-trivial and non-*ad hoc* resolution to the paradoxes can be so difficult as to seem insoluble. But, as they also recognized, this is often only seeming, and the task is not truly insoluble. Despite their name, *insolubilia* were considered to have difficult, but

[12] See footnote 10.

not impossible, solutions.[13] One insoluble statement discussed in a large number of the extent treatises on *insolubilia* was the Liar Paradox.[14]

6.4.1 Naïve restriction strategy solutions

Spade in [Sp87] distinguishes five different medieval solutions to the Liar Paradox:

1. The Aristotelian solution, that is, classification under the fallacy *secundum quid et simpliciter*.
2. The *transcasus* theory.
3. The distinction between the *actus exercitus* and the *actus significatus*.
4. The *restrictio* theory.
5. The *casatio* theory.

For our purposes, we are interested in solutions of type 4, and we say nothing about the others. A *restrictio* solution to the Liar Paradox is one which restricts the possibility of self-reference. Two types of *restrictio* can be identified: Strong *restrictio* forbids self-reference of any type, and weak *restrictio* forbids self-reference only in certain circumstances and situations.

Strong *restrictio* is in general of little interest because in many cases it is clearly too strong. One may agree that there is something problematic with the sentence "This sentence is false" without agreeing that there is anything problematic with the sentence "This sentence has five words" or "This sentence is not in German". When one recognizes that strong *restrictio* is too strong, it is natural to try to weaken the restrictions. But weak *restrictio* comes with the problem we mentioned in §6.2, namely that of giving a principled restriction. The natural restriction, that self-reference is allowable except in cases where it leads to paradox, is both trivial and *ad hoc*. Finding a happy medium between being so strong as to be false and being so weak as to be uninteresting was hence the task of the *restringentes*.

In [Sp75], Spade discusses 71 different texts dealing with the Liar Paradox.[15] Of these 71 texts, fourteen espouse some type of *restrictio* theory, either explicitly or implicitly, by limiting the cases where self-reference can be applied. We do not include in our discussion here authors who mention the *restrictio* solution but which ultimately reject it in favor of another. These fourteen treatises are the following:

[13] Cf. [Re90, pp. xii–xiii].

[14] According to Spade, the earliest known medieval formulation of the paradox is in Adam of Balsham's *Ars disserendi*, dating to 1132. However, Adam "says nothing whatever to indicate that he was aware of the very special problems they pose, that they were current topics of philosophical discussion in his day, or how one might go about trying to answer those questions" [Sp87, p. 25].

[15] Though, as he says in his preface, his catalogue "does not pretend to be complete".

II Anonymous, *Puncta de insolubilibus Hollandrini* [c. 1400–1420]. This text has not been edited, to our knowledge.

VIII Anonymous, *Insolubilia* [1368]. This text is edited in [Sp71].

X Anonymous, *Insolubilia* [after 1335]. This text is edited in [Sp69].

XX Anonymous, *Insolubilia* [mid-13th C]. This text is edited in [Bra67].

XXX Henry of England (Harvey of England, Berotus of England), *Insolubilia* [1395]. This text has not been edited, to our knowledge.

LI Peter of Ailly, *Insolubilia* [1372]. This text is edited in [PoA95].

LIV Richard of Campsall, *Quaestiones super librum priorum analyticorum* [before 1307]. This text is edited in [Sy68].

LXI Roger Nottingham, *Insolubilia* [1343]. This text is edited in [Sy64].

LXII Roger Roseth, *Quaestiones super Sententias* [before 1337]. This text has not been edited, to our knowledge.

LXVII Walter Burley, *Insolubilia* [before 1320]. This text is edited in [Ro70].

LXVIII Walter Sexgrave, *Insolubilia* [before 1333]. This text has not been edited, to our knowledge.

LXX William of Ockham, *Summa logicae* III, 3, c. 38, 45 [1324–1327]. This text has been edited in numerous places.

LXXI William of Ockham, *Tractatus super libros elenchorum* [before 1328]. This text is edited in [WoO79].

We briefly summarize of the positions in these texts.[16] One text grants the viability of both strong and weak *restrictio*:

VIII The anonymous author notes that some people reject self-reference and that some allow it, and that "*ambae opiniones possunt salvari, quia, ut 'falsum' est pars, non supponit pro toto; sed, ut totum praedicabile de illa et de aliis, supponit pro parte, et sic diversimode*" [Sp71, ¶14].[17]

Seven texts in the above list appear to espouse a strong *restrictio* theory:

II The author distinguishes two ways to solve an insoluble. The first is to allow self-reference, in which case insolubles falsify themselves. The second, which is described as the "easier" way, is to disallow self-reference altogether.

[16] Where we have not been able to obtain access to the relevant treatise, we have based our summaries on Spade's information.

[17] "Both positions may be held, when 'false' is a part which does not supposit for the whole; but when the whole is predicable of this and the other, it supposits for the part, and so on diversely".

X The author rejects self-reference in all of mental, spoken, and written propositions.

XX The author gives a proof that "*terminus non supponit pro oratione cuius est pars*" [Bra67, p. 132].[18] This restriction on self-reference is combined with a *transcasus* theory: a present-tense verb in an insoluble doesn't in fact refer to the present time, but to a previous one.

XXX Henry says that in no proposition is it possible for something to supposit for itself (that is, the proposition).

LI Peter says that parts of mental propositions cannot supposit for the proposition of which it is a part.

LXI Spade summarizes Nottingham's view and says "If this summary is an accurate one, it indicates that Nottingham allowed self-reference, but held that all affirmative self-referential statements are false." While this is not technically a *restrictio* position, the result is much the same.

LXVIII Sexgrave rejects all self-reference.

Five of the texts reject strong *restrictio* but accept a weaker version. These texts may be further divided into two types: Those which reject self-reference only for *insolubilia*, and those which do not. Of the people who argue for an *ad hoc* theory of weak *restrictio* we may place Roger Roseth [LXII] and William of Ockham [LXX] and [LXXI]:

LXII Roger denies self-reference in insolubles.

LXX, LXXI Ockham's discussion in the *Summa Logica* of insolubles falls within a discussion of *institutio* obligations. He says that in this type obligation, there is a rule which disallows parts suppositing for wholes of which they are parts. He also notes exceptions to this rule. In the *Tractatus*, the position is much the same as above: Self-reference is allowable except in insolubles.

The two remaining texts, Richard of Campsall [LIV] and Walter Burley [LXVII], espouse a form of weak *restrictio* which does not immediately appear to be trivial:

LIV Richard allows self-reference but only in innocuous cases: "*dicendum quod subiectum aliquando potest supponere pro toto cuius est pars, et hoc est ubi non accidit refleccio in indiffinita . . . et ideo subiectum in ista: 'proposicio est vera,' indifferenter supponit pro ista, sicud pro quocunque alia.*" [Sy68, ¶10.46].[19]

[18] "A term does not supposit for the statement of which it is a part".

[19] "It must be said that the subject sometimes is able to supposit for the whole of which it is a part, and that is when reference does not occur to something indefinite. . . and for this reason the subject in this: 'a proposition is true', indeterminately supposits for this [proposition], just as for any other whatever."

LXVII Burley explicitly rejects strong *restrictio* but accepts a weakened version: "*Ad responsiones restringentium dicendum quod non est inconveniens quod idem sit pars integralis alicuius totius et tamen totum universale respectu eiusdem, sicut patet sic: dicto 'omnis propositio est vera'; subjectum huius est pars integralis huius propositiones, et tamen est totum universale ad hanc propositionem, quia est totum universale ad omnem proposition*" [Ro70, ¶2.06].[20] Unfortunately, Burley does not expand on when such reference suitable and when it is not.

6.4.2 Roger Swyneshed's solution

One text which was omitted from [Sp75] (not surprisingly since it was not edited until 1979) is a treatise on *insolubilia* by the fourteenth-century English logician Roger Swyneshed, edited in [Sp79] and discussed in [Sp83]. Swyneshed, who is not to be confused with Richard Swyneshed, a Merton mathematician and author of the *Liber calculationum*, was born some time before the turn of the 14th century, and died around 1365, a master of theology and a Benedictine monk at Glastonbury. Three treatises can be confidently ascribed to him: a tract, likely written before 1335, on *insolubilia* and *obligationes*, which we discuss here and which became a standard university text in continental universities in the next century and a half; a treatise on consequences (which has not been identified); and a text on natural philosophy [Wei64, pp. 243–245].[21]

In his treatise on *insolubilia*, Swyneshed gives a solution to the Liar Paradox which was "one of the influential and most controversial medieval theories of the semantic paradoxes" [Sp83, p. 105]. His solution is that the Liar Paradox is not a paradox, it is simply false. Before we discuss his solution in detail, we must point out a way in which medieval semantics differ from modern semantics. Swyneshed follows the standard line of thought among medieval logicians in taking not sentence-types (what modern logicians usually call "propositions") as the bearers of truth value but rather sentence-tokens, that is specific instances of mental, spoken, or written sentences. A sentence which is neither thought, spoken, or written does not exist, and a sentence which does not exist cannot have a truth value. This point will be important below, and should be kept in mind in all of the definitions.[22]

Swyneshed constructs his solution to the Liar Paradox by distinguishing be-

[20] "To respond to the *restringentes* [i.e., those that reject self-reference completely] it must be said that it is not unsuitable that the same thing may be an integral part of some whole and yet a universal whole with respect to that very thing, just as is clear thus: I say 'every proposition is true'; the subject of this is an integral part of the proposition, and yet it is a universal whole for that proposition because it is a universal whole according to every proposition."

[21] For more details of Swyneshed's life, and useful commentary distinguishing him from his slightly younger contemporaries Richard Swyneshed and John Swyneshed, see [Wei64].

[22] For more details, see [Nu73].

tween the notions of truth and correspondence. He defines the truth and falsity of sentences as follows:

> *Secunda est haec: Propositio vera est propositio non falsificans se principaliter sicut est significans naturaliter aut ex impositione vel impositionibus qua vel quibus ultimo fuit imposita ad significandum.*
>
> *Tertio definitio: Propositio falsa est oratio falsificans se vel oratio non falsificans se principaliter aliter quam est significans naturaliter, ex impositione, vel impositionibus qua vel quibus ultimo fuit imposita ad significandum* [Sp79, p. 185].[23]

To understand these definitions we must unpack the various types of signification and imposition which are being used here. The signification of a sentence is (roughly speaking) what the sentence says. Note that the signification of a sentence should not be identified with the proposition expressed by the sentence, since the proposition expressed does not vary from hearer to hearer, whereas the different types of signification can.

Principal signification is to be contrasted with partial signification. The sentence "One cat chased the other cat down the hall" principally signifies that one cat chased the other cat down the hall, and partially signifies that one cat chased the other cat. We disregard partial signification here, because, according to Spade, "throughout his *Insolubilia* Swyneshed seems to be concerned with principal signification only; partial signification plays no role" [Sp83, p. 106].

There are three ways that a sentence may (principally) signify, and these ways depend on the type of sentence involved:

1. A mental sentence signifies by nature, e.g., it is not a matter of choice or convention.

A written or spoken sentence, on the other hand, signifies artificially, in one of two ways:

2. Their signification may be arbitrarily or conventionally imposed by the speaker or writer, or
3. Their signification may be unintended by the speaker or writer; this is still artificial, but it is not imposed.

[23] 'The second (definition) is this: A true sentence is a sentence that does not falsify itself and that principally signifies as is the case, either naturally or from the imposition or impositions by which it was last imposed to signify.

'The third definition: A false sentence is an expression that falsifies itself, or else an expression that does not falsify itself and that principally signifies otherwise than is the case, either naturally or from the imposition or impositions by which it was last imposed to signify" [Sp83, p. 105].

Clearly, a sentence can have more than one type of signification for different hearers or readers at the same time. However, in the determination of the truth value of a sentence, only types (1) and (2) are relevant.

The next distinction found in the definitions is that of 'signifying as is the case' as opposed to 'signifying as is not the case'. Unfortunately, Swyneshed gives no clear definition of what it means to 'signify as is the case' or to 'signify as is not the case'. Spade puts forward an explication of 'signifying as is the case' as signifying "in a way that corresponds to some fact that really obtains" [Sp83, p. 107], and 'signifying as is not the case' as signifying "in a way that denies or contradicts some fact that really obtains" [Sp83, p. 107]. However, as even he admits, this definition is not adequate to all cases which Swyneshed considers.[24] However, the intricacies of the cases when this description fails are not relevant for our purposes here.

To complete his solution to the Liar Paradox, Swyneshed introduces the notion of self-falsification. A sentence can be self-falsifying either mediately or immediately:

> *Propositio falsificans se ipsam est duplex. Quaedam falsificat se mediate, quaedam immediate. Propositio falsificans se mediate est propositio significans principaliter sicut est vel aliter quam est et ipsa sic significando falsificat propositionem aliam a se falsificantem se...*
>
> *Propositio falsificans se immediate est propositio significans principaliter sicut est vel aliter quam est pertinens ad inferendum se ipsam fore falsam* [Sp79, ¶¶4–5].[25]

The Liar self-falsifies itself immediately, whereas the Nested Liar:

The next sentence is false.
The previous sentence is true.

is an example of sentences which are self-falsified mediately, because the sentences are only paradoxical in conjunction with each other, and not on their own.

Sentences which falsify themselves immediately can be divided into two further groups, those that are sufficient for falsifying themselves and those which

[24] [Sp83, p. 108]. Additionally, it is important to note that 'signifying as is the case' and 'signifying as is not the case' are not mutually exhaustive; there are some sentences which neither signify as is the case nor signify as is not the case. This could be because the sentence doesn't signify anything, or because there is no fact of the matter which the sentence signifies as being the case or not being the case.

[25] "Propositions which falsify themselves are twofold. Some falsify themselves mediately, some immediately. A proposition falsifying itself mediately is a proposition signifying principally as is the case or otherwise than is the case and it itself by signifying in such a way falsifies a proposition other than itself which falsifies itself [that is, the original proposition]...

A proposition falsifying itself immediately is a proposition signifying principally as is the case or otherwise than is the case and is pertinent for inferring that it is false."

are insufficient. It is in trying to account for this latter group of sentences that Swyneshed's explication of what it means for something to be "relevant for inferring that it is false" breaks down. A sentence p is *pertinent* or *relevant* for inferring "q is false" if there is a valid argument from p (perhaps in conjunction with other premises) to "q is false", such that p's presence is necessary for the argument's validity.[26] For the case of self-falsifying sentences which are immediate and sufficient, this simply means that given the existence of p there is an inference from p to "p is false" which requires no other premises. We make this notion formal in §6.5.

However, this definition of relevance does not work for immediately self-falsifying sentences which are insufficient for inferring that they are false. For, take any innocuous sentence which signifies as is the case, say $\varphi := 2 + 2 = 4$. There is a valid argument with φ as a premise and "φ is false" as the conclusion where φ is required for the validity of the argument, namely the argument which has $\neg\varphi$ as a second premise. The reason that φ is required as a premise in the argument is that from $\neg\varphi$ alone we cannot conclude "φ is false", because we do not know whether φ exists. The addition of φ as a premise guarantees that it does exist. Swyneshed does not address this point, and, as Spade concludes [Sp83, pp. 109–110], it's not clear whether he would have been able to give a consistent definition of relevance had he realized the problem.

We leave this problem for the moment, and restrict our attention to the unproblematic case, namely mediately self-falsifying sentences and immediately self-falsifying and sufficient sentences.

6.5 Announcement pointer semantics

In this section we combine the ideas of Gaifman's pointer semantics (introduced in §6.2.2) and dynamic epistemic logic (discussed in §6.3.1). We fix a set of propositional letters P and define the language $\mathcal{L}^* := \langle P, \neg, \wedge, \mathrm{K}, T, [\,] \rangle$. The set Fml^* of well-formed formulas of $\mathcal{L}^*$ is defined by:

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid \mathrm{K}\,\varphi \mid T(p) \text{ for } p \in P \mid [p]\psi \text{ for } p \in P$$

Note that the language $\mathcal{L}_{\mathrm{L}}$ used for the Liar Paradox is a sublanguage of $\mathcal{L}^*$ (i.e., $\mathrm{Fml}_{\mathrm{L}} \subseteq \mathrm{Fml}^*$), and thus the notion of a system (i.e., a function $\Sigma : P \to \mathrm{Fml}$) still makes sense. Given a system Σ, we shall now define a semantics $\models_\Sigma$ for announcement pointer models $\mathfrak{M}$, using the notion of restriction of an epistemic model $\mathfrak{M}{\restriction}\varphi$ from dynamic epistemic logic (see §6.3.1):

[26] See [Sp79, p. 181, fn. 37] and [Sp83, p. 109].

Definition 6.5.1 (Announcement pointer semantics)**.**

$$\begin{array}{lll} \mathfrak{M}, w \models_\Sigma \mathrm{K}\,\varphi & \text{iff} & \text{for all } w' \text{ such that } w \sim w',\ \mathfrak{M}, w' \models_\Sigma \varphi \\ \mathfrak{M}, w \models_\Sigma T(p) & \text{iff} & \mathfrak{M}, w \models_\Sigma \Sigma(p) \\ \mathfrak{M}, w \models_\Sigma [p]\psi & \text{iff} & \mathfrak{M}, w \models_\Sigma \Sigma(p) \text{ implies } \mathfrak{M}{\restriction}\Sigma(p), w \models_\Sigma \psi \end{array}$$

With these semantics, we can give a definition in complete analogy to van Benthem's notion of self-refutation:

Definition 6.5.2. A proposition p in a system Σ is called *immediately relevant to inferring that it itself is false* if for all announcement pointer models $\mathfrak{M}$, we have $\mathfrak{M} \models_\Sigma [p]\,\mathrm{K}\,\neg p$.

This corresponds (except for the use of the K operator) directly to Swyneshed's informal definition of the notion given in the previous section.[27] Note that this is again an instance of a semantic solution with the $\mathcal{L}^*$-formula scheme

$$\Phi(x) := [x]\,\mathrm{K}\,\neg x.$$

This is literally the same formula as was used in van Benthem's solution.

Proposition 6.5.3. The instance λ is immediately relevant to inferring that it itself is false.

Proof. Consider a arbitrary announcement pointer model $\mathfrak{M}$ and fix a node w. If $\mathfrak{M}, w \models_\Lambda p$, then $\mathfrak{M}, w \not\models_\Lambda \Lambda(p)$, and thus any formula beginning with $[p]$ is trivially true at w. However, $\mathfrak{M}{\restriction}\Lambda(p)$ has only nodes in which $\Lambda(p) = \neg p$ is true, so obviously $\mathfrak{M}{\restriction}\Lambda(p) \models_\Lambda \mathrm{K}\,\neg p$. □

Swyneshed's solution thus corresponds to restricting the set of possible instances to the set of those systems that have no propositional letter that is immediately self-falsifying.

6.6 Conclusion

We have seen that there is a structural similarity between the solutions of van Benthem and Swyneshed. Both are *semantic solutions* in the sense that they exclude instances on the basis of their semantical behavior, and both can be represented in a dynamic setting with only slightly differing semantics for the announcement operators (for van Benthem's solution, the standard semantics are adequate, for Swyneshed, we needed the pointer semantics, i.e., the update according to the *intended meaning* of the proposition).

[27] The use of the K operator does not affect our analysis here: the definition could have been given as "for all epistemic models $\mathfrak{M}$, we have $\mathfrak{M} \models_\Sigma [p]\neg p$", but we wanted to keep the formal description structurally as similar to van Benthem's solution as possible.

Moreover, we see that when presented in a formal dynamic setting, both solutions use the same formula in order to do the semantic restriction for excluding problematic cases, namely

$$\Phi(x) := [x]\,\mathrm{K}\,\neg x.$$

The choice of $\Phi(x)$ was not tied specifically to the problematic instances ι_{Fitch} and λ, but was rather grounded on other, independently interesting and acceptable criteria. Hence, the solutions of both Swyneshed and van Benthem fall in the class of interesting, non-trivial, and non-*ad hoc* solutions to their respective paradoxes.

Chapter 7

A logic for reasoning about the trinity

7.1 Paralogisms of the trinity

Despite the fact that identity is often taken to be, by definition, reflexive, transitive, and symmetric, the following syllogism about the trinity appears to have true premises and a false conclusion:

> The Father is God.
> God is the Son.
> Therefore, the Father is the Son.

Other examples can easily be given, and the conclusion that many people, both modern and medieval, draw is if we want to speak rationally about the trinity, ordinary logic with ordinary identity will not suffice. Some even draw the further conclusion that we cannot reason about the trinity at all, and that the concept of three persons in one is, at worst, self-contradictory, or, at best, simply ineffable. An example of someone who held the later view was Boëthius, who allowed that there was a fundamental difference between 'being' as that term is used of created things and as it is used of the members of trinity, but he held that philosophy (or logic) could say nothing about the divine use of the term.[1] But for those who do believe logic can be applied to the trinity, there are two options: The first is to show that the standard logic can be extended to a logic which adequately addresses trinitarian issues and the second is to deny that the standard logic is universally applicable, and to develop instead a separate logic for trinitarian reasoning.

[1] Boëthius's views should not be confused with the stronger, and wide-spread, medieval view of negative theology. Negative theology is the view that positive predications cannot be made of God; we can only say what God is *not*. This view is usually most closely associated with Pseudo-Dionysius the Areopagite; see [CorHa08]. Boëthius thinks that we *can* make positive predications about God, but that philosophy cannot tell us what the nature of the terms being predicated of God is. (For example, when we say "God is rational" and "Man is rational", the term *rational* may be equivocal; if it is, only theology can tell us, philosophy cannot.).

In this chapter, we investigate a medieval text which offers a solution of the first type. [Mai88] contains an edition of an anonymous logical treatise found in MS. Munich, Bayerische Staatsbibliothek, lat. 17290, ff. 136r–145v (all references will be to this edition). The treatise discusses modes of predication and syllogistic reasoning in the trinity. Few details about the authorship or localization of the text are known. Because Thomas Aquinas is referred to as a saint, the text was almost certainly written after his canonization in 1323. On the basis of other textual and conceptual references, a composition date of the late 14th or early 15th century can be postulated, possibly in a Germanic setting.[2] This manuscript is the only known manuscript containing this text. The text is, unfortunately, incomplete; in paragraph 106 an objection is introduced, and the text breaks off directly after, leaving the objection unaddressed. A translation of this text into English is given in Appendix C of this dissertation.

In the text, the author argues that we must make distinctions in modes of speaking and modes of predication when we are talking about divine things, and these are distinctions which collapse when we talk about created things. The author notes that Aristotle ignored the modes of being which could be found in divine things, and concentrated only on the modes of being found in created things, and for this reason many philosophers and theologians following Aristotle took the same route. Furthermore, even if the philosophers had disagreed about whether there were any objects which were personally identical, they were all agreed that the only type of *predication* that was possible was formal predication (we discuss this further in §7.4.1). This means that one could assume that in a syllogism all the propositions were formal predications, and there was no need to specify this explicitly, and as a result, there was no need for a logic which could account for all types of predication. And, as noted above, even those like Boëthius and Richard of Saint Victor who did recognize a distinction between the types of being had independent reasons for thinking that logic could say nothing about these distinctions.[3]

Because traditional Aristotelian and Boëthian logic was developed to reason about created things, we are often misled into thinking that there is just one mode of speaking and one mode of predication. The paralogisms of the trinity arise when we try to reason about the trinity using the standard definition (three distinct persons who are yet one indivisible God), with just this single mode; reasoning about the one simple God and the three distinct persons simply cannot be done in the same way that reasoning about non-divine, created things is done. We can solve these paralogisms by making explicit the modes of being that are in use in various predications concerning the trinity, and develop a sound logic for reasoning about these modes, a logic from which ordinary logic, for reasoning about created things, can be recovered. If this solution is possible, it certainly

[2]For further discussion of this, see Maierù's introduction [Mai88, pp. 251, 255–257].

[3]This is discussed in [¶¶1–2].

should be preferred: We neither have to reject the application of reason to the trinity, nor insist upon any fundamental difference between trinitarian logic and creation logic.

In §7.2 we summarize the contents of the text. In §7.3 we discuss the theoretical background with which the author was working, introducing the different modes of being and modes of speaking, supposition theory and its relation to the distribution of terms, and the expository syllogism. This theoretical apparatus is formalized in §7.4; we then prove certain features about the resulting system. In the final section, we show how this system can be used to resolve trinitarian paralogisms.

7.2 The text

The text can be divided into three main parts, each of which builds upon the previous one:

1. a discussion of modes of being.
2. a discussion of modes of predication.
3. a discussion of syllogistic reason.

According to the author, the first of these is properly within the scope of philosophy (or, when it concerns the trinity, theology); the latter two make up the scope of logic. The author opens his text by pointing out that:

> *Sicut in divinis est quidam modus essendi qui non est actualiter in creaturis, ita circa eadem divina ut videtur debet esse quidam modus predicandi ac sylogizandi qui in creaturis non est necessarius* [¶1].

This is offered both as a starting point for the investigations of the rest of the treatise and as an apology for Aristotle: Because Aristotle in his philosophical writings focused on the mode of being or existence as it is found in created things, this explains why his syllogistic system, which is based on predications expressing the mode of being in created things, does not accommodate reasoning about non-created, i.e., divine, things. But because Aristotle's philosophical focus was what it was, the author argues that we cannot fault him for not recognizing that his logic was limited in scope.

