



The double nature of Maxwell's physical analogies

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ABSTRACT

Building upon work by Mary Hesse (1974), this paper aims to show that a single method of investigation lies behind Maxwell's use of physical analogies in his major scientific works before the *Treatise on Electricity and Magnetism*. Key to understanding the operation of this method is to recognize that Maxwell's physical analogies are intended to possess an 'inductive' function in addition to an 'illustrative' one. That is to say, they not only serve to clarify the equations proposed for an unfamiliar domain with a working interpretation drawn from a more familiar science, but can also be sources of defeasible yet relatively strong arguments from features of the more familiar domain to features of the less. Compared with the reconstructions by Achinstein (1991), Siegel (1991), Harman (1998) and others, which postulate a discontinuity in Maxwell's approach to physical analogy, the account defended in this paper *i*) makes sense of the continuity in Maxwell's remarks on scientific methodology, *ii*) explains his quest for a "mathematical classification of physical quantities" and *iii*) offers a new and more plausible interpretation of the debated episode of the introduction of the displacement current in Maxwell's "On Physical Lines of Forces".

1. Introduction

James Clerk Maxwell's "On Faraday's Lines of Force" (FLF, 1855–1856) is a landmark of nineteenth-century electromagnetism.¹ For historians and philosophers interested in scientific methodology, FLF is also notable for Maxwell's first exposition of the method of "physical analogy" (I:156), a mode of investigation that he will invoke repeatedly over the course of his scientific works. It involves borrowing the notions and tools employed in more well-established sciences to use in unfamiliar territories of physical investigation. For instance, FLF compares an electric field with a system of connected tubes carrying an incompressible fluid, in which each tube corresponds to the direction of a 'line of force' and the intensity of the force corresponds to the cross-section of a tube (or, equivalently, to the velocity of the flow along the direction of the line of force). By adapting well-known notions in fluid dynamics to a much younger science, Maxwell sheds new light on electromagnetism and makes a crucial step towards a re-conceptualization of the physical world in accordance with the vision of Michael Faraday.

According to FLF, the use of analogy is preferable to two other methods of investigation commonly employed at the early stages of physical research. One is the "mathematical method" (I:155), which

consists in formulating equations from which the known experimental results can be deduced; its main defect being that, in the attempt to find the right formula:

we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexions of the subject. (I:155)

At the other extreme is the method of "physical hypothesis" (I:155), which consists in guessing the micro-level mechanism that lies behind the phenomena. In this case, however:

we see the phenomena only through a medium, and are liable to that blindness to facts and rashness in assumptions which a partial explanation encourages. (I:156)

Neither of these defects is shared by Maxwell's preferred methodology. As FLF contends, by approaching a new domain through a physical analogy, we eschew rash assumptions about the micro-level mechanical causes; at the same time, by allowing "the mind at every step to lay hold of a clear physical conception" (I:156) drawn from some familiar physical domain, we maintain an independent grasp on the meaning of the mathematical formulas. These features make the analogical method

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¹ References to Maxwell's work will be as follows: "I" and "II" refer to, respectively, volume I and II of *The Scientific Papers of James Clerk Maxwell*, W.D. Niven (ed.) (Maxwell, 1890); "LP" refers to *The Scientific Letters and Papers of James Clerk Maxwell*, P. Harman (ed.) (Maxwell, 1995); "T" refers to Maxwell's *A Treatise on Electricity and Magnetism* (Maxwell, 1873).

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especially suited for situations in which evidence about a target system is sparse and insufficient, where a primary concern of the investigator is to achieve a form of:

simplification and reduction of the results of previous investigations to a form in which the mind can grasp them. (I:155)

For instance, through the analogy with the flow of an incompressible fluid, FLF shows how:

the connexion of very different orders of phenomena [in electricity and magnetism] may be clearly placed before the mathematical mind. (I:158)

An important puzzle that interpreters of Maxwell's method have faced is how to reconcile FLF's allusions to the 'illustrative' function of analogy, as an aid to the understanding ("to lay hold of a clear physical conception", I:156), with the audacity of the conclusions that Maxwell draws from them, especially in his later works. There is perhaps no better example than in his "On Physical Lines of Force" (PLF, 1861–1862). Having compared the electromagnetic field to an imaginary system of quickly rotating vortices (Fig. 1), Maxwell goes on to argue that the interstices between the vortices must be, partly for mechanical reasons, in some elastic state in between rigid and fluid. Based on this conjecture, he rewrites Ampere's law by adding a term on its right-hand side for the 'displacement current' and presents the resulting equation as the correct electromagnetic law. As we now know, the introduction of the displacement current was the right move: among other things, Maxwell's equation predicted electromagnetic radiation, which was verified experimentally in 1887. But how could Maxwell maintain, in FLF, that physical analogies are ways of "arranging and interpreting [previous] results" (I:159) without "adding anything to that which has been already proved by experiment" (I:159) and, in PLF, rely so heavily on the molecular vortices analogy to defend his new electromagnetic law?

In the wake of Achinstein's (1991) and Siegel's (1991) authoritative reconstructions, a common way of resolving the interpretative puzzle has been to insist that the method of analogy that Maxwell presented in FLF is not the same as the one used in PLF and other later works (cf. Harman, 1998; Cat, 2001; Achinstein, 2003, 2019; Hon & Goldstein, 2012, 2020). On Achinstein's view, there is evidence of a bolder use of physical analogy already in "Illustrations of the Dynamical Theory of Gases" (DTG, 1860), where Maxwell compares a gas with an imaginary system of "perfectly elastic spheres [...] in rapid motion" (I:377). For Achinstein, the DTG analogy is no longer a mere device for illustration but a

full-fledged theory that can explain the observations about gases. In important works, Siegel (1991:39), Harman (1998:101), Cat (2001:422), Hon and Goldstein (2020:227) have similarly contended that the PLF molecular vortices model was an explanatory hypothesis disguised as a mere analogy and that "evidence of a hypothetico-deductive character was accepted as providing support for [its] realistic status" (Siegel, 1991:168). If correct, the thesis of Maxwell's shift in approach would account for the extra confidence displayed in DTG and PLF compared to FLF's more modest attitude.

The aim of this paper is to subvert the received wisdom just outlined and to develop a different interpretation of Maxwell's methodology based upon the contribution of Mary Hesse (1974). On this view, a single method of analogy unites FLF, DTG, PLF and other major research works before the *Treatise on Electricity and Magnetism*.² Key to understanding the operation of this method is to recognize that, for Maxwell, physical analogies possess a double nature: illustrative and inductive at the same time. They not only serve to illustrate the equations proposed for an unfamiliar domain with a working interpretation drawn from a more familiar science, but can also be sources of relatively strong arguments from features of the more familiar domain to features of the less. If correct, the 'double nature' interpretation proposed in this paper will not only uncover an entirely different way of reading Maxwell's early work FLF, but also put an end to recurrent attempts at identifying discontinuities in his later use of physical analogy or at crediting some of his main scientific breakthroughs to hypothetico-deductivism instead.

The defense of this novel outlook, which expands on and partly revises Hesse's (1974) relatively brief discussion, will proceed as follows. Section two will challenge the common view that the method of physical analogy outlined in FLF was abandoned in the writings of the early 1860s, such as DTG and PLF. Section three will then introduce the double nature interpretation and distinguish it from other readings (e.g., Nersessian's 2008) that are only superficially similar. Section four will address some of the most immediate historical problems that the double nature interpretation faces, tackling the case-studies of the fluid analogy in FLF and the flywheel analogy in "A Dynamical Theory of the Electromagnetic Field" (DTE, 1865). Section five will consider the objection that attributing an inductive function to physical analogies is problematic from an epistemological standpoint, showing that Maxwell himself addressed this pressing issue through his discussion on the "mathematical classes of physical quantities" (II:237). Finally, section six will offer a defense of a double nature reading of the molecular vortices analogy in PLF. This reading does justice to the insights behind two prominent interpretative approaches to PLF, namely Siegel's (1991) and Nersessian's (2008); but, unlike them, it shows that Maxwell's remarks on the method of analogy are fully consistent with his use of the method in practice.

2. A methodological Odyssey?

This section will first present (2.1) and then criticize (2.2) the considerations often adduced in support of the popular 'discontinuity narrative', whose defenders include Achinstein (1991, 2003, 2019), Siegel (1991), Harman (1998), Cat (2001), and Hon and Goldstein (2012, 2020). Among other things, the discussion below will help bring out some of the terminological nuances in Maxwell's writing that defenders of that common narrative systematically downplay or neglect.

