Introduction to Santilli Iso-Mathematics

Christian Corda[a]

[a]*Institute for Theoretical Physics and*
Advanced Mathematics (IFM) Einstein-Galilei, Via Santa Gonda, 14 -
59100 Prato, Italy.

Abstract. Santilli iso-mathematics, which represents the mathematical framework of Hadronic Mechanics and Hadronic Chemistry, is introduced and reviewed at a semi-popular level for the participants to the Workshop on Iso-mathematics at ICNAAM 2013.

Keywords: Iso-Mathematics, Hadronic Mechanics, Hadronic Chemistry.
PACS: 02.10.De

In the late 1970s, Santilli generalized conventional mathematics in an axiom-preserving framework which is today known as Santilli iso-mathematics [1, 2, 3, 4]. We stress that the prefix "iso" in the term "isotopic" has the Greek meaning of preserving the original axioms, while the prefix "geno" in the term "genotopic" has the Greek sense of including new axioms. In this extended abstract, only the most relevant aspects of Santilli iso-mathematics can be introduced and reviewed. Researchers interested in further details of Santilli iso-mathematics should study [1, 2, 3, 4] and references within. From a technical point of view, iso-mathematics only concerns reversible structures, for example hydrogen's structure and water molecules. Instead, Santilli geno-mathematics is broader and more general, and results necessary for the representation of irreversible processes, for example complex chemical reactions [5], which we do not consider in this proceeding-paper.

The first, basic step of iso-mathematics started when Santilli [1, 2, 3, 4] lifted the standard multiplicative unit $I = +1$ into a generalized new unit 1, called iso-unit, which is positive (to admit the inverse $1 = 1 / t > 0$, where t is called the isotopic element, again a positive definite quantity), can be N -dimensional and in general can depend on various variables like the time, the position vector, the velocity vector, the momentum vector, the temperature function, the wave function and its transposte, and the corresponding partial differentials of such variables. The iso-numbers are, in turn, generated by Santilli [1, 2, 3, 4] as $n \equiv n \times 1$. The definition of 1 constitutes a non-unitary transform. In order to preserve such a non-unitary transform, Santilli [1, 2, 3, 4] expressed 1 as $1 = U\times U^{\dagger} = 1 / t > 0$, where the superscript (†) denotes the transpose or "hermitean", and hermiticity of 1 is guaranteed by $U\times U^{\dagger} = (U\times U^{\dagger})^{\dagger}$. Hence, $U\times U^{\dagger}$ is a positive definite, non-singular N-dimensional matrix for which $\mathrm{Det}\,(U\times U^{\dagger})>0$. In this way, the lifting to the iso-unit can be represented as $I = +1 \rightarrow U\times I\times U^{\dagger} = 1$, representing the non-canonical, non-unitary transform of the conventional unit.

Santilli then constructed the so-called "regular realization of iso-mathematics" via the systematic application of a non-unitary transform to the totality of the quantities and their operation of conventional 20th century applied mathematics, with the understanding that Santilli identified another form, valled "irregular realization of iso-mathematics" that cannot be reached via non-unitary transforms. In this way, Santilli regular iso-mathematics is characterized by the above indicated non-unitary transform of the trivial unit +1, as well as of the conventional associative product

$$a\times b\rightarrow U\times(a\times b)\times U^{\dagger} = (U\times a\times U^{\dagger})\times(U\times U^{\dagger})^{-1}\times(U\times b\times U^{\dagger})\; a \times t \times b = a \times b \,, \tag{1}$$

In other words, Santilli [1, 2, 3, 4] obtained the two isotopic-liftings:

$$\times\rightarrow\times = \times U\times U^{\dagger}\times = \times t\times \quad \text{and} \quad a\rightarrow UaU^{\dagger} = a \tag{2}$$

The iso-products of iso- and conventional numbers are expressed as

$$a \times b = c = a \times 1\times t \times b \times 1 = (a \times b) \times 1 = c \times 1 \qquad \text{and} \qquad a\times b = a\times t \times b, \tag{3}$$

AIP Conf. Proc. 1558, 685-687 (2013); doi: 10.1063/1.4825584

respectively. As 1 is a multiplicative iso-unit, Santilli [1, 2, 3, 4] get

$$1 \times a = a \times 1 = a \text{ and } 1 \times a = a \times 1 = a. \tag{4}$$