Before any discussion of modes of being or predication can be made, we must first establish certain essential facts about our topic of inquiry, and this is what the author does starting in ¶4. He makes a very brief and elliptical comment on the nature of the trinity, from a theological point of view. The author takes his cue from Athanasius, saying:

> *Modus essendi in divinis est quod tres persone sunt una essencia simplicissima et eadem simplicissima essencia tres persone et quelibet earum* [¶4].

This view of Athanasius's is expressed by the Church in the Athanasian Creed, which was adopted in the 6th century:

> *Unum Deum in Trinitate, et Trinitatem in Unitate veneremur; neque confundentes personas: neque substantiam separantes... Sed Patris et Filii et Spiritus Sancti una est divinitas: aequalis gloria, coaeterna majestas... Aeternus Pater: aeternus Filius: aeternus [et] Spiritus Sanctus. Et tamen non tres aeterni: sed unus aeternus.*[4]

In taking this as his basis, our author is making as few controversial assumptions about the nature of the trinity as possible.[5] As Friedman in [Fr99] notes, divergent medieval theories of the trinity all "have in common the claim that the three divine persons share everything—they are, in medieval terms, 'essentially identical', i.e., the same absolutely simple God—apart from one minimal difference, a 'property' or special characteristic that makes each of the persons distinct from the other two" [p. 14]. What the nature of this special property is and how it is to be interpreted is a matter for the theologians to debate, and our author does not address the ontological issue in his text; it suffices for the purpose of logic that there is a way to distinguish the persons from each other, as we'll see in more detail below.

This short summary of the properties of the trinity is followed, in ¶¶6–24, by a discussion of the modes of being which can be found in the trinity, and the connection that these modes of being have to what we can call 'modes of speaking'. In ¶25, the author notes that, modes of being having been spoken of, we can now move to a discussion of modes of predication and syllogistic reasoning, for, as he says, *[l]ogica vero, quantum ad propositum sufficit, in modis predicandi ac sylogizandi consistit.* Because predications are predications in some mode of being, before logic proper is discussed it is first required that the philosophical issues of modes of being be covered. Speaking very anachronistically, we can say that the first 24 paragraphs were setting up the semantics of our system, explaining the underlying factors which will make certain predications true or false, and that

[4] "We worship one God in Trinity, and Trinity in Unity; neither confounding the Persons: nor dividing the Substance... But the Godhead of the Father, of the Son, and of the Holy Ghost, is all one: the Glory equal, the Majesty coeternal... The Father eternal: the Son eternal: and the Holy Ghost eternal. And yet they are not three eternals: but one eternal."

[5] In particular, in accepting this view, the author is rejecting the heretical view of modalism (also known as Sabellianism (named for its 3rd-century founder Sabellius), Patripassianism, or aspectualism), which says that ontologically there are not three persons, only one, and that the persons of the Father, Son, and Holy Spirit are just different modes, aspects, or roles of the one person of God. For more information on modalism, see [Will$_1$24, pp. 302–307] and [Ne01, ch. 1, §5].

starting in ¶25 we are now being given syntax. Facts about generating modes of predication from the modes of being are discussed in ¶¶25–32, and the discussion of syllogisms, which makes up the rest of the text, begins in ¶33. In presenting his syllogistic system, our author makes use of two typically medieval developments in logic: supposition theory and expository syllogisms. From standardly accepted facts about the supposition of terms and the reduction of certain classes of general syllogisms to expository syllogisms, the author is able to isolate a class of divine syllogisms which are valid, and to give a justification for their validity. Rules governing the validity of categorical syllogisms with mixed premises are given in ¶51 (for affirmative syllogisms) and ¶¶57–60 (for negative syllogisms). After a discussion of how these rules relate to expository syllogisms, the class of valid syllogisms which have two positive premises is summarized in ¶¶93–96, and the class of valid syllogisms which have a negative premise summarized in ¶¶98–105. Unfortunately, ¶106 provides a counterexample to the system which has just been outlined, and as the text breaks off we are left with no indication as to how the author would have resolved this problem.

In the next section, we will cover, from an informal point of view, the three theoretical building blocks which we will use as the foundation of the formal system that we'll construct in §7.4. These are: modes of being and modes of speaking (§7.3.1), supposition theory, as used by this author (§7.3.2), and expository syllogisms and their relationship to standard syllogisms (§7.3.3).

7.3 Background theory

7.3.1 Modes of being and speaking

When our author discusses the so-called modes of being (*modi essendi*) of an object (divine or created), it is clear that what he is speaking of is modes of identity, that is, different ways that two objects can be identical. This is because he never speaks of an object simply *existing* in one of these modes of being, but rather he speaks of one object *being the same as* another object in one of these modes of being.[6] The author distinguishes three modes of being, that is, three ways in which two things can be identical with each other:

- *Essencialiter*
- *Personaliter/Ydemptice*
- *Formaliter/Proprie*

[6]In this way we have further evidence, that despite talk of 'modes', we are not dealing with any form of theological modalism; at all times, we are talking about the identity of two ontologically distinct objects, not the identity of two different ways of speaking about one ontological unitary object.

This distinction of types of identity can be found as early as Abelard.[7] In his *Theologica 'scholarium'* II.95–99, Abelard distinguishes three ways that things can be the same [Ab87, pp. 454–456]:

- *Essentialiter siue numero*
- *Proprietate seu diffinitione*
- *Similtudine*

Roughly speaking, two things are essentially the same if they share the same essence; but things which are essentially the same may still yet differ in the accidental properties that they share or in the definitions which define them.[8]

Abelard's three ways of being the same correspond to the three modes of being in the anonymous text we're considering.[9] Abelard's essential identity is also called *idem quod* sameness, and Knuuttila glosses it as "[t]he sameness pertaining to the subject and predicate of a singular proposition in the sense that there is a third of which both are said". This is distinguished from *idem qui* sameness, glossed as "the sameness between the meanings of terms". This *idem qui* sameness covers both personal and formal (or proper) identity [Knu07, p. 193]. Basically, if two things are essentially identical, then they share the same essence. If they are personally identical, then they share the same properties and definitions. Finally, if two things are formally identical, then they share sufficient similarity that they can be placed under the same genus, or form.[10]

This gives us an idea of what is meant when we say that two objects are personally the same, or that they are formally distinct, but it does not give us

[7]For further discussion of Abelard's views, see [Knu99], especially p. 242.

[8]The paralogisms of the trinity that arise when the type of identification or distinction in the premises of a syllogism is not fully specified are related to the 'Leibniz's law' arguments that Schnieder discusses in [Schn06], that is, arguments of the form [p. 40]:

(i) x is thus and so
(ii) y is not thus and so
Ergo $x \neq y$.

Schnieder argues that this type of argument is only acceptable when the negation in (ii) is what he calls the 'ordinary use of negation' [p. 49; cf. also p. 45]. When the negation is being used not to indicate that the proposition expressed by the sentence is false, but rather that the sentence is defective in other ways, then we cannot make the inference from (i) and (ii) to the conclusion. Similarly, our author is arguing, as we'll see in more detail later, that from premises of the form (i) and (ii) one can only draw a conclusion if the type of denial used in (ii) is the same as the type of predication in (i).

[9]As Knuuttila notes, "The originally Abelardian distinction between intensional (personal) and extensional (essential) identity was widely employed in later medieval Trinitarian theology and influenced late medieval logic" [Knu07, p. 195].

[10]In ¶32 the author says that *est quidam modus essendi quo aliqua sunt formaliter idem, ita quod in quocumque est unum in eodem est et alterum.* It is not clear whether this condition is a sufficient or necessary condition for two things being formally identical.

information about the nature of the relationships 'being essentially the same as', 'being personally the same as', and 'being formally the same as'. If they are all equivalence relations, then they cannot all hold universally of the trinity, because then there would be no distinguishing between them, and we would be back in the situation that we started out in, namely that of the paralogisms. No clear statement of the properties of these relations is given in the text, but we can extract some of them by looking at the examples of identities and distinctions that the author makes.

For ease of proceeding, we introduce some notation. We let $=_e$, $=_p$, and $=_f$ be the three relations listed above, respectively. We let F stand for the Father, S for the Son, and HS for the Holy Spirit. But we need more than this: our author's examples make use of five further aspects[11] of the trinity. These are the essence, E, and the substance, Su, and the three things which are called by our author the "personal properties", following Peter Lombard (¶¶23–24): the fatherhood (or paternity), P; the wisdom, Wi; and the charity (or love), C. These properties are the distinguishing properties of the persons of the Father, the Son, and the Holy Spirit, respectively.[12]

At the most basic level, the quotes from Athanasius (¶¶4–5) tell us that all three persons of the trinity are essentially the same, but they are distinguished from each other personally (this latter fact is reiterated in ¶21). The persons are personally the same as the essence (¶17), but formally distinguished from the essence because we can predicate one name of the essence which is not true of all the persons (e.g., 'the essence is the Son' but 'the Father is not the Son'; cf. ¶¶7, 22.[13]). The author later says:

> *In trinitate autem divina non est distinccio essencialis; ideo de ea hic non curo* [¶20].

From this it is clear that essential identity is, we could say, the *real* identity in the trinity: that is, it behaves as we expect identity to behave, being reflexive, transitive, and symmetric, and covering all members. And indeed we will see that not much more beyond this is said about $=_e$.

Each of the personal attributes, P, Wi, and C, is formally the same as the persons F, S, and HS, respectively (¶¶18, 24), and each personal attribute is personally distinct from the others (¶21). Determining further the relationship

[11] We use the term 'aspect' in a loose, informal sense, without intending any implication of an aspectual or modal theory of the trinity.

[12] The essence also has essential attributes, namely *sapiencia ⟨et⟩ essencialiter dicte, iusticia, bonitas, etc.* [¶19]. But we need not introduce new terms for these essential attributes, since they are all formally identical with the essence [¶¶19, 32], and the author makes no further mention of them.

[13] This is reiterated in ¶48 when the author says: *Nec obstat hoc quod essencia distinguitur formaliter a persona patris et a qualibet alia persona, quoniam hoc nichil plus est dicere nisi quod essencia, que est penitus et realiter eadem cum persona, eciam est persona filii, que persona filii non est persona patris.*

between the persons and the three attributes is complicated by the fact that we can speak of these attributes in two different ways. The author distinguishes between, e.g., the 'wisdom essentially speaking' or 'wisdom taken essentially' and the 'wisdom which is properly the Son'. (We anticipate §7.4 by noting now that we will distinguish these two different ways of speaking by functions on terms, indicated by the subscripts $_{\mathsf{es}}$ and $_{\mathsf{fs}}$, for 'essentially speaking' and 'formally speaking', respectively.)

The author points out the distinction between these two ways when he says:

> *verbum in divinis est proprie sapiencia et tamen pater et spiritus sanctus sunt sapiencia. Sed, ut videtur, hoc non est aliud dicere quam quod verbum est proprie, idest formaliter, sapiencia, scilicet genita, et pater et spiritus sanctus sunt sapiencia, non tamen sunt sapiencia genita que proprie, idest formaliter, est verbum, sed sunt sapiencia essencialiter dicta, que communis est tribus personis* [¶10].[14]

What he is saying is that when we want to speak properly or formally, the personal attributes should be (formally) identified only with the person, but if we are using the personal attributes in reference to the underlying essence, then because the essence is essentially shared by all the persons, their attributes are also all shared, in so far as those attributes are identified with the essence. This is further exemplified in ¶12 when he says:

> *Patet eciam quod sapiencia essencialiter dicta distinguitur formaliter a sapiencia que est proprie verbum, sicut essencia distinguitur formaliter a filio; patet, quia sapiencia essencialiter dicta est tres persone, sed sapiencia que est proprie verbum non est tres persone, quia solum est persona filii.*

For the same reason, even when the personal attributes are taken essentially, they can each be formally identified with only one of the persons:

> *Nec tamen pater, aut spiritus sanctus, est proprie seu formaliter sapiencia que est communis tribus personis, sed solum per ydemptitatem, alioquin pater, aut spiritus sanctus, esset ydemptice tres persone, quod verum non est. Ex quibus patet quod pater, aut spiritus sanctus, distinguitur formaliter a sapiencia essentialiter dicta, quia sapiencia ut sumitur esssencialiter est tres persone et quelibet earum, sed nec pater nec spiritus sanctus est tre persona, etc.* [¶11].

[14]The *verbum in divinis* is the Son; this is a reference to John 1:1, "In the beginning was the word, and the word was with God, and the word was God". That the *verbum* mentioned here is the Son is verified in John 1:14, "And the Word became flesh and lived among us, and we have seen his glory, the glory as of a father's only son, full of grace and truth."

Understanding the argument here is easier once we read in ¶19 that *[e]ssencia etiam et attributa essencialia. . . sunt formaliter idem, quia in quocumque supposito est essencia, in eodem sunt eciam cetera attributa essencialia*, namely that $\mathsf{E} =_f \mathsf{Wi}_{\mathsf{es}}$, and likewise for P_{es} and C_{es}. However, the author says in ¶23 that $\mathsf{E} \neq_f \mathsf{Wi}_{\mathsf{fs}}$, and likewise for P_{fs} and C_{fs}, from which we can conclude that, as is done in ¶12, that $\mathsf{Wi}_{\mathsf{es}} \neq_f \mathsf{Wi}_{\mathsf{fs}}$. Briefly, these cannot be formally the same because the $\mathsf{Wi}_{\mathsf{es}}$ is the essence, but the $\mathsf{Wi}_{\mathsf{fs}}$ is the Son, and the essence and the Son are formally distinguished; the same argument can be given for the Father and paternity and the Holy Spirit and charity (cf. ¶15). However, when the personal properties are taken essentially, they are personally identified with the all of the persons, because the essence is personally the same as each of the persons:

> *Aliqua vero sunt ydemptice idem, sicut essencia et persone, atque sapiencia essencialiter sumpta et persone; similiter caritas essencialiter sumpta et persone* [¶17].

A comment on the use of *ydemptice* instead of *personaliter* here. Given that $=_p$ is clearly not an equivalence relation[15], it is strange to the modern ear to say that being *personaliter idem* is being *ydemptice idem*. The author never explicitly equates being *ydemptice idem* and being *personaliter idem*. However, the textual evidence indicates that this must be the case. In ¶¶16–18, the author gives a summary of the material from Peter Lombard that he has just presented; in ¶16 he discusses things being *essencialiter idem*, in ¶17 things being *ydemptice idem*, and in ¶18 things being *formaliter idem*, and in each case the discussions are being contrasted with each other. Later, in ¶30, he uses *ydemptice* and *personaliter* as if they are synonymous, and where *ydemptice* is again contrasted with *essencialiter*:

> *[Q]uidam modus essendi in divinis est quo aliqua sunt idem non solum essencialiter, sed eciam ydemptice et personaliter.*

And again in ¶65 *personaliter* is used as a synonym of *ydemptice*. From this we can conclude that when our author speaks of two things being *ydemptice idem*, he is speaking of personal, not essential, identity.

The substance of the trinity, Su, is discussed primarily in the context of the author summarizing the views of Peter Lombard, in ¶9. From this summary, it is clear that Su stands in the same formal relation to P, Wi, and C that E does to F, S, and HS. As a result, Su is formally distinguished from E, because the persons and the personal properties are formally distinct from the essence (cf. ¶23).

We summarize the discussion of formal and personal identities in the trinity found in ¶¶5–24 in Table 7.1 (note that some of the cells are not wholly filled in because the text is underspecific). Since $=_e$ is an equivalence class of which all parts of the trinity are members, we omit it from the table since it would appear in every cell. We also omit the subscripted terms; it will become clear where they fit once we give the formal definition of the functions $_{\mathsf{es}}$ and $_{\mathsf{fs}}$ in §7.4.

[15] The author gives a counterexample to the transitivity of personal identity in ¶52.

	E	F	S	HS	P	Wi	C	Su
E	$=_f$	$\neq_f,=_p$	$\neq_f,=_p$	$\neq_f,=_p$	$\neq_f$	$\neq_f$	$\neq_f$	$\neq_f$
F	$\neq_f,=_p$	$=_p,=_f$	$\neq_p,\neq_f$	$\neq_p,\neq_f$	$=_p,=_f$	$\neq_f$	$\neq_f$	$\neq_f$
S	$\neq_f,=_p$	$\neq_p,\neq_f$	$=_p,=_f$	$\neq_p,\neq_f$	$\neq_f$	$=_p,=_f$	$\neq_f$	$\neq_f$
HS	$\neq_f,=_p$	$\neq_p,\neq_f$	$\neq_p,\neq_f$	$=_p,=_f$	$\neq_f$	$\neq_f$	$=_p,=_f$	$\neq_f$
P	$\neq_f$	$=_p,=_f$	$\neq_f$	$\neq_f$	$=_p,=_f$	$\neq_p,\neq_f$	$\neq_p,\neq_f$	$\neq_f$
Wi	$\neq_f$	$\neq_f$	$=_p,=_f$	$\neq_f$	$\neq_p,\neq_f$	$=_p,=_f$	$\neq_p,\neq_f$	$\neq_f$
C	$\neq_f$	$\neq_f$	$\neq_f$	$=_p,=_f$	$\neq_p,\neq_f$	$\neq_p,\neq_f$	$=_p,=_f$	$\neq_f$
Su	$\neq_f$	$\neq_f$	$\neq_f$	$\neq_f$	$\neq_f$	$\neq_f$	$\neq_f$	$=_f$

Table 7.1: Formal and personal identity in the trinity

7.3.2 Supposition theory and the distribution of terms

In the previous section we discussed the author's presentation of what we can anachronistically call the semantic theory of the trinity. Once the presentation is completed, we can move to questions of predicate and syllogistic reasoning because that, as noted earlier, is what logic consists in. In ¶¶27–32 the author tells us that because we have identified three different modes of being which can be found in the trinity, we need to be able to make predications which express these three different modes of being. Because according to the *consuetum usum theologorum*, '*est*' is generally taken to indicate not only essential identity but also personal and formal identity (¶28), we must make our language more specific in order to be able to keep our predicative sentences from being ambiguous. This is done through the addition of adverbs modifying the copula, adverbs that indicate that only one of the modes of being is in use:

> *Vel si quis diceret: pater est* a *filius, ita quod* a *sit signum ydemptitatis solum essencialis, tunc similiter credendum quod ista proposicio esset vera, scilicet: pater est* a *filius* [¶28].

This move is extremely interesting from the perspective of a modern logician because it appears to be nothing so much as a first step towards a semi-formalized language. Here *a* is being used to indicate the adverb '*essencialiter*', and later on the author uses *b* for '*personaliter*' or '*ydemptice*' and *c* for '*formaliter*' or '*proprie*'.

We identify how to construct predications which indicate the different modes of being because we will use these predications within syllogisms. But before we can do so we need to consider how the divine terms such as F, S, etc., function within these predications. The study of the properties of terms as they occur within propositions falls under supposition theory. Our author takes supposition theory as one of his basic working tools; that is, he uses it as he requires without giving a full exposition of the theory behind his usage. We do not attempt to

reconstruct the author's full theory of supposition from his use of it in his text, but instead just give an overview of the important aspects of the theory that he uses. We invite the reader to compare what is developed here with Lambert's theory of supposition as outlined in §5.2 of Chapter 5.

The first point the author makes on this subject, in ¶33, is that terms which have only one formal suppositum differ from terms which have more than one formal supposita, both in how the distribution of the terms function and in how the propositions in which these terms occur function within syllogisms.[16] A subject term is distributed in a predication when it is modified by a syncategorematic term (such as *omnis* 'all') which divides the term into different pieces, to each of which the predicate applies equally well, and vice versa for predicate terms.[17] Some syncategorematic terms have distributed force and some do not. In categorical propositions, *omnis*, *nullus*, and *non* all have distributive force; *quidam* and *aliquid* do not. The universal quantifier has the effect of distributing the subject term of a proposition, and the negative quantifier has the effect of distributing the predicate term. This means that the subject terms of A and E statements[18] are distributed, the predicate terms of E and O statements are distributed, and in an I statement, no term is distributed.

When Aristotle presented his syllogistic theory in the *Prior Analytics*, he said that a perfect syllogism is a deduction "which needs nothing other than what has been stated to make the necessity [of the inference] evident" [Ar84, Bk. 1 24b27–24b30, p. 40]. Because the perfect syllogisms are self-evident, he believes that no proof can be given of them. Medieval authors who inherited Aristotelian syllogistics realized that arguments for the validity of the perfect syllogisms could be given on the basis of the distribution of the terms in the premises and the conclusion [Lag04, §8]. The following three conditions are individually necessary and jointly sufficient for a valid syllogism:

1. The middle term must be distributed at least once.
2. Any term distributed in the conclusion must be distributed in at least one premise.
3. At most one premise can be negative, and if one premise is negative, the conclusion must be negative.

These three conditions are usually expressed in medieval texts by a regulative principle or rule called the *dici de omni et nullo*. This principle is often split into two, the *dici de omni* for affirmative syllogisms, and the *dici de nullo* for negative syllogisms. Our author gives the rule for affirmative syllogisms:

[16]The division of supposition which Lambert calls 'accidental' is called by other authors 'formal' [Re08, §3]. This use of 'formal' is to be contrasted with 'material', not with 'essential' and 'personal'.

[17]For this reason, earlier authors, such as William of Sherwood, call this feature both 'distribution' and 'division' [WoS66, p. 29, fn. 31].

[18]See Appendix A.1.

Rule 7.3.1 (*Dici de omni*)**.** *Quandocumque aliquod predicatum dicitur de aliquo subiecto distributo, tunc de quocumque dicitur tale subiectum distributum de eodem eciam dicitur tale predicatum* [¶36].

For example, in the proposition

All dogs are mammals.

dogs is a distributed subject term. Therefore anything to which this subject applies, the predicate also applies. So, if

Fido is a dog.

we can conclude

Fido is a mammal.

Our author does not explicitly give the rule for negative syllogisms. As this rule, the *dici de nullo*, was widely used and would have been familiar to his readers, there was no harm in his omitting explicit reference to it. We give a standard formulation of it:

Rule 7.3.2 (*Dici de nullo*)**.** Whenever some predicate is denied of some distributed subject, then of whatever is said to be of such a distributed subject, of the same thing indeed it is denied to be of such a predicate.

Both of these rules come with a caveat. They can only be applied if the conditions of the copulae in the premises and the conclusion are the same. The conditions referred to here are things such as tense and mood. For example, if the first premise is about the present situation and the second about a potential situation, one cannot draw any conclusion. From

Every running thing is a donkey.

and

Every man can be a running thing.

We cannot conclude either

Every man can be a donkey.

or

Every man is a donkey.

Likewise, if the second premise were "Every man will be a running thing", we could not draw any conclusion, either present-tensed or future-tensed.

Similarly, if the conditions of the copulae of the premises are modified by the presence of an adverb such as *essencialiter*, *personaliter*, and *formaliter*, then the *dici de omni* and the *dici de nullo* can be applied only if the same adverb is used in both of the premises and the conclusion. As many paralogisms of the trinity can be resolved by making explicit which different adverbs identifying different types of identity are modifying copulae of the premises and the conclusion, we need to have different rules telling us how to handle cases where we have mixed propositions in the premises and the conclusion. Once we have the modified rules, we can then make clear why certain paralogisms look valid but in fact are not.

The revised rule for affirmative syllogisms comes in two parts:

Rule 7.3.3 (*Dici de omni* for mixed affirmative syllogisms)**.**

- *Quandocumque aliquod predicatum dicitur formaliter de aliquo subiecto distributo, tunc de quocumque predicabitur tale subiectum ydemptice, de eodem predicabitur et tale predicatum ydemptice* [¶51a].
- *Quandocumque aliquod predicatum predicatur ydemptice de aliquo subiecto distributo, tunc de quocumque predicabitur tale subiectum formaliter, de eodem predicabitur tale predicatum ydemptice* [¶51b].

For mixed negative syllogisms—that is, syllogisms with at least one negative premise—our rule is split into four parts:

Rule 7.3.4 (*Dici de nullo* for mixed negative syllogisms)**.**

- *Quando aliquod predicatum negatur formaliter de aliquo subiecto distributo, tunc non oportet quod de quocumque predicatur ydemptice tale subiectum, quod de eodem negatur ydemptice vel formaliter tale predicatum* [¶57].
- *Quando aliquod predicatum negatur ydemptice de aliquo subiecto distributo, tunc non oportet, si tale subiectum predicatur ydemptice de aliquo termino, quod de eodem negatur ydemptice tale predicatum* [¶58].
- *Si aliquod predicatum negatur formaliter, idest in predicacione formali, de subiecto distributo, de quocumque predicatur formaliter tale subiectum distributum, de eodem negatur in predicacione formali tale predicatum* [¶59].
- *Quandocumque aliquod predicatum negatur ydemptice de aliquo subiecto distributo, tunc de quocumque dicitur tale subiectum formaliter, de eodem negatur tale predicatum ydemptice* [¶60].

These rules, like much of the text, leave much to be desired in terms of clarity. We will address this drawback in §7.4 where we develop a formal system which we can use to model the semantic features discussed in §7.3.1 and the rules given above.