2.1. The discontinuity narrative

A recurrent concern in FLF is that the fluid analogy should not be understood as a "mature theory, in which physical facts will be physically explained" (I:159), but as a "temporary instrument of research" (I:207).

² As the *Treatise* includes significant parts of review of previous investigations, intended for didactic purposes, a discussion of its use of analogy is better left for a separate occasion; cf. also Hon and Goldstein (2020:228).

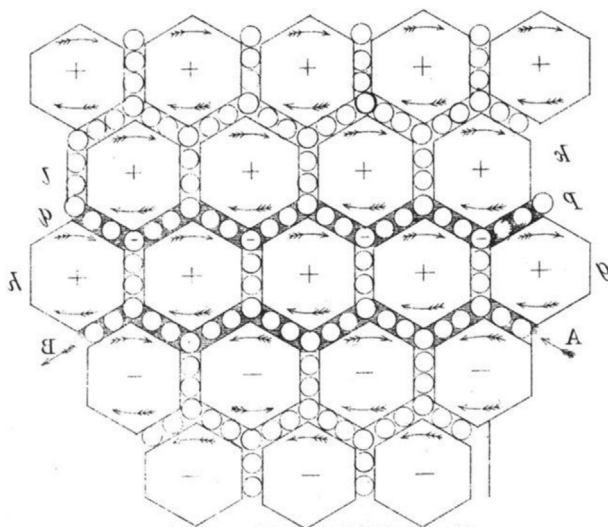


Fig. 1. Maxwell's molecular vortices model of the electromagnetic field in PLF (1861–62).

In order to “obtain physical ideas without adopting a physical theory” (I:156), the investigator begins by making some *generic* assumptions, in the form of given equations, about the physical conditions instantiated in the target system: in FLF’s case, this is the assumption that “the forces of which we treat ... may be represented in magnitude and direction by the uniform motion of an incompressible fluid” (I:159). This assumption is ‘generic’ in the sense that it is compatible with many distinct realizations of the micro-level mechanisms at work. Then, drawing upon one’s firm grasp of the source domain, in which the same conditions possess a materially different realization, one gradually begins to deduce from the equations the various portions of the observations made thus far about the target, without introducing *ad hoc* hypotheses or adding anything to the model that cannot be justified on independent grounds.

As a simple example of how experimental evidence can be recovered through a physical analogy without thereby being “physically explained” (I:159), consider Maxwell’s point that the capacity of a given body to produce electrostatic induction in another depends, among other things, “on the nature of the interposed medium, or dielectric” (I:177). This can be seen as analogous to the behavior of a fluid, where the quantity of flow in a tube depends, among other things, on the nature of the resisting medium: hence, “by making the resistance less we obtain the analogue to a dielectric which more easily conducts Faraday’s lines” (I:177). Importantly, even though in this case electric phenomena behave ‘as if’ they are correctly described by the fluid analogy, the similarity is only at the level of macro-behavior: as we know from other cases where materially distinct domains satisfy formally similar laws, the micro-level realization of electric forces need not have anything in common with fluids. By subsuming portions of the available evidence under the fluid analogy, then, one achieves a form of “simplification” (I:156) of what is known without embracing premature explanations as to the cause of electricity.

According to the received wisdom, however, dissatisfaction with FLF’s non-committal approach must soon have started to grow in Maxwell. As evidence for this claim, Achinstein (1991) points to those DTG passages following equation 24, where Maxwell derives the law $M = 1/3 PLV$, where M is gas viscosity, P the density, L the mean free path of the molecules and V the mean velocity. Having deduced the law from his molecular model, Maxwell notes:

A remarkable result ... in equation 24 is that if this explanation of gaseous friction be true, the coefficient of friction is independent of the density. (I:391)

For Achinstein, Maxwell’s use of ‘explanation’ signals a clear departure from FLF’s analogical approach: the goal has become to account for the phenomena related to gases rather than merely offering analogies. Siegel (1991) similarly finds evidence of a bold explanatory goal in PLF:

I propose ... to determine what tensions in, or motions of, a medium are capable of producing the mechanical phenomena observed. If, by the same hypothesis, we can connect ... magnetic attraction with electromagnetic phenomena and those of induced currents, we shall have found a theory which, if not true, can only be proved erroneous by experiments which will greatly enlarge our knowledge of this part of physics. (I:452)

On the discontinuity narrative, the quoted passages demonstrate two points. First, they show that Maxwell’s “position was not complete epistemic neutrality” (Achinstein, 1991:228) towards the assumptions of his new mechanical models: e.g., the assumption that a gas *just is* a system of particles in motion, or that magnetism *just is* the motion of invisible molecular vortices. Second, they show that, insofar as Maxwell did not have sufficient empirical evidence for making those identifications, he must have advanced them as *physical hypotheses* to explain the

phenomena related to gases (such as gaseous friction) or electromagnetism (such as induced currents).³ It follows that Maxwell’s approach had become practically indistinguishable from the method of hypothesis criticized in FLF: the one whereby some mechanism is postulated in order to deduce the observations. Accordingly, Siegel (1991) writes that PLF “Maxwell embraced the hypothetical method of ... the Cambridge school” (54), a version of hypothetico-deductivism (H-D; Hon & Goldstein, 2020:211). Achinstein (2003), instead, takes DTG and PLF to use a third method— “an exercise in mechanics” (144)— close to H-D in its postulation of mechanism, but where the verification of a hypothesis’ prediction does not automatically yield confirmation.

To clarify, defenders of the discontinuity narrative continue to assign an ‘illustrative’ role to the analogies in DTG and PLF in addition to the explanatory one. That is to say, they recognize that the novel representational structures resulting from a given analogy can have, among other functions, a didactic purpose or suggest (in a merely heuristic sense) new hypotheses to test.⁴ However, it is their view that DTG’s and PLF’s mechanical analogies are mainly intended as explanatory models; as Hon and Goldstein (2020) write: “analogy as the principal tool of research is abandoned” (227). It is precisely this historical allegation that the next sub-section will focus on, with the aim of showing that it lacks adequate basis in the textual evidence.

2.2. No change in method

A much-understated problem for the discontinuity narrative is that Maxwell presents both DTG and PLF as methodologically continuous with FLF. In the introduction to DTG, for instance, he explains his motivation for studying the properties of a system of colliding spheres as follows:

If the properties of such a system of bodies are found to correspond to those of gases, an important *physical analogy* will be established (I:378, our emphasis)

Announcing DTG in a letter to G. Stokes, Maxwell claims to be avoiding physical hypotheses:

I intend to arrange my propositions about the motions of elastic spheres in a manner independent of the speculations about gases. (LP, 1:169)

Similar remarks apply to PLF, whose mode of investigation is introduced in the following terms:

My object in this paper is to clear the way for speculation ..., by investigating the mechanical results of certain states of tension and motion in a medium, and comparing them with the observed phenomena of electricity and magnetism. (I:452)

Save for the fact that PLF’s perspective is an explicitly mechanical one, the plan of action set forth in this PLF passage is the same as the one that we find in FLF’s introduction, which was to:

trace the consequences of assuming certain conditions of motion, and to point out the application of the method to ... phenomena of electricity, magnetism and galvanism (I:159)

These passages indicate that Maxwell’s intent in DTG and PLF is to be consistent with the method of analogy. The textual evidence supporting discontinuity is slimmer in comparison. The passages from DTG that Achinstein (1991:219) highlights, for instance, are occurrences of the terms ‘explanation’ and ‘hypothesis’ in Maxwell’s writing: “if this explanation of gaseous friction be true..” (I:391). Although these

³ Cf. Siegel (1991:39), Achinstein (1991:226), Cat (2001:419) on the passage from illustration to explanation.

⁴ Here and elsewhere, ‘illustrative role’ will continue to be understood as including both the *pedagogical* function of serving as aid to teaching and the *heuristic* function of being a device for scientific discovery.

occurrences may suggest a change in approach, they do not force this reading. First, even if Maxwell was using the terms in a strict sense, the passages are consistent with the FLF method insofar as they express a merely conditional fact: “if this explanation ... be true” (I:391, our emphasis). This is not sufficient evidence that Maxwell meant to identify gases with systems of colliding spherical particles. Moreover, terms such as ‘explanation’, ‘theory’ and ‘hypothesis’ also appear in FLF, where they are intended in a sense that implies neither a commitment to mechanical explanation nor to a hypothetical methodology.⁵ For instance, Maxwell refers to the application of the fluid model to current electricity as a “Theory of the Conduction of Current Electricity”; in another FLF passage, he refers to his investigation as “searching for the explanation of the phenomena” (I:193).

