Symmetry Analysis of Differential Equations with *Mathematica*®

Gerd Baumann

Symmetry Analysis of Differential Equations with *Mathematica*®

Springer Science+Business Media, LLC

Gerd Baumann
Department of Mathematical Physics
University of Ulm
Ulm D-89069
Germany

Library of Congress Cataloging-in-Publication Data
Baumann, Gerd.
Symmetry analysis of differential equations with Mathematica /
Gerd Baumann.
p. cm.
Includes bibliographical references and indexes.
Additional material to this book can be downloaded from http://extras.springer.com.

ISBN 978-1-4612-7418-6 ISBN 978-1-4612-2110-4 (eBook)
DOI 10.1007/978-1-4612-2110-4

1. Differential equations—Numerical solutions—Computer programs.
2. Symmetry (Physics) 3. Mathematica (Computer program language)
I. Title
QA371.B36 1998
515′.35—dc21 98-26975

Printed on acid-free paper.

© 2000 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. 2000
Softcover reprint of the hardcover 1st edition 2000

TELOS®, The Electronic Library of Science, is an imprint of Springer-Verlag New York, Inc.

This Work consists of a printed book and a CD-ROM packaged with the book, both of which are protected by federal copyright law and international treaty. The book may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC), except for brief excerpts in connection with reviews or scholarly analysis. For copyright information regarding the CD-ROM, please consult the printed information packaged with the
CD-ROM in the back of this publication, and which is also stored as a "readme" file on the CD-ROM. Use of the printed version of this Work in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known, or hereafter developed, other than those uses expressly granted in the CD-ROM copyright notice and disclaimer information, is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone. Where those designations appear in the book and Springer-Verlag was aware of a trademark claim, the designations follow the capitalization style used by the manufacturer.

Production managed by Steven Pisano; manufacturing supervised by Jacqui Ashri.
Typeset by Integre Technical Publishing Co., Inc., Albuquerque, NM.

9 8 7 6 5 4 3 2 1

Dedicated to my beloved one and only

Preface

The purpose of this book is to provide the reader with a comprehensive introduction to the applications of symmetry analysis to ordinary and partial differential equations. The theoretical background of physics is illustrated by modern methods of computer algebra. The presentation of the material in the book is based on *Mathematica* 3.0 notebooks. The entire printed version of this book is available on the accompanying CD. The text is presented in such a way that the reader can interact with the calculations and experiment with the models and methods. Also contained on the CD is a package called *MathLie*—in honor of Sophus Lie—carrying out the calculations automatically. The application of symmetry analysis to problems from physics, mathematics, and engineering is demonstrated by many examples.

The study of symmetries of differential equations is an old subject. Thanks to Sophus Lie we today have available to us important information on the behavior of differential equations. Symmetries can be used to find exact solutions. Symmetries can be applied to verify and to develop numerical schemes. They can provide conservation laws for differential equations. The theory presented here is based on Lie, containing improvements and generalizations made by later mathematicians who rediscovered and used Lie's work. The presentation of Lie's theory in connection with *Mathematica* is novel and vitalizes an old theory. The extensive symbolic calculations necessary under Lie's theory are supported by *MathLie*, a package written in *Mathematica*.

Each chapter of the present book includes theoretical considerations and practical applications of *MathLie* and *Mathematica*. The *Mathematica* examples range from simple definitions to complete notebooks discussing specific problems. The examples include definitions of general derivatives, derivations and solutions of determining equations, drop formations in liquids, and the first atomic explosion.

The end of a definition and a theorem in the text is indicated by ○. The end of an example is indicated by □. On the CD, *MathLie* and *Mathematica* notations in the text are denoted by the color dark red. *Mathematica* input is given in red while the output is in blue.

I wish to express my gratitude to Peter Olver, Willy Hereman, and Mike Mezzino for reading the manuscript. My appreciation goes to Gerda Göler and Joachim Engelmann for proofreading the text. I also acknowledge contributions by Gernot Haager, Gerald Landhäußer, and Ronald Schmid.

Any suggestions and comments related to the book or to *MathLie* are most appreciated. Please send your e-mail to Gerd.Baumann@physik.uni-ulm.de or visit my home page at http://www.physik.uni-ulm.de/math/gbaumann/bau.html.

Gerd Baumann

Contents