Logical Machines: Peirce on Psychologism

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Critical notice

Logical machines: Peirce on psychologism

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Abstract This essay discusses Peirce's appeal to logical machines as an argument against psychologism. It also contends that some of Peirce's anti-psychologistic remarks on logic contain interesting premonitions arising from his perception of the asymmetry of proof complexity in monadic and relational logical calculi that were only given full formulation and explication in the early twentieth century through Church's Theorem and Hilbert's broad-ranging *Entscheidungsproblem*.

In *Gulliver's Travels*, Jonathan Swift relates that in his voyage to *Balnibarbi* Gulliver comes across a professor of the grand academy of *Lagado* who shows him a machine capable of improving and extending knowledge by 'mechanical operations.' (Swift 1735: 195) The academician explains that 'by his contrivance, the most ignorant person at a reasonable charge, and with a little bodily labour, may write books in philosophy, poetry, politicks, law, mathematicks and theology, without the least assistance from genius or study.' (Swift 1735: 196) So far as the other voyages are concerned, it is clear whom Swift meant to ridicule, the follies and foibles he wished to expose. *Balnibarbi*'s target is rather less obvious. Why was Swift so acutely roiled by mathematicians and natural scientists in a period when mathematics was being enriched by major discoveries, when mathematical physics was being systematically extended, and when experimental sciences were flourishing in almost every department? Nonetheless, the target of the above narrative is apparently to deride the Organons of Aristotle and Bacon by exposing the inanity of supposing that any machine or instrument can do the work of the mind.

The ambition to mechanise logic, in particular, goes back to Leibniz's dream of *a lingua characterica* – a logical language into which every clear question could be translated and then settled by calculation. Yet, in his *Metalogicon* John of Salisbury reports about one of his students, William of Soissons, that:

> … he invented a machine for the purpose of subduing by force the old established principles of logic, for constructing unbelievable consequences and destroying the theories of the ancients. (Martin 1986: 565)

William and Martha Kneale present this passage as the source for the recent suggestion that William's machine was possibly a physical device perhaps anticipating nineteenth century 'invention of a logical machine'. (Kneale and Kneale 1988: 201) Explicit mechanisation in logic finally emerged in the construction of logical machines by Stanley Jevons and Allan Marquand. (Burks and Burks 1988; Gardner 1958) Jevons was the first pioneer to realise mechanisation of logic and in 1869 he succeeded in constructing a logical machine he exhibited next year to the Royal Society of London. (Gardner 1958) In one description, its appearance was like that of a very small upright piano which was somewhat whimsically referred to as the 'Logical Piano'. (Ketner 1984: 188) Marquand, of Princeton, who had been in correspondence with Jevons about his works, designed in 1881 'a device somewhat similar to that of Jevons', and in the next year built an improved model, using as a basis his own multi-variable diagram'. (Mays and Henry 1953: 504) The upshot of this, not only *possible* but also *actual*, mechanisation was to set off many philosophical discussions on the nature of logical reasoning and its relation to mechanics[1], among whose early discussants one encounters Charles S. Peirce.[2]

[1] For a historical survey of the wider impact of the emergence of logical machines, for example, on economics, see Maas 1999.

[2] It should be noted that Peirce had more than a philosophical interest in the issue of logical machines. In fact, Marquand was Peirce's student at Johns Hopkins University before moving to Princeton, and Marquand's first published design for a logical machine appeared in *Studies in Logic* which was edited by Peirce. (Ketner 1984) Moreover, Peirce was apparently the first person who suggested to Marquand to use electricity for computing logical operations. (Peirce 1993: xliv, 421-23, and 482-3)

Peirce was against psychologism in logic and expressed his qualms about it as early as 1865[3], which somewhat predates Fregean and Husserlian assaults, with the aim of avoiding 'all possible entanglement in the meshes of psychological controversy.' (Peirce 1982: 308)[4] However, unlike Francis H. Bradley yet closer in spirit to Frege's attempt to prove the existence of *Gedanken*, Peirce launched his criticism by rejecting the frames of reference in which the controversy was being conducted. To set the new boundaries, Peirce states that 'in logic we are not occupied with *cognition* or *the mode of cognition*, but only with the forms of representation', *i.e.*, the concern is with formal relations between symbols. (Michael and Michael 1979: 85; my emphasis) To justify this change of arena, Peirce appeals to the following consideration: let there be an argument in a recently recovered ancient tablet in an *undeciphered* language. Obviously one would not say that such an argument was valid when it was understood and thought, and now that its language is undeciphered and not understood, it is no longer valid. The argument is valid, Peirce holds, in virtue of the relations between symbols irrespective of their being understood and thought. Thus, he remarks:

> … the unpsychological view makes that systematically evident, which it would seem were otherwise sufficiently axiomatic, that these laws apply not merely to what can be thought but to whatever can be symbolized in any way. (Peirce 1982: 166-7)

This change of the frame of reference paves the way for Peirce to state his direct argument against logical psychologism by considering the implications of the existence of logical machines.