A *prima facie* more compelling passage is the one that Siegel highlights from PLF (quoted above): “If, by the same hypothesis, we can connect the phenomena [of electromagnetism] we shall have found a theory”, (I:452). However, it is not obvious that by “the same hypothesis” Maxwell means the molecular vortices model (as Siegel, 1991:36 contends). Another reading treasures Maxwell’s distinction in part II of PLF between making a hypothesis about the “condition” that a medium must be in to produce a given set of forces (e.g., a state of stress) and making a hypothesis about the “cause” of that condition (e.g., a difference in pressure between the vortices: I:467; see fn.5). The reading whereby “the same hypothesis” refers to the state of stress in the medium (the condition, not the cause) is fully in line with the FLF method of analogy, which was “to trace out the consequences of assuming certain conditions of motion ... without making any assumptions as to the physical nature of electricity” (I:159). Moreover, such a state of tension is clearly capable of “producing” (I:452) the macro-level observations (*pace* Siegel, 1991:36, Hon & Goldstein, 2020:100, for whom “producing” postulates a mechanism).

Other passages produced in defense of discontinuity are similarly indecisive. For instance, Hon and Goldstein (2020) signal several places where Maxwell’s “physical” language is apparently inconsistent with the FLF approach, such as the use of ‘system’ in the PLF passage:

We have now shewn in what way electro-magnetic phenomena may be imitated by an imaginary system of molecular vortices (I:451)

As Hon and Goldstein (2020) write, “the term ‘system’ ... makes clear that the hypothesis is not, in fact, an analogy at all” (101). Another reading of that passage, however, does not highlight the term “system” (which also occurs in FLF to refer to the fluid model: I:161, I:169, I:187), but the terms “imitated” and “imaginary”: the result that Maxwell is emphasizing is the discovery of a *physical analogy* between the laws of electromagnetism and those governing a mechanically conceivable apparatus. This language is especially appropriate given that, by Maxwell’s own admission, many parts of that PLF model were not even plausible *candidates* for reality, as witnessed by his explicit remarks on the ‘awkwardness’ of the resulting mechanical image:

The conception of a particle having its motion connected with that of a vortex by ... rolling contact may appear ... awkward. I do not bring it forward as a mode of connexion existing in nature, or even as that which I would willingly assent to as an electrical hypothesis. (I:486)⁶

⁵ We must distinguish Maxwell’s ‘hypothesis’, which is used as synonymous with ‘assumption’, and ‘physical hypothesis’, which is a claim about unobservables; see, e.g., Maxwell’s use of ‘electrical hypothesis’ in I:486.

⁶ Note that this passage is especially hard to square with Maxwell’s alleged hypothetico-deductivism: he is deducing the observations about electricity from the sliding particle assumption without thereby regarding the latter as any more well-confirmed. This is one of Achinstein’s (2003:153) reasons for seeing both DTG and PLF as employing a third method in between analogy and H-D. As previously discussed, however, the textual evidence in support of Achinstein’s attribution is slim (see, e.g., Maxwell’s letter to G. Stokes quoted above).

Finally, a more general problem for the discontinuity narrative is that, in works *after* DTG and PLF, Maxwell distanced himself clearly from the hypothetico-deductive methodology:

The reason why so many of our physical theories have been built up by the method of hypotheses is that the speculators have not been provided with methods and terms sufficiently general to express the results of their induction in its early stages. They were thus compelled either to leave their ideas vague ... or to present them in a form the details of which could be supplied only by the illegitimate use of the imagination. (II:419)

Indeed, looking back at the molecular vortices analogy in the *Treatise*, Maxwell clarified that his intent had not been to advance a physical hypothesis about the cause of electromagnetism:

The attempt which I then made to imagine a working model of this mechanism must be taken for no more than it really is, a demonstration that mechanism may be imagined capable of producing a connexion mechanically equivalent to the actual connexion (T:470)

To account for these remarks, some defenders of the discontinuity narrative suppose that, after embracing the method of physical hypothesis in PLF, Maxwell regretted it and returned to his FLF steps – a “methodological Odyssey”, as Hon and Goldstein (2020) put it. But a simpler explanation is that Maxwell never embraced the method of physical hypothesis in the first place. The fact that “Maxwell referred to [the molecular vortices analogy] as a hypothesis, whereas here, in [the *Treatise*], he called it a ‘working model’” (Hon & Goldstein, 2020:182) is not sufficient to justify the interpretative epicycle: first, as previously stressed, the ‘hypothesis’ that PLF often refers to is not a specific mechanical explanation (*viz.*, that the cause of magnetic action is the rotation of molecular vortices) but a generic assumption about the condition of the imaginary medium (*viz.*, that it is in a state of stress)⁷; moreover, as will be discussed later in 6.2, PLF Maxwell did not *treat* the molecular vortices analogy as an actual physical hypothesis.

In summary, the claim that DTG and PLF aimed at elaborating micro-level explanations in the style of a hypothetical methodology is questionable on textual grounds. This is not to deny that the mechanical models of DTG and PLF differ from the “geometrical construction” (I:162) of FLF; nor that those mechanical models had at least some claim to reality, in that a subset of their assumptions, e.g., the vortex representation of magnetism, were regarded by Maxwell as “probable” (I:468). But it does not follow that Maxwell’s *method* has changed. First, Maxwell was no less partial to FLF’s suggestion that electromagnetism belongs to the same class of continuous-action phenomena as fluid flow and heat than to PLF’s suggestion that the cause of magnetic action is rotating vortices in the aether. Second, even though PLF’s goal has shifted to providing a micro-level perspective on electromagnetism, it is not obvious that a change in method must follow suit.⁸ If anything, Maxwell’s talk of ‘analogy’ and ‘imitation’ in DTG and PLF signals that the same epistemological concern raised in FLF, of being “carried beyond the truth by a favorite hypothesis” (I:156), is still present when one turns to the micro-level. Altogether, these considerations put considerable pressure on the discontinuity narrative.

⁷ Cf. Maxwell’s letter to Tait in 1867: “[the vortex theory] is built up to show that the phenomena are such as can be explained by mechanism. The nature of this mechanism is to the true mechanism what an orrery is to the Solar system” (LP, 1:337). That is, the molecular vortices as a ‘working model’ is one of many possible mechanical realizations of the physical laws compatible with the ‘hypothesis’ of a medium in a state of stress.

⁸ Cf. Nersessian (2008:40) on PLF: “What is driving the analysis is the assumption that the *dynamical* relations between the idle wheels [i.e., the particles that play the electricity role] and the vortices are of the *same kind* as those between current and magnetism, *not that the specific mechanisms of each are the same*” (our emphasis).

3. The double nature interpretation

Can any sense be made of Maxwell's use of physical analogy in light of the textual evidence indicating continuity in his methodology? In 3.1, two readings will be distinguished both of which are compatible with the claim of methodological continuity, namely the 'illustrative-only' and the 'double nature' interpretations, and an argument will be offered in favor of the latter. In 3.2, some passages from PLF and later works will be highlighted which, though falling short of explicitly endorsing the double nature interpretation, offer important hints for clarifying Maxwell's considered views. The distinctions introduced in 3.2 will prove useful in subsequent sections, where a systematic defense of the double nature interpretation will be provided.

3.1. Two continuity narratives

If the idea of a discontinuity in method is abandoned, what else can explain the confidence that Maxwell often displays in the conclusions drawn from his physical analogies? One view, most notably defended by Nersessian (1984; 2008), denies that the audacity is even one of our data. On this reading, Maxwell consistently *refrained* from drawing the inferences that his analogies suggested; the method of physical analogy therefore remains, from FLF onwards, an illustrative one throughout. Nersessian (2008) offers a nuanced reconstruction of PLF along these lines. She carefully discusses the theoretical and empirical constraints that guide Maxwell's investigation. At the same time, she denies that the molecular vortices analogy justifies any inferences about yet unobserved features of the target (2008:48). The aim is rather the elaboration of "novel representational structures ... through the integration of the various constraints" (29) determined by the specific problem that Maxwell aims to solve – in PLF's specific case, the construction of a mechanically conceivable model consistent with all known electromagnetic phenomena.