The reason why our author introduces supposition theory and discusses the distribution of terms is that when a syllogism is made up out of distributed terms which have only one formal suppositum, the validity of the syllogism doesn't depend on the distribution of the terms but rather on the singularity of the suppositum:

> *Ubi notandum quod omnis sylogismus in quo distribuitur aliquis terminus habens solum unum suppositum formale, non tenet virtute distribucionis eiusdem termini (idest, quod conclusio non probatur virtute distribucionis precise), sed tenet ut sylogismus expositorius. Verbi gracia, iste sylogismus: omnis essencia divina est pater, et sapiencia est essencia divina, igitur etc., non tenet virtute distribucionis huius termini 'essencia', sed racione singularitatis divine essencie; quod patet, quia in talibus eodem modo valet expositorie arguere sicut distributive. Verbi gracia, nec plus nec minus probatur per dictum sylogismum quam per istum: ista essencia divina est pater, et bonitas est ista essencia, igitur* [¶33; see also ¶56].

Both of these arguments are valid, but their validity does not rest on the *dici de omni*, but on the immediate validity of the expository syllogism. Thus, when we have syllogisms which contain terms which are distributed for one single formal suppositum, then we can disregard the rules of distribution given above, and instead look at the corresponding expository syllogism. We discuss expository syllogisms in the next section.

7.3.3 Expository syllogisms

If some distributed term in a proposition has only one formal suppositum, then this proposition is of a special kind, a kind which can be distinguished from both universal and particular propositions, namely singular propositions (cf. p. 71). Some authors (e.g., Jean Buridan, John Dorp) claimed that singular propositions were merely a special case of particular propositions, for a particular proposition refers to "at least one" thing of some kind, and a singular proposition refers to "exactly one". Other authors (e.g., Lambert of Lagny) said that singular propositions can be reduced to universal propositions, because we can convert a proposition such as "Socrates is white" to one of the form "Everything which is Socrates is white" or, to use modern parlance, "All Socratizers are white".[19] But even though not everyone agreed on the nature of singular propositions, a special branch of syllogistics was developed, namely the theory of expository syllogisms.

An expository syllogism is one where one or both of the premises is a singular proposition. The following syllogism is expository:

[19] For further discussion, see [As08, §7].

This Father generates.
The essence is this Father.
Therefore, the essence generates.

The reason we introduce expository syllogisms is because, as our author notes, any syllogism with distributed terms having just one formal suppositum can be rephrased as an equivalent expository syllogism (cf. ¶¶33, 71). If a term, such as "thing which generates", stands or supposits formally for only one thing, then there is no difference between a syllogism using the general term and one which replaces the general term with a definite description referring to the single suppositum of the term:

> *Igitur qui taliter vult sylogizare, scilicet universaliter, in terminis non habentibus plura supposita formalia, non debet respicere ad distribucionem ut distribucio est, sed ut in ea includitur syngularisacio termini distributi. Ex distribucione igitur non debet assignari defectus, quia statim iste sylogismus reduci posset ad expositorium sylogismum. Verbi gracia, si quis istum sylogismum: omnis pater generat, essencia est pater, igitur essencia generat, negaret propter distribucionem, tunc potest reduci ad expositorium sic: iste pater generat, et essencia est iste pater, igitur essencia generat* [¶34].

If a distributed term has only one formal suppositum, then the conclusion of the syllogism holds not in virtue of the distribution rules discussed earlier, but rather immediately, without recourse to any further proof or rule.

One such term which has only one formal suppositum in a universal affirmative categorical is *essencia*. This term has, in divine contexts, only one suppositum for which it can be formally distributed, namely, the essence of the trinity. That *essencia* can supposit for the essence is obvious, but that this is the only thing it can stand for is not. Our author argues for this:

> *[Q]uod solum habeat unum suppositum pro quo sit distribuibilis patet, quia quicumque terminus est distribuibilis pro pluribus, eadem significat per modum plurium, quia supposicio et per consequens distribucio est inferior ad significacionem: si ergo aliquid distribuitur pro aliquibus per modum plurium, ipsum significat ea per modum plurium, alias posset de eis dici pluraliter, et sic pater et filius essent plures essencie, quod est inconveniens* [¶72].

Furthermore,

> *Item huic termino 'essencia' ex modo sue imposicionis non significat patrem et filium distincte. Item iste terminus 'essencia' non est predicabilis cum ly 'alia' de patre et filio, ut: pater est essencia et filius est alia essencia; taliter autem predicari contingit termino pro pluribus distributo, ut: Sortes est homo, Plato est alius homo* [¶73].

Valid expository syllogisms are said to be valid immediately, or obviously, without any recourse to further proof or reductions. The regulative principle that governs expository syllogisms is the following:

Rule 7.3.5 (Regulative principle of expository syllogisms)**.** *[Q]uecumque uni et eidem sunt eadem, inter se sunt eadem* [¶62].

This rule can be understood in two ways. In one way, it is making a statement about words, and this way is called *realiter*; in the other way, it is making a statement about the relationships between terms, and this is called *loycaliter*, and is the interpretation that we will make use of. When the rule is understood logically, it is as follows:

> *[Q]uicumque termini coniunguntur cum aliquo termino tento singulariter et univoce, illi eciam coniunguntur inter se* [¶62].

There are, not surprisingly, three ways that two terms are able to be called the same: essentially, personally or identically, and formally [¶¶64–66]. These correspond to the three modes of being and the three ways the copulae of categorical propositions can be modified to indicate those modes of being. When the predications involved are all essential, this is the case: Since essential identity is an equivalence relation, and the $=_e$ equivalence class contains all divine terms, any syllogism whose premises and conclusion assert the essential identity between any three divine terms cannot fail to be valid [¶67]. Similarly, because formal identity is an equivalence relation, if two terms are formally identical with a third, through transitivity they will be formally identical with each other [¶68]. From these two facts, it is easy to see how the validity of expository syllogisms with either only essential or only formal predications is immediate.

But this is not the case with personal identity, because personal identity is not transitive; it is possible for both the father and the son to be personally the same as the essence, but to be personally distinct from each other,

> *quia filius non est idem cum essencia eadem ydemptitate personali seu ydemptica qua pater est idem cum essencia. Hoc patet, quia pater et essencia sunt eadem res ydemptitate personali, que ydemptitas est persona patris; filius autem et essencia sunt eadem res ydemptitate ⟨personali⟩, que ydemptitas est persona filii* [¶69].

Thus, our author gives us two ways of determining the validity of syllogisms containing divine terms: We must either pay attention to the distribution of the terms, and then their validity is governed by Rules 7.3.1 and 7.3.2, or we are able to singularize the subject terms and consider the corresponding expository syllogisms, whose validity is immediate. The author discusses the relative merits of both approaches after he has given more specifics about what types of inferences can be drawn from what types of predications. We will do the same; in the next

section we introduce a formal system for modeling the material just presented, and then in §7.5 we show how this system can be used to resolve the paralogisms of the trinity, and the benefits and drawbacks of doing it this way.

7.4 The formal system

We present our logical model of the trinitarian syllogistics in three parts: First, in §7.4.1, we define our model formally and specify the construction and truth conditions of well-formed formulas in our logic. In §7.4.2, we give formal interpretations of the rules of inference discussed in §7.3.3 and prove certain features about the resulting system. Finally, in §7.5, we apply the formal system back to the text and show how we can resolve the paradoxes.

At the outset we need to stress that the formal system we present here was developed in order to be able to model reasoning within a particular natural language, namely medieval Latin as it was used by logicians. This fact is the cause of certain otherwise non-standard modeling choices that we make. In particular, we have designed our system to deal with ambiguous natural language statements such as

Homo est animal.

Because Latin does not have an indefinite or definite article, this sentence is ambiguous between the reading *omnis homo est animal* and *quidam homo est animal*.[20] Another way that features of our formal model will be determined by features of Latin is in the use of context-dependent indexicals like *hoc* ('this'). When we say things such as *haec tabula est viridis*, we are saying something more than 'some table is green' but something less than 'all tables are green'. We will introduce specific operators into our language to be able to deal with issues surrounding the use of indexicals in this manner.

7.4.1 Language and models

We use the language $\mathcal{L}_{\text{trin}}$ consisting of a set of terms $\mathfrak{T}$; the relations $=_e$, $=_p$, $=_f$ and their negations $\neq_e$, $\neq_p$, $\neq_e$; the functions es and fs; the quantifiers $\mathtt{A}$, $\mathtt{E}$, and !; and two punctuation symbols, [and]. $\mathfrak{T}$ contains all of E, Su, F, S, HS, P, Wi, and C, and potentially other terms, e.g., 'man', 'cat', 'Socrates'. We use t as a variable ranging over $\mathfrak{T}$, and we use $=_*$ as a meta-variable over $=_e$, $=_p$, $=_f$ when we need to make statements about all three relations. $!t$ is to be read 'this t'

[20] When the sentence is literally translated into English, this ambiguity manifests itself in questionable grammar: "Man is animal." A more natural translation would add definite or indefinite articles or quantifiers, e.g., 'the essence is the father' for *essencia est pater*, which adds two definite articles which are not present in the Latin.

(English) or *hoc t* (Latin). This operator will be used in formalizing ambiguous natural language sentences such as the ones just discussed.

Traditional Aristotelian syllogistic logic is a term logic, not a predicate or propositional logic. This means that the formal system we develop will be neither a predicate nor a propositional logic, though, as we'll see below, we will use predicate logic as a meta-logic when giving the truth conditions for formulas in models. Instead we will develop a logic whose basic constituent is the categorical proposition, though we will go a step beyond traditional medieval syllogistics by allowing boolean combinations of these categorical propositions. We begin by giving a definition of the set of *basic terms* and the set of *quantified terms* in our language:

Definition 7.4.1 (terms)**.** The set $\mathsf{T}_{\text{trin}} = \mathsf{T}_{\text{trin}}^{\text{basic}} \cup \mathsf{T}_{\text{trin}}^{\text{quant}}$ is the set of *terms of* $\mathcal{L}_{\text{trin}}$ where

- $\mathsf{T}_{\text{trin}}^{\text{basic}}$ is the set of *basic terms of* $\mathcal{L}_{\text{trin}}$, defined recursively as follows:
  - If $t \in \mathfrak{T}$, then $t, t_{\mathsf{es}}, t_{\mathsf{fs}} \in \mathsf{T}_{\text{trin}}^{\text{basic}}$.
  - If $t \in \mathsf{T}_{\text{trin}}^{\text{basic}}$, then $[t]_{=_*} \in \mathsf{T}_{\text{trin}}^{\text{basic}}$.
  - Nothing else is in $\mathsf{T}_{\text{trin}}^{\text{basic}}$.

  We call terms of the form $[t]_{=_*}$ *equivalence terms.*
- $\mathsf{T}_{\text{trin}}^{\text{quant}}$ is the set of *quantified terms of* $\mathcal{L}_{\text{trin}}$ defined as follows:

$$\{\mathtt{A}t : t \in \mathsf{T}_{\text{trin}}^{\text{basic}}\} \cup \{\mathtt{E}t : t \in \mathsf{T}_{\text{trin}}^{\text{basic}}\} \cup \{!t : t \in \mathsf{T}_{\text{trin}}^{\text{basic}}\}$$

Definition 7.4.2 (categorical propositions)**.** The set $\mathsf{CAT}_{\text{trin}}$ of *categorical propositions of* $\mathcal{L}_{\text{trin}}$ is defined as follows:

- If $t, t' \in \mathsf{T}_{\text{trin}}$, then $t =_e t'$, $t =_p t'$, $t =_f t' \in \mathsf{CAT}_{\text{trin}}$. We call categorical propositions of this type *affirmative.*
- If $t, t' \in \mathsf{T}_{\text{trin}}$, then $t \neq_e t'$, $t \neq_p t'$, $t \neq_f t' \in \mathsf{CAT}_{\text{trin}}$. We call categorical propositions of this type *negative.*
- Nothing else is in $\mathsf{CAT}_{\text{trin}}$.

Note that all categorical propositions are of the form $Qt =_* Q't'$ for terms t, t' and quantifiers (possibly null) Q, Q'. If φ is a categorical proposition, then we indicate the type of identity in φ by φ_*, and we call the term on the left-hand side of the identity sign the 'subject' and the term on the right-hand side the 'predicate'.

Definition 7.4.3 (WFFs)**.** The set of $\mathsf{WFF}_{\text{trin}}$ of *well-formed formulas* of $\mathcal{L}_{\text{trin}}$ is defined recursively:

- if $\varphi \in \mathsf{CAT}_{\text{trin}}$, then $\varphi \in \mathsf{WFF}_{\text{trin}}$.

- if $\varphi \in \mathsf{WFF}_{\text{trin}}$, then $\neg\varphi \in \mathsf{WFF}_{\text{trin}}$.
- if φ, $\psi \in \mathsf{WFF}_{\text{trin}}$, then $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi \in \mathsf{WFF}_{\text{trin}}$.
- nothing else is in $\mathsf{WFF}_{\text{trin}}$.

In order to prove some of the theorems in §7.4.2, we need to isolate a special class of terms called *divine terms*; we'll use the distinction between divine and created (non-divine) terms in our proof.

Definition 7.4.4 (divine terms)**.** The set $\mathsf{T}_{\text{div}} \subseteq \mathsf{T}_{\text{trin}}$ of *divine terms of* $\mathcal{L}_{\text{trin}}$ is the set of all terms $t \in \mathsf{T}_{\text{trin}}$ such that t only contains $\mathsf{E}, \mathsf{Su}, \mathsf{F}, \mathsf{S}, \mathsf{HS}, \mathsf{P}, \mathsf{Wi}, \mathsf{C}$ and nothing else.

We define the sets $\mathsf{CAT}_{\text{div}}$ and $\mathsf{WFF}_{\text{trin}}$ from Definitions 7.4.4, 7.4.2, and 7.4.3 by replacing $_{\text{trin}}$ with $_{\text{div}}$ throughout.

These formulas gain meaning when they are interpreted in models.

Definition 7.4.5. A structure $\mathfrak{M}_{\text{trin}} = \langle O, I, \{\dot{t} : t \in \mathfrak{T}\}, \doteq_e, \doteq_p, \doteq_f, \ddot{\mathsf{es}}, \dot{\mathsf{fs}}\rangle$ is a *trinitarian model* iff:

1. O is a set of objects such that $\dot{\mathsf{E}}, \dot{\mathsf{Su}}, \dot{\mathsf{F}}, \dot{\mathsf{S}}, \dot{\mathsf{HS}}, \dot{\mathsf{P}}, \dot{\mathsf{Wi}}, \dot{\mathsf{C}} \in O$. We use o, x, y, z, etc., as meta-variables ranging over O.
2. $I : \mathfrak{T} \rightarrow 2^O$ associating a set of objects with each term of $\mathfrak{T}$, such that $I(\mathsf{E}) = \{\dot{\mathsf{E}}\}, I(\mathsf{Su}) = \{\dot{\mathsf{Su}}\}, I(\mathsf{F}) = \{\dot{\mathsf{F}}\}, I(\mathsf{S}) = \{\dot{\mathsf{S}}\}, I(\mathsf{HS}) = \{\dot{\mathsf{HS}}\}, I(\mathsf{P}) = \{\dot{\mathsf{P}}\}, I(\mathsf{Wi}) = \{\dot{\mathsf{Wi}}\}, I(\mathsf{C}) = \{\dot{\mathsf{C}}\}$. I can be extended to I' which covers equivalence terms: $I'([\dot{t}]_{=_*}) = \{x \in O :$ there is a $y \in I(t)$ and $x =_* y\}$.
3. $\doteq_e$ is a binary equivalence relation on O such that if $o \notin I(t)$ for all $t \in \mathsf{T}_{\text{div}}$, then for all $o' \in O$, $\langle o, o'\rangle \notin \doteq_e$.
4. $\doteq_p$ is a symmetric binary relation on O satisfying the conditions in Figure 7.1 such that if $o \notin I(t)$ for all $t \in \mathsf{T}_{\text{div}}$, then for all $o' \in O$, $\langle o, o'\rangle \notin \doteq_p$.
5. $\doteq_f$ is an equivalence relation on O satisfying the conditions in Figure 7.1.
6. $\ddot{\mathsf{es}}, \dot{\mathsf{fs}}$ are partial unary functions such that if $o \in I'([\dot{\mathsf{E}}]_{\doteq_e})$ then $o_{\ddot{\mathsf{es}}} = \dot{\mathsf{E}}$; if $o \in [\dot{\mathsf{F}}]_{=_f}$ then $o_{\dot{\mathsf{fs}}} = \dot{\mathsf{F}}$; if $o \in [\dot{\mathsf{S}}]_{=_f}$ then $o_{\dot{\mathsf{fs}}} = \dot{\mathsf{S}}$; if $o \in [\dot{\mathsf{HS}}]_{=_f}$ then $o_{\dot{\mathsf{fs}}} = \dot{\mathsf{HS}}$; and undefined otherwise.

Conditions 3, 4, and 5 of Definition 7.4.5 capture the fact that when we are reasoning about non-divine things, we can only make formal predications. When explaining why essential and identical predications do not show up in Aristotelian syllogistics, our author notes that though the terminists and the realists may disagree about whether there are only formal identities between created objects, or whether there are also personal identities, nevertheless they agree that all predications are predications of formal identity:

> *Et quia in creaturis omnes predicaciones sunt formales, quia iuxta opinionem communem terministarum omnia que sunt idem in creaturis sunt formaliter idem, ideo non fuit necesse in creaturis modus sylogizandi per proposiciones de predicacione ydemptica.*
>
> *Secundum modum autem realistarum, secundum quem non omnia in creaturis que sunt idem sunt formaliter ⟨idem⟩, adhuc omnes predicaciones sunt formales, quod patet, quia que non sunt formaliter idem secundum realistas, secundum ipsos necessario negantur de semetipsis si eciam ydemptice sint idem* [¶¶39–40].
>
> *[I]n creaturis omnes predicaciones sunt formales secundum omnes, scilicet tam realistas quam terministas* [¶49].

We now give the truth conditions of the members of $\mathsf{WFF}_{\text{trin}}$ in a trinitarian model. Boolean combinations of categorical propositions are as expected:

Definition 7.4.6 (Truth conditions of boolean formulas)**.**

$$\begin{array}{lll} \mathfrak{M} \vDash \neg\varphi & \text{iff} & \mathfrak{M} \nvDash \varphi \\ \mathfrak{M} \vDash \varphi \wedge \psi & \text{iff} & \mathfrak{M} \vDash \varphi \text{ and } \mathfrak{M} \vDash \psi \\ \mathfrak{M} \vDash \varphi \vee \psi & \text{iff} & \mathfrak{M} \vDash \varphi \text{ or } \mathfrak{M} \vDash \psi \\ \mathfrak{M} \vDash \varphi \to \psi & \text{iff} & \mathfrak{M} \vDash \neg\varphi \text{ or } \mathfrak{M} \vDash \psi \end{array}$$

For the categorical statements, we correlate the quantifiers of $\mathcal{L}_{\text{trin}}$ with quantifiers in ordinary mathematical logic via an interpretation function int. Two of the quantifiers are standard—$\mathsf{int}(\mathtt{A}) = \forall$ and $\mathsf{int}(\mathtt{E}) = \exists$. As we noted earlier, indexical pronouns like '*hoc*', which we formalize with !, indicate something more than existence but something less than universality. Pronouns like '*hoc*' are essentially context-dependent choice functions that, given a term, will pick out an appropriate witness for that term, given the context. We capture these two facts by interpreting ! with a generalized quantifier.[21] For a term t, we indicate such a context-dependent choice function as $\chi_!(t)$, which means we can define $\mathsf{int}(!)$ as $\big\{\{\chi_!(t)\}\big\}$ for appropriate t. This leaves us with the empty quantifier, which shows up in formalizations of Latin sentences such as *essencia est pater* and *homo est animal*, which, as we noted above, are essentially ambiguous. Our author does not say how these sentences should be interpreted, but, given how his discussion of modes of being mirrors Abelard's three ways of being identical, it's reasonable that he would also subscribe to Abelard's view of predication. Knuuttila summarizes Abelard's view thus:

> In his *Logica Ingredientibus* Abelard argues that the simple affirmative statement 'A human being is white' [*homo est albus*] should be

[21] Generalized quantifiers were first introduced in [Mos57]; for a general overview see [Wes05].

> analysed as claiming that that which is a human being is the same as that which is white (*idem quod est homo esse id quod album est*) [Knu07, p. 192].

It is natural to read 'that which is a human being' universally, and 'that which is white' particularly. Thus, for statements of identity, involving $=_*$, we stipulate that the int of the empty quantifier of a subject is $\forall$, and the int of the empty quantifier of a predicate is $\exists$. For statements of non-identity, involving $\neq_*$, we stipulate that the int of the empty quantifier on either side of $\neq_*$ is $\forall$. The difference in how the empty quantifier is treated when it appears in a predicate is a result of the distributive force of negation; see Definition 7.4.9.

Given these preliminaries, we can now give a uniform truth condition for categorical sentences:

Definition 7.4.7 (Truth conditions for categorical formulas)**.** Let Q, Q' be (perhaps empty) quantifiers, and $t, t' \in \mathfrak{T}$. Then,

$$\mathfrak{M} \vDash Qt =_* Q't' \quad \text{iff} \quad \mathsf{int}(Q)x \in I(t)\left(\mathsf{int}(Q')y \in I(t')\left(\langle x, y\rangle \in \doteq_*\right)\right)$$

We will see examples of these conditions in the next section when we discuss the formalization of natural language sentences concerning the trinity. Note that defining the truth conditions for the empty quantifiers in this way automatically deals with the issue of existential import, by allowing the inference, regularly accepted in the Middle Ages, from *omnis homo est mortalis* to *quidam homo est mortalis*, but not automatically allowing the inference, which is not so readily accepted by the medieval logicians (cf. [Par08, §1.2]), from *nullus homo est immortalis* to *quidam homo non est immortalis*, because $\mathfrak{M} \vDash \mathtt{A}t \neq_f t'$ when both $I(t) = \emptyset$ and $I(t') = \emptyset$.

7.4.2 Properties of the system

In order to properly define a trinitarian logic, we would need to give axioms and rules of inference in addition to syntax and semantics. It is clear how one would go about doing this—to cover the propositional cases of Definition 7.4.6, we need all axioms of propositional logic, plus *modus ponens*. We also need transitivity, reflexivity, and symmetry axioms for $=_e$ and $=_f$, and reflexivity and symmetry axioms for $=_p$. We would also need axioms to cover the specifically trinitarian part of our logic, namely, the properties laid out in Table 7.1, and then we could prove soundness and completeness results for this logic with respect to the semantics given in the previous section.

However, doing this would obscure the more interesting approach, which is to give formal analogs of the informal rules from 7.3, and prove meta-properties about the class of trinitarian models. For this reason, we will not give a complete axiomatization of the syntactic side of the logic, but continue to take a semantic

approach and concentrate on properties of trinitarian models. Before we continue, we note that it doesn't really make sense to talk of axioms in the context of a syllogistic logic. This is because what is valid in a syllogistic logic is not sentences, but arguments, which means that the 'axioms' are simply rules for moving from two premises to a conclusion. In ordinary, non-divine, syllogistics, these rules are the perfect syllogisms, Barbara, Celarent, Darii, and Ferio (see Appendix A.1). That is, if $t, t', t'' \notin \mathsf{T}_{\text{div}}$, then we have:

Rule 7.4.8.

Barbara:	If	$\mathfrak{M} \vDash \mathtt{A}t' =_f t$	and	$\mathfrak{M} \vDash \mathtt{A}t'' =_f t'$,	then	$\mathfrak{M} \vDash \mathtt{A}t'' =_f t$
Celarent:	If	$\mathfrak{M} \vDash \mathtt{A}t' \neq_f t$	and	$\mathfrak{M} \vDash \mathtt{A}t'' =_f t'$,	then	$\mathfrak{M} \vDash \mathtt{A}t'' \neq_f t$
Darii:	If	$\mathfrak{M} \vDash \mathtt{A}t' =_f t$	and	$\mathfrak{M} \vDash \mathtt{E}t'' =_f t'$,	then	$\mathfrak{M} \vDash \mathtt{E}t'' =_f t$
Ferio:	If	$\mathfrak{M} \vDash \mathtt{A}t' \neq_f t$	and	$\mathfrak{M} \vDash \mathtt{E}t'' =_f t'$,	then	$\mathfrak{M} \vDash \mathtt{E}t'' \neq_f t$

The admissibility of these four syllogisms follows straightforwardly from the fact that $=_f$ is an equivalence relation:

Proof.

Barbara Assume $\mathfrak{M} \vDash \mathtt{A}t' =_f t$ and $\mathfrak{M} \vDash \mathtt{A}t'' =_f t'$. Then by Definition 7.4.7, the following two formulas hold:

$$\forall x \in I(t')(\exists y \in I(t)(\langle x, y\rangle \in \doteq_f)) \tag{7.1}$$

$$\forall z \in I(t'')(\exists w \in I(t')(\langle z, w\rangle \in \doteq_f)) \tag{7.2}$$

Take arbitrary $x \in I(t'')$. From (7.2) it follows that there is a $y \in I(t')$ such that $\langle x, t\rangle \in \doteq_f$. From (7.1), we know that there is some $z \in I(t)$ such that $\langle y, z\rangle \in \doteq_f$. Since $=_f$ is transitive, we can conclude that $\langle x, z\rangle \in \doteq_f$. Since x was arbitrary, we have shown that the following holds:

$$\forall x \in I(t'')(\exists z \in I(t)(\langle x, z\rangle \in \doteq_f)) \tag{7.3}$$

and hence that $\mathfrak{M} \vDash \mathtt{A}t'' =_f t$.