It should be noted that Jevons' and Marquand's machines were not designed to check whether a given logical argument is valid, but to indicate implicitly conclusions that *could* be derived from given

[3] For an account of how Peirce's anti-psychologism can be reconciled with the *apparently* psychologistic proclivities shown in his later influential papers such as 'The Fixation of Belief' and 'How to Make Our Ideas Clear,' see Kasser 1999.

[4] This, rather *accidentally*, reinforces what George Boolos phrases as 'the iconoclastic tendency' of Putnam's article, 'Peirce the Logician', in which he attempts to reinstate Peirce, as opposed to Frege, as the real precursor of modern logic. (Boolos 1994: 31) Putnam states his aim thus: 'to show that much that is quite familiar in modern logic actually became known to the logical world through the efforts of Peirce and his students.' (Putnam 1992: 252)

premises which alone were fed into them. Bradley, however, pressed the point that

> The result that comes out and is presented by the machine, is not really the conclusion. The process is not finished when the machinery stops; and the rest is left to be done by the mind. What is called *"reading"* the conclusion is to some extent *making* it. (Bradley 1922: 384)

Bradley thus denied that machines *can* perform inferences, but as he could see that Jevons' machine clearly did perform logical operations, he conceded that 'it performs mechanically an operation which, if performed ideally, would be an inference.' (Bradley 1922: 383) Obviously the issue turns on the nebulous phrase *ideal performance*. But, so far as one can gather from the text, what Bradley means by an ideal performance is 'an operation performed in the mind' which renders Bradley's position rather question-begging. What goes on in the mind during logical operations is rather obscure, but Peirce, contrary to Bradley's approach, urges that 'needed light on the nature of the reasoning process' is bound to be shed by studying 'how much of the business of thinking a machine could possibly be made to perform, and what part of it must be left for the living mind'. (Peirce 1887: 165)

What is, nonetheless, interesting about Peirce's position is that he goes on to stress that no such light could illuminate *logic* itself, for it does not in actual fact depend on 'the nature of reasoning process', but rather on *truth* which is quite independent of any thinking process. 'How we think', Peirce writes, 'is utterly irrelevant to logical inquiry.' (Peirce 1960, II: 31) He supports this attack on psychologism in logic by arguing that its laws apply equally well to human or mechanical reasoning. If it be objected that machines do not think, Peirce's response is that thinking 'has nothing to do with logical criticism, which is equally applicable to the machine's performances and to the man's.' (Peirce 1960, II: 33) Indeed, it should be admitted that the machines of Jevons and Marquand, as well as Charles Babbage's analytical engine, all perform inferences and thus should be regarded as reasoning, since

> If from true premisses they always yield true conclusions, what more could be desired? Yet those machines have no souls that we know of. They do not appear to think, at all, in any psychical sense; and even if we should discover that they do so, it would be a fact altogether without bearing upon the logical correctness of their operations … (Peirce 1960, II: 31-32)

Peirce, therefore, uses *logical machines* to argue against psychologism in logic. There is, however, a problem of demarcation here. That is, if calculating machines also reason and, as Peirce states, 'Babbage's analytical engine would perform considerable feats in mathematics' (Peirce 1960, II: 31), where should the line be drawn? For instance, can one say that at every revolution a steam engine works its problem in thermodynamics? In the context of contemporary cognitive science, John Searle thinks that this type of question parallels ones like 'Does the visual system *compute* shape from shading?' or 'Does the visual system *compute* object distance from size of retinal image?'[5] To make his point more striking, Searle claims that those questions are of a piece with a *fallacious* one like: 'do nails compute the distance they are to travel in the board from the impact of the hammer and the density of the wood?' (Searle 1992: 214)

Notwithstanding the Searlian demur, one may attempt to sharpen the question in the case of logic, for example, by considering the issue of what exactly constitutes a *logical* machine. The answer to such a question lies in what Peirce phrases as the 'secret of all reasoning machines'. (Peirce 1887: 168) That is:

> It is that whatever relation among the objects reasoned about is destined to be the hinge of a ratiocination, that same general relation must be capable of being introduced between certain parts of the machine. (Peirce 1887: 168)

Peirce illustrates this abstract statement with the following concrete example. A machine can argue *syllogistically in Barbara* if there is a connection 'such that when one event A occurs in the machine, another event B must also occur. This connection being introduced between A and B, and also between B and C, it is necessarily virtually introduced between A and C.' (Peirce 1887: 168)

Then, Peirce argues that it must be admitted that *every* machine is really a reasoning machine, for in every machine certain relations between its parts will 'involve other relations that were not expressly intended.' (Peirce 1887: 168) Peirce extends this line of thought even to apparatus used in scientific experiments by calling them likewise 'reasoning machines', yet noting that such instruments do not depend

[5] For an extension of the concept of logical machine to sense perception in Peirce and its epistemological implications, see Nesher 2002.

on the laws of human mind 'but on the objective reason embodied in the laws of nature.' (Peirce 1887: 168) Thus he remarks,

> Accordingly, it is no figure of speech to say that the alembics and cucurbits of the chemist are instruments of thought, or logical machines. (Peirce 1887: 168)

Interestingly enough, Peirce's view appears to be an elaboration of a position expressed more than two years earlier, in 1885, by Bernard Bosanquet that all instruments of measurement and observation have a right to be called 'reasoning machines'. Bosanquet writes:

> It has always appeared to me that the element of knowledge incorporated in our instruments of measurement and observation has met with insufficient recognition from logical theory … I think that a spectroscope, or a fine compound microscope with all sorts of illuminating devices, or even a first-rate chronometer, is perhaps as truly a reasoning machine as any logical apparatus that has been devised. … Professor Jevons … with his ingenious logical machine … has called special attention to the principles by help of which our instruments furnish us with exact measurements. (Bosanquet 1885: 327-28)

A logical machine, Peirce holds, reasons no more than any other machine, but differs from others 'merely in working upon an excessively simple principle which is applied in a manifold and complex way, instead of upon an occult principle applied in a monotonous way.' (Peirce 1960, II: 32-3) The special truth-conditions satisfied by logical machines are not the exclusive mark of all reasoning which Peirce considers as no more than a terminological issue. What he emphasises on is that a man

> … may be regarded as a machine which turns out, let us say, a written sentence expressing a conclusion, the man-machine having been fed with a written statement of fact, as premiss. Since this performance is no more than a machine might go through, it has no essential relation to the circumstance that the machine happens to work by geared wheels, while a man happens to work by an ill-understood arrangement of brain-cells … (Peirce 1960, II: 33)

Nevertheless, Peirce's attack on psychologism stops short of endorsing the Hobbesian stance that 'all reasoning is computation'. (Peirce 1960, II: 31)

Hobbes maintained that all thought or reasoning is just *ratiocination*. The underlying nominalism is inspired by the 'Latines' who called accounts of money

> ... *Rationes*, and accounting, *Ratiocinatio*: and that which we in bills and books of accounts call Items, they called *Nomina*; that is, *Names*: and thence it seems to proceed, that they extended the word *Ratio*, or the faculty of Reckoning in all other things. (Hobbes 1651: 106)

To reinforce his nominalism by appealing to authority, Hobbes notes that the Greeks had but one word, *logos*, for both speech and reason, because they believed that reason presupposed speech. He had meanwhile become convinced that *reason* 'is nothing but *Reckoning* (that is, Adding and Substracting) of the Consequences of general names agreed upon, for the *marking* and *signifying* of our thoughts'. (Hobbes 1651: 111) The poignancy of this conviction becomes more striking when read against the following statement: '... in what matter soever there is place for *addition* and *substraction*, there also is place for *Reason*; and where these have no place, there *Reason* has nothing at all to do.' (Hobbes 1651: 110-11) He found support for this thesis in Pascal's construction of the first adding machine. (Boden 1964: 333-34) In the words of Pascal's sister, Madame Perier, the arithmetic machine

> ... was an entirely new thing in nature, by which he reduced to mechanism a science which resides in the mind, and by which he found the means of carrying out all the operations with a complete certainty *without recourse to reasoning*. (Jaki 1969: 22-3; my emphasis)

Thus, for Hobbes, reasoning is just a computational process whose mechanical nature is confirmed by such constructions as Pascal's machine. However, it should be said that Hobbes was in no position to show that all computation can be reduced to mechanism; nor could he show that, since a machine can do what had previously required a mind, a mind then is a machine.