A virtue of Nersessian's reading is its compatibility with the evidence indicating continuity in Maxwell's method. The idea of a physical analogy giving rise to "novel representational structures" resonates well with FLF's innovative way of partitioning space into "unit tubes" (I:162) as well as its imagery of the lines of magnetic forces "embracing" (I:184) electric currents.⁹ However, the costs which Nersessian incurs for denying Maxwell's audacious use of physical analogy in PLF are significant. Consider, for instance, how Maxwell presents one of the main conclusions drawn from the PLF molecular vortices analogy in a letter to Faraday:

I think we have now strong reason to believe, whether my theory is a fact or not, that the luminiferous and electromagnetic medium are one. (LP, 1:686)¹⁰

Maxwell's confidence in this central derivation from the PLF molecular vortices model is at odds with the reading whereby "he was avoiding the inference about the possible identity of the two media" (Nersessian, 2008:47). What is missing is an account of Maxwell's grounds for taking the PLF analogy as providing a "strong reason to believe" (LP 1:686) the identity claim.

The supposition that the physical analogies possess an inductive function in addition to the broadly 'illustrative' one that Nersessian defends has the potential to solve the interpretative puzzle. The idea was put forward by Hesse (1974) in a relatively short discussion. On Hesse's view, Maxwell's approach to electromagnetism consists in a "generalized method of analogy and induction" (98) seen as an alternative to the

⁹ The specific image of the "mutually embracing curves" is the subject of the classic study by Wise (1979).

¹⁰ In PLF's published text (see 6.1), Maxwell was slightly more cautious: "we can scarcely avoid the inference that ..." (I:500). However, this is still far from supporting Nersessian's claim that the inference was unjustified.

predominant H-D methodology. Exactly under what conditions can a physical analogy possess such an inductive function will be discussed in detail below (as we will see, the reconstruction offered in this paper diverges from Hesse's on this issue). For now, it is important to stress that the 'inductive' role of physical analogy differs from the 'explanatory', since the latter *presupposes* (as a physical hypothesis) an identification of the properties of the model with those of the target (cf. Siegel, 1991:39). On the double nature view, a physical analogy can underwrite defeasible arguments to yet unknown similarities in kind between a model and a target without making use of physical hypotheses; as Hesse (1974) writes, "no hypothetical concepts or entities are postulated" (98) in the investigation.

A comparison with Nersessian's view is useful for clarifying the distinctive features of the double nature reading. The two accounts agree with regards to Maxwell's consistent refusal to embrace a H-D methodology. As Nersessian (2008) notes, "Maxwell's problem-solving was by reasoning through modelling processes" (49), which means engaging in a complex activity of building, refining and extending a model (or a series of them) so as to satisfy what Nersessian calls the "empirical, theoretical and mathematical constraints" (2009:28) of the investigation. For instance, FLF set out to show that Faraday's 'local action' picture of electromagnetic action could be given mathematical consistency and its unifying concept of the 'lines of force' be made applicable to "some of the less complicated phenomena of electricity, magnetism and galvanism" (I:159). To tackle this problem, Maxwell considers the properties of a generic incompressible fluid flow – a paradigmatic example of continuous-action phenomenon – and recovers from that model and its extensions an important portion of the experimental results in electromagnetism.

The point of divergence between the two readings concerns the status of the representation obtained through this "incremental modelling process" (Nersessian, 2008:55). On both readings, it is correct to say that the resulting model is a "hybrid" (Nersessian, 2008: 52) object: not a literal description of the target, but also not a description of an actually existing source. On the double nature interpretation, however, a key aspect of Maxwell's method is that, precisely because the resulting model satisfies in a unique way the empirical and theoretical constraints set out at the beginning of the investigation, it stands in a "very different scientific position" (II:223) from all sorts of rival models aiming simply at accommodating the available empirical evidence. It is therefore reductive to talk, as Nersessian does, of the "heuristic value of [Maxwell's] method" (1984:80). The relative confidence that Maxwell displays in the conclusions drawn from the analogies indicates that he assigns them a capacity to *justify* those conclusions to some extent.

Of course, such an interpretation of the method remains a mere theoretical possibility (a rational reconstruction of the use of physical analogy in his works) unless we can show that it reflects Maxwell's actual views on the subject. The next subsection will discuss some telling passages in Maxwell's texts that give historical substance to the double nature interpretation.

3.2. Mathematical coincidence and physical similarity

That the epistemological problem raised by the inductive use of physical analogy was indeed a live issue for Maxwell is shown clearly by his mentioning it at the end of part II of PLF:

The facts of electro-magnetism are so complicated... that the explanation of any number of them by several different hypotheses must be interesting ... to all who desire to understand how much evidence the explanation of phenomena lends to the credibility of a theory, or *how far we ought to regard a coincidence in the mathematical expression of two sets of phenomena as an indication that these phenomena are of the same kind* (I:488, our emphasis)

This passage is important not only because it shows that Maxwell was aware of the question whether a physical analogy could be an

“indication” of physical similarity, but also because it simultaneously reminds us that a physical analogy remains a “coincidence in the mathematical expression of two sets of phenomena” (I:488; cf. a “partial similarity in the laws” in I:155).

Could a physical analogy so understood be evidence of yet unobserved physical similarities? A seemingly compelling answer to this question is negative. Although the analogy may be useful heuristically, to motivate new research, it may seem epistemically irresponsible to have any extra faith in the new hypotheses that it suggests, based merely on “a coincidence in the mathematical expression” of the laws of two domains. After all, as Maxwell often stresses (e.g., I:208; II:418), rival accounts of the empirical evidence are always conceivable on which the analogy’s model turns out to be physically entirely distinct from the target. Yet, Maxwell’s subsequent remarks in the PLF passage indicate that the seemingly obvious negative answer may be too quick:

Partial coincidences of this kind have been discovered; and the fact that they are only partial is proved by the divergence of the laws of the two sets of phenomena in other respects. We may chance to find, in the higher parts of physics, instances of more complete coincidence, which may require much investigation to detect their ultimate divergence. (I:488)¹¹

That this passage merely hints at the possibility that some mathematical coincidences be physically significant should not be a cause of concern: Maxwell is known for “keeping his philosophical cards close to his chest” (Hacking, 1996:62). What is more interesting is his use of the word “investigation”. We find a similar term in the later Address to the British Society (ABS, 1870). Discussing “the Relation of the two branches (Mathematics and Physics), their action and reaction upon one another” (II:216), Maxwell notes that, when a physical analogy is discovered, where each term of the target “retains all the formal relations to the other terms of the [source]”:

it becomes an important philosophical question to determine in what degree the applicability of the old ideas to the new subject may be taken as evidence that the new phenomena are physically similar to the old. (II:227)

This passage echoes the epistemological problem raised in PLF regarding the “coincidence in mathematical expression” but adds that “it is an important philosophical question” (II:227) to determine whether a resemblance in the equations can be evidence of physical similarity.

Despite the seemingly aporetic nature of the PLF and ABS passages, a close historical analysis reveals that, in concomitance with his major scientific works before the *Treatise*, Maxwell had in fact been elaborating an epistemological account of inference from physical analogy that could vindicate its inductive use. In outline, the view that lies behind Maxwell’s allusions in PLF and ABS is that satisfaction of various experimental and theoretical constraints can determine whether, and to what extent, a given physical analogy may conceal physical similarity – in this sense, the question of physical similarity is a “philosophical” one (II:227). Moreover, – and this is where the double nature interpretation turns out to be especially illuminating – determining this question is “important” (II:227) because, precisely to the extent that one physical analogy is more well-supported than potential rivals, arguments drawn from it are going to be *stronger* than

¹¹ Note how Maxwell’s interest is clearly captured by the problem about whether “mathematical coincidences” can be evidence of physical similarity rather than the other problem mentioned in the passage, “how much evidence the explanation of phenomena lends to ... a theory” (I:488). Indeed, if we take seriously what Maxwell says about the method of physical hypothesis in FLF, the answer to the question to what extent explanation is evidence for a hypothesis’ *truth* is: ‘not much’ – and especially so for mechanical hypotheses. Cf. Hon and Goldstein (2020:111) for a different way of reading the passage along the lines of the discontinuity narrative.

those drawn from the rivals. Hence, the favored analogy can be used not only as an illustration, but as a “metaphor of a bolder kind” (II:227) – as a source of defeasible yet relatively strong arguments to unobserved features of the target system.

The following sections aim to defend the reconstruction of Maxwell’s methodology just outlined in a systematic way. Although the focus will be on the consistency with which Maxwell pursues his program in methodology and on the coherence of the underlying epistemological picture, even the more technically-minded readers should find sufficient detail in the following sections to see the promise of the historical reconstruction. What deserves emphasis above all is that, if the interpretation offered below corresponds even roughly to what lies behind the labyrinth of Maxwell’s methodological allusions throughout his works, it would uncover an entirely new picture of the intellectual development that led to Maxwell’s ground-breaking discoveries. Rather than seeing those breakthroughs as the result of methodological opportunism, the double nature interpretation views them as the consequence of Maxwell’s observance of a carefully articulated program in methodology seen as alternative to hypothetico-deductivism. This makes the double nature account interesting not only as a historical hypothesis in Maxwell scholarship, but also as a source of reflection for contemporary philosophy of science.