Celarent Assume $\mathfrak{M} \vDash \mathtt{A}t' \neq_f t$ and $\mathfrak{M} \vDash \mathtt{A}t'' =_f t'$. Then by Definition 7.4.7, (1) for every $x \in I(t')$ and $y \in I(t)$, $\langle x, y\rangle \notin \doteq_f$, and (2) for every $z \in I(t'')$ there is a $w \in I(t')$ such that $\langle z, w\rangle \in \doteq_f$. Take arbitrary $x \in I(t'')$. By (2) there is some $y \in I(t')$ such that $\langle x, y\rangle \in \doteq_f$. By (1), for all $z \in I(t)$, $\langle y, z\rangle \notin \doteq_f$. Now, suppose that there is a $w \in I(t)$ such that $\langle x, w\rangle \in \doteq_f$. Since $\langle x, w\rangle \in \doteq_f$ and $\langle x, y\rangle \in \doteq_f$, by transitivity and symmetry of $=_f$, this means that $\langle y, w\rangle \in \doteq_f$, which is a contradiction. Since $x \in I(t'')$ was arbitrary, we can conclude that the following holds:

$$\forall x \in I(t'')(\forall y \in I(t)(\langle x, y\rangle \notin \doteq_f)) \tag{7.4}$$

and hence $\mathfrak{M} \vDash \mathtt{A}t'' \neq_f t$.

Darii Assume $\mathfrak{M} \vDash \mathtt{A}t' =_f t$ and $\mathfrak{M} \vDash \mathtt{E}t'' =_f t'$. Then by Definition 7.4.7, (1) for every $x \in I(t')$ there is a $y \in I(t)$ such that $\langle x, y\rangle \in \doteq_f$, and (2) there is a $\hat{z} \in I(t'')$ and $w \in I(t')$ such that $\langle \hat{z}, w\rangle \in \doteq_f$. (1) and (2) together give immediately that there is a $y \in I(t)$ such that $\langle \hat{z}, y\rangle \in \doteq_f$, and hence there exists a $z \in I(t'')$ and a $y \in I(t)$ such that $\langle z, y\rangle \in \doteq_f$, which is the same as saying that $\mathfrak{M} \vDash \mathtt{E}t'' =_f t$.

Ferio Assume $\mathfrak{M} \vDash \mathtt{A}t' \neq_f t$ and $\mathfrak{M} \vDash \mathtt{E}t'' =_f t'$. Then by Definition 7.4.7, (1) for every $x \in I(t')$ and $y \in I(t)$, $\langle x, y\rangle \notin \doteq_f$, and (2) there exists $z \in I(t'')$ and $w \in I(t')$ such that $\langle z, w\rangle \in \doteq_f$. Suppose that there is a $y \in I(t)$ such that $\langle z, y\rangle \in \doteq_f$. Then by symmetry and transitivity, we would have $\langle w, z\rangle \in \doteq_f$ and hence $\langle w, y\rangle \in \doteq_f$, which violates (1), and hence $\mathfrak{M} \vDash \mathtt{E}t'' \neq_f t$.

□

A corollary of this is that Rules 7.3.1 and 7.3.2 are both sound.

We are then left with the cases where the terms do fall in T_{div}. The admissibility of the essential analog of Rule 7.4.8 follows immediately from the proof of the admissibility of that same rule, by substitution of $=_e$ for all occurrences of $=_f$. For the other cases, as we discussed in §7.3.2, the standard *dici de omni* for affirmative syllogisms only holds when the propositions in the premises and the conclusion are all of the same type. First we give a formal definition of a term being distributed in a formula:

Definition 7.4.9 (Distribution)**.** A term t is in the scope of $\forall$ iff one of the following holds:

1. $t \in \mathsf{T}^{\text{quant}}_{\text{trin}}$ and is of the form $\mathtt{A}t'$.
2. $t \notin \mathsf{T}^{\text{quant}}_{\text{trin}}$ and is a subject.
3. $t \notin \mathsf{T}^{\text{quant}}_{\text{trin}}$ and is a predicate of a negative categorical.

If t is in the scope of $\forall$ in a categorical proposition φ, then we say that t is *distributed in* φ.

We can now give a formal statement of Rules 7.3.3 and 7.3.4.

Rule 7.4.10 (*Dici de omni* for mixed affirmative syllogisms)**.** If t and t' are the terms of φ and t is distributed in φ, and Q is any quantifier, then

- If $\varphi = \varphi_f$ and $\mathfrak{M} \vDash \varphi$, then if $\mathfrak{M} \vDash Qt'' =_p t$, then $\mathfrak{M} \vDash Qt'' =_p t'$.
- If $\varphi = \varphi_p$ and $\mathfrak{M} \vDash \varphi$, then if $\mathfrak{M} \vDash Qt'' =_f t$, then $\mathfrak{M} \vDash Qt'' =_p t'$.

Proving the admissibility of this rule is straightforward:

Proof. Assume $\varphi = \varphi_f$, $\mathfrak{M} \vDash \varphi$, and $\mathfrak{M} \vDash Qt'' =_p t$. Since t is distributed in φ and φ is affirmative, we know that φ is either of the form $\mathtt{A}t =_f Q't'$ or $t =_f Q't'$, for some possibly empty quantifier Q'. Looking at Table 7.1, the only formal identities (other than those which fall out of the reflexivity of $=_f$) are between the persons and their personal properties, and since the persons are personally identical with both themselves and their personal properties, it follows that $\mathfrak{M} \vDash Qt'' =_p t'$. The other case follows similarly. □

In Rule 7.4.8, there are only two syllogistic forms which have only affirmative premises, Barbara and Darii. For both of these, there are four possible ways to form a divine syllogism: either both premises are formal, both are personal, the major is personal and the minor formal, or the major is formal and the minor personal (¶54). In the first case, the syllogism is valid, because:

> *Secundo dico quod si aliquod predicatum dicitur formaliter de subiecto distributo, tunc de quocumque dicitur formaliter tale subiectum distributum de eodem eciam dicitur formaliter tale predicatum* (¶53).

Which is to say that the traditional *dici de omni* remains valid when considering categorical propositions with divine terms, not just ones containing only created terms.

In the second case, the syllogism is not valid, because:

> *Dico igitur primo. . . quod quando aliquod predicatum predicatur ydemptice de subiecto distributo, et si tunc tale dicitur ydemptice de aliquo tercio termino, tunc non oportet quod tale predicatum eciam dicatur ydemptice de eodem tercia termino* (¶52).

The third and fourth cases are covered by Rule 7.4.10.

Now for the negative syllogisms, Celarent and Ferio. Again we have four cases—the major premise is formal and the minor personal, the major premise is personal and the minor formal, both are personal, or both are formal. All four are expressed explicitly in the rule:

Rule 7.4.11 (*Dici de nullo* for mixed negative syllogisms)**.** If t is a distributed subject in φ and Q is any quantifier, then

1. If $\varphi = \varphi_f$ and $\mathfrak{M} \vDash \varphi$, then if $\mathfrak{M} \vDash Qt'' =_p t$, then neither $\mathfrak{M} \vDash Qt'' \neq_p t'$ nor $\mathfrak{M} \vDash Qt'' \neq_f t'$ follows necessarily.
2. If $\varphi = \varphi_p$ and $\mathfrak{M} \vDash \varphi$, then if $\mathfrak{M} \vDash Qt'' =_p t$, then $\mathfrak{M} \vDash Qt'' \neq_f t'$.
3. If $\varphi = \varphi_f$ and $\mathfrak{M} \vDash \varphi$, then if $\mathfrak{M} \vDash Qt'' =_f t$, then $\mathfrak{M} \vDash Qt'' \neq_f t'$.
4. If $\varphi = \varphi_p$ and $\mathfrak{M} \vDash \varphi$, then if $\mathfrak{M} \vDash Qt'' =_f t$, then $\mathfrak{M} \vDash Qt'' \neq_p t'$.

Again, proving the admissibility of these rules is straightforward:

Proof.

1. We can prove this case by noting that $\mathfrak{M} \vDash \mathsf{E} \neq_f \mathsf{F}$ and $\mathfrak{M} \vDash \mathsf{F} =_p \mathsf{E}$, but $\mathfrak{M} \vDash \mathsf{F} =_p \mathsf{F}$ and $\mathfrak{M} \vDash \mathsf{F} =_f \mathsf{F}$.
2. This follows from the fact that, per Table 7.1, personal identities and non-identities only occur between the persons and their personal properties or between the persons and the essence, and that each person is formally distinct from both the essence and the personal properties which are not his characteristic property.
3. This valid case is identical with Celarent or Ferio (*Istud autem tercium dictum non facit mixtionem* [¶59]).
4. This case follows from (2) by contraposition.

□

We are now in a position to show that the four rules characterizing valid mixed affirmative syllogisms given in ¶¶93–96 and the eight rules for mixed negative syllogisms given in ¶¶98–105 are correct.

Lemma 7.4.12 (Rules for affirmative syllogisms)**.**

- *[P]rima regula: quando predicaciones sunt formales, scilicet tam conclusionis quam premissarum, sylogismus est bonus* [¶93].
- *Secunda regula: quando ambe premisse sunt de predicacione ydemptica, nec oportet sequi conclusionem de predicacione formali neque de predicacione ydemptica* [¶94].
- *Tercia regula: si proposiciones in quibus copule solum denotarent ydemptitatem essencialem essent in usu, tunc ex duabus premissis de predicacione ydemptica sequeretur conclusio de tali predicacione, scilicet in qua denotaretur solum ydemptitas essencialis rerum pro quibus formaliter supponunt extrema* [¶95].
- *Quarta regula: si ⟨solum⟩*[22] *una premissarum est de predicacione ydemptica, conclusio debet esse de predicacione ydemptica* [¶96].

Proof.

Prima This follows directly from Rule 7.4.8.

Secunda $\mathfrak{M} \vDash \mathsf{E} =_p \mathsf{F}$ and $\mathfrak{M} \vDash \mathsf{S} =_p \mathsf{E}$, but $\mathfrak{M} \vDash \mathsf{S} \neq_p \mathsf{F}$; and $\mathfrak{M} \vDash \mathsf{F} =_p \mathsf{E}$ and $\mathfrak{M} \vDash \mathsf{P} =_p \mathsf{F}$, but $\mathfrak{M} \vDash \mathsf{P} \neq_f \mathsf{E}$

[22] This is my addition; it has to be added, otherwise this fourth rule contradicts the previous one.

Tercia Because all members of the trinity are essentially identical with all others, and are so necessarily, a proposition asserting the essential identity of any two divine terms will follow from any set of premises.

Quarta This is a consequence of Rule 7.4.10.

□

This gives us an immediate corollary:

Corollary 7.4.13. *Et sic conclusio de predicacione formali solum sequitur ex ambabus premissis de predicacione formali. Item ex duabus premisses de predicacione formali ⟨sequitur⟩ conclusio de predicacione ydemptica, sicut conclusio particularis sequitur ex premissis universalibus. Item conclusio de predicacione denotante solum ydemptitatem essencialem nata est sequi ex quibuscumque premissis, sive ambe sint de predicacione ydemptica sive formali, sive una sit de predicacione ydemptica et alia de formali* [¶97].

Lemma 7.4.14 (Rules for negative syllogisms)**.**

- *[P]rima talis: quandocumque conclusio est negativa de predicacione ydemptica, oportet unam premissarum esse affirmativam de predicacione formali* [¶98].
- *Secunda regula: quando conclusio est negativa de predicacione ydemptica, oportet premissam negativam esse de predicacione ydemptica* [¶99].
- *[T]ercia regula: quandocumque conclusio esset de predicacione ydemptica negativa, oportet unam premissam esse de predicacione ydemptica negativa et aliam de predicacione formali affirmativa* [¶100].
- *Quarta regula: ex negativa de predicacione formali et affirmativa de predicacione ydemptica non oportet sequi conclusio de predicacione formali negativa* [¶101].
- *Quinta regula: ex negativa de predicacione ydemptica et affirmativa de predicacione formali sequitur conclusio negativa de predicacione formali* [¶102].
- *Sexta regula: ex duabus premissis de predicacione ydemptica bene sequitur conclusio negativa de predicacione formali* [¶103].
- *Septima regula: ex duabus premissis de predicacione formali sequitur conclusio negativa de predicacione formali* [¶104].
- *Alia regula: conclusio negativa de predicacione essenciali, idest cuius contradictorie copula solum denotat ydemptitatem essencialem, sequitur ex negativa de predicacione essenciali, idest cuius contradictorie etc., et affirmativa de quacumque predicacione* [¶105].

Proof. First, note that from two negative premises, no conclusion can be drawn. This is obvious from checking the four perfect syllogisms in Rule 7.4.8.

Prima If the conclusion is negative, then at most one of the premises is negative. The negative premise is either formal or identical. If it is identical, then by cases 2 and 4 of Rule 7.4.11, the affirmative premise must be formal. If the negative is formal, then there are two cases:

(a) The affirmative is identical. But this case is not sound, since $\mathfrak{M} \vDash \mathsf{P} \neq_f \mathsf{E}$ and $\mathfrak{M} \vDash \mathsf{E} =_p \mathsf{F}$, but $\mathfrak{M} \vDash \mathsf{P} =_p \mathsf{F}$.

(b) The affirmative is also formal. This case is also not sound, since $\mathfrak{M} \vDash \mathsf{S} \neq_f \mathsf{E}$ and $\mathfrak{M} \vDash \mathsf{E} =_f \mathsf{E}$, but $\mathfrak{M} \vDash \mathsf{S} =_p \mathsf{E}$.

Hence, the negative premise cannot be formal.

Secunda This is an immediate corollary of the first rule.

Tercia This is an immediate corollary of the first and second rules.

Quarta This is case 1 of Rule 7.4.11.

Quinta This follows from case 4 of Rule 7.4.11 and the fact that $\neq_p$ implies $\neq_f$, as can easily be seen by inspecting Table 7.1.

Sexta This follows from contraposition of case 4 of Rule 7.4.11.

Septima This is case 3 of Rule 7.4.11.

Alia This follows from the fact that, by definition, all members of the trinity are in $[\mathsf{E}]_{=_e}$. This means that when t and t' are divine terms, $t \neq_e t'$ is a necessary falsehood, and thus any conclusion follows.

□

All of these proofs allow us to now state the main theorem of this chapter:

Theorem 7.4.15. Aristotelian syllogistic logic is a special case of the logic defined in §7.4.1.

We discuss the import of this theorem at the end of the next section.

7.5 Resolving the paradoxes

In the previous section we introduced the ! quantifier but didn't say much about its usage. The ! quantifier is used when we formalize natural language sentences about the trinity in order to make their import explicit.

A paralogism arises when a syllogism appears to be sound but where the conclusion is intuitively false. These paralogisms can be blocked by recognizing the various ways that categorical propositions containing divine terms can be ambiguous. There are two main ways that categorical predications like this can be ambiguous. First, the type of identity being expressed by *est* is not made explicit. Paralogisms that arise from this type of ambiguity make up a large percentage of the fallacious arguments concerning the trinity:

> *[M]ulte ⟨fallacie⟩ que fiunt in divinis, fiunt ex coniunctione ydemptica extremorum cum medio, et propter hoc creduntur inter se posse coniungi ydemptice; vel ex coniunctionibus ydemptica et formali, propter quas coniunctiones creduntur inter se posse formaliter coniungi* [¶75].

As a result, to avoid paralogisms of this type we need to make explicit the type of identity (cf. ¶81). If we make explicit which type identity is being expressed by *est* (for purposes of examples we will take it to be $=_f$), then we still have a potential ambiguity, because there are two ways that we can interpret the sentence *essencia est formaliter pater*. By the default interpretation of the empty quantifiers that we introduced in the previous section, this sentence should be interpreted as *omnes essencia est pater*. But since in *omnes essencia est pater*, *essencia* is a distributed term which has only one formal suppositum (namely $\dot{\mathsf{E}}$), by ¶33, this means that a syllogism with this as a premise can be reduced to an expository syllogism which has *hoc essencia est pater* as a premise. That is, if we singularize the subject term, we do not change the truth conditions of the sentence (cf. ¶34).

However, there is a second way that we could interpret *omnis essencia est pater*, namely by generalizing the subject term, e.g., *omnis res que est essencia est pater* (cf. ¶¶56, 74). The two interpretations are not equivalent, and they do not have the same signification:

> *Sed breviter dico quod iste due propociones: omnis essencia est pater, et: omnis res que est essencia est pater, ex modo significacionis et imposicionis non habent easdem mentales, nisi velis abuti terminis; et subiectum istius: omnis res que est essencia est pater, summendo primum 'est' ydemptice, supponit pro pluribus formaliter, scilicet pro tribus personis; subiectum autem illius: omnis essencia est pater, supponit pro uno solo formaliter, scilicet pro essencia, et indistincte vel ydemptice pro tribus personis* [¶83].

The truth conditions for both versions are intuitive. *Hoc essencia est essencialiter pater* is a singular proposition, whose truth conditions are governed by Definition 7.4.7, that is, it is true iff the particular, singular thing which is the essence stands in the essential identity relation with [something that is] the father. *Omnis res que est essencia est essencialiter pater* is true iff everything which is the essence stands in the essential identity relation with [something that is] the father. Formally, the distinction is between:

$$!\mathsf{E} =_f \mathsf{F}_{/=_f}$$

and

$$\mathsf{E}_{/=_*} =_f \mathsf{F}_{/=_f}$$

Notice the introduction of $=_*$ into the first term; as our author notes, if we want to expound *essencia* as *omnis res que est essencia*, we need to ask which type of identity is being expressed by this *est*:

> *[V]erbi gracia resolvendo hanc: omnis essencia est pater, dupliciter: omnis res que est essencia est pater. Quero ergo qualem ydemptitatem importent ambe copule. Si dicis: ambe important formalem, ut sit sensus: omnis res que est formaliter essencia est formaliter pater,—ecce copule sunt stricte et modificate, et ymo ista resolvens non plus nec minus valet quam ista: essencia est pater, capta secundum predicacionem formalem, ut patet intuenti. Si autem dicis ambe copule important ydempitatem non formalem sed ydempticam... Si autem una predictarum copularum dicit ydemptitatem formalem, alia vero non, ut sit sensus: omnis res que est ydemptice essencia est formaliter pater, vel: omnis res que est formaliter essencia est ydemptice pater...* (¶82).

In ¶¶84–88, the author argues in favor of interpreting *omnis essencia est pater* as only *hec essencia est pater*, and not as *omnis res que est essencia est pater*. While if we interpret it as *omnis res que est essencia*, then we can reason according to Rules 7.3.3 and 7.3.4, if we do so, then *non salvabis omnes modos Aristotelis, ut patet de disamis* [¶84]. Instead, if we singularize the subject terms and pay attention to the modification of the copulae introduced by *essencialiter*, *personaliter*, and *formaliter*, then *solves omnes paralogismos; salvabis eciam omnes modos Aristotelis* [¶84]. Taking this route, we will see that *multa apparencia distorta ipsis infidelibus sequuntur ad modum de completa ⟨distribucione⟩, quorum nullum sequitur ad modificacione copularum predictam* [¶89]. And thus we are able to resolve the paralogisms.

In the beginning of his text, our author makes an apology for Aristotle, explaining why the Philosopher apparently 'missed' the modes of syllogistic reasoning that are to be found in divine things. Towards the end, the author returns to Aristotle and says that:

> *Et puto ⟨quod⟩ si Aristoteles adhuc viveret et proponerentur sibi illi modi, scilicet tam essendi rerum quam modificandi copularum, ipse concederet talem modum modificandi copularum necessarium ad loquendum exquisite de predicto modo essendi rerum* [¶91].

Those who believe that logic can be applied to the divine terms have two options: The first is to show that the standard logic can be extended to a logic which adequately addresses trinitarian issues and the second is to deny that the standard logic is universally applicable, and to develop instead a separate logic for trinitarian reasoning. Interestingly, our author is often classed with people who fall in the second category, that is, the category of people who don't accept a single logic (see, e.g., [Hal03, p. 86]). What we have seen above is that this is not the case.[23] We can extract Aristotelian syllogistics from within the framework that we have provided, as we have shown in Theorem 7.4.15. This allows

[23] An example of someone who really does fall in this category is Jean Buridan, who says

us to say that reasoning about the trinity is not a "special case" which cannot be handled by regular syllogistic logic. Instead the situation is almost the other way around: Reasoning about creation is just a special case or a reduction of trinitarian syllogizing. We can do all of our logical reasoning within one formal system that handles propositions about divine and created things equally well.

The fact that the predications used in syllogisms about the trinity can be formal, identical, or essential explains why we have paralogisms. The expository syllogism

Hoc essencia divina est pater.
Filius est essencia divina.
Igitur, pater est filius.

is valid and sound if the statements are all taken to be essential predications. The paralogism arises when we interpret the conclusion as making a personal or formal predication. Once this misinterpretation is cleared up, by making the type of predication explicit via our formal system and reasoning with expository syllogisms, then the paralogisms disappear.

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in Book III, Part I, ch. 4 of his book on consequences, "*Sed diligenter aduertendum est quod hae regulae non tenent in terminis diuinis, qui supponunt pro re una simplicissima simul et trina. Vnde licet deo simplici sit idem pater et eidem deo sit idem filius, tamen filius non est pater; et licet idem pater sit deus et non filius, tamen falsum est quod filius deo non sit idem*" [Bu76, p. 85] ("But it should be carefully noted that these rules do not hold in the case of God, [the terms for Whom] supposit for a simple thing one and triune at the same time. Whence although the Father is the same as the simple God and the Son is the same as the simple God, the Father is nevertheless not the Son; and although the same Father is God and not the Son, it is false nevertheless that the Son is not the same as God" [Bu85, 3.4.8, p. 265].)

Appendix A

Logical preliminaries

A.1 Categorical syllogisms

In this section we give a brief refresher course on basic (non-modal) Aristotelian syllogisms.

Syllogistic reasoning was first introduced by Aristotle in the *Prior Analytics* [Ar84], and his method for proving validity of classes of syllogisms was to show how they could be reduced, through a series of different types of conversion and *reductio*, to one of a single set of four intuitively and obviously valid syllogism types (called the "perfect syllogisms"). Syllogisms are ordered sets of three categorical claims, satisfying certain constraints which we will discuss below. A categorical claim is one of the following four types of sentences:

A: All S are P **E**: No S is P

I: Some S is P **O**: Some S is not P

where S is a variable for a subject-term, such as 'cat', 'horse', etc., and P is a variable for a predicate-term, such as 'blue', 'running', 'horse', etc. The first two categorical claims in the ordered set are the *premises*, the last the *conclusion*.

In order for a set of three categorical claims to be called a syllogism there must be exactly three distinct subject- and predicate-terms occurring in the set: One must occur only in the premises, and the other two must occur in the conclusion and exactly one premise. Take the following syllogistic form as an example:

All S are P.
All P are Q.
Therefore, all S are Q.

The predicate term of the conclusion, Q, is called the *major term*. The subject term of the conclusion, S, is called the *minor term*. The term not occurring in the conclusion is called the *middle term*. It is a convention that the premise

1st figure			2nd figure		
$S\_\_P$,	$P\_\_Q$:	$S\_\_Q$	$P\_\_Q$,	$P\_\_S$:	$S\_\_Q$
$Q\_\_P$,	$S\_\_P$:	$S\_\_Q$	$P\_\_S$,	$Q\_\_P$:	$S\_\_Q$
3rd figure			4th figure		

Figure A.1: The four figures

containing the major term is always written first, though from a logical point of view, the order of the premises makes no difference.

Aristotelian syllogisms can be divided into four figures[1]; the figure determines the order of the terms in the premises and conclusion (see Figure A.1). Moods are created from the figures by inserting one of the four copulae A, E, I, O into each categorical proposition.

Since each figure has three slots and there are four different copulae, this means there are 64 moods. Only 24 of these moods are valid. The medievals gave mnemonic names to 19 of the 24 valid moods, where the vowels indicate the copulae of the major premise, the minor premise, and the conclusion (in that order), and the consonants indicate which of the four basic syllogism moods it is to be converted into, and by which conversion methods. This list has been extended in modern times to include names for all 24 of the valid moods. These are [Lag04, §1]:

1st figure Barbara, Celarent, Darii, Ferio, Barbari, Celaront

2nd figure Cesare, Camestres, Festino, Baroco, Cesaro, Camestrop

3rd figure Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison

4th figure Bramantip, Camenes, Dimaris, Fesapo, Fresison, Camenop

A.2 Kripke semantics

In this section we give a general introduction to Kripke semantics.

Kripke semantics formalize our informal ideas about both modality and tense with the same type of structure, the Kripke frame.

Definition A.2.1. A *Kripke frame* is a pair $\mathfrak{F} = \langle W, R \rangle$, where W is a set and R is a binary relation on W, called an accessibility relation.

[1]Both Aristotle and medieval authors considered there to be only three figures; the fourth can be derived from the third through rearranging of the premises, and so is included for completeness's sake.