Now, returning to Peirce's argumentation against psychologism, although he explicitly agrees with Hobbes that 'numerical computation is reasoning' (Peirce 1960, II: 31), Peirce disavows the stronger flat theory of ratiocination in which all operations of the mind are traced back to addition and subtraction, *i.e.* all reasoning is computational. The reason for rejecting the latter claim lies in what Peirce perceives as the *essential* differences in reasoning between humans and machines.

Peirce claims that all logical machines suffer from 'two inherent impotencies.' (Peirce 1887: 168) The first impotence that Peirce finds in a logical machine is that it is

> … destitute of all originality, of all initiative. It cannot find its own problems; it cannot feed itself. It cannot direct itself between different possible procedures. (Peirce 1887: 168)

To give an example, Peirce remarks that how can a machine automatically thread its way through such a labyrinth as von Staudt's long proof for Desargues' theorem – a simple proposition in projective geometry starting from a few premises but involving 'no less than 70 or 80 steps in the demonstration.' (Peirce 1887: 169) But, even if the machine can be programmed to do it,

> … it would still remain true that the machine would be utterly devoid of original initiative, and would only do the special kind of thing it had been calculated to do. (Peirce 1887: 169)

Yet, Peirce, like many of his contemporaries, seems to have been rather insensitive to the fact that Babbage had already conceived, if not built, a machine which could execute *all* 'possible procedures', and not just some 'special thing'. It is now part of the common lore that Babbage's vision of universal machines which can 'direct themselves' between different programmes is readily realisable. However, it should be noted that Turing's argument that a universal machine must be considered to have 'originality' if any machine has it begs the question whether any machine *does* have it. (Turing 1950: 450 *ff*.)

In Peirce's understanding, the second inherent inadequacy of any machine is that its capacity 'has absolute limitations; it has been contrived to do a certain thing, and it can do nothing else.' (Peirce 1887: 169) *Prima facie*, this sounds like the first one all over again, but Peirce's reason for this impotence, as distinct from the first one, is that existing logical machines can only deal with a limited number of symbols. This explanation immediately prompts Peirce to note a comparable limitation on human beings which apparently undermines the force of his diagnosis. He writes,

> The unaided mind is also limited in this as in other respects; but the mind working with a pencil and plenty of paper has no such limitation. It

> presses on and on, and whatever limits can be assigned to its capacity to-day, may be over-stepped to-morrow. (Peirce 1887: 169)

However, contrary to Peirce's claim about logical machines, this is merely a matter *of external memory* capacity which is assumed to be unlimited and potentially infinite for a universal machine. This is obviously an *idealisation*. 'There is no theoretical difficulty', Turing writes, 'in the idea of a computer with an unlimited store. … Such computers have special theoretical interest and will be called infinitive capacity computers.' (Turing 1950: 438-39) And, as Roger Penrose points out, the advances in 'modern computer technology have provided us with electronic storage devices which can, indeed, be treated as *unlimited*'. (Penrose 1990: 35; my emphasis)

The failure to fathom the full force of the idea of universal machines, as broached by Babbage's analytical engine, is also evident in the way that Peirce differentiates 'non-relational' logic from the 'logic of relatives'. The former, in Peirce's view, has misled logicians into believing that deductive (necessary) reasoning was all a matter of following rigid rules: that is, rules which infer just *one* conclusion from given syllogistic premises. This has in turn led to the conclusion that machines might therefore perform *all* such reasoning. Peirce, however, thinks that this is not borne out by relational logic where 'from any proposition whatever … an endless series of necessary consequences can be deduced'. (Peirce 1960, III: 407) This way of characterising relative logic also encouraged Peirce to attack Kant's view of logic as analytic in the sense that it 'only elicits what was implicitly thought in the premisses' (Peirce 1960, III: 407), and consequently his famous analytic/synthetic distinction in judgements.[6] He argues that in the logic of relatives

> Matter entirely foreign to the premisses may appear in the conclusion. (Peirce 1960, III: 408)

What is noteworthy here about Peirce's conception of relational logic is that despite the fact that he lacks the terminology to express his conjecture precisely, he intimates the idea that the addition of relations to a monadic logic puts statements, generally, beyond the ability of

[6] However, for the overall positive influence of Kantian conception of logic on Peirce's early metaphysics, see Forster 1997.