4. Defending the double nature: historical problems

An important difficulty for the double nature interpretation is to defend the idea that, among the functions that Maxwell attributes to his physical analogies, there is a distinctly inductive one. As this claim may easily strike as a complete non-starter for a reconstruction of Maxwell’s method, it is imperative to address some of the more urgent historical problems with it. The following two sub-sections will therefore bracket the epistemological problems and tackle, respectively, the case-studies of the fluid analogy in FLF and the flywheel analogy in DTE. In both cases, there exists a long tradition of interpreting the physical analogies employed in those works as purely illustrative devices. As will be discussed below, however, the traditional account overlooks the distinctive ways in which the physical analogies support arguments to novel conclusions.

4.1. The fluid analogy

A common view in Maxwell scholarship sees FLF as expression of Maxwell’s “initial reliance on ... heuristic physical analogies – in the Scottish, skeptical vein” (Siegel, 1991:28) apparently acquired in the years of his Edinburgh training. This verdict is partly suggested by FLF’s almost proverbial modesty, as when Maxwell contrasts his “temporary instrument of research” with the electromagnetic theory by M. Weber, based on the concept of action at a distance, a “physical theory ... so elegant, so mathematical, and so entirely different from anything in this paper” (I:207). These and similar passages have induced many notable interpreters to conclude that the FLF analogy was “purely illustrative” (Siegel, 1991:29; cf. also Harman, 1998:98; Cat, 2001:416). These interpreters concede that Maxwell was partial to the idea that electromagnetic phenomena belonged to the same continuous-action phenomena as fluid motion; however, as Achinstein (1991) explains: “from the fact that the fluid ... has certain properties and satisfies certain laws analogous to those of the electromagnetic field, [Maxwell] does not conclude that there is any reason to suppose that ... the electromagnetic field has analogous micro-properties” (246).¹²

¹² An exception may be Hon and Goldstein (2020:221), at least if one ought to interpret their term “strong analogy” as equivalent to the idea that the analogy offers non-negligible *inductive support* (or ‘confirmation’) to hypotheses about the yet unknown – as opposed to merely *heuristic support*, which is a pragmatic notion.

Yet, several considerations can be brought in support of a double nature reading of FLF. First, it must be noted that Maxwell's expressions of modesty are exclusively concerned with denying that the fluid analogy is intended as an *explanation* of electromagnetic action:

it is not even a hypothetical fluid which is introduced to explain physical phenomena. It is merely a collection of imaginary properties ... (I:160)

In itself, this is not yet an admission that the analogy is purely illustrative: while “imaginary”, the model retains a connection to real-world fluids, in that its behavior is governed by the same laws of continuous media. It is in virtue of this fact that the analogy helps us “lay hold of a clear *physical* conception” (I:156, our emphasis). Accordingly, it is far from clear that the defeasible arguments one might draw from the fluid analogy are epistemically on a par with the merely heuristic inferences one might draw from an entirely fictional (possibly un-physical) model.

Second, there is the fact that potential allusions to the double nature of physical analogies appear throughout FLF. Commenting on the analogy between “light and the vibrations of an elastic medium” (I:156) giving rise to the wave theory of light, Maxwell notes that:

by stripping it of its physical dress and reducing it to a theory of ‘transverse alternations’, we might obtain a system of truth strictly founded on observation, but probably deficient both in the vividness of its conception *and the fertility of its method*. (I:156, our emphasis)

The same idea of the ‘fertility’ of physical analogies comes back in many other FLF passages:

We may however obtain a different view of [electromagnetism], *and one more suited to our more difficult inquiries*, by adopting for the definition of the forces of which we treat, that they may be represented ... by the uniform motion of an incompressible fluid. (I:159, our emphasis)

Introducing his formalization of Faraday's laws of the electro-tonic state, Maxwell adds:

If it should then appear that these laws, originally devised to include one set of phenomena, may be generalized so as to extend to phenomena of a different class, these mathematical connexions may suggest to physicists the means of establishing physical connexions. (I:189)

While for defenders of the traditional reading Maxwell's “suggest” (and similar allusions to the “fertility of the method”) was meant in a heuristic way, it is at least conceivable that a stronger reading was alluded to in those passages: namely, that Maxwell was referring to the potential of physical analogies to offer defeasible support to novel empirical hypotheses. As the last passage quoted clarifies, the potential of a physical analogy to “suggest physical connexions” is intimately tied to its capacity to work as an independently plausible organizing principle for the experimental evidence about the target: of “extending to phenomena of a different class” (I:189) than the one it was originally meant for. This is consistent what Maxwell will say in PLF with regards to the potential of some “coincidences in the mathematical form” to be “an indication that these phenomena are of the same kind” (II:488) and suggests that the conception of the double nature of physical analogies may well exist at least *in nuce* at the time of FLF.

Third, and most importantly, the traditional reading overlooks the underlying *argumentative* structure of FLF, which Maxwell also expressed in a letter to Tait: “the analogical argument on Faraday's Lines of Force is in the Press” (LP, I:495; cf. also [Hon & Goldstein, 2020:76](#)). With hindsight, the structure of this argument is especially noteworthy as it will return practically unchanged in PLF and DTE. It begins in “Application of the Idea of Line of Force” with Maxwell's showing how the facts of static electricity, which had partly inspired Faraday's talk of ‘lines of force’, could be recovered in accordance with the fluid analogy (I:177). This is,

as it were, the ‘paradigmatic application’ of the physical analogy. In subsequent sections, Maxwell then shows how one could extend the same idea of line of force to derive the correct phenomenological laws for various facts about magnets (I:179), crystals in magnetic fields (I:180), the connection between static and current electricity (I:181) and the magnetic forces produced by closed currents (in accordance with Ampere's original law for electromagnetism).

Having explained the application of the fluid analogy to “some of the less complicated phenomena” (I:156), Maxwell moves to some more speculative issues. In particular, he mentions Faraday's observation that a conductor moving transversely through lines of magnetic force produces an “electro-motive force” (I:189) and that this force is due to the conductor's “cutting” (I:190) the magnetic lines. Like Faraday, Maxwell is tempted to carry the analogy further and to suppose an underlying state of matter measured by the number of tubes enclosing it:

It is natural suppose that a force of this kind, which depends on a change in the number of lines, is due to a change of state which is measured by the number of these lines. (I:187)

Maxwell was aware that a verification of this “electro-tonic state” would have established what [Siegel \(1991\)](#) calls the “primacy of the field” (55) approach – the Faradayan conception of electromagnetism based on local action. For this reason, Maxwell noted regretfully that:

[the idea of the electro-tonic state] has not yet presented itself to my mind in such a form that its nature and properties may be clearly explained without reference to mere symbols (I:188)

However, this issue did not stop him from completing the argument for the electrotonic state. Having compared the electromagnetic laws that he derived from the fluid analogy with the “elegant” (I:207) theory by Weber, based on action-at-a-distance, Maxwell first notes that the experimental evidence is insufficient to tell in favor of either: it is not only “a good to have two ways of looking at a subject”, but we must “admit that there *are* two ways of looking at it” (I:208). The argument for the local action picture is completed when, in the final section of FLF, Maxwell adds that Weber's alternative theory violates the principle of energy conservation:

There are ... objections to making any ultimate forces in nature depend on the velocity of the bodies between which they act ... the principle of the Conservation of Force requires that these forces should be in the line joining the particles and functions of the distance only (I:208)

Maxwell's objection had faults; but this is not the point. What should make us pause is the naturalness with which FLF is read as accumulating a complex set of experimental and theoretical considerations (e.g., conservation principles) to justify a form of partiality towards the fluid analogy to electromagnetism and the predictions that follow from that model. As Maxwell must have realized, the arguments that he could draw from the fluid analogy were defeasible; in this sense, the modesty that he displays in many FLF passages is no mere rhetoric.¹³ But it would be too quick to conclude that the physical analogy that FLF had so carefully developed was purely illustrative. In addition to Maxwell's describing FLF as an “analogical argument” in the letter to Tait, there is correspondence with Maxwell's later remarks with regards to it being a subject of “much investigation” (I:488) and “an important philosophical question” (II:227) to determine if a physical analogy conceals physical similarity; as will be discussed momentarily, the match with DTE's mode of investigation is also impressive. A double nature reading of Maxwell's method in FLF must therefore be regarded as a serious historical possibility.