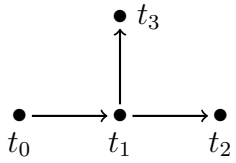

Figure A.2: An example Kripke frame.

The elements of W can be interpreted as times or as possible worlds, depending on the context. The accessibility relation indicates which instants of time are past or future with respect to the current instant or which worlds are considered "possible" with respect to a given world. We can represent Kripke frames schematically, as in Figure A.2; the nodes are the members of W and the arrows indicate R.

Kripke frames are turned into models by the addition of a valuation function, V. This function maps propositions (which we indicate by lower case Roman letters p, q, r, etc.) to subsets of W. If $w \in W$ and $w \in V(p)$, then we write $w \models p$ and say that p is true at w. While V only gives information about the truth values of simple propositions at worlds or instants, we can easily extend this to give the truth values of more complex propositions, as follows:

$$\begin{array}{lll} w \models \neg p & \text{iff} & w \notin V(p) \\ w \models p \wedge q & \text{iff} & w \models p \text{ and } w \models q \\ w \models p \vee q & \text{iff} & w \models p \text{ or } w \models q \\ w \models p \rightarrow q & \text{iff} & w \models \neg p \text{ or } w \models q \end{array}$$

These definitions all involve only one world and thus correspond exactly to the classical truth conditions for the boolean connectives. We can move beyond propositional logic by adding modal operators of various kinds.

The truth conditions of the modal operators $\Box$ 'necessarily' and $\Diamond$ 'possibly' are defined as:

$$\begin{array}{lll} w \models \Diamond p & \text{iff} & \text{there is a } v \text{ such that } wRv \text{ and } v \models p \\ w \models \Box p & \text{iff} & \text{for all } v \text{ such that } wRv,\ v \models p \end{array}$$

And those of the temporal operators, F (at some time in the future), P (at some time in the past), G (at all times in the future), and H (at all times in the past) are defined as:

$$\begin{array}{lll} w \models Fp & \text{iff} & \text{there is a } v \text{ such that } wRv,\ v \models p \\ w \models Pp & \text{iff} & \text{there is a } v \text{ such that } vRw,\ v \models p \\ w \models Gp & \text{iff} & \text{for all } v \text{ such that } wRv,\ v \models p \\ w \models Hp & \text{iff} & \text{for all } v \text{ such that } vRw,\ v \models p \end{array}$$

We call P and H *past-tensed* operators and F and G *future-tensed* operators. Notice that P and H are just the symmetric versions of F and G, and that H and G can be defined as $\neg P \neg$ and $\neg F \neg$, respectively.

property	**formula**	**name**
$\forall x(xRx)$	$\Box p \to p$	reflexivity
$\forall x, y(xRy \to yRx)$	$\Box\Diamond p \to p$	symmetry
$\forall x, y, z(xRy \wedge yRz \to xRz)$	$\Box p \to \Box\Box p$	transitivity
$\forall x, y, z((xRz \wedge yRz) \to (xRy \vee yRx \vee x = y))$	$FPp \to (Pp \vee p \vee Fp)$	backwards linearity

Figure A.3: Some formulas which correspond to frame properties

A formula which is true at every world in a frame or a class of frames all sharing a certain property regardless of the valuation function V is called *valid* on that class of frames. The validity of certain modal formulas on a frame or class of frames can be connected to properties of the R-relation on these frames. A list of some common properties of R, their names, and formulas which correspond to them can be found in Table A.3. The proofs of the correspondence between the properties and the formulas can be found in any standard modal logic text book, such as [ChaZa97, §3.5].

A.3 Quantified modal logic

Basic modal and temporal logics are propositional. In this section we consider quantified modal logic. Fitting and Mendelsohn's book [FittMe98] is the standard text on the subject, and we follow their presentation below.

To extend the semantics given in the previous section, we extend Kripke frames to quadruples $\mathfrak{F}^{\mathrm{Q}} = \langle \mathfrak{F}, O, D, I \rangle$ where O is a set of objects, (D) is a domain function assigning non-empty sets of objects to each world, and I is an interpretation function which assigns each constant in the language to an object in each world and each n-ary predicate to a set of n-tuples of objects in each world. Extended frames $\mathfrak{F}^{\mathrm{Q}}$ are turned into models with the addition of a valuation function V assigning values to the free variables. (D) can either be a constant function on W, in which case $\mathfrak{M}^{\mathrm{Q}}$ is called a *constant-domain* model, or it can differ from w to w, in which case it is called a *varying-domain model*. We stipulate that every object in a world has a constant which is interpreted as that object; if we work with finite domains, then we can always augment our language if required.

The two types of models each capture a different type of quantification, the *possibilist* view of quantification and the *actualist* view, respectively. The question immediately arises of which of these two types of models should be preferred, and the answer is, as Fitting and Mendelsohn point out, that "it doesn't matter. We can formalize the same philosophical ideas either way, with a certain amount of care" [FittMe98, p. 105]. Two pragmatic reasons to prefer constant-domain models are that they are formally simpler to define and to use, and they also "model our intuitions about modality most naturally if we take the domain to consist

of possible existents, not just actual ones, for otherwise we would be required to treat every existent as a necessary existent" [FittMe98, p. 94]. If one wishes to use varying-domain models, it is necessary, in order to address this problem, to introduce an existence predicate, and to relativize all the quantifiers to only the objects which exist at a certain world. It can be shown that if you introduce a relativization of this kind, if a formula is a theorem of a varying-domain model, its relativization is a theorem of a constant-domain model [FittMe98, prop. 4.8.2].

The interesting questions in quantified modal logic involve how the quantifiers and the modal operators interact. Two different proposals for this interaction are the Barcan formula and the Converse Barcan formula:

Barcan formula	$\forall x \Box \varphi x \rightarrow \Box \forall x \varphi x$
Converse Barcan formula	$\Box \forall x \varphi x \rightarrow \forall x \Box \varphi x$

(Their names come from the person to first investigate them in detail, Ruth Barcan Marcus. Technically speaking, these are not formulas but formula schemes, with φ being substitutable for any formula, though we will continue to speak of the singular "the Barcan formula".) These formulas, like the ones in Figure A.3, correspond to the frame properties of upwards monotonicity of D (the Converse Barcan formula) and downwards monotonicity of D (the Barcan formula).

On variable-domain models, the Barcan formula is valid iff the domains are anti-monotonic, and the Converse Barcan formula is valid iff the domains are monotonic (cf. [FittMe98, props. 4.9.6, 4.9.8]). A logic which validates both the Barcan formula and the Converse Barcan formula will have only constant-domain models, with no object coming into or going out of existence.

Appendix B

On modal propositions

In this appendix we give translations of two 13th-century works on modal logic that we refer to in Chapter 4. In §B.1 we translate *De propositionibus modalibus* [Aq06], which is presumed to be an early work of St. Thomas Aquinas. In §B.2 we translate the portions of *Summa totius logicae Aristotelis* [Pse06] which deal with modal propositions, conversions, and syllogisms. This work was previously attributed to Aquinas, but that attribution has been questioned.

B.1 Aquinas, *On modal propositions*

(1) Because a proposition (*propositio*) is said to be modal because of a mode, in order to understand what may be a modal proposition it is necessary to know first what a mode may be. (2) A mode is a determining attribute of a thing, which indeed is made by an addition of an adjective word, which determines a substantive, such as when it is said 'man is white', or by an adverb, which determines a verb, as in 'a man runs well'. (3) Indeed it must be understood that modes are threefold. (4) Some determine the subject of a proposition, as in 'a white man runs'. (5) Some determine a predicate, as in 'Socrates is a white man', or 'Socrates runs swiftly'. (6) Some determine the composition itself of the predicate with the subject, as when it is said 'that Socrates runs is impossible', and by this mode alone is a proposition called modal. (7) But the other propositions which are not modal, they are called assertoric (*de inesse*). (8) But the modes which determine composition are six, namely: true, false, necessary, impossible, possible, contingent. (9) True and false, however, attach nothing above the significations of the assertoric propositions: indeed the same thing is signified when it is said 'Socrates does not run' and 'that Socrates runs is false', and 'Socrates runs' and 'that Socrates runs is true'. (10) That does not appertain to the other four modes, because the same [thing] is not signified when it is said 'Socrates runs' and 'that Socrates runs is possible'.

(11) And therefore, passing over true and false, let us consider the other four. (12) But because a predicate determines the subject, and not conversely, for that reason for this, [that is] which propositions may be modal, it is necessary that the four aforesaid modes are made known, and the word causing the composition may be placed before the subject: because indeed it happens if an infinitive verb is assumed for the indicative verb of the proposition, and the accusative case [is assumed] for the nominative; then the *dictum*[1] of the proposition is expressed, just as of this proposition 'Socrates runs', the dictum is 'that Socrates runs'. (13) Therefore when the *dictum* is fixed as the subject, and a mode for the predicate, then the proposition is modal, as when it is said 'that Socrates runs is possible'. (14) If however it is converted, [then] it will be assertoric, as in 'it is possible that Socrates runs'. (15) Moreover, of modal propositions, some are *de dicto*[2], some are *de re*[3]. (16) Modality is *de dicto* in which the whole *dictum* is made the subject and the mode is predicated, as in 'that Socrates runs is possible'; modality is *de re* in which the mode is inserted into the *dictum*, as in 'Socrates is possibly running'.

(17) However it must be understood that all modals *de dicto* are singulars, because a mode is predicated of this or that in the same way as of a certain singular. (18) On the other hand modality *de re* is distinguished: universal, particular, indefinite or singular according to the subject of the *dictum*, just as concerning assertoric propositions; whence this 'that every man is possibly running' is universal, and so concerning the others.

(19) Likewise it must be understood that a modal proposition is called affirmative or negative according to the affirmation or negation of the mode, and not of the *dictum*: whence this 'that Socrates doesn't run is possible' is affirmative; but this 'that Socrates runs is not possible' is negative. (20) However, it must be understood that 'necessary' has a likeness with a universal affirmative sign, because what is necessary, always is; [and] 'impossible' [has a likeness] with a universal negative sign, because what is impossible, never is. (21) But 'contingent' and 'possible' have a likeness with a particular sign: because what is contingent and possible, sometimes is, sometimes isn't.

(22) And for that reason is-necessary and is-impossible are contraries; and is-possibly and is-possibly-not are subcontraries; is-necessary and is-possible are subalterns, and similarly is-impossible and is-possible-not; but is-necessary and is-possible-not are contradictories, similarly is-impossible and is-possible, as it makes clear in the following figure.[4] (23) It must be noted that all propositions which are in the same order are equipollent. (24) Moreover, a rule and mode of proving in contraries, subcontraries, and contradictories in the same mode may be applied in these just as in assertoric propositions. (25) Further, it must be known

[1] lit. 'what is said'

[2] lit. 'of the *dictum*'.

[3] lit. 'of the thing'.

[4] My source text has omitted the figure.

concerning equipollent modals, that the same negation put down near a mode works, just as in assertoric propositions. (26) Negation in a proposition makes equipollent its contradictory in mode, as in 'is not necessary' and 'is possibly not' [are] equipollent; but negation apposite the *dictum* makes equipollent a contrary: however negation apposite either [makes equipollent] a subaltern. (27) From which it must be understood that 'possible' and 'contingent' signify the same thing: whence propositions of possibility and contingency are similarly ordered [as] equipollents. (28) However all equipollent propositions are by themselves in one order. (29) Whence it is clear that there are four orders. (30) The first order is: is-possible with its equipollents. (31) Second: is-possible-not with its equipollents. (32) Third: is-impossible with its equipollents. (33) Fourth: is-necessary with its equipollents. (34) From this it is clear that the fourth order is contrary to the third, therefore the first is subcontrary to the second, the third is contradictory to the first, the fourth is contradictory to the second, the first is subaltern to the fourth, and the second to the third.

(35) Whence the verse: The third order is always contrary to the fourth. (36) The second disputes with the fourth in contradiction. (37) For you, the first may be the subcontrary line to the second. (38) The third order is contradictory to the first. (39) The first may be subsumed by the fourth, having particularity. (40) But it has by this rule a line following from the second. (41) Or the subaltern order may be the first or the second. (42) The first is 'amabimus', the second 'edentuli'. (43) The third is 'illiace', and the remaining one 'purpurea'. (44) U destroys the whole, but A establishes one or the other, E destroys the *dictum*, and I destroys the mode.

B.2 Pseudo-Aquinas, excerpts from *Summa totius logicae Aristotelis*

Tract. 6, cap. 7

(1) It remains to speak of the third division, which is according to quantity. (2) Whereby you must note that of categories of assertions certain [ones] are assertoric (*de inesse*), certain [others are] in fact modal. (3) Moreover, an assertion is called assertoric which is the simple inherence of a predicate to a subject, as in 'man is animal'. (4) But it is modal in which the inherence of the predicate to the subject is modified, as in 'that Socrates runs is possible' or 'man is necessarily animal'. (5) Whence in the first place it must be spoken concerning quantity, equipollents, oppositions, which accompany quantity in assertoric assertions; in the second place [is] concerning the same in modal assertions. (6) However in order to see the quantity of the assertoric assertions themselves, it must be understood that those which the intellect apprehends, certain [ones] are universals, namely, what are not suited to be found begotten in several: certain [others] are singulars,

namely, what are not suited to be found except in one. (7) However a universal is able to be considered in two ways: in one mode about the separation from singulars, namely, according to being which presides objectively in the intellect: in another mode according to being which presides in singulars. (8) On the first mode of considering a universal, of that something is able to be asserted in two ways. (9) In one mode, when something is attributed to it because it pertains to a single action of the intellect; as when we say 'man is predicable of many things' or 'man is universal' or 'man is a species': indeed the intellect forms this kind of intentions, and attributes the same to the natured intellect, for example to man, according to which it is compared that very thing to those which are outside the soul. (10) In the other mode something is asserted concerning a universal so supposed when something is attributed to itself, exactly as the natured intellect is apprehended by the intellect as one; nevertheless that which is attributed to it, does not pertain to the act of the intellect, but to the being which has that very natured intellect in things which are outside the soul; you must consider whether it is said 'man is the most dignified of creatures': indeed this agrees with human nature according to which it is in singulars: for any single man whatever is more dignified than all irrational creatures; but nevertheless all single men are not one man outside the soul, as when it is said in the aforementioned assertion 'man is the most dignified of creatures' here the word 'man' stands for all singulars, but is one only in the receiving of the mind. (11) And because it is not generally agreed that universals subsist apart from singulars; for this reason common usage of speaking does not have some word or sign which attaches to universals according to the appointed modes by which something is predicated of the same. (12) But Plato, who posited that universals subsist apart from singulars, invented certain figures of speech, which attached to universals in such ways of predication: he said indeed: through itself man is a species, or: predicable man is a species.

(13) By the second mode something is asserted concerning universals, according to what is in singulars themselves; and this is twofold. (14) In one way when something is attributed to itself by reason of its universal, which clearly pertains to its own essence or which acquires its own principle essence; as when it is said 'man is animal' or 'man is risible'. (15) In the other way when something is attributed to the same by reason of a singular in which it is found; namely, when some one individual accident is attributed to itself, as when it is said 'man moves'. (16) And because this mode of asserting something of a universal fails in the common apprehension of men; for that reason certain actions for a mode that must be designated to be attributed to a universal accepted in this way are found. (17) Whence if something is attributed to itself in the first way, namely, by reason of that very thing, as in a universal; because this is something to be universally predicated of itself, for that reason this sign 'every' was invented which designates, that the predicate is attributed universally with respect to all that which is contained underneath the subject. (18) In fact in negative predications near the same invention is this word 'nullus': through which it is signified that

the predicate is removed universally from the subject according to every thing which is contained in it. (19) If in fact something is attributed to it by the second mode, namely, by reason of a singular; a particular sign is found near that which designates in an affirmative, namely, this word 'certain' or 'some', through which it is designated that a predicate is attributed by a universal to a subject by reason of its particular. (20) But because 'indeterminate' signifies the form of some singulars; for that reason it designates a universal under a certain indetermination. (21) Whence it is said an individual is vague. (22) But in negatives some word is not found, or some other sign: but we say that a certain man does not run or some man runs.

(23) Therefore in this way there are three genera of affirmations in which something is predicated of a universal. (24) First, in which something is predicated universally of a universal, as in 'every man is an animal'. (25) Second, in which something is predicated particularly of a universal as in 'some man is white'. (26) Third, in which something is predicated of a universal without a universal or a particular determination, as in 'man is animal'. (27) The first assertion is called universal: the second particular: the third indefinite. (28) To which if a singular is added, in which something is predicated of a singular, as in 'Socrates runs', there will be four modes of assertion, which can be negative just as they are affirmative. (26) And in such a way it is clear et cetera.

Tract. 6, cap. 11

(1) Now it remains to put down or to speak concerning modal propositions. (2) Moreover, a mode, as here supposed, is an adjoining determination of a thing; that is, a determination made through an adjective. (3) However, adjectives are twofold:namely, of nouns, as in white and black, and of verbs, which type are adverbs. (4) Because indeed an adverb stands beside a verb and always depends on a verb, for that reason it is said to be an adverbial adjective, as when it is said 'he runs swiftly'. (5) It should be noted that adverbs are able to determine a verb in many ways: certain ones determine it by reason of the action or passion which the verb signifies, as in, 'I run swiftly' or 'I act bravely': and these make qualitative adverbs. (6) Certain others [determine] in truth by reason of time, as in temporal adverbs. (7) Others in truth [determine] by reason of a mode, as in adverbs of naming and desiring. (8) Certain others in truth [determine] a verb by reason of a composition which is made in speech: and those are six: namely, necessarily, impossibly, possibly, contingently, truly, falsely. (9) Indeed when it is said 'Socrates runs quickly', it is signified that his running is fast: but when it is said 'Necessarily Socrates runs', it is not signified that his running is necessary, but that the composition, namely, 'Socrates runs' is necessary: and in this way concerning the other five adverbs already mentioned. (10) It must be known that the aforementioned six adverbs make true modal assertions: because they are able to make modal propositions understood adverbially, when it is said,

'Socrates runs necessarily', and by name, as when it is said 'that Socrates runs is necessary': and the same concerning the others.

(11) Moreover it is true that two of these modes, namely, truly and falsely, do not change the assertion according to oppositions, equipollents, and suchlike, but by the same mode we consider it, or by the same mode it is assigned in it just as in assertoric categoricals: for this reason we may pass over these. (12) But because the other four adverbs, namely, possibly, impossibly, necessarily, contingently, change the aforementioned assertions, for this reason we may concentrate on these. (13) However concerning these for the time being we will consider four: namely, quantity, quality, corruptions, and equipollents, because of conversion of these it will be said in the tract concerning syllogisms, where indeed it will be said of conversions of assertoric assertions. (14) However for knowing the quantity of them, it must be noted that certain ones are modal propositions *de dicto*, as in 'that Socrates runs is necessary', in which clearly the *dictum* is made subject, and a mode is predicated: and these are truly modals, because a mode here determines the verb by reason of composition, as in what's said above. (15) However certain others are modal [propositions] *de re*, in which a mode is interposed in the *dictum*, as in 'Socrates is necessarily running': indeed by this mode the sense is not that this *dictum* is necessary, namely, 'that Socrates runs', but of this the sense is that in Socrates is necessity for running. (16) And it is more clearly apparent concerning 'possible'. (17) Indeed when it is said 'that Socrates runs is possible', the sense is that this *dictum*, namely, that Socrates runs, is possible; but when it is said 'Socrates is possibly running', the sense is that in Socrates it is possible for running to be. (18) However there are other assertions which appear to be modal, and are not: because clearly the mode is made subject and the *dictum* predicated: as in, 'it is possible that Socrates runs'. (19) The reason for this is, because the denomination [of a proposition as modal] should be obtained by form: however, the formal [thing] in the assertion is the predicate, and for that reason it ought to be denominated by the predicate. (20) When therefore in an assertion a mode is predicate, it will be modal: but when the *dictum* is predicated, it will not be modal.

(21) It must be known that all modal *de dicto* assertions are singulars, although in it might be a universal sign. (22). Whence this 'that every man runs is possible', is singular: and in this way concerning all the others. (23) And this is the reason of it. (24) For, as was said above, an assertion is said to be singular because in it a singular or a singular term is made subject, as in 'Socrates runs'. (25) But in such assertions this signified *dictum* is made subject, namely, 'that every man runs', because the whole is accepted for one signified term. (26) Therefore all assertions of such a kind are singulars. (27) In fact in modals *de re*, and in those which seem to be modal and are not, quantity is assigned according to what terms and signs are in the *dictum*. (28) Whence this 'it is possible that all men run' is universal: and this 'it is possible that some man runs' is particular, and so it stands concerning modals *de re*. (29) Therefore it is clear concerning their quantity.

Tract. 6, cap. 12

(1) It follows concerning their quality. (2) Whereby it should be noted that in assertoric assertions three things are perceived: namely, the subject, the predicate, and the composition of the two: which thus stands in a certain way just as in natural things. (3) Indeed in men body and soul and humanity are observed: the body is the matter: the soul is the form, which is a part of composition, whence it is the form with respect to the body; but humanity is the form with respect to both, namely, with respect to the body and the soul. (4) Just as in what was just said, in an assertion the subject is like the matter, the predicate is in fact like the form, which is a part of composition, whence it is a quasi-form with respect to the subject: but, the composition is the form of both. (5) Whence in the same affirmation and negation are obtained according to composition or division, in which is negation. (6) Whence when a negation is not a composition, it will be an affirmative assertion: if in fact in it is a negation, then it is a negative assertion. (7) However in modals, just as the predicate stands in those assertoric so stands the mode: because it is as a form with respect to the *dictum*. (8) And for this reason, if a mode is composed with an affirmative *dictum*, the modal will be affirmative: if in fact it is negative, the proposition will be negative. (9) Indeed this 'that Socrates is not running is possible' is affirmative, because the composition of the mode with the *dictum* is affirmed. (10) However this 'that Socrates runs is not possible' is negative, because such composition is negated.

(11) And this is clearly apparent in the truth and falsity of them. (12) Indeed an affirmation of the same singular is opposed to the contradictory of the negation; and consequently, if one is true, the remaining one is false. (13) But these 'that Socrates runs is possible', 'that Socrates does not run is possible', both are true, because Socrates is able to run and able not to run, and concerning both dicta the possibility is verified. (14) Therefore it is not the case that one is affirmative and the other negative. (15) It must be understood that it is permitted that a modal assertion is called affirmative and negative by an affirmative mode and by a negative mode; nevertheless, any of these is able to be varied in four ways: because either it will have both, namely, an affirmative *dictum* and mode, as in 'that Socrates runs is possible': or both negative, as in 'the Socrates does not run is not possible': or a negative *dictum* and an affirmative mode, as in 'that Socrates does not run is possible': or an affirmative *dictum* and a negative mode, as in 'that Socrates runs is not possible'. (16) And in this way it is clear concerning the quality of them.

Tract. 6, cap. 13

(1) Now it must be considered concerning the opposition of them. (2) Whereby it should be noted, that modals in this mode are varied according to affirmation and negation in the *dictum* and in the mode, just as was immediately said, and in

this way oppositions are made between them. (3) But because by a diverse mode they are opposed mutually to each other, for that reason in the first place it must be spoken of opposition of modals according to diverse modes; afterwards we will reduce one mode to another through equipollents; and in this way oppositions of all will be accessible. (4) It should be noted, that 'possible' is able to be obtained in two ways: either in its whole signification, and then it includes necessary and contingent: and in this way what is necessary to be, is possible to be, and what is contingent to be, is possible to be. (5) In the other mode it is accepted only for a contingent; and in this way it is accepted in such oppositions. (6) Whence it is permitted that modal assertions be four, however three of them make diversity in oppositions and equipollents, because those of a contingent and those of a possible are on behalf of the same. (7) Therefore concerning such we may speak of three modes; namely, necessary, impossible, possible. (8) And it is permitted, as when it is said when a mode subsists and a *dictum* is predicated, the assertion is not modal, however it is easily reduced to a modal. (9) And in this way we will make use of such assertions. (10) It must be known that just as it is said in assertoric assertions, that sign 'every' designates that the predicate of the assertion is attributed to the subject with regard to all that which is contained beneath it, in fact this sign 'not any' removes from the subject the whole of what is contained under it; and because of this the universal affirmative and the universal negative are contraries. (11) In this way in such modal assertions, this mode 'necessary' functions to signify total inherence of the subject to the predicate, because what belongs necessarily belongs to all such; and for this reason it represents the same category, namely, an affirmative modal of necessity and a universal affirmative assertoric. (12) And just as 'not any' removes the whole, indeed therefore [it is] impossible; because what is impossible to belong, belongs to no such; and for this reason a proposition of impossibility represents the category of a universal negative. (13) And just as '*quidam*' in having to be affirmed does not indicate total inherence, and also similarly in having to be denied it does not remove the whole of what is contained under the subject, in this way is the mode 'possible': because what is possible to belong, does not belong to all, and what is possible to not be in, is not in none; and for that reason 'possible' being affirmed represents the category of a particular affirmative; and 'possible' being denied represents the category of a particular negative. (14) Therefore according to what was said, these phrases 'is necessary to be' and 'is impossible to be' are contrary: 'is necessary to be' and 'is impossible to be' are contradictory: 'is impossible to be' and 'is possible to be' are contradictory: 'is possible to be' and 'is possible not to be' are subcontrary: 'is necessary to be' and 'is possible to be' are subaltern: 'is impossible to be' and 'is possible not to be' are subaltern: as is clear in the following figure.[5] (15) By considering these, it will immediately be clear concerning their equipollents and concerning

[5]The figure was not included in my source text.

the oppositions of them, because they are varied by negations put down in the *dictum* or in the mode. (16) For equipollents of modals, are made in the same way as in assertoric assertions, clearly according to that verse, 'before Contradic., after Contra., before and after Subalter.'. (17) Indeed, negation put before the mode makes equipollent its contradictory; whence this 'is not necessary to be' is equipollent with this 'is possible not to be'. (18) And this 'is not impossible to be' is equipollent with this 'is possible to be'. (19) But negation put after the mode of course makes equipollent its contrary: whence this 'is necessary not to be' is equipollent with this 'is impossible to be'. (20) And this 'is impossible not to be' is equipollent with this 'is necessary to be'. (21) 'Before or after Subaltern.', that is negation put before and put after the mode makes equipollent its subaltern: whence this 'is not necessary not to be' is equipollent with this 'is possible to be'; and similarly 'is not impossible not to be' is equipollent with this 'is possible not to be'. (22) From what is said it is possible to make clear in what way oppositions of the same propositions are varied in mode through negations: for example concerning assertions of necessity, that 'is necessary to be' and this 'is not necessary to be' are contradictories, and those, namely, 'is not necessary not to be' and 'is necessary to be' are subalterns; and so it stands concerning other singular modal assertions, through this verse: amabimus, edentuli, purpurea, which makes it sufficiently clear.