In general, Santilli [1, 2, 3, 4] also obtained

$$n \times a = n \times 1 \times t \times a = n \times a . \tag{5}$$

Iso-products are not generally iso-commutative , i.e. $a \times b \neq b \times a$. Instead, they are iso-associative, i.e. $a \times (b \times c)=(a \times b)\times c$. The iso-sum is simply given by $+ = +$, and it is commutative and associative, i.e. $a + b= b + a$ and $a + (b + c)=(a + b) + c$. Santilli [1, 2, 3, 4] introduced also an additive iso-unit (iso-zero) 0 which implies

$$a + 0= 0 + a = a. \tag{6}$$

In that way, the lifting to iso-zero reads $0 \rightarrow U\times 0\times U^{\dagger} = 0$, and the negative iso-number $-a$ holds a relationship with its positive counterpart as $a + (-a) = 0$. The iso-products and iso-sums obey both the right and the left iso-distributive laws, i.e.

$$a \times (b + c) = a \times b + a \times c \text{ and } (a + b) \times c= a \times c + b \times c. \tag{7}$$

Such liftings imply $a^n=a^n 1$ and $a^n=a^n t$, see [1, 2, 3, 4] for details. The iso-squareroot reads $a^{1/2}= a^{1/2}\, 1^{\ 1/2}$. Thus, for the iso-unit Santilli [1, 2, 3, 4] obtained $1^{1/2}=1 = 1^{\ 1/2}$. Hence, iso- and conventional products of iso-unit are generally represented as $1^n=1= 1^{\ n}$, where $0= 0 \times n= 0 \times n$ has been used. Then, Santilli [1, 2, 3, 4] also obtained $t^n=t= t^{\ n}$. Moreover, as 1 is an iso-unit, Santilli [1, 2, 3, 4] obtained $a^1= a =a \times 1$.

Santilli [1, 2, 3, 4] lifted the conventional division to the iso-division as $a/b = U\times(a/b)\times U^{\dagger} = a / b$. In this way, he obtained the correspondent non-unitary transform as $/ \rightarrow / = / \times 1$. Then, the iso-inverse is given by

$a^{-1} = (1/a^1)\times 1=a^{-1}\times 1$. As 1 is the iso-unit, Santilli [1, 2, 3, 4] get $1 \times 1 = 1 / 1 =1$. The iso-product between an iso-quantity and its iso-inverse is obviously equal to the iso-unit 1. Thus, Santilli [1, 2, 3, 4] obtained the iso-rule that anything raised to 0 is equal to 1 . The same remains obviously correct for conventional numbers, i.e. $a^0= 1$. In the same way, it is $a^0=1$.

Santilli [1, 2, 3, 4] found the iso-lifting of a complex number c in analogous way to the one of a real number, i.e.

$$c=(a+ib)\rightarrow c=(a+ib), \tag{8}$$

where i is the usual imaginary unit $i = (-1)^{1/2}$.

Santilli [1, 2, 3, 4] also represented the iso-norm as $|\ a\ | = |a|\times 1$. The iso-norm of iso-products reads

$$|\ a \times b\ |= |\ a\ | \times |\ b\ |. \tag{9}$$

Setting $1= i$, an iso-number a transforms as

$$a = a \times 1=ai \text{ and } \quad t = 1^{\ -1}=1/i=-i. \tag{10}$$

Then, Santilli [1, 2, 3, 4] transformed the iso-product of iso-quantities as $a \times b=abi=ab1 = ab$. The iso-division of iso-quantities transforms in turn as [1, 2, 3, 4] $a / b=-ai/b$.

Santilli [1, 2, 3, 4] represented the iso-functions and the iso-logarithm as

$$f(r)=f(r\times 1) \times 1 \quad \text{and} \quad \log_e a=1 \times \log_e a,\ \ \log_e e=1\ ,\ \ \log_e 1=0, \tag{11}$$

respectively. Other iso-operations on iso-logarithms have been introduced by Santilli [1, 2, 3, 4], i.e. $\log_e (a \times b)= \log_e a+ \log_e b$, $\log_e (a / b)= \log_e a - \log_e b$, $\log_e (a^{-1})=- \log_e a$, $b \times \log_e a = \log_e (a^{-b})$, together with the iso-exponentiation $(e^{\,a \times t}) \times 1= 1 \times(e^{\,a \times t})$.