¹³ It is also worth stressing that, on the double nature account, a physical analogy is typically insufficient to *license* empirical conclusions; the claim is only that the analogy can *support* those conclusions to some extent.

4.2. The flywheel analogy

Another long-held doctrine in Maxwell scholarship (e.g., Siegel, 1991:50; Harman, 1998:116) is that DTE (1865) marks the replacement of the mechanical account of electromagnetism of PLF (1861–62) with a theory based on Lagrangian dynamics, stating the general laws that govern the evolution of an electromagnetic field over time. The contraposition of the “concrete and pictorial” approach in PLF with the “abstract and general” (Siegel, 1991:50) investigation in DTE has resulted in a neglect of the mechanical analogy that Maxwell develops in 1865, that between the electromagnetic field and a ‘flywheel’ – a rotating device often used in mechanical systems to improve stability and store rotational energy. A re-evaluation of the role of the flywheel analogy has only recently begun in the secondary literature, most notably in works by Lazaroff-Puck (2015) and Hon and Goldstein (2020). The following discussion will draw upon those recent contributions to demonstrate that the double nature view offers the most plausible reconstruction of Maxwell’s reasoning in this crucial pre-*Treatise* work. The continuity of Maxwell’s approach in 1855 and 1865 reinforces the main historical claim of this paper.

The mechanical apparatus that Maxwell introduces in DTE is depicted in Fig. 2. Two wheels A and B (the ‘driving-points’) attach by separate axles to a central object C – the ‘flywheel’. The latter is constituted by two metal rods at right angles to each other with a weight attached to both ends of each of the rods. Two bevel gears connect the axles between A and C and B and C, respectively. The two bevel gears need not have the same gear ratios. A string attached to a weight hangs from each of the two driving-points, acting as friction. If A is made to rotate while B is at rest, A’s motion imparts a rotation of B in the opposite direction to A’s as well as a rotation of C around the axis between A and B. The angular speed of C’s rotation in A’s direction is equal to half of the difference between A’s and B’s linear speeds multiplied by half the length of C’s rods. As A is set in motion, then, we have a transfer of momentum to C and then to B; if a new force is then applied to B so as to accelerate it in the opposite direction of A, the effect will be another change in C’s momentum. These changes in momentum due to forces being applied on A and B constitute what Maxwell calls “Reduced Momentum” (I:538).

The notion of reduced momentum exemplified by the flywheel is immediately put to use in the paradigmatic case of electromagnetic induction. As DTE explains, increasing the strength of a current produces changes in the surrounding electromagnetic field in an analogous way to

how changes in the velocity of the driving-point in the machine brings about rotation of the flywheel around the axis between A and B. The result of this connection is, as Maxwell writes:

to endow the current with a kind of momentum, just as the connexion between the driving-point of a machine and a fly-wheel endows the driving-point with an additional momentum, which may be called the momentum of the fly-wheel reduced to the driving-point (I:467-8)

The effects of this are visible in the case of two neighboring currents. If the strength of one current is increased, the effect is transferred to the electromagnetic field and then onto the neighboring current in a way that is analogous to those by which an acceleration exerted on driving-point A produces motion onto the flywheel C and from this onto the driving-point B.

Having explained the application of the flywheel analogy to induced currents (including the case in which a current is affected when the position of the conductor of a neighboring current changes) in the language of Lagrangian dynamics, Maxwell goes on to make an important suggestion as to how the flywheel analogy could be extended to phenomena beyond those of induced currents (I:471). As Lazaroff-Puck (2015) puts Maxwell’s idea: “If he has been justified in using the mechanical illustration and thus ... its accompanying flywheel analogy to investigate how electromotive forces arising from changes in circuit affect the electromagnetic field, then such reasoning should work equally well in reverse” (474). The ‘reverse’ is an account of how electromotive forces arising from changes in the electromagnetic field affect currents (and, specifically, the magnetic properties of conductors carrying them). In this way, Maxwell explains: “both induction of currents and electromagnetic attractions may be proved by mechanical reasoning” (I:542). As Hon and Goldstein (2020) have noted, “the structure of the argument is analogical [...] The illustration functions then as a plausibility argument” (134).

The structure of Maxwell’s argument is indeed not far from the one that, as discussed in 4.1, runs through FLF’s discussion. In both papers, Maxwell starts from a paradigmatic phenomenon of which the proposed physical analogy can be considered a clear illustration: in FLF, this is static electricity; in DTE, it is the phenomenon of induced currents. Moreover, in both papers the analogy is shown to extend to closely connected phenomena to those which serve as primary applications: in FLF, these are (among others) the facts about the distribution of magnetism and the generation of currents; in DTE, these are (among others) the effects on currents due to the change in position of a neighboring circuit. Finally, in both papers the applicability of the analogy to phenomena of one kind (e.g., induced currents) is used as defeasible support for the idea that the analogy extends beyond the phenomena it was originally devised for: in FLF, this is the idea of the electrotonic state; in DTE, it is the construction of generalized equations for electromotive force that covers both induced currents and electromagnetic attractions.

It is therefore plausible that, despite the different goals of the two electromagnetic papers, the use of physical analogy in FLF and DTE is similar. This conclusion partially overlaps with those reached by Lazaroff-Puck (2015) and Hon and Goldstein (2020). Specifically, it agrees with Lazaroff-Puck’s (2015) idea that the FLF and DTE share “similar methods” (484) and with Hon and Goldstein’s (2020) view that the physical analogies in both papers play a crucial role in Maxwell’s reasoning. At the same time, there are also important differences between the above reconstructions. Specifically, the double nature reading makes a bolder claim about DTE’s use of physical analogy than Lazaroff-Puck’s view that “the fluid and flywheel analogies ... allow for the construction of the electrotonic state in 1855 and a generalized equation for induced electromotive force in 1865” (485, our emphasis). Instead, the role of the physical analogy is both one of ‘allowing for conceptually’ and of *supporting* the inferences that Lazaroff-Puck mentions.¹⁴ Moreover,

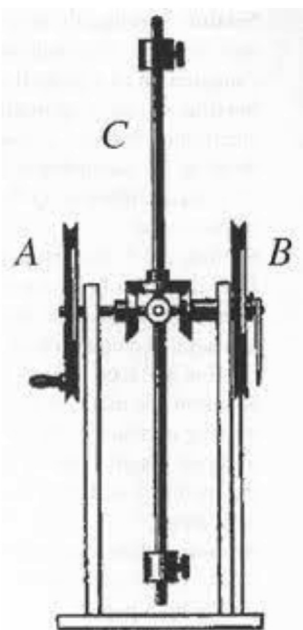


Fig. 2. The ‘flywheel’ mechanical apparatus mentioned in DTE (1865).

¹⁴ Cf. Hon and Goldstein (2020): “[Maxwell] needed a plausibility argument, for it was not immediately obvious that a current can have momentum ... since the nature of electricity ... was not specified” (139).

the double nature reading categorically rejects Hon and Goldstein's (2020:167) allegations regarding the difference in method between DTE and PLF. As will be discussed in 6.2, PLF does not mark any surrendering to a hypothetical methodology.

In summary, an analysis of the two papers FLF and DTE makes the double nature reading a serious interpretative hypothesis. It is especially remarkable that, although the “geometrical construction” (I:160) of FLF has been replaced by the “dynamical” (I:466) outlook of DTE, based on the Lagrangian formalism, there is an impressive match in the use of physical analogy in the two papers. The observation that the same method of physical analogy is invoked to develop two such different perspectives on electromagnetism (viz., the geometry of the lines of force in FLF and the dynamics of the field's evolution over time in DTE), and at such a wide temporal distance from one another, is evidence of an underlying continuity in Maxwell's approach to physical inquiry. Before extending this historical analysis further, to see how it applies to the debated works of the central years 1860–62, it will be useful to get clearer on the epistemological foundations of Maxwell's method. Specifically, one must address the problem of how a physical analogy can provide non-negligible inductive support to novel hypotheses.

