MAGNETIZATION OF AN ELECTRON PLASMA BY MICROWAVE PULSES:
FARADAY INDUCTION

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It has been argued theoretically that the recently proposed vacuum field $B^{(3)}$ is not accompanied by a real electric field $E^{(3)}$. Experimental evidence for this inference is available in the data reported by Deschamps *et al.* [10], using micro-wave magnetization of an electron plasma set up in helium gas. Faraday induction due to $B^{(3)}$ does not occur in the inert gas and is not observed experimentally in the absence of free electrons. Whenever $B^{(3)}$ interacts with free electrons, however, Faraday induction occurs through a pulse of induced magnetization (i.e., induced orbital electronic angular momentum).

Key words: $B^{(3)}$ field, Faraday induction.

1. INTRODUCTION

The type of cyclically symmetric, non-Abelian, Lie algebra used to define the novel vacuum field $B^{(3)}$ [1–8] indicates that it is accompanied in the vacuum by an imaginary and unphysical electric field $iE^{(3)}$. The *non*-existence of a real electric field in the defining non-Abelian algebra is indicated simply by considering the elementary property that the cross product of two polar vectors (two electric fields) cannot produce another polar vector (another electric field). It produces an axial vector (a magnetic field). On the other hand, the cross product of two axial vectors (two

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magnetic fields) can produce another axial vector.   In consequence, the magnetic field algebra [1–8] is

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*}, \qquad \mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = iB^{(0)}\mathbf{B}^{(1)*},$$
$$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = iB^{(0)}\mathbf{B}^{(2)*}, \tag{1}$$

which is cyclically symmetrical in the indices (1), (2) and (3) of the circular basis.   Here "*" denotes as usual "complex conjugate".   In this algebra, $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ are the ordinary vacuum plane waves, but $\mathbf{B}^{(3)} = \mathbf{B}^{(3)*}$ is real and phase free.   It is a spin field which propagates at the speed of light through the vacuum and can be detected experimentally through its characteristic square root power density profile ($I_0^{1/2}$) when it magnetizes an electron plasma.

The corresponding algebra [1–8] for electric fields is, on the other hand,

$$\mathbf{E}^{(1)} \times \mathbf{E}^{(2)} = -E^{(0)}(i\mathbf{E}^{(3)})^*, \qquad \mathbf{E}^{(2)} \times (i\mathbf{E}^{(3)}) = -E^{(0)}\mathbf{E}^{(1)*},$$
$$(i\mathbf{E}^{(3)}) \times \mathbf{E}^{(1)} = -E^{(0)}\mathbf{E}^{(2)*}, \tag{2}$$

which is symmetric in $\mathbf{E}^{(1)}$, $\mathbf{E}^{(2)}$, and $i\mathbf{E}^{(3)}$.   The algebra shows that $(i\mathbf{E}^{(3)})^* = -i\mathbf{E}^{(3)}$ is pure imaginary and unphysical, and formally takes the symmetry of an *axial* vector.   Such a result is the outcome of special relativity.   The vectors $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ (the ordinary plane waves) are mathematically complex quantities (with real and imaginary parts).

In this Letter, O(3) gauge geometry [5, 9] is used to show that there is a formal equation linking $\mathbf{B}^{(3)}$ and $i\mathbf{E}^{(3)}$ in the vacuum, one which is structurally identical with a Faraday induction equation.   In the absence of matter, however, $\mathbf{B}^{(3)}$ can never be accompanied by a real $\mathbf{E}^{(3)}$, and so no Faraday induction occurs in the vacuum of a real $\mathbf{E}^{(3)}$ by $\mathbf{B}^{(3)}$ of a pulse of radiation.   This inference is supported experimentally by the results of Deschamps *et al.* [10]. These authors observed Faraday induction due to an induced magnetization ($\mathbf{M}^{(3)}$) in a plasma, but when the plasma was removed, no induction occurred in the residual inert gas (helium). $\mathbf{M}^{(3)}$ is induced by $\mathbf{B}^{(3)}$ and is proportional to powers of e, the electronic charge.   If there is no plasma there are no free electrons, and in consequence $\mathbf{M}^{(3)}$ (the induced orbital electronic angular momentum) is zero for all $\mathbf{B}^{(3)}$.   Faraday induction occurs [10] due to an $\mathbf{M}^{(3)}$ pulse, which sets up an electromotive force and Ohm's law current in

a 100 turn induction coil [10] through the Faraday law of induction,

$$\nabla \times \boldsymbol{E}_{\boldsymbol{T}} = -\mu_0 \frac{\partial \boldsymbol{M}^{(3)}}{\partial t} . \tag{3}$$

Here μ_0 is the vacuum permeability in S.I. units, and $\boldsymbol{E}_{\boldsymbol{T}}$ is the transverse induced electric field. In the absence of a plasma, this field is zero experimentally [10]. Therefore, the field $\boldsymbol{B}^{(3)}$ does *not* produce induction in the absence of electrons. The reason for this is developed in Sec. 2.