Tract. 7, cap. 2

(1) But conversion of propositions, as it is taken here, is to make a predicate of the subject and a subject of the predicate; thus with respect to this whenever true conversion is existing, likewise it will be true in what it is converted into. (2) For example this proposition, every man is animal, if it is converted into this one, every animal is man, a subject is correctly made of the predicate, and a predicate of the subject; nevertheless the first proposition is true, but the second is false: for this reason such a conversion is not valid. (3) However in propositions of finite terms, of which we here consider, conversion is twofold: namely, simple and *per accidens*. (4) Moreover it is called simple conversion, whenever a subject is made of the predicate, and the predicate of the subject, with the second proposition remaining the same in quality and quantity with the first. (5) But it is called *per accidens*, whenever a subject is made of the predicate, and conversely, with the same quality of the proposition remaining, but the quantity changing. (6) In the first way are converted universal negative and particular affirmative propositions: in the second way are converted the universal affirmative, and, as some say, the universal negative. (7) Nevertheless, it is not necessary to put down this: for if, from this, no man is a stone, it follows, no stone is a man, and this is true; by necessity it follows that this is true: some stone is not a man. (8) For as was said above, whenever true universals are existing, the particulars are always true, although not conversely. (9) First let us prove simple conversion, and first

of universals. (10) And because, as was said, in such syllogisms and of those propositions it does not matter in what the matter may be; for that reason I will use transcendent terms, in the place of which they are able to put down any terms whatever.

Tract. 7, cap. 3

(1) Now it must be spoken of conversion of modal propositions. (2) It must be understood that propositions of necessity and impossibility are converted in the same way as assertoric propositions, and are proved through the same principle. (3) But propositions of possibility and contingency are not converted in the same way. (4) But because oppositions are not obtained in modal propositions as in assertoric propositions; for that reason it is necessary to reveal proofs of the aforementioned propositions. (5) And in such a way as we will make it clear concerning propositions of necessity, so we will concerning propositions of impossibility. (6) Therefore such a proposition may be converted, 'it is necessary for no b to be a' which is converted into this 'it is necessary for no a to be b', because from the opposite of the second proposition it can be inferred the opposite of the preceding: indeed the opposite of that, 'it is necessary for no a to be b' is that, 'it is not necessary for no a to be b'; but that is equipollent to this 'it is possible for some a to be b': for 'it is not necessary not to be' is equipollent with this 'what is impossible to be': because 'not none' is equipollent with this 'what is something': therefore that, 'it is not necessary for no a to be b' is equipollent with this, 'it is possible for some a to be b'. (7) However it follows from this, 'it is possible for some b to be a': which is possible to prove through an expository syllogism, as was said above concerning a particular affirmative. (8) But this, 'it is possible for some b to be a', is a contradictory of the antecedent, which was 'it is necessary for no b to be a'. (9) Therefore from the opposite of the consequent it is inferred the opposite of the antecedent. (10) Therefore the first consequence or conversion was good.

(11) A particular affirmative is converted in the same way, and thus is proved through the same principle. (11) It is necessary for a certain b to be a, therefore it is necessary for a certain a to be b. (13) The opposite of this is 'it is not necessary for some a to be b' which is equipollent with this, 'it is possible for no a to be b', which is converted into this, 'it is possible for no b to be a'. (14) Indeed, it is not assumed here that 'possible' is exactly the same as what is contingent, because it is not converted, as will be made clear below; but 'possible' is assumed only in its whole significant, as it comprehends necessary and contingent, as is said above in another tract.[6] (15) However this, 'it is impossible for no b to be a' is contradictory to this, 'it is necessary for a certain b to be a', which was the antecedent. (16) Indeed in the same way the conversion *per accidens* of

[6]This discussed in tract. 6, cap. 13, 4–5.

a universal affirmative is proved, namely, through conversion made through an accident. (17) It must be noted that this is different between concrete accidental and substantial predicates, and their subjects. (18) For they call a form according to a part of the predicate, but [it is] according to the subject [that] they name that which is that form. (19) Indeed, when I say 'Socrates is white', this word 'white', names only the form of whiteness; but when I say 'a certain white thing is Socrates' the word 'white' names what has whiteness. (20) Therefore in this kind of matter conversion of modal affirmatives fails: for this is true 'it is necessary for a certain white thing to be a body'; and yet this is false, 'it is necessary for a body to be white': because this is contingent. (21) And in this way it is clear concerning conversions of propositions of necessity. (22) In the same way those of impossibility in their whole signification are made.

(23) However conversions of propositions of contingency, and of possibility, as is the same as what is contingent, are made in terms, such as 'it is contingent for no b to be a' is converted into this 'it is contingent for all b to be a'. (24) Whence these conversions in another and opposite mode stand in themselves for conversions of assertoric propositions, and of modal dicta: for in these the predicate always takes the place of the subject, and conversely; and in these their quality is always preserved, although not always their quantity. (25) But in those which were subject or predicate, remain in the same mode and the quality is changed. (26) But the reason of these is: because, as was said, this is a good conversion, in which just as it is true in a converted proposition, thus it is among those according to which it is converted. (27) If however a proposition of contingency is converted in this way, because the predicate took the place of the subject, and conversely; it was not found in all proper matter, but in such matter the antecedent was true and the consequent false, therefore the conversion was bad. (28) For example in significative terms: 'it is contingent for no man to be white', that is true: because this was able to be contingent: if it is converted in this way, 'it is contingent for no white thing to be a man', that is false. (29) Therefore let it be put in this way: that Socrates was white; never will it be contingent for Socrates not to be a man. (30) However this conversion is made in a separate quality. (31) For contingency is trifold: either bilateral[7], as in 'it is contingent for man to be white': or plural (*ut in pluribus*), as in 'it is contingent for man to have two eyes': or paucal (*ut in paucioribus*), as in 'it is contingent for man to be one-eyed'. (32) However, something is called a bilateral contingent, because however many they are able to be in, from so many they are able to be removed. (33) Therefore a negative and an affirmative form are true at the same time. (34) However a plural contingent cannot be converted into a plural contingent, but into a paucal contingent. (35) Whence that 'it is contingent for no man to be blind' is converted into this 'it is contingent that all men are blind'. (36) Indeed the first is a plural contingent but the second is paucal. (37) In the

[7]Following Mark Thakkar's translation of *contingens ad utrumlibet* in [Tha05].

same way a paucal contingent is converted just as a plural contingent is, and in this way the conversion of them is obtained just as was said concerning bilateral contingents. (38) For if it happens in few then it ceases in many. (39) And in this way it is clear concerning conversions of modals. (40) It must be noted that some propositions are put down which are free from conversion, when nothing may be made from the proposition, because none of them are able to be put in any syllogisms which are reducible, therefore clearly in this way these lack conversion: and in such a way of the aforesaid it is superfluous to speak.

Tract. 7, cap. 13

(1) Now it must be spoken of modal syllogisms. (2) Whereby it should be noted that, because propositions of necessity and impossibility and those of possibility and contingency are obtained in the same way, as was said above in the tract on conversions[8]; for that reason modal syllogisms in two ways are made differing between them. (3) Firstly we will speak of syllogisms of necessity, to which those of impossibility are able to be reduced. (4) Secondly we will speak of syllogisms of contingency, to which those of possibility contingently supposed are reduced. (5) It must be known that some syllogisms of necessity have both propositions[9] necessary; and then in whatever figure or mode they are made, the conclusion is always necessary. (6) For example. (7) It is necessary for every man to be an animal. (8) It is necessary for every risible thing to be a man. (9) Therefore it is necessary for every risible thing to be an animal: and so concerning each individual et cetera. (10) It must be noted that necessity is twofold: namely, simple necessity, when it belongs to something simply, and not according to some time or place, or this sort of thing: like 'it is necessary for man to be an animal'. (11) Another is necessity according to some thing, or according to time; like when we say that anything which is, when it is, it is necessary for it to be; or according to place or according to some such sort of thing: and by this mode a necessary proposition is not obtained. (12) Indeed when Socrates runs, he necessarily runs, and yet this proposition 'Socrates runs' is not necessary, but contingent. (13) But if the syllogisms of necessity have one proposition necessary and the other assertoric: although always we may conclude the major extreme to be in the minor, still we cannot always conclude so out of necessity: but at some times it is thus and at other times not. (14) Whereby it must be known that in the first figure, with major propositions being necessary and minor assertoric, it always follows a necessary conclusion. (15) But with the majors being assertoric in so far as the minor may be necessary still the conclusion will not be necessary: because terms are found where it is so, and where it is not so. (16) For example: every man is an animal. (17) Every risible thing is necessarily a man. (18) Therefore every

[8]Tract. 7, cap. 3.

[9]Recall that 'proposition' is given as a synonym for 'premise' in Tract. 7, cap. 3, sen. 4.

risible thing is necessarily an animal. (19) Here are terms where it is so. (20) But the terms where it is not so, are these. (21) Every man is white. (22) Every risible thing is necessarily a man. (23) Therefore every risible thing is necessarily white. (24) However the conclusion is not simply necessary. (25) Thus it is concerning the other moods of the first figure in affirmatives as in negatives. (26) But in the second figure in the first three modes when the negative proposition is necessary, whether it is the major or the minor, the conclusion will be necessary, this being assertoric. (27) But when the affirmative is necessary, and the negative assertoric, a necessary conclusion does not follow. (28) And this is able to be clearly seen by reducing the aforesaid syllogisms to moods of the first figure: for always in the reduced syllogisms the negative will be the major. (29) For, in the first figure, as was said, if the major is necessary, then the conclusion is; but if not, neither is the conclusion.

(30) But in the fourth mood of the second figure, when either proposition is necessary, whether affirmative or negative, provided that the other is assertoric a necessary conclusion does not always follow: for if the universal affirmative is not necessary, in the same terms a syllogism will be made in which it will be made the second mood of the same figure when a universal affirmative is necessary. (31) But as was said, if the affirmative is necessary, a necessary conclusion doesn't follow: which is clear through reduction of it to the second mood of the first figure: therefore this also doesn't follow. (32) But if a particular negative is necessary, a necessary conclusion doesn't always follow, as is clear in these terms, namely: every white thing is a man: some ass is necessarily not a man: but it does not follow: therefore a certain ass is necessarily not white. (33) But in the third figure are affirmative syllogisms and negative. (34) However of the affirmative ones, some have both propositions universal, but some [have] one. (35) All that have both propositions universal, whichever one being necessary, whether the major or the minor, a necessary conclusion follows; that is clear through reduction of it to the first figure. (36) In truth all that have [just] one universal, when it is necessary, whether the major or the minor, a necessary conclusion follows. (37) But if the particular is necessary, a necessary conclusion does not follow: because if it is reduced to the first figure, the major is assertoric: whereby, as was said, a necessary conclusion does not follow. (38) But of negative [syllogisms] certain ones have a negative universal proposition, however certain [others have] a particular. (39) Whence all that have a universal negative proposition, when it is necessary, a necessary conclusion doesn't follow. (40) In truth those having a negative particular and an affirmative universal, whenever one of them is necessary, a necessary conclusion doesn't follow. (41) For example thus: A certain man is not vigilant. (42) Every man is necessarily an animal. (43) Therefore, a certain animal is necessarily not vigilant. (44) And this is when the affirmative is necessary. (45) However, when the negative is necessary, as in this 'a certain white thing is necessarily not an animal', 'every white thing is vigilant': it does not follow (46) therefore, some vigilant thing is necessarily not an animal: seeing that every

vigilant thing may necessarily be an animal. (47) And so it is clear concerning modal syllogisms of necessity.

Tract. 7, cap. 14

(1) It follows concerning syllogisms of contingency. (2) Whereby it should be noted that contingency is twofold: namely, contingency which is in and contingency which is able to be in. (3) Moreover it is said that this contingency which is in, at any time it is predicated, although it may not be of the essence of the subject or a property of it, nevertheless it is a term which is able to be predicated and denied of the subject, although now it will actually be predicated of the same: and such a proposition, although it may be of contingent matter, still it is called of contingency which is in. (4) However, contingency which is able to be in, is designated when the predicate is of contingency, just as concerning white with respect to man: however in a proposition it is not said to be in actually, but possibly: as when I say 'it is contingent for man to be white', the sense is not that 'man might be white' but that he is able to be white.

(5) It must be known that in the first figure, when both propositions are of contingency, likewise the conclusion will be of contingency, as follows: (6) It is contingent for all white things to be musical. (7) It is contingent for all men to be white. (8) Therefore it is contingent for all men to be musical. (9) And it is likewise concerning the other moods.

(10) But in the second figure if both propositions are of contingency, whether they may be affirmative, or one of them be negative, whether they may be universal, or one of them be particular; the conclusion will never be of contingency. (11) For example: it is contingent for no man to be white. (12) It is contingent for all risible things to be white. (13) Therefore, it is contingent for no risible thing to be a man. (14) It does not follow: because it is necessary for every risible thing to be a man. (15) And likewise concerning the others.

(16) But in the third figure, when the premises are of contingency, whether they may be affirmative, or one of them be negative, whether universals, or particulars; a conclusion of contingency always follows, as in: it is contingent for all men to be white. (17) It is contingent for all men to be musical. (18) Therefore, it is contingent for a musical thing to be white.[10] (19) And likewise in the other moods.

[10]This is very curious: The conclusion here is not a standard modal categorical: there is no quantifier. However, there is a deeper problem: one can create a syllogism of the same form where the conclusion is impossible, not contingent: It is contingent for all men to be white, it is contingent for all men to be black, therefore it is contingent for a white thing to be black. If we translate the ⟨*modal*⟩ + ⟨*quantifier*⟩ '*A esse B*' construction in the equally correct 'that ⟨*quantifier*⟩ *A* is *B* is ⟨*modal*⟩', then our argument is: That all men are white is contingent, that all men are black is contingent, therefore that a white thing is black is contingent, and here the conclusion is clearly not contingent but necessarily false.

(20) But if one premise is of contingency and the other assertoric; a conclusion of contingency doesn't always follow. (21) Whence in the first figure when the major is of contingency, whether it may be affirmative, or negative, but the minor is simply assertoric, then a conclusion of contingency always follows. (22) As in: It is contingent for all men to be white.[11] (23) Every risible thing is a man, therefore it is contingent for every risible thing to be white. (24) But when the major is assertoric and the minor of contingency, a conclusion of contingency doesn't always follow, even if the syllogism is perfect, as follows. (25) Every sound thing is an animal. (26) It is contingent for every horse to be sound: it does not follow; therefore, it is contingent for every horse to be an animal. (27) It must be understood, because in any proposition of contingency whatever the *dici de omni* is able to be saved in two ways, according to the two senses discussed above. (28) Whence when I say 'it is contingent for every man to be white', I compose 'man' to what was said, of which it is able to be predicate, either according to the existence of its whiteness, to be this sense: everything which is a man, it is actually contingent for it to be white. (29) Or according to possible inherence, to be this sense: everything which is a man, it is contingent for it to be able to be white. (30) An assertoric proposition has only one *dici de omni*, namely, according to the composition of the subject to its inferiors according to the actual inherence of the predicate: and from this it follows that by virtue of the *dici de omni* that an assertoric [proposition] is able to subsume that of contingency. (31) Indeed one *dici de omni* underneath two is a consequence, because it is in at least one. (32) But that of contingency is not able to subsume that which is assertoric: indeed two *dici de omni* are not contained beneath one. (33) And this is the cause whereby when the major is of contingency but the minor assertoric, a conclusion of contingency doesn't follow in virtue of syllogistics, although it may follow by grace of the matter.

(34) It must be known that an assertoric proposition is twofold: namely, assertoric as of now (*ut nunc*), that is, when the predicate is in the subject only as of now, as in Socrates runs, and simply assertoric. (35) Whence according to this because a conclusion of contingency may follow when the minor is assertoric, it [the minor] must be simply assertoric, because the predicate always follows the subject: for this reason it follows that whatever is contingently in the predicate is contingently in the subject. (36) But because in an assertoric as of now proposition the predicate does not always follow from the subject, it will not always be true that whatever is contingently in the predicate is contingently in the subject. (37) Neither is it the case that if the minor is of contingency, which is as valid as that which is assertoric as of now, and the major is of contingency, the conclusion follows. (38) For if the minor is of contingency, the predicate is

[11] Here we see a distinction between syntax and semantics. Syntactically, this is not a modal proposition, but an assertoric proposition. But semantically, it is a modal proposition, in that it is a necessary truth.

always contingently in the subject; for this 'it is contingent for man to run' is true, because it is always contingent for a man to run, nevertheless this 'man doesn't run' is not always true: for that reason it stands in itself otherwise than of bilateral [contingency].

(39) However in the second figure in the first three moods, the negative being contingent but the affirmative being assertoric does not make a syllogism. (4) And the cause is because every syllogism in so far as they imply [something] to that extent they are reduced to the first figure, where is the *dici de omni* and the *dici de nullo*. (41) However such syllogisms are not able to be reduced to the first figure: indeed a universal negative of contingency is not converted, when the predicate is made from the subject and the subject is made from the predicate, as was noted above: but it is converted in terms. (42) Whence this 'it is contingent for no man to be white' is converted into 'it is contingent for every man to be white'. (43) Whence following this, when a universal negative is of contingency, it is evidence that in the three moods a syllogism isn't made, because it is not able to be proved through the *dici de omni*. (44) But when the affirmative is of contingency, and the negative assertoric a syllogism is made, because such a syllogism is able to be reduced to the first figure, through negative propositions similarly converted: still a conclusion of contingency will not be, as was said above. (45) However in the fourth mood, in no way is a syllogism made, whether the major of the minor is of contingency. (46) Indeed such a syllogism is not able to be reduced to the first figure through conversion, but through a converted syllogism. (47) But whenever one is of contingency and the other assertoric it is not possible to make such reduction: therefore nothing follows from it. (48) However because when the major is of contingency such a reduction is not able to be made, it is clear. (49) Such a syllogism might be: it is contingent for every man to be white: a certain stone is not white: therefore a certain stone is not a man; the opposite of which is 'every stone is a man'. (50) Therefore a syllogism may be made in the first figure in this way: it is contingent for all men to be white. (51) Every stone is a man, therefore it is contingent for every stone to be white. (52) But this is not opposed to this 'a certain stone is not white' which was the minor: for as was said, in a proposition of contingency the *dici de omni* may be saved through possible inherence: indeed in the future it will be able to be true that every stone is made white, but presently a certain stone is not white: and thus in this mood the combination is not valid. (53) And yet it is apparent that such a syllogism may not be valid: for when the propositions are true, sometimes a false conclusion follows: as thus: 'it is contingent for all men to be white'. (54) A certain risible thing is not white: it follows: therefore it is contingent for a certain risible thing not to be a man: which is simply false. (55) Similarly if the minor is of contingency, nothing follows, and a syllogism is made in this way: every man is an animal. (56) It is contingent for a certain white thing not to be an animal. (57) Therefore it is contingent for a certain white thing not to be a man: as for example an egg, which is white, it is contingent that sometimes it will be

an animal and sometimes it will not be, if this follows assertoric, namely, that 'a certain white thing is not a man', it does not hold. (58) Indeed the opposite of this is 'every white thing is a man', therefore every white thing is an animal, which is not opposed to this 'it is contingent for a certain white thing not to be an animal', as was said above. (59) But if a conclusion of contingency is obtained, namely, 'it is contingent for a certain white thing not to be an animal', the opposite of which is 'it is necessary for every white thing to be an animal', the same follows: and a syllogism is in the first figure so: every man is an animal. (60) It is necessary for every white thing to be a man. (61) It follows: therefore every white thing is an animal: as was said above concerning mixtures of necessary and *de inesse* [propositions]: which is not opposed to that minor, namely, 'it is contingent for a certain white thing to be an animal': and in this way it is a useless combination. (62) But in the third figure, when the major is of contingency but the minor assertoric a conclusion of contingency follows: for when the minor is converted in five moods of the syllogisms, a reduction to the first figure is made. (63) For example, a syllogism is made this way: it is contingent for every man to be white. (64) Every man is an animal. (65) Therefore it is contingent for a certain animal to be white: when the minor is converted *per accidens*, a third mood of the first figure is made in this way: it is contingent for every man to be white. (66) Every animal is a man. (67) Therefore it is contingent for a certain animal to be white: and it is likewise concerning the other four moods.

(68) However the fifth mood of the third figure is reduced to the first through a converted syllogism, so: it is contingent for a certain man not to be white. (69) Every man is an animal. (70) Therefore it is contingent for a certain animal not to be white. (71) The opposite of which is: it is necessary for every animal to be white. (72) Then the minor of the first syllogism may be put down beneath it, namely, every man is an animal: it follows: therefore it is necessary for every man to be white: which is the contradictory of the major of the first syllogism. (73) And so it is clear concerning a mixture of propositions of contingency and assertoric in the third figure, etc.

Tract. 7, cap. 15

(1) It follows concerning the mixture of contingency and necessity. (2) It should be noted that in the first figure with regard to affirmatives, when the major is of contingency but the minor is of necessity, the syllogism will be perfect, and will imply a conclusion of contingency. (3) For example: it is contingent for all animals to be white. (4) It is necessary for every man to be an animal. (5) Therefore it is contingent for all men to be white. (6) But if it is conversely, namely, when the major is of necessity and the minor of contingency, no syllogism will be made. (7) The cause is, because a necessary proposition can have one *dici de omni*, namely, according to the actual inherence of the predicate to the subject, and to what is contained beneath it. (8) But that [proposition] of

contingency has a double *dici de omni*, as was said above. (9) Therefore this is not able to be subsumed of necessity under that by virtue of the principle which is the *dici de omni*. (10) It should be known that in the aforesaid syllogism, although it could be concluded through the *dici de omni*, nevertheless a conclusion of contingent possibility was able to be concluded, which clearly holds indiscriminately of necessity and contingency, in virtue of this rule: if some subject is essentially beneath some predicate, whatever is contingently beneath the subject is contingently beneath the predicate.

(11) However in a negative syllogism, when the affirmative proposition is necessary, but the negative is of contingency, a conclusion of contingency follows in this way: it is contingent for no man to be white. (12) It is necessary for every risible thing to be a man: it follows: therefore, it is contingent for every risible thing to be white. (13) But if the negative proposition is necessary, but the affirmative contingent, two conclusions follow: namely, sometimes of contingency, sometimes assertoric; which is able to be seen: for from the opposite of either conclusion with one of the premises, the opposite of the other premise is inferred. (14) However the reason by which one conclusion may follow from one negative [proposition] of necessity is because not only is the predicate not said to be in the subject, but indeed neither is it able to be in it. (15) Whence the syllogism signifies that the predicate may be not in the subject, and even is not able to be in it. (16) Whence when something is contingently put under such a subject, it is signified that such a predicate is actually removed from that which is contingently put beneath the subject: and this will be a conclusion of contingency; because what is actually removed contingently makes a contingent assertion, and it is signified that in no way is it able by itself to be in: and in this way not only is it not in contingently, indeed in no way is it in it, if it is an assertoric assertion.

(17) However in the second figure in the first three moods when the negative is of necessity but the affirmative of contingency, indeed a twofold conclusion follows: namely, of contingency and assertoric: because through conversion of necessary propositions they are reduced to the first figure. (18) But in the fourth mood when the affirmative is of contingency but the negative of necessity, or conversely, the combination will be useless. (19) But in the third figure with regard to affirmative syllogisms, those having universal propositions or some universal [proposition], when the major is of contingency but the minor of necessity, a conclusion of contingency follows. (20) Therefore when the minor is converted a syllogism of the first figure is made. (21) However if the major is of necessity but the minor of contingent possibility, because propositions of contingent possibility are converted just as those of necessity, when the minor is converted, a first figure [syllogism] is made just as was said, with respect to which when the major [sic][12] is of necessity but the major of contingency, a conclusion of contingent possibility follows, although not through the *dici de omni* but through the first rule. (22)

[12]This is clearly supposed to be 'minor', so that this case differs from the previous.