5. Defending the double nature: epistemological problems

Suppose that one concedes that Maxwell's physical analogies in works such as FLF and DTE are intended to provide some inductive support to inferences about unfamiliar targets. The question remains: how *strong* can these arguments ever be? For a comparison, consider the analogical reasoning used, in medical contexts, to infer from the effects of a certain drug on mice to its similar effects on humans. In those cases, we know that mice resemble humans in *causally relevant* features: for instance, the processes of assimilation of chemicals are roughly the same. By contrast, by adopting a physical analogy Maxwell claims to be making no hypotheses *whatsoever* as to the causes operating in the target; indeed, he often stresses that the analogy is merely a “partial similarity *in form*” (I:157). How could an analogy so construed ever afford, let alone certainty, anything close to the confirmation afforded by animal models?¹⁵ We come back, in other words, to the “philosophical question” that PLF had posed. In 5.1, an outline will be given of the answer that Maxwell eventually reached, drawing from work in the early 1870s. In 5.2, this account will be specified and distinguished from Hesse's (1974) reconstruction of it.

5.1. Mathematical classes

How could a “geometrical construction” (I:162) or an “imaginary system” (I:451) provide anything more than merely heuristic support to novel hypotheses about real-world targets? The epistemological problem is so serious that, even when Maxwell gives signs of having considered it, one still finds a certain resistance to accepting his answer. This is precisely the state of puzzlement caused by the criterion of ‘correctness’ for physical analogies in his 1870 ABS:

The correctness of [a physical analogy] depends on whether the two systems of ideas which are compared together are really analogous in form, or whether, in other words, the corresponding physical quantities really belong to the same mathematical class. When this condition is fulfilled, the illustration is not only convenient for teaching science ..., but the recognition of the formal analogy between the two systems of ideas leads to a knowledge of both, more profound than could be obtained by studying each system separately. (II:219)

¹⁵ For nuanced discussions on the epistemological problem raised by inference from physical analogy, see Achinstein (1991:221–3) and the discussion in Steiner (1998) on the role of mathematical similarity in physics. Cf. also Bartha (2009), ch. 6.

The question that this passage raises is: if it is true (as the double nature reading suggests) that the distinctively epistemic payoff of ‘correct’ physical analogies that Maxwell emphasizes in this passage is to be spelled out at least partly in terms of their underwriting relatively strong inductive arguments from model to target, how could the mere belonging of the respective quantities to the same ‘mathematical classes’ yield this payoff? Would we not need some more concrete resemblance – ‘causal’ rather than ‘mathematical’ – to justify an inductive use?¹⁶

Maxwell's puzzling remarks in ABS about the criterion of “correctness” (II:219) of physical analogies are elaborated in the paper “On the Mathematical Classification of Physical Quantities” (MCQ, 1871). As Maxwell explains the object of his investigation:

[the classification in question is] founded on the mathematical ... analogy of the different quantities, and not on the matter to which they belong. (II:237)

Although Maxwell does not provide a complete list, he offers three examples of mathematical classes of physical quantities. One is the distinction between *scalar* and *vector* quantities. For instance, volume is a scalar quantity, since it has magnitude but no direction; pressure, instead, is a vector quantity since it has both. Among vectors, Maxwell distinguishes *forces* and *fluxes*. The magnitude of a force is defined along a line, whereas a flux's is defined in terms of an area. In the theory of heat, temperature gradient is a force; heat flow is a flux. A third distinction is that between vectors possessing a *linear* versus *rotational* character. For instance, on the PLF model, electric currents have a linear character, since they produce their effect along a straight line, whereas magnetic forces are rotatory, since they produce their effect through circular motion.

A crucial distinction that Maxwell draws in the course of MCQ is between merely describing a given physical quantity as belonging to a certain mathematical class and that quantity “really belonging” (II:261) to that class. For instance, Maxwell explains that, up to a certain level of detail, fluid velocity can be treated “equally well” either as a force (“with reference to the unit of length”) or as a flux (“with reference to the unity of area”, II:261). But he adds that:

if we endeavour to develop a more complete theory of fluids, which shall take into account the facts of diffusion, where one fluid has a different velocity from another in the same place; or if we accept the doctrine, that the molecules of a fluid, in virtue of the heat of the substance, are in a state of agitation; then, though we may give a definition of the velocity of a single molecule with reference to unit of length, we cannot do so for the fluid; and the only way we have of defining the motion of the fluid is by considering it as a flux. (II:261)

This passage clarifies that the ‘real belonging’ of a quantity to a mathematical class depends on non-trivial considerations of both the experimental (‘facts of diffusion’) and the theoretical kind (such as compatibility with the “doctrine that the molecules of a fluid ... are in a state of agitation”, which Maxwell regards as a plausible hypothesis about the composition of fluids).

Why is it so important, according to Maxwell, to establish to which class a physical quantity ‘really belongs’? As Hesse (1974) pointed out, an important part of the answer lies with the fact that, by identifying a physical quantity with a mathematical class, one thereby locates its role within a relatively stable and well-understood system of relations. These relations are specified by mathematical operations on those classes of quantities. An example is the Hamiltonian operator ∇ , which, when

¹⁶ According to an epistemological doctrine still popular today (e.g., Cartwright, 2009; Hesse, 1963; Sugden, 2009), no observations about a model (whether real or imaginary) can justify empirical conclusions about a target to a non-negligible extent unless there is evidence of similarities in *causally relevant* features between model and target. Cf. Nappo (2020) for a contemporary critique of this epistemological doctrine.

applied to a scalar (e.g. temperature), yields a gradient, but when:

σ represents a vector function, $\nabla\sigma$ may contain both a scalar and a vector part, which may be written $S\nabla\sigma$ and $V\nabla\sigma$. I propose to call the scalar part the *Convergence* of σ [...] But $\nabla\sigma$ has, in general, also a vector portion, and I propose ... to call this vector the *Curl*. (II:265)

For instance, if we imagine the vector field σ to represent the flow of a liquid or gas, then $S\nabla\sigma$ will be scalar quantity representing the tendency of the fluid to collect at a point (today more commonly measured in terms of its opposite, namely ‘divergence’), and $V\nabla\sigma$ will be another vector component standing for the tendency of the fluid to rotate around the same point. What matters for the classification is that such relations are preserved by systems of physical quantities that are mathematically analogous (though materially very different from) those of fluids.

Putting the pieces together, an important answer begins to emerge as to why Maxwell would draw a connection between the issue of the ‘correctness’ of a physical analogy and the ‘real belonging’ of the respective physical quantities to the same mathematical classes. The central idea is that real membership of a given physical quantity to a mathematical class is associated with genuine ‘constraints’ on the kinds of physical interactions there can be in the assumed three-dimensional space. Therefore, when a novel domain is presented to us, it is important to determine (on the basis of experimental and theoretical considerations) to what mathematical classes its quantities ‘really belong’ because this will help us determine what ‘kind’ of physical system the domain in question is and hence what kinds of yet unobserved physical features we can reasonably infer it will have. Through his idea of the mathematical classification, then, Maxwell has thereby laid out the needed epistemological foundations for an *inductive* use of ‘correct’ physical analogies. The next sub-section will discuss this answer in more detail, paying particular attention to the nature of the ‘constraints’ that distinct mathematical classes identify.

5.2. The nature of the constraints

As a brief recap, according to our interpretative hypothesis the methodological papers of the early 1870s are the culmination of Maxwell’s long-standing concern with the epistemological foundations of the method of physical analogy – a concern that arguably traces back to FLF and that Maxwell developed in concomitance with his scientific works of the 1860s. This historical account, which is suggested in broad outline in Hesse (1974), finds significant support when considering the details of Maxwell’s claims about the mathematical classification, and specifically his distinction between a quantity’s merely being describable by a mathematical class and that quantity’s ‘really belonging’ to that class.¹⁷ On the view indicated above, determining the correct classification is important because mathematical classes tend to identify genuine respects whereby two physical systems can be ‘of the same kind’; accordingly, physical analogies that are ‘correct’ in ABS’s sense can be used inductively in physical investigation.

An important question that remains to be addressed concern the nature of the ‘constraints’ that mathematical classes identify, such that the ‘real belonging’ of some quantities to the same classes can be evidence of similarity in kind. While the analysis of Maxwell’s account suggested thus far has been roughly in line with Hesse’s, our respective answers to this

¹⁷ Some interpreters miss the centrality of this distinction by understanding the notion of two quantities belonging to the same mathematical class as equivalent to their being ‘isomorphic’ (see Cat, 2001:417). This obscures Maxwell’s point that two quantities can be isomorphic yet fail to ‘really belong’ to the same class. Cf. also Hon and Goldstein (2020), for whom MCQ ‘‘introduced some of the mathematical techniques that he later put to use in [the *Treatise*]’’ (165). This is a dubious reconstruction because most of MCQ’s distinctions were already present in FLF and PLF. There is more to MCQ’s concern than introducing mathematical techniques.

question diverge. On Hesse’s view, those constraints result from the broadly *dynamical* features of the domains being classified. As she writes: ‘‘The mathematical entities involved – scalars, vectors, forces, fluxes, translations, rotations – are not uninterpreted symbols but have at least a *spatio-temporal* interpretation which is identical in all physical systems to which they apply’’ (1974:93, our emphasis). Bokulich (2015) has recently embraced Hesse’s idea that the real belonging of the respective quantities to the same mathematical classes is an indication that the two domains are dynamically similar insofar as the mathematical classes carry with them ‘‘thin’’ physical interpretations’’ (34), where ‘‘the emphasis is ... on the general *dynamical* relations and properties, which can be instantiated in a number of different systems’’ (30, our emphasis).