2. ABSENCE OF FARADAY INDUCTION DUE TO A PULSE OF $\boldsymbol{B}^{(3)}$

The existence of $\boldsymbol{B}^{(3)}$ is indicated by the algebra (1) and means that the gauge geometry of vacuum electromagnetism must be extended from the conventional, planar, $O(2)$ to the three-dimensional $O(3)$. If this is carried out self-consistently [5], using the rigorous theory of gauge geometry, the result is that the ordinary vacuum Faraday law equations:

$$\nabla \times \boldsymbol{E}^{(1)} = -\frac{\partial \boldsymbol{B}^{(1)}}{\partial t} , \tag{4a}$$

$$\nabla \times \boldsymbol{E}^{(2)} = -\frac{\partial \boldsymbol{B}^{(2)}}{\partial t} , \tag{4a}$$

are supplemented by an equation which can be written formally as

$$\nabla \times (i\boldsymbol{E}^{(3)}) = -\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = \boldsymbol{0} . \tag{5}$$

Eq. (5) does not exist on $O(2)$ electrodynamics, whereas Eqs. (4) refer as usual to plane waves

$$\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{1} - i\boldsymbol{J})e^{i\phi},$$

$$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*} = \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{1} + \boldsymbol{J})e^{i\phi}, \tag{6}$$

where ϕ denotes the electromagnetic phase. It can be seen that Eq. (5) formally relates $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ of Eqs. (1) and (2). This means that $\partial \boldsymbol{B}^{(3)}/\partial t$ can never produce a physical $\boldsymbol{E}^{(3)}$, i.e., Faraday induction due to $\boldsymbol{B}^{(3)}$ cannot occur in the

absence of matter. In the presence of free electrons, on the
other hand, a $B^{(3)}$ pulse produces a pulse of magnetization,
$M^{(3)}$, which results in an E_T pulse through Eq. (3).

3. ANALYSIS OF EQUATION (5)

Equation (5) means that a hypothetically non-
zero $\partial B^{(3)}/\partial t$ (e.g., postulated to exist in a pulse of micro-
wave radiation lasting a microsecond [10]) formally produces
the curl of an imaginary electric field $iE^{(3)}$ in *the same*
longitudinal polarization, (3). Therefore, a pulse of $B^{(3)}$ in
the vacuum cannot produce a transverse electric field; the
latter is produced as usual by a transverse magnetic field as
described in the ordinary Eqs. (4a) and (4b) of vacuum
electromagnetism. On the other hand, a pulse of longitudinal
magnetization, $M^{(3)}$, produces a transverse E_T as in Eq. (3),
an E_T that is picked up [10] by the induction coil. Evident-
ly, E_T is not a plane wave propagating at the speed of light.
        The physical (i.e., real) part of Eq. (5) is

$$\frac{\partial B^{(3)}}{\partial t} = 0, \tag{7}$$

which must be interpreted to mean that $\partial B^{(3)}/\partial t$ in tesla per
second cannot produce the curl of a physically meaningful
electric field strength. Such a field would have to be
perpendicular to $B^{(3)}$, but these fields are transverse plane
waves in axes (1) and (2).

4. DISCUSSION

The $B^{(3)}$ field from Eqs. (1) is real and physical and is
formed in electromagnetic radiation propagating at the speed
of light. It is proportional [1–8] to the angular momentum
of the classical field, an angular momentum in the (3) axis
which is relativistically invariant. In the quantized field
this becomes the Dirac constant $\hbar$ for each photon, a univer-
sal, unchanging property. If we consider in a thought
experiment a beam of radiation of a given cross section to
contain only one photon, then the $B^{(0)}$ of that photon is also
fixed, being the scalar amplitude per photon of the magnetic
flux density carried by the beam. In the quantum field
theory it is not possible for one photon to carry less or
more inductance. Thus $B^{(3)}$ per photon, being $B^{(0)}$ per photon

multiplied by a unit vector $e^{(3)}$, cannot be changed in time
for a given beam cross section. If now another photon is
added to the beam, a similar situation holds; we have $2B^{(3)}$,
which cannot be changed with time. This argument extends to
all photons of the beam, leading to $\partial B^{(3)}/\partial t = 0$, as in Eq.
(7). It is always possible to change the beam cross section,
by focusing, in which case $\nabla B^{(0)}$ is not zero. In this
eventuality, however, we must consider the convective
derivative [11],

$$\frac{dB^{(0)}}{dt} := \frac{\partial B^{(0)}}{\partial t} + \mathbf{v} \cdot \nabla B^{(0)},\tag{8}$$

where $\mathbf{v}$ is the linear velocity in axis (3) of the $B^{(3)}$ field,
the velocity of light. The Faraday law of induction then
takes the integral form given by Jackson [11], for example,

$$\frac{d}{dt}\int_s \mathbf{B}^{(3)} \cdot \mathbf{n}da = \int_s \frac{\partial \mathbf{B}^{(3)}}{\partial t} \cdot \mathbf{n}da + \oint_c \mathbf{B}^{(3)} \times \mathbf{v} \cdot d\mathbf{l},\tag{9}$$

and since $B^{(3)}$ is parallel to $\mathbf{v}$ the second term on the right
hand side disappears. We have argued that the intrinsic
$\partial B^{(3)}/\partial t$ is zero, and so again there is no Faraday induction
by $B^{(3)}$ in the vacuum.

Whenever $B^{(3)}$ produces $M^{(3)}$ in free electrons, however,
there is induction, because $M^{(3)}$ is proportional to induced
electronic orbital angular momentum, which is a circulating
current of electrons, classically with no linear momentum.
This pulse of current produces Faraday induction in another
coil. Clearly, $M^{(3)}$ has no classical linear momentum relative
to the induction coil, whereas $B^{(3)}$ is travelling at the speed
of light with respect to it.

ACKNOWLEDGEMENTS

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