However with respect to affirmative syllogisms having some premise particular, if the major is universal, it follows the aforementioned rules. (23) But if the major is particular, it follows the second rule. (24) However with respect to negative syllogisms, I say that with respect to the two moods having a universal negative major, when the major is of contingency but the minor of necessity, a conclusion of contingency follows, and is it reduced to the first figure through conversion of the minor. (25) If however the major will be of necessity, but the minor of contingency, a syllogism is not made: indeed it is not able to be reduced to the first figure. (26) But if the minor is of contingent possibility, because it is able to be converted in the same way as those of necessity, a syllogism could be made in the first figure, whereby a twofold conclusion could be concluded: namely, assertoric and of contingency, as was said above. (27) However a negative syllogism, the major of which is a particular negative, when it is of contingency, and when the major [sic][13] is of necessity, entails a conclusion of contingency, and it is reduced to the first figure through a converted syllogism. (28) But when the major is of necessity, a syllogism is not made. (29) And in this way it is clear concerning modal syllogisms.

[13]As above, this is an error for 'minor'.

Appendix C

On the mode of predication and syllogistic reasoning in divine things

In this appendix we give a translation of the anonymous late medieval text *De modo predicandi ac sylogizandi in divinis* [Mai88], which we discuss in Chapter 7.

Translation

(1) Just as in divine things [there] is a certain mode of being which is not present in created things, so concerning the same divinity it is seen that there should be a certain mode of predication and syllogistic reasoning, which in creatures is not necessary. And for that reason, as I think, Aristotle and the other philosophers, ignoring this special mode of being in divinity, did not consider this special mode of predication and syllogistic reasoning in divine things.

(2) But Boëthius said in the first book of the *Consolation of philosophy* that philosophy herself was being seen to strike the heavens with her head; and thence [her] head rising up higher was penetrating the heavens and frustrating the view of gazing men. This Boëthius [says]. Then indeed philosophy is seen to penetrate the heavens with her head whenever celestial things are investigated in her; then in truth the head rising up higher penetrates the heavens as well as frustrating the view of men any time in which divine things are investigated in her, of which the mode of being is above all modes of being of creatures, and the mode of knowing this (the divine things) is above reason.

(3) Whence Richard[1] in the little book of the mystical muse, putting down six steps (grades) or six genera of contemplations, said that 'the fifth type of contemplation is above reason, yet nevertheless is not beyond reason'; but the 'sixth genus of contemplating' he says dwells in those 'which are above reason and are perceived to be beyond reason or even contrary to reason'. And after a few words he says: 'Such kinds are nearly all which we are told to believe of

[1]Richard of Saint Victor, a 12th-century Scottish mystical theologian from Paris.

the persons of the divine trinity; concerning these things when human reason is considered, nothing else is seen to be against [reason]'. This Richard [says] in the above [cited] place, in book one chapter 6. Of these grades of contemplation Saint Thomas treats in *Secunda Secunde* q. 180, article 4° in the solution of the 3^e reason, and this is alleged by Richard.

(4) The mode of being in divinity is that three persons are one most simple essence and likewise the most simple essence [is] three persons and each (*quaelibet*) of them. Whence certain things in this divine trinity are essentially the same, but notwithstanding this the names of them are denied of them; just as the three persons are one and the same being (as in the *Symbolus* of Athanasius: "But of the father and of the son and the holy spirit the divinity is one" etc.), and still the names of those persons are denied concerning them, because the father is not the son nor the son the father, and the father is not the holy spirit nor the holy spirit the father; and the same of the son and the holy spirit.

(5) Whence these three persons are essentially the same but are distinct personally. Whence Athanasius: "Different is the person of the father, different of the son, different" etc. For indeed between these persons is relative opposition, and because of this the names of the persons are denied of them: not because of an essential distinction, but because of a personal distinction.

(6) Put another way by the doctor in divinity, [it is] identical identity but not formal, just as the essence is the same as the persons. Whence the name of essence is truly and affirmatively predicated of the names of the persons and vice versa; for indeed these are true predications: the essence is the father, and: the father is the essence.

(7) However as many modern teachers say, the essence is not formally the persons in the same way, because even if the names of the essence and the persons may be predicated truly and affirmatively of each other, yet because of the name of the essence and of the name of the person the same [thing] may be predicated through an opposite mode (*per modem contradiccionis*[2]), for this reason the aforementioned teachers put down a formal distinction between essence and person.

(8) For example in these: the essence is the son, and: the father is not the son, concerning the name of the essence and the name of the father, the same thing, namely, the name of the son, is predicated through an opposite mode, because it is predicated negatively of the name of the father and affirmatively of the name of the essence; or if it pleases you more, you may express it thusly: because the father is not the son out of him because he is opposed relatively to him, and generates him, and essence is in reality the son and not opposed to him because he doesn't generate and doesn't produce him [i.e., the father], hence it is that the

[2]The choice of 'opposite mode' rather than the more literal 'mode of contradiction' was influenced by the fact that there is a very specific, technical meaning of 'contradiction' in modern logic, and it's not clear that that modern meaning is what is being invoked here.

essence and the father are distinguished formally, and consequently of the names of the father and the essence the same [thing] is predicated through an opposite mode, because the father generates and the essence doesn't generate.

(9) This is seen to agree with that which is said by Augustine and the Master of the Sentences in dist. 10 of the first [book] of the sentences, namely that 'properly the word of God is further said to be the wisdom of God, while both the father and the holy spirit are the wisdom. If therefore properly any of these three ought to be named 'charity', who more properly than the holy spirit? So that, namely, in that most high and simple nature one thing is not the substance and another the charity, but the substance itself is the charity and the charity itself is the substance, whether in the father, in the son, or in the holy spirit, and nevertheless the holy spirit properly is named charity'. Thus the words of the Master are followed: 'Behold, with these words Augustine openly makes clear that in the trinity, charity sometimes refers to the substance which is common in the three persons and whole in each; sometimes specially to the person of the holy spirit; just as the wisdom of God is sometimes accepted for the substance of the divinity, and sometimes properly accepted for the son and this is found to be [realized] in many [properties]'. And beneath dist. 32 the Master says: 'And just as in the trinity there is a love which is the father and the son and the holy spirit, which is itself the essence of the divinity; and yet the holy spirit is the love which is not the father or the son, and not therefore are there two loves in the trinity, because the love which is properly the holy spirit is the love which is the trinity, and yet is not itself the trinity, just as the holy spirit is the essence which the trinity is, yet is not itself the trinity; in the same way there is wisdom in the trinity which is the father and the son and the holy spirit which is the divine essence, and yet the son is the wisdom which is not the father or the holy spirit, and not for this reason are there two wisdoms there, because the wisdom which is properly the son is the wisdom which the trinity is; but itself is not the trinity; just as the son is the essence which is the trinity, yet he himself is not the trinity'. And below dist. 34 the Master says: And yet we do not deny a distinction will have [to be made][3] following rational intelligence, when it is said substance and when it is said essence, because in there [the latter] is signified what is common to the three, but in the former not. Yet the substance is the essence and conversely'.

(10) From which words of Augustine and the Master it is evident first that the word in divinity is properly wisdom and yet the father and the holy spirit are the wisdom. But, as it is seen, this is not something other than to say that the word is properly, that is, formally, wisdom, namely the generated [wisdom], and the father and the holy spirit are the wisdom, yet [they] are not the generated wisdom which properly, that is formally, is the word, but are the wisdom essentially speaking, which is common in three persons.

[3]The text has *distinccionem habendam* whereas Lombard's original has *aliquam distinctionem habendam fore*.

(11) However the father, or the holy spirit, is not properly or formally the wisdom which is common in the three persons, but only through identity, otherwise the father, or the holy spirit, might be three identical persons, which is not true. It is clear from this that the father, or the holy spirit, is distinguished formally from the wisdom essentially speaking, because the wisdom when taken essentially is the three persons and all of them, but neither the father nor the son is the three persons, etc.

(12) Furthermore it is evident that the wisdom essentially speaking is distinguished formally from the wisdom which is properly the word, just as the essence is distinguished formally from the son; it is evident because the wisdom essentially speaking is three persons, but the wisdom which is properly the word is not three persons, because single is the person of the son.

(13) Moreover it is clear that the father is not only distinguished formally from the wisdom which is properly the son, but also identically and personally; it is clear because the father is not the wisdom which is properly the word, and the wisdom which is properly the word is not the father, just as the father is not the generated wisdom and the generated wisdom is not the father.

(14) Finally it is evident that the wisdom which is properly the word is formally the same with the son; it is evident because it is properly the word, therefore it is properly and formally the son.

(15) However these things which are said concerning the wisdom which is properly the word and the wisdom common of the three persons, similar things are able to be said of the charity which is properly the holy spirit and concerning the charity common to the three, concerning the identity and the distinction of which obviously I omit at present because of brevity.

(16) From this it is clear that in divinity, certain things are essentially one and the same, such as the persons: for example the father and the son and the holy spirit are one God and one essence and consequently are essentially the same.

(17) But some things are identically the same, such as the essence and the persons, as well as the wisdom essentially speaking and the persons; similarly the charity essentially speaking and the persons.

(18) However, some things are formally the same, just as a person and his property, as in the father and fatherhood (*paternitas*); similarly the son and the generated wisdom are the same formally, and further the proceeding love and the holy spirit.

(19) Furthermore, the essence and the essential attributes, as in the wisdom [and] charity essentially speaking, justice, goodness, etc. are formally the same, because the essence is in whatever is subordinated, in the same way are moreover the other essential attributes.

(20) However in the divine trinity there is no essential distinction; for this reason I do not trouble myself concerning it.

(21) Nevertheless there is a personal distinction in which the persons are distinguished from each other: indeed the father is not the son, and the son

is not the father. In a similar mode the personal properties are distinguished, because fatherhood is not sonhood and sonhood is not fatherhood.

(22) Another [distinction] is a formal distinction in divinity because it is between the essence and the persons: indeed it is possible that the essence may be the father, nevertheless, furthermore because the essence is the son and the father is not the son, for this reason the essence and the father are distinguished formally.

(23) Furthermore, the formal distinction is between the essence and the personal properties: although indeed the essence may be the fatherhood, nevertheless, moreover because the essence is the sonhood and the fatherhood is not sonhood, for this reason the essence is distinguished formally from them; because just as was recounted above, the Master [in] 34 dist.: And yet we do not deny a distinction will have [to be made] following rational intelligence, when it is said substance and when it is said essence, and yet we do not disavow that a distinction must be made following rational intelligence when it is called 'substance' and when it is called 'essence', indeed in the same way I am able to say that the distinction according to rational intelligence is required when it is called 'fatherhood' and when it is called 'essence': indeed fatherhood is [said] of the father only, but the essence is [said] of the father and of the son and of the holy spirit.

(24) From this, as is seen, it is possible for a question which troubled the Master to be resolved, he reckons [it] difficult and it troubles him [in] dist. 33 of the first [book] in these words: 'But if by chance you will ask since these properties cannot be in the persons and not determine them, in what way are they all to be in the divine essence, in such a manner as not to determine them'. And the Master immediately adds: 'I respond to you, and this with Hilarius: I do not know, do not ask me again and I will be relieved yet. The Archangels do not know, etc.' In truth I say that the properties, although they may be the persons and may be furthermore the essence, nevertheless determine the persons and not the essence, because properly and formally they are the persons and not properly and formally the essence. For example, fatherhood determines the person of the father and not the essence, because fatherhood properly and formally is the father and is not properly and formally the essence, because fatherhood is only the father and neither the son nor the holy spirit; in truth the essence is the father, and the son, and the holy spirit. Whence fatherhood is properly and formally the father, but is not properly and formally the essence, but only [is the essence] through identity. Hence it is that fatherhood determines and denominates the person of the father and not the essence.

(25) Now, because what was said is concerning the mode of being in the divine trinity, what needs to be said is concerning the mode of predication and syllogistic reasoning: for after the treatment concerning the lady, namely philosophy or wisdom, it needs to be followed by a discussion concerning the maidservant, namely logic, for while wisdom penetrates the heavens, she can remain by herself,

but her handmaiden, namely logic, can also ascend with her. In truth logic, in so far as it suffices for the present purpose, consists in modes of predication and syllogistic reasoning.

(26) But because according to the doctors 'who is' is properly a name of God, because he himself alone truly and immutability is and has in himself the extent of infinite essence, therefore, as is seen, in divine predications the attention is to the substantive word, namely 'is' and its declined forms.

(27) However just as in divinity there is a certain mode of being in which some things are essentially one and the same, in this manner they are able to be formed in propositions which only denote the essential identity. For example, the father and the son are essentially the same; therefore the formation is one proposition or predication in which is denoted only the identity of essence, just as this: the father is essentially the same with the son.

(28) And if this predication and similar things are not suited for syllogistic reasoning, then as I judge it is possible to form simpler predications in which from supposition or through a special sign being attached to the copula only essential identity is denoted. For example, as you will, following the habitual custom of the theologians in this proposition: the father is the son, essential identity is not only denoted, but indeed identical or personal identity is denoted, and for this reason this must be denied, nevertheless, if someone says: I suppose that in this: the father is the son, essential identity alone is denoted, then it must be allowed that this may be true, because through it nothing more is signified than that the father may be essentially the same with the son. Or if someone says: the father is *a* son, in this manner because *a* may be a sign of only essential identity, then similarly it must be allowed that this proposition may be true, namely: the father is *a* son.[4]

(29) A corollary follows that every term in divinity is able to be predicated of each other truly in an affirmative essential predication; it is clear, because these predications are all true: fatherhood is *a* sonhood; active generation is *a* passive generation; active spirit is *a* passive spirit. It is clear from something common, because in divinity inwardly nothing is an essential distinction, but all things are essentially the same.

(30) Nevertheless if someone has come upon an objection to this corollary, either he corrects himself or rejects it totally. In a similar way, if this mode of predication in which only essential identity is denoted is not necessary or is not useful for syllogistic reasoning, it can be rejected.

(31) Now because as was previously said, a certain mode of being in divinity exists in which some things are the same not only essentially, but furthermore identically and personally, for that reason a mode of predication must be fixed in which not only essential identity but indeed identical or personal identity is denoted; it is clear, because from the fact that the essence and the father are

[4]Here *a* is being used as a variable for adverbs.

identically the same, for this reason the name of the father and the essence are predicated of each other identically. For these are predications of identity: the father is the essence, and: the essence is the father.

(32) However it was said previously that in divinity [there] is a certain mode of being in which some things are formally the same, on the condition that in whatever way one is the other is also in the same way. For this reason I say thirdly that a mode of predication must be fixed in which not only essential identity is denoted, or identical or personal, but indeed identity in every way and formal is denoted. For example, essence and goodness are in every way and formally the same, because in everything [which] is essence in the same is also goodness, because just as the essence is the father and the son and the holy spirit, so indeed goodness is the father and the son and the holy spirit. For this reason formal predictions must be fixed in which formal identity is denoted: for truly from the fact that the essence and goodness are formally the same, the names of the essence and the goodness are able to be formally predicated of each other, as in: the essence is formally the goodness.

(33) Now with the holy spirit supporting, I may say something concerning the mode of syllogistic reasoning in divinity. Whereby it is noted that every syllogism in which is distributed some term having only one formal suppositum does not hold in virtue of the distribution of this term (that is, that conclusion is not proved by precise virtue of the distribution), but it holds as an expository syllogism. For example, such a syllogism: Every divine essence is the father, and wisdom is a divine essence, therefore etc., does not hold in virtue of the distribution of this term 'essence', but by reason of the singularity of the divine essence; that is clear, because in such circumstances in the same mode it is valid to argue expositorily just as distributively. For example, neither more nor less is proved through the said syllogism than through such: This divine essence is the father, and goodness is this essence, therefore. Whence, just what was said is in the suppositions, whether a distributive sign is added to a term of such a kind[5], or not, this does not make a difference.

(34) And briefly a universal or distributive mode of speaking is given from nature and art when anything is said concerning any plural thing; however when there are not more, this is not able to be used. Therefore anyone who wishes to syllogize in such a way, that is, universally, in terms not having many formal supposita, he is not required to consider the distribution as the distribution is, but as the singularization of a singular term is included in it. Then from the distribution, it is not necessary to distribute the deduction because this syllogism can be reduced immediately to an expository syllogism. For example if someone refuses this syllogism: Every father generates, the essence is a father, therefore the essence generates, because of distribution, then it is able to be reduced to this expository syllogism: This father generates, and the essence is this father,

[5]That is, one with only one formal suppositum.

therefore the essence generates. And so it behooves you thereupon to recognize the deduction.

(35) Consequently, I say that in syllogisms where terms having many formal supposita are distributed, it is possible to consider the distribution. For example, in this syllogism: Every thing which is the essence is the father, and the son is a thing which is the essence, therefore the son is the father, there the concern is for distribution, because the major subject has many formal supposita; and there it is important to add or not to add a distributive sign, because a distributive sign produces more there than a discrete or singular sign. However in terms having one formal suppositum a distributive sign produces nothing more than a singular [one].

(36) Now what will be seen is concerning the regulative principles of syllogisms. And the first is the *dici de omni* which is concerning the middle common feature[6] of affirmative syllogisms. And it is such: Whenever some predicate is said of some distributed subject, then of whatever is said to be of such a distributed subject, of the same thing indeed it is said to be of such a predicate.

(37) However this will be understood to be according to the certain condition of the copula. Whence it is not necessary that, if some term is said of a distributed subject by means of a copula of the present and assertoric (*de inesse*), and if such a subject is said of some term by means of a copula of the possible or another ampliative, that then such a predicate is said of the same term by means of a copula of the possible or of the present. From which this syllogism is not valid: Every running thing is a donkey, every man can be a running thing, therefore every man can be a donkey, or indeed just concluding of the present, as in: Every man is a donkey.

(38) Therefore I say that for formally inferring a single proposition concerning the present it is necessary that both premises be of the present and that the copulae of the premises induce the same time with the copula of the conclusion; or it is necessary for the minor premises that the time of the copula of the conclusion may be included in the time of the premises etc. And this [is how it is] in syllogizing in creation. However [it is] in the same manner in divine syllogizing: If all copulae, namely of the premises and the conclusion, are formal, that is because formal predications constitute [it], then I say that every affirmative syllogism is valid in every figure.

(39) And because in creation all predications are formal, because according to common opinion of the terminists all the things which are the same in creation are formally the same, therefore the mode of syllogizing through propositions concerning identical predications is not necessary in creation.

(40) However, according to the mode of the realists, according to which not all things in creation which are the same are formally [the same], still all predications are formal, which is clear because what is not formally the same according to

[6]Nowadays we'd say 'middle term'.

the realists, according to they themselves must necessarily be denied of each other if indeed they are identically the same. For example, realists refuse this *simpliciter*: Socrates is either humanity or animality, not in opposition to the fact that they permit identifying humanity in this Socrates. However, alongside this mode, Aristotle speaking in many places says propositions are false in which an abstract is predicated of a concrete; however, this is only true according to the intention likewise of Aristotle and the realists, following a predication formal and not identical, following which certainly the realists have to concede this: man is humanity, or: Socrates is humanity. And indeed just as Albert the Great in his *Metaphysics* says that a universal form, such as humanity or animality, is nothing except what is put down in his definition. However, indeed, according to the intention of Albert this sole truth is following from formal predication.

(41) Whence according to identical predications of such a kind, a universal form is certainly a suppositum or a singular because this universal form is identified with singulars. However, equally that may be said of this syllogism: Every man runs, and humanity is a man, therefore humanity runs (where the conclusion is most fallacious, because according to Aristotle in the introduction of the *Metaphysics* acts and operations are singular: a doctor does not heal men, but Callia and Socrates. Therefore a universal form, such as humanity, does not run, but rather a suppositum or some particular man; on the contrary Aristotle is seen to desire that same thing concerning the singular forms of supposita, because he says in the third [book] of *De anima*: Whoever says that an animal understands say that it is fashioned or is edified); therefore it is said for the above mentioned syllogism, that if all propositions are of formal predications, the consequence is valid, but the minor premise is false, namely: humanity is a man. If however the same minor premise is of identical prediction, then, following the middle term the same conclusion is not able to be only of formal predication, because in it adjectival word is predicated, and for that reason the consequence must be denied because such a mixture is not valid, as will be shown later.

(42) Therefore because according to the realists as well as the terminists all predications are formal, as has already been shown anyhow in creation, therefore neither was nor is necessary to speak of the mode of syllogistic reason which is found [to be] necessary in divinity and which is made mixed through formal and identical predications.

(43) Moreover from which Aristotle and other natural philosophizers have not credited the trinity of the person in divinity and have not counted some distinction concerning the inner being, but have said that everything in every mode is formally the same in divinity (indeed on the contrary, some moderns and Christians say that God does not participate with creation in universal forms; therefore as you can see through Saint Thomas in *Concerning the faith against the Gentiles*, they put him [God] down in no genus and in no predicament; they say that he is above every genus and predicament), therefore Aristotle and other natural philosophizers have not devised such a mode of syllogizing in divinity

which nevertheless is necessary for Christians to have, because in him nothing was necessary or useful besides their modes.

(44) Nevertheless, if the realists and Aristotle were indeed admitting identical predications in creation, so that you were wondering whether, from their words, that occasionally it is apparent that such a kind must be conceded where some act or operation may be predicated of a universal form, or where an abstract is predicated of a concrete, then indeed it is necessary to admit for them such a mode of syllogizing, or a similar one, of which afterwards may be spoken, which is had in divinity.

(45) Wherefore natural philosophizers deny, in creation, propositions in which a form is predicated of a suppositum or an act of a form, and [others] of this sort, I believe this to be because of this, because created supposita are essentially distinct, and the universal form which is identified in them is not one singular thing and not numerically one, but specific, although according to the realists, it may be simply true that this is one; for that reason a universal form is denied of a suppositum and acts and operations are denied of a universal form.

(46) However, in divinity it is not thus, because that, namely the essence which is identified with the supposita, is singular entirely and numerically, and for this reason it itself is truly predicated of a suppositum.

(47) Likewise in creation a universal form, when it is conceived in such way as if it was a part of a suppositum, it is permitted in reality to be identified with the suppositum. Indeed, conceived as abstracted from matter, the suppositum in truth is conceived as a whole and in some mode as material. And for this reason if this was being conceived: a man or Socrates is humanity, it was being seen in a certain way conceded that a whole was its part. However, that this may be true is clear through Albert the Great in his *Metaphysics* in the fifth [book] putting down differences between a whole and a universal; whence among several differences he puts one which is of this sort: that a universal is a part of its parts; but I understand this: that is, a universal form is conceived in some mode when a part of its suppositum that is conceived with matter, universally in truth [is conceived] without matter. Consequently, Albert says something different in concluding: in truth the whole is not a part of its parts.

(48) In divinity, however, it is neither whole nor material. Whence divine essence is not a part of a person, and a person is not any material thing nor must it be conceived in some mode as material, but any person is to be conceived by pure abstraction from all matter, namely, as a most pure act. Similarly, essence must not be conceived as a material part, nor as a formal part, in the way it is conceived to be a universal form in creation, when this may be a most singular thing numerically one, as it is said; but it must be conceived that the total essence, as will be said as follows, may be this person and that nothing is in the person that may not be thoroughly in this essence, because any person is altogether most simple and indivisible. And briefly, in a divine person nothing inner is included beyond the essence which existing most simply is only one in any person. And

this does not oppose that the essence is distinguished formally from the person of the father and from any other person, because this is saying nothing more except that the essence, which is thoroughly and really the same with the person, indeed is the person of the son, which person of the son is not the person of the father. Therefore because of that and similarly, the essence is able to be truly predicated of a divine suppositum. With which it remains that a universal form in creation is not truly predicated of a created suppositum.

(49) Therefore it is plausibly thought, from the aforesaid, that in creation all predications are formal according to everyone, namely the realists as well as the terminists. For that reason the aforesaid principle rule of syllogisms, namely the *dici de omni*, does not need much commentary.

(50) When however it happens concerning divine things that whereas this is made through propositions of a formal predication, at some times [they are] truly identical, but sometimes they are made mixed, for this reason in syllogistic reasoning in divinity since the premises and the conclusion are of formal predications, and then the aforesaid principle again does not need much commentary, because then such kinds of syllogism are simply good provided that they are made in mode and in figure. But whenever it is possible to syllogize through propositions which all are of identical predication, then such syllogisms are not necessarily valid of form, just as will be made more clear in rules. Therefore about to such syllogizatings again it is not necessary to limit the aforesaid rule, namely the *dici de omni*. But whenever it is possible to make in some syllogism a mixture of propositions formal and identical, and in this mode whenever syllogisms are made good and formally correct, just as in mixing propositions of possibility or necessity and of being; but at these times syllogisms are made not valid in form.

(51) And therefore because of such syllogisms so mixed the principle rule of affirmative syllogisms must be modified. And it is divided into two principles, or in one twofold copulative, namely of such a kind:

(a) whenever some predicate is said formally of some distributed subject, then of whatever such a subject is predicated identically, of the same it is predicated and such a predicate identically.