Santilli [1, 2, 3, 4] defined the iso-differentiation as follows. A transformations for the iso-differential is

$$dr= t \times (dr \times 1) \rightarrow t \times d(r \times 1)=d\, r;\ \ 1=\text{constant}. \tag{12}$$

Now, assuming $1\neq$ constant leads to

$$d\, r=t\times d\, r\hat{\ } =t\times [t \times d(r \times 1))] \neq d\, r \tag{13}$$

Similarly, Santilli [1, 2, 3, 4] obtained the iso-derivatives as $d / d\, r = 1 \times (d/d\, r)$, while the iso-integration was represented as $\int d\, r= 1 \times \int dr$.

Very important and also largely used in the fields of Hadronic Mechanics [5] is the lifting by Santilli [1, 2, 3, 4] of the Schrodinger equation to the iso-Schrodinger equation,

$$H \times |\Psi\rangle = E_0|\Psi\rangle \quad \rightarrow \quad U\times(H \times |\Psi\rangle) = H \times |\Psi\rangle = E|\Psi\rangle, \tag{14}$$

where $E \neq E_0$, because the new iso-Hamiltonian operator iso-operates on the iso-wavefunction (defined by Santilli [1, 2, 3, 4] as $|\Psi\rangle=U\times|\Psi\rangle$) which is bound to produce a different value of corresponding energy eigenvalue. Other important iso-quantities used by Santilli [1, 2, 3, 4] , which permits a generalization of Quantum Mechanics in the framework of Hadronic Mechanics, are the iso-inner product, defined as

$$\langle\Psi \mid \varphi\rangle = 1 \times \int dr^3 \, \Psi^\dagger \, t\,(r, ...)\, \varphi \,, \tag{15}$$

the iso-normalization defined as $\langle\Psi \mid \Psi\rangle = 1$, and the iso-uncertainties that Santilli [1, 2, 3, 4] defined in general as:

$$1=\hbar, \;\; \Delta x \, \Delta k \geq (1/2)1, \;\; \Delta x \approx (a/ t^{1/2}), \Delta k \approx 1/(a/ t^{1/2}). \tag{16}$$

A first application of Santilli iso-mathematics is that of turning complex non-linear formulations on conventional spaces over conventional fields into identical reformulations verifying the axioms of linearity on iso-spaces over iso-fields, called iso-linearity. This reformulation is simply achieved by embedding all non-linear terms in the new iso-unit and then reconstructing fields and spaces with respect to the new unit.

Finally, it is worth mentioning that Santilli iso-mathematics allows the lifting of conventional divergent series into strongly convergent forms under the assumption of a sufficiently large (sufficiently small) iso-unit (iso-topic element)

$$A(w) = A(0) + w\,(AH - HA)\,/1! + ... \Rightarrow \text{infinity}, \;\; w > 1 \Rightarrow$$
$$\Rightarrow A(w) = A(0) + w\,(A\, t\, H- H\, t\, A)\, /\, 1! + ... = N < \text{infinity}, \; |t\,| \ll w \tag{17}$$

Even for conventional convergent series, Santilli iso-mathematics can accelerates their convergence to such a degree to reach a good approximation via only the first terms, thus reducing rather substantially computation or computer times, as shown in various applications, such as those for the binding energy of the Hydrogen and water molecules.

In summary, in this extended abstract we have introduced and reviewed the most relevant aspects of Santilli iso-mathematics. Researchers interested in further details of Santilli iso-mathematics should study [1, 2, 3, 4] and references within and also carefully follow the Workshop on Iso-mathematics at the 2013 ICNAAM Conference [6,7].

ACKNOWLEDGMENTS

The Ruggero Maria Santilli Foundation has to be thanked for financing this short review paper.

REFERENCES

1. R. M. Santilli, Circolo matematico di Palermo, (Special Issue on Santilli Isotopies) Supp.,Vol. 42 , 742(1996).
2. R. M. Santilli, Alg. Gr. Geom. 10, 273-321 (1993) .
3. C. X. Jiang, *Foundations of Santilli's Isonumber Theory, with Applications to New Cryptograms, Fermats Theorem and Goldbachs Conjecture,* International Academic Press, America-Europe-Asia, 2002.
4. C. Corda, AIP Conf. Proc. 1479, 1013 (2012) .
5. R. M. Santilli, "Hadronic Mathematics, Physics and Chemistry", Vols. I [4a], II [4b], III [4c], IV [4d] and V [4e] , New York : International Academic Press , also in http://www.i-b-r.org/Hadronic-Mechanics.htm .
6. http://www.icnaam.org/sessions_minisymposia.htm
7. http://www.santilli-foundation.org/isomathemat-work.php