machines to classify as theorems or non-theorems. Peirce's perception of an asymmetry of proof complexity between monadic and relational logical systems appears to be premonitory of certain metalogical theorems that were only developed later in the twentieth century. In hindsight, his claim that machines could generally classify statements in non-relational monadic logic, but the deductive theory of relations requires deductions that are sometimes regulated by 'choice and a deliberate plan', was only given full formulation and explication by Church's Theorem, or Church's Theorem together with the Turing-Church Thesis, a few decades later. Indeed, by emphasising the intrusion of 'choice and a deliberate plan' in the proof constructions of relational logics, in the following passage dating back to 1883, Peirce comes close to one of the consequences of Church's Theorem that such proofs might require creativity for their discovery:

> The logic of relatives is highly multiform; it is characterized by innumerable immediate inferences, and by various distinct conclusions from the same set of premisses. …
>
> The effect of these peculiarities is that this algebra cannot be subjected to hard and fast rules like those of the Boolian calculus; and all that can be done in this place is to give a general idea of the way of working with it. (Peirce 1960, III: 200)

Yet, one has to admit that Peirce's remarks here are not unequivocal as they can be read in two different ways: either he could be saying that the logical theory with relations cannot be recursively axiomatised, or he could be saying that there are no 'rules' for discovering proofs and disproofs. However, in an article entitled 'On the Algebra of Logic' published in 1885, two years after the above remarks, Peirce attempts to offer clear rules for such discoveries. It, therefore, seems that he had either changed his views by 1885 or intended the 1883 statements in the first sense. Nonetheless, despite the presence of ambiguity and absence of precise conceptual vocabulary in his claims, as Randall Dipert observes, for Peirce to have made these observations, even vaguely, was a remarkable achievement as 'most interesting metalogical results were beyond the vision of all other 19th-century logicians, including Frege.' (Dipert 1984: 58)

In another piece of relatively the same period as the earlier claim that matter 'entirely foreign to the premisses may appear in the conclusion', however, Peirce admits that even the logic of relations has failed to 'eradicate' completely the idea that all necessary reasoning could conceivably be left to a machine. But, he continues, 'it does show that much *unexpected truth* may often be brought to light by the repeated reintroduction of a premiss already employed'. (Peirce 1960, IV: 506; my emphasis) Again in retrospect, Peirce's point here may be made more precise by drawing on the undecidability of Hilbert's broad-ranging *Entscheidungsproblem*. The problem can be couched in the following question: Is there some mechanical procedure for answering all mathematical problems, belonging to some broad, but well-defined class? The problem became poignant and part of the established lore only after Turing's demonstration that a machine such as Babbage's is able to find every such truth while remaining *itself* undecidable. (Turing 1936/37) Any method for predicting what such a machine will do in general will be confronted with 'unexpected truth' about its *halting* behaviour, so unexpected indeed that it will not predict them at all. Peirce, in fact, did not see any real difficulty in constructing a machine to work the 'logic of relations' even with a large number of terms. His conjecture was that:

> … owing to the great variety of ways in which the same premisses can be combined to produce different conclusions in that branch of logic, the machine, in its first state of development, would be no more mechanical than a hand-loom for weaving in many colors with many shuttles. The study of how to pass from such a machine as that to one corresponding to a Jacquard loom, would be likely to do very much for the improvement of logic. (Peirce 1887: 170)

Although this was a fecund formulation of what was needed, Babbage had already done quite literally this by applying the punched cards of the Jacquard loom to the problem of mechanising arithmetic, while Turing later established that it could indeed 'work the logic of relatives'.

In conclusion, though Peirce's prognostication about machine impotencies fails to bear scrutiny, his use of logical machines to argue against psychologism was a novel and interesting approach. Also, it is worth noting that there appears to be an aura of paradox around logical machines *vis-à-vis* the status and nature of logical principles. On the one hand, one sees how Peirce argues that since logical operations (reasonings) can be adequately performed by logical machines, logic cannot be exhausted by such mechanical operations. The nature of logic *still* needs to be illumined. On the other hand, one encounters the Hobbesian, or in Searle's jargon the 'strong AI' (Searle 1991: 28), camp that since 'brass and iron of machines have been invested with reasoning functions and instructed to perform some of the most difficult operations of the mind,' then there is *nothing* over and above these computational manipulations to logic and logical principles. One thus finds oneself with the following bewildering situation*: the same premise but contradictory conclusions*. Consequently one may adopt the conservative position that the premise in question, *i.e.* the existence of logical machines, on its own is insufficient for drawing any conclusion either way. The battle for determining the nature of logic should largely be fought on grounds and premises other than the existence of logical machines.[7]

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