(b) and whenever some predicate is predicated identically of some distributed subject, then of whatever such a subject is predicated formally, of the same such a predicate is predicated identically.

(52) Therefore I say firstly that for investigation of these sorts of principles that when some predicate is predicated identically of a distributed subject, and if then such [subject] is said identically of some third term, then it is not necessary that such a predicate indeed may be said of the same third term. Whence this does not follow: Every essence is the father, and the son is an essence, therefore the son is the father; just as this doesn't follow: every common human nature is identically Socrates, and Plato is identically common human nature, therefore Plato is identically Socrates.

(53) Secondly I say that if some predicate is said formally of a distributed

subject, then of whatever thing such distributed subject is said formally, of the same indeed such a predicate is said formally, as in this good consequence: Every father generates, and fatherhood is the father, therefore the father generates; similarly: every divine essence is goodness, and wisdom is an essence, therefore wisdom is goodness.

(54) Therefore because the premises of a syllogism are able in themselves to be disposed fourfold in Barbara or Darii, we see that either both are of formal predications, and then it is a good mixture according to the second thing said, evidently immediately; or both are of identical predication, and then it is not a universal of form according to the first thing said; or the major is of a formal predication and the minor is of an identical predication, and this is regulated through the first part of the copulative; or the major is of identical and the minor of formal, and this is regulated by the second part of the copulative.

(55) An example of the first part of the copulative: Every father is fatherhood, and the essence is a father, therefore the essence is fatherhood. And the first part must be understood copulatively: unless it may be made an impediment because of the mode of signifying being grammatical, namely if such a predicate may not be able to be predicated except formally, as in an adjective word. Wherefore this doesn't follow: Every father generates, the essence is a father, therefore the essence generates, because the conclusion is a formal predication, therefore it is not subject to the aforesaid principle. If however through any mode of conceiving whatever the conclusion may have been able to be of identical predication, as in a pure [syllogism] which might have been valid only in the same way: The essence is what generates, then the syllogism was good and regulated by the aforesaid principles.

(56) An example of the second part of the copulative: Every essence is identically the father, and goodness is formally the essence, therefore goodness is identically the father. And that should be noted, because what was said in the principle that in syllogisms in which is distributed a term having one suppositum only the attention is not for the distribution, but for the singularization, and for this reason it must be seen that for some examples either those conclude something true or something false, as in: Everything which is the father generates, and the essence is a thing which is the father, therefore the essence generates; and in such a way for similar things. Indeed it should be noted that until now it was possible to make a variety of combinations by reason of the conclusions, as, for example, in concluding from two premises of formal predication a conclusion of identical [predication] and the same of others, but I will touch on [this] completely in rules.

(57) I will speak of the mode of the *dici de nullo*. And firstly that when some predicate is formally denied of some distributed subject, then it is not necessary that of whatever such a subject is predicated identically that of the same thing such a predicate is denied identically or formally, as in this bad sequence: No essence is formally the father, paternity is identically the essence, therefore

fatherhood is not formally the father, it does not follow: Therefore the essence is not identically the father.

(58) Secondly I say: whenever some predicate is denied identically of some distributed subject, then it is not necessary, if such a subject is predicated identically of some term, that of the same such a predicate is denied identically, as in this bad sequence: No father is identically the son, and essence is identically the father, therefore essence is not identically the son; nevertheless the conclusion follows formally from formal predication, as this follows: therefore the essence is not formally the son.

(59) Thirdly I say that if some predicate is denied formally, that is in formal predication, of a distributed subject, of whatever such distributed subject is formally predicated, of the same such a predicate is denied in formal predication, as in this good sequence: No essence is formally the father, and goodness is formally the essence, therefore goodness is not formally the father. However that third utterance is not made mixed.

(60) Therefore I say fourthly for mixed negative syllogisms: whenever some predicate is denied identically of some distributed subject, then of whatever such a subject is said formally, of the same such a predicate is denied identically, as in: No son is the father, and the word is the son, therefore the word is not the father.

(61) It should be noted that where in the earlier examples I put down 'formally' and 'identically' next to the copulas, it was not necessary that the parts of the proposition exist there, but I put [it] down for clear designation the predication being identical or formal, etc.

(62) For the final demonstration of what has been spoken, and indeed for the greater completion of the mode of syllogistic reasoning, it must be spoken consequently of expository syllogisms, and then indeed at the same time a mode of common syllogisms will be made known.

Therefore I say that the regulative principle of expository syllogisms is such: whenever one and the same things are the same, then they are mutually the same. However, this is able to be understood in two ways: namely really, how the words are placed, and also logically, namely thus: any terms whatever which are conjoined with some term in connection singularly and univocally, those are indeed conjoined mutually. However, that second understanding of this principle follows in some way from the first, because from the same that some things are the one and the same in the same way, which because of this mutually are the same, it follows that the terms which are connected with a term maintained discretely, in that way mutually denote the same thing; and consequently this conjunction is good. For this reason, moreover, it must be seen first of the understanding of the real aforesaid principle, in order that next a logical understanding may be had, according to which affirmative expository syllogisms are regulated.

(63) And first I say that some things are in many different ways able to be called the same, namely in genus or species, of which identity I will not worry about at present.

(64) In another way some things are able to be called the same only essentially, such as those which have the same essence, which nevertheless are distinguished personally in number, and thus the father and the son are the same in some mode.

(65) Something is able to be called the same as something [else] only personally or identically, as when something is the same person with something, granted that that something may be formally distinct from that other thing; and thus the divine essence is the same as the father identically only.

(66) In another mode some thing is formally the same as something [else], thus certainly because whatever thing is that thing, the same thing is indeed that of which it itself is said to be formally the same; and thus the father and fatherhood are formally the same, because whatever thing is the father, indeed the same thing is fatherhood; similarly the essence and goodness are formally the same, because whatever thing is the essence the same thing is indeed goodness.

(67) Presently I say to the point concerning expository syllogisms, that that principle does not require some gloss concerning essential identity: because, I mean, that the father and the son and the holy spirit are essentially the same with the essence, in themselves they are mutually essentially the same. And as luck would have it they demonstrate this about all affirmative paralogisms concerning divinity, as in that: This essence is the father and this essence is the son, therefore the son is the father: indeed those premises are not intended to imply anything more than that the father and the son are essentially the same. Nevertheless sometimes by chance indeed personal or identical identity is proved.

(68) Indeed I say that no gloss is required for the aforesaid principle concerning the third identity, namely formal: indeed whatever things are formally the same with some third thing, they are formally the same between each other, speaking for good sense, as in: active generation : father : fatherhood, similarly: essence : goodness : potency, potency : wisdom : goodness.

(69) However, of the second mode of identity, this is not necessary, whence it is not necessary that if the father and the son are the same in the mode with the essence, that because of this they be thus between them; and the reason is, because the son is not the same by identical or personal identity with the essence in the same way which the father is the same with the essence. This is clear, because the father and the essence are the same things in personal identity, which identity is the person of the father, however the son and the essence are the same thing in personal identity, which identity is the person of the son. Therefore it is clear that the father is the same with the essence in another identity than the son; because of this it is not necessary that the son and the father may be the same in the second aforesaid mode. If however the father and the son were the same with the essence in the same way by personal identity as in the father and the active generation, then the father and the son would have been such between themselves by the second mode, or indeed by the third mode.

(70) From those things it is clear that, if two terms are conjoined only with a third maintained discretely and univocally, it is not necessary that these two

terms are conjoined mutually identically, although by chance it is necessary to conjoin them essentially. The second part of this must not be made known, but the first part is clear, because it is not necessary, if two things may be the same identically with a third, that because of this they are identically the same with each other, as has been made clear; therefore it is not necessary that propositions representing in such a way are verified, as is noted.

(71) From those things I return to one which I spoke of in principle, namely that in syllogisms in which a term having one sole formal suppositum is distributed, attention need not be paid to the distribution of such a term, but instead attention must be given to the singularity or unity of the supposition of such a term and to a conjunction of extremes with such a term. For example, in this syllogism: Every divine essence is the father, and the son is a divine essence, therefore the son is the father, from the part of the distribution some fallacy is not committed, but a fallacy is committed there because by means of an identical conjunction of extremes, namely 'father', 'son', with a middle [term], namely 'essence', such extremes are supposed to be connected to each other identically, nevertheless it is not necessary for them to be [so] connected, as has been seen.

(72) Neither in the major of that syllogism, namely: every essence is the father, is the distribution of this term 'essence' incomplete, on the contrary I say that there is the most complete distribution of it: which is clear, because this term 'essence' has only one suppositum in divinity for which it is able to be distributed and for that it is distributed, therefore it is completely distributed. That it may be distributed for that suppositum is clear, because the essence is distributed; but that it may have only one suppositum for which it may be distributed is clear, because whatever term is distributable for many, the same term signifies through a plural mode, because the supposition and consequently the distribution is inferior to the signification: therefore if something is distributed for things through a plural mode, the very same thing itself signifies through a plural mode. But this term 'essence' does not signify the father and the son through a plural mode, otherwise it is able to be said of those things plurally, and thus the father and the son are plurally the essence, which is undesirable. Therefore that term 'essence' signifies the father and the son as one indistinctly, therefore it supposits for them indistinctly. However a term which is distributed for many, is distributed and supposits for them distinctly and as if they were distinct, because the distribution (*distribucio*) is diverse distribution (*tribucio*).

(73) Likewise by this term 'essence' from its mode of imposition does not signify the father and the son distinctly. In the same way this term 'essence' is not predicable with this 'other, another' (*alia*) of the father and the son, as in: the father is essence and the son is another essence; however in this way it happens that a term is predicated for plural distribution, as in: Socrates is a man, Plato is another man.

(74) Similarly it is not complete distribution of this term 'essence' when it is said: everything which is the essence is the father; it is clear, because this

term 'essence' in the aforesaid proposition is not distributed, but stands merely confusedly; which is clear because elsewhere this term 'man' could not have been being distributed in this: everything which is a man is rational; and then the sense would have been: everything which is every man is rational; and then it would have been false, because of its subject, namely 'thing which is every man', would have been suppositing for nothing.

(75) Therefore I say finally that many [fallacies] which are made in divinity, are made from identical conjunction of extremes with a middle, and because of this they are believed to be able to be connected with each other identically; or from identical and formal conjunctions, because of which conjunctions they are believed to be able to be connected with each other formally.

(76) However for excluding those fallacies I say first that the propositions are twofold, namely modal and nonmodal. However nonmodal [propositions] are trifold: some are of the present, such as: 'man is animal', and this signifies *simpliciter* being or in-being; but some are of the past, such as: 'man was animal', and such propositions signify *simpliciter* past being and past being-in; but others are of the future, as in: 'man will be animal', and such propositions signify *simpliciter* future being.

(77) Second I say that for the first member of the aforesaid division, that the aforementioned trifold propositions are able to be modified by at least four modes. Examples of propositions of the present include: 'man is of necessity an animal'; 'man impossibly is a donkey'; 'man possibly is an animal'; 'man possibly is not an animal'; or other examples can be selected. Similarly these are of the past: 'Adam having been man is necessary'; 'Adam having been man is impossible', etc. And in the same way for the future.

(78) Furthermore I say that propositions of the present and of the past differ by modification by the very same modes; for example, this is true: 'Adam not having existed is impossible', taking this 'impossible' strictly. And nevertheless this is false: 'Adam not existing is impossible.'

(79) Consequently I say that out of a complex mode of being of things comes forth multiple ways of modifying the copula of a proposition. For example, some things do not exist, but possibly exist, such as a man which God is able to create, which he nevertheless neither created nor creates nor will create, and from this it is possible to modify this proposition: such a kind of man is possible to be; but some things exist, and exist of necessity, such as the three divine persons, and from this it is possible to modify this proposition: the father is of necessity God; but some things exist and do not exist of necessity, and from this such a proposition is modified: 'a man not existing is possible'.

(80) Therefore from what is it that propositions are modified? Because some thing is possibly the same as another thing, but one is of necessity and another is not of necessity. But because some existing things are only essentially the same, such as the father [and the] son; but some are only identically [the same] such as the essence [and the] father; but some formally, such as fatherhood [and the]

father, the essence [and the] goodness; therefore indeed in such a way the copulae are able to be modified for the denoting of such identities, or at least the second and third. Therefore if there was some copula which was modified in this way because it was denoting only essential identity, then this 'father' was predicated by means of such copula in such a way as to be a true modification of this 'son', as in: the son is *a* father—and this '*a*' is such a mode—which proposition would've been true.

(81) Indeed, with regard to second and third modes of identity, the modes are not considered in the use of such modifications, but whenever the words, from the common use of the theologians (*usus loquendi*) bring of themselves such or such identity: moreover in this way adjectival words always bring about formal identity. However that this may be true is clear, because the essence and the wisdom are formally the same; however for this being denoted, it does not suffice to say: the essence is the wisdom, because from this predication it does not say whether the essence is the wisdom formally or only identically; therefore it is required for a true, modified copula, as in: the essence is *c* the wisdom, or: the essence is formally the wisdom.

(82) However, it is necessary to preserve these modes of modification in the said complete distribution, for example in resolving this: every essence is the father, twice over: everything which is the essence is the father. Therefore I ask what kind of identity both copulae cause. If you say: both cause formal, to be the sense: everything which is formally the essence is formally the father,—behold! the copulae exist strictly and modified, and on the contrary this resolution is neither more nor less valid than this: the essence is the father, taken according to formal predications, as is clear by the intention. If however you say that both copulae import identical and not formal identity, to have the sense: everything which is identically the essence is identically the father—behold the copulae are more relaxed than in the preceding resolution. Therefore either here or in the preceding resolution or in both they are modified. Unless for any reason whatever you don't desire of me to make only goodwill, because you wish to appeal against these modifications. If however one of the aforementioned copulae asserts formal identity, but the other does not, to have this sense: everything which is identically the essence is formally the father, or: everything which is formally the essence is identically the father, I will argue similarly as in the previous.

(83) Briefly I say that these two propositions: every essence is the father, and: every thing which is the essence is the father, by the mode of signification and imposition do not have the same mentals (*mentales*), unless you want to abuse the term; and the subject of this: everything which is the essence is the father, taking the first 'is' identically, supposits formally for many things, namely for the three persons; however the subject of this: every essence is the father, supposits formally for one thing alone, namely for the essence, and only indistinctly and identically for the three persons.

(84) Therefore you must accept this mode of modifying copulae and syllogistic reasoning and you will solve all paralogisms; you will even save all the modes of Aristotle, as will be clear; or you must accept the mode of complete distribution and you will not save all the modes of Aristotle, as is clear of Disamis [cf. Appendix A.1].

(85) You must accept this mode, and you will not worry about the complete distribution of terms, which is nothing; or you must accept the mode of complete distribution and it will be necessary for you to care about the modifications of copulae, as has been made clear.

(86) You must accept this mode and not make use of a new imposition of nominal propositions, but only of special modification of the copulae because of special modes of being of things; or you must accept the mode of complete distribution and it will be necessary for you to impose on propositions a special imposition beyond the imposition of the terms, as is clear concerning this proposition and similar: every essence is the father, which according to the mode of complete distribution beyond the imposition of the terms is imposed for the signification only in so far as this: everything which is the essence is the father.

(87) You must accept this mode and you will say nothing is distributed except what is distributed; or you must accept the aforementioned mode of complete etc. and you will say a completely and most perfectly distributed term is not distributed: in which, however, is laughter!

(88) You must accept this mode and you will say all syllogisms formed by all philosophers to be formal are not to have added this 'which is', which is clear, because in every such syllogism are pure formal predications, as was made clear; or you must accept this mode of complete etc. and you will say no syllogism is formal except when this 'which is' is added.

(89) And thus many apparent distortions in the infidels themselves follow according to the mode of complete [distribution], of which nothing follows from the aforementioned modification of the copulae; which is clear: apparently the greatest inconvenience which is in the aforesaid mode, is forgotten or is a certain new modification of the copulae or predication of terms. However that is most convenient, which is clear, because in what way things exist, in such a way they are predicated and are recognized, and in that way grasped and affirmed in the mind. Indeed in some way the mind divides things and refuses being, just as it knew them not to exist.

(90) From which, however, some things are essentially the same, therefore in such a way the mind is able to grasp and affirm them; but others [are the same] essentially and personally or identically according to good sense, therefore in such a way the mind is able to grasp or predicate them; others finally are formally the same, again according to good sense, therefore in such a way the mind combines them. However some things are distinguished essentially, but others personally and not essentially, but others formally only and not personally: therefore the mind in such way divides, combines, or negates them. However such affirmation

or negation is made through modified copulae.

(91) And I believe [that] if Aristotle hitherto was living and was putting forward each of those modes, namely as the being of things so the modification of copulae, he himself would concede such a mode of modification of the copulae necessary for talking of the search concerning the aforementioned modes of being of things.

(92) And just now, I say that, although it may be of the nature of adjectival speech in common modes of speaking to predicate formally, nevertheless [it is] substantive, whether they are simply substantive or adjectivally substantive, they are able to predicate formally or identically indifferently. For example: the essence is the father, or: the essence is generating. If the predication is obtained purely adjectivally it is a formal predication and the proposition is false. If however the predication is obtained substantively, then as yet it is able to be a formal predication, and the sense will be: the essence is formally the hypostasis of the father, or of the generating [thing], or: the essence is formally the same as the hypostasis of the father, or the generating [thing]. If however it is an identical predication, the sense will be: the essence is identically the hypostasis of the father, or the generating [thing], etc.

(93) Consequently rules which must be observed in syllogistic reasoning in divinity must be put down.

And the first rule is: Whenever the predications are formal, namely as the conclusion is so are the premises, the syllogism is good: this is correct, just as from two universals follows a universal conclusion. An example of the rule: every generator is a producer, the father is a generator, therefore the father is a producer.

(94) The second rule: Whenever both premises are of identical predications, it is not necessary for a conclusion of formal predication nor of identical predication to follow: this is correct, just as when both premises are particular, neither a universal nor a particular conclusion necessarily follows. An example of the rule, which does not follow: every essence is the father, the son is the essence, therefore the son is the father, whether the conclusion is a formal predication or indeed an identical predication.

(95) The third rule: If propositions in which the copula denotes only essential identity were being in use, then from two premises of identical predication it would follow a conclusion of such predication, namely in which was denoted only the essential identity of things the terms supposit formally for them. For example, if this proposition: the son is the father, was denoting only the essential identity of the father and the son, then this syllogism would be good: every essence is the father, and the son is the essence, therefore the son is the father; or such an example may be put down: every essence is b the father, and the son is b the essence, therefore the son is a the father,—and this 'a' is a sign or mode of essential identity only, and this 'b' is a sign of personal and not only formal or identical identity—: then the syllogism is formal, because in a similar form an

instance is never ascribed.

(96) The fourth rule: If one of the premises is of identical predication, the conclusion is required to be identical predication; this is correct, just as if the premise is particular, the conclusion is required to be particular. Whence this does not follow: every essence is *c* goodness, and the father is *b* the essence, therefore the father is *c* goodness—and this '*c*' may be a sign or mode of formal identity. Or such an example may be put down: not following: every essence is formally goodness, and the father is identically the essence, therefore the father is formally the goodness.

(97) And thus a conclusion of formal predication only follows from both premises of formal predication. Likewise from two premises of formal predication [follows] a conclusion of identical predication, just as the conclusion of a particular follows from universal premises. Likewise a conclusion of a predication denoting only essential identity is designed to follow from any propositions whatever, whether both are of identical predication or formal, or one is of identical predication and the other formal.

(98) Now I put down the rules of the negative syllogisms.

And the first of such is: Any time the conclusion is a negative of identical predication, it is necessary for one of the premises to be an affirmative of formal predication. Whence this does not follow: every father is not the son, and the essence is identically the father, therefore the essence is not identically the son. Likewise it does not suffice that the negative premise may be of formal predication provided that the affirmative is of identical predication, because this does not follow: the essence is not formally the father, and the fatherhood is identically the essence, therefore the fatherhood is not formally the father.

(99) The second rule: Whenever the conclusion is a negative of identical predication, it is necessary for the negative premise to be of identical predication. Whence this does not follow: the essence is not formally the father, and the goodness is formally the essence, therefore the essence is not identically the father.

(100) And a corollary from these rules is able to be the third rule: Any time the conclusion is of negative identical predication, it is necessary for one premise to be of negative identical predication and the other of affirmative formal predication; this is clear from the preceding two rules.

(101) The fourth rule: From a negative of formal predication and an affirmative of identical predication it is not necessary for a conclusion of negative formal predication to follow; for example this does not follow: the father is not formally the essence, and the goodness is identically the father, therefore the goodness is not formally the essence.

(102) The fifth rule: From a negative of identical predication and an affirmative of formal predication follows a negative conclusion of formal predication; as this correctly follows: the father is not identically the son, and the fatherhood is

formally the father, therefore the fatherhood is not formally[7] the son.

(103) The sixth rule: From two premises of identical predication a negative conclusion of formal predication correctly follows; as this follows well: the father is not identically the son, the essence is identically the father, therefore the essence is not formally the son.

(104) The seventh rule: From two premises of formal predication follows a negative conclusion of formal predication, as in: the father is not formally the essence, and the active generation is formally the father, therefore the active generation is not formally the essence; similarly: the essence is not formally the father, and the wisdom is formally the essence, therefore the wisdom is not formally the father.

(105) Another rule: A negative conclusion of essential predication, that is of which the copula of the contradictory denotes only essential identity, follows from a negative of essential predication, that is of which of the contradictory etc., and an affirmative of whatever predication. An example: the father is not a, that is essentially, the body, and the holy spirit is a, that is essentially, the father, therefore the holy spirit is not essentially, or a the body. Similarly: the father is not a body, and the essence is b, that is identically, the father, therefore the essence is not a, or essentially, the body. Similarly: the father is not a body, and the active generation in divinity is c, that is formally, the father, therefore the active generation in divinity is not a the body.

(106) But against these rules it is argued in this way: it does not follow: the father is not a, or essentially, a man, and the son in divinity is a, or essentially, the father, therefore the son in divinity is not essentially a man; similarly it does not follow: the father is not a, that is essentially, a man, and the essence is b, that is identically, the father, therefore the essence is not a, that is essentially, a man, because each consequence is false and each antecedent is true.

[7]The edition notes that the MS has *ydemptice* here, not *formaliter*, but this is clearly an error.

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Samenvatting

Dit proefschrift is een oefening in conceptuele archeologie. Met behulp van instrumenten uit de moderne logica analyseren we teksten uit de middeleeuwse logica en reconstrueren hun logische theorieën door formele systemen op te bouwen waarin deze passen. Onze nadruk ligt op middeleeuwse teksten die verscheidene modaliteiten behandelen: de geschriften over alethische modaliteiten van Willem van Sherwood, Pseudo-Aquinas, en Thomas van Aquino in de 13e eeuw; de geschriften van Anselmus van Canterbury over *facere* en *debere* in de late 11e eeuw; Lambert de Lagny's 13e-eeuwse verhandeling over *suppositio* en het verband met moderne temporele logica; Roger Swyneshed's dynamische modaliteit van zelf-weerlegging, geschreven in de vroege 14e eeuw; en de verschillende wijzen van het Zijn die worden genoemd in opmerkingen over de Drieëenheid in een anonieme tekst uit de late Middeleeuwen. We vullen onze discussie over deze middeleeuwse teksten aan met een hoofdstuk over de betrekking tussen de kerk en de ontwikkeling van modale en temporele logica in de 13e en 14e eeuw, en twee bijlagen met Engelse vertalingen van verschillende bronteksten.

We laten zien dat we met behulp van logische instrumenten die in de laatste drie decennia zijn ontwikkeld meer inzicht kunnen verkrijgen in de theorieën van de middeleeuwse logica, in het bijzonder die van de middeleeuwse modale logica, dan we vijftig of zeventig jaar geleden konden, toen het voornaamste instrument van de logicus nog de wiskundige logica van Frege en Russell was. Uiteraard werpt deze onderneming ook in de andere richting vruchten af: Wij geven aan waar middeleeuwse benaderingen van bepaalde filosofische of theologische problemen meer op hun plaats zijn dan gangbare hedendaagse benaderingen.

Abstract

This dissertation is an exercise in conceptual archeology. Using the tools of contemporary logic we analyse texts in medieval logic and reconstruct their logical theories by creating a formal framework which models them. Our focus is medieval texts which deal with various modalities: the writings on alethic modalities by William of Sherwood, Pseudo-Aquinas, and St. Thomas Aquinas in the 13th century, St. Anselm of Canterbury's writings on *facere* and *debere* in the late 11th century; Lambert of Lagny's 13th-century treatise of supposition and its connection to modern temporal logic; Roger Swyneshed's dynamic modality of self-falsification, written in the early 14th century; and the different modes of being which are expressed in statements about the Trinity, from an anonymous, late-period text. We supplement our discussion of these medieval texts with a historical chapter discussing the relationship between the church and the development of modal and temporal logic in the 13th and 14th centuries, and two appendices containing translations into English of various source texts.

We demonstrate that by using logical tools which have been introduced in the last quarter-century we can make better sense of the theories of medieval logic, particularly medieval modal logic, than we could 50 or 75 years earlier, when the logician's primary tool was the mathematical logic of Frege and Russell. The venture is also fruitful in the other direction: We point to places where medieval responses to certain philosophical or theological problems seems more apropos than favored modern responses.

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