A close look at the textual evidence shows that this specific view about the mathematical classes is only partially correct. Maxwell makes it clear that the notions of lines and surfaces, of translation and rotation are *dimensional* and *topological* notions, resulting purely from the features of the three-dimensional space in which (according to Maxwell) any physical system must be located. For instance, in defending his coinage of the term ‘curl’, Maxwell notes:

I have sought for a word which shall neither, like Rotation, Whirl, ... connote motion, nor, like Twist, indicate a helical or screw structure which is not of the nature of a vector. (II:265)

That the features being classified are sometimes purely dimensional and topological is clear from Maxwell’s noting that the mathematical classes for physical quantities that he had been able to list tended to make no reference to the passage of time. This is why Maxwell writes:

We may imagine another step in the advancement of science to be the invention of a method, equally appropriate, of conceiving dynamical quantities. (II:259)

Maxwell’s idea, then, is that to identify a given quantity with its correct mathematical class is to locate it within a relatively stable system of relations among quantities, which can sometimes be characterized purely in terms of their dimensional and topological features. Despite sometimes failing to individuate similarities in dynamical features, these identifications are still capable of making some physical analogies underwrite relatively strong inductive arguments. For instance, by determining that a given quantity in the target belongs to the class of vectors with curl equal to zero (so-called ‘irrotational vectors’) we may be led to infer an accompanying quantity in a larger vector field belonging to the class of vectors with divergence equal to zero (so-called ‘solenoidal vectors’). An argument of this kind (see also section 6.2) would be exploiting the fact that (*modulo* certain smoothness and decaying conditions) a vector field can be decomposed into the sum of an irrotational and a solenoidal component. Importantly, even though this fact is a mathematical theorem, the argument from the evidence of an irrotational vector to the postulation of an accompanying solenoidal vector is an inductive one, based merely on the expectation that the assumptions of the mathematical theorem that are already known to hold in a more familiar physical domain will also be observed to hold in the unfamiliar target.

In summary, we are in a position to appreciate how Maxwell’s account of the mathematical classification aims to vindicate the inductive use of physical analogy in his scientific works.¹⁸ The coherence of the epistemological account with Maxwell’s actual practice is remarkable. As discussed in the previous sections, his scientific works systematically invoke a physical analogy’s satisfaction of empirical and theoretical constraints as basis for its use in inductive argumentation. On the account that MCQ develops, the empirical and theoretical considerations function as

¹⁸ See also the discussion of ‘Helmoltz’s mistake’ in Bokulich (2015). Maxwell’s attitude is fully explicable on a double nature reading: which mathematical classes we attribute to physical quantities matters *inductively*.

reasons for identifying an unfamiliar target as belonging to a particular ‘kind’ of physical system; consequently, a mere “coincidence in the mathematical expression of two sets of phenomena” (I:488) can be turned into a source of relatively strong arguments about a given target. Such an inductive function can be fulfilled without making any physical hypothesis as to the mechanisms operating in the target, and therefore without the risk of incurring in that “blindness to facts or rashness in assumptions which a partial explanation encourages” (I:156).

Let’s now take a look at how this reading of Maxwell’s methodological project, which makes sense of such a large portion of his texts, also accounts for the use of physical analogy in PLF.

6. The displacement current

To summarize the interpretative hypothesis so far, a single method of investigation lies behind Maxwell’s major scientific works before the *Treatise*. As section four has shown, this view accounts for Maxwell’s use of physical analogy in the early work FLF (1855–6) as well as in the late DTE (1865). Moreover, as section five has urged, Maxwell’s remarks on the mathematical classification of physical quantities in the years 1870–72 can be understood as offering the outline of an epistemological account that vindicates the inductive use of physical analogy. If we can now link this account to Maxwell’s use of analogy in the central years 1860–62, which are commonly regarded as the years of his departure from the FLF method, we would have thereby established the double nature reading as a serious contender in Maxwell scholarship. In what follows, the focus will be on explicating the use of the molecular vortices analogy in PLF.¹⁹ Among other things, accounting for PLF’s reasoning to the displacement current and the subsequent identity of the optical and electromagnetic media is standardly regarded as a test for any interpretation of Maxwell’s method, which justifies the focus on this specific work.

6.1. The historical debate

Here is the episode in brief. Based on Faraday’s observation that a light beam entering a magnetic field is subject to rotation of its plane of polarization, PLF’s part I develops an analogy between magnetic action and a system of quickly rotating vortices (I:459–461). The need to account for the action of electricity leads him to imagine, in part II, a thin layer of particles sliding in the interstices between the vortices (I:469). In part III, Maxwell argues that the resulting mechanical construction must be endowed with a kind of elasticity to preserve motion. This leads to the introduction of the ‘displacement current’ (I:491). Although the name suggests an actual current, the formula that Maxwell used shows that he intended it as a measure of the rate of change of an electric field over time: to Ampere’s law $\text{curl } \mathbf{B} = 4\pi\mathbf{J}$, where \mathbf{B} is a magnetic field and \mathbf{J} a current, Maxwell adds $+\Delta\mathbf{E}/\delta t$ on the right-hand side, where \mathbf{E} is the electric field. The most striking consequence of this correction is that the velocity of propagation of the transverse waves in the medium proves to be so close to the measured speed of light that:

we can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena* (I:500)

The way in which Maxwell reaches the conclusion about the identity of the electromagnetic and optical media led [Duhem \(1902\)](#) and later [Chalmers \(1986\)](#) to contend that Maxwell had in fact worked out the equations already with an eye to reducing optics to electromagnetism; the analogy with molecular vortices was only a later addition used as a cover. However, historical analysis by [Nersessian \(1984\)](#) and [Siegel](#)

¹⁹ The double nature reading of PLF below will diverge significantly from [Hesse’s \(1974\)](#) reconstruction, whose brief disparaging remarks on the displacement current episode appear uncharacteristically rash.

(1991) has called into question these allegations. In particular, [Siegel \(1991\)](#) shows that the seemingly *ad hoc* assumptions about the molecular vortices were actually approximations of the values of some of the model’s parameters to probable values. The approximations concerned specifically the shape of the vortices:

The actual form of the [molecular vortices] probably does not differ from that of a sphere sufficiently to make much difference in the numerical result. (I:492)²⁰

[Siegel \(1991\)](#) admits that Maxwell had made an error in the calculations of the velocity of the elastic medium, but that he had probably been misled by expecting a simple relation between the two main quantities of the elastic medium he was describing (density and torsion modulus). On Siegel’s view, then, we can trust Maxwell when he claimed to have “worked out the formulae in the country before seeing Weber’s number [the measured speed of light]” (LP, 1:685).

While recent scholarship has departed from Duhem’s and Chalmers’ allegations, an open question remains about the function of the PLF model. It is significant that two of the most prominent interpreters, Nersessian and Siegel, diverge sharply on this issue.²¹ For Nersessian, PLF’s reasoning ultimately illustrates: “the heuristic value of [Maxwell’s] method. In his attempt to extend the analogy beyond his original intentions ... he was led to hope for – what he himself called ‘surprising’-results.” (1984:80). The reference is specifically to Maxwell’s conjecture that the electromagnetic and optical media are one. On Nersessian’s reconstruction, because “the value of the transverse velocity ... was determined *directly and only* from specific suppositions of [the molecular vortices analogy]” (2008:48), Maxwell must have realized that he had no ground to infer the identity of the electromagnetic and optical media: “however tempting it was, he did ‘avoid the inference’ ... Maxwell’s reticence reinforces my interpretation” ([Nersessian, 2008:48](#)).

[Siegel \(1991\)](#) takes the opposite position on what Maxwell meant by “*we can scarcely avoid the inference*” (I:500). He connects this passage to a letter to Faraday where he announces PLF:

This coincidence [between the velocity of transverse waves in the model and the speed of light] is not merely numerical ... I think we have now strong reason to believe, whether my theory is a fact or not, that the luminiferous and electromagnetic medium are one. (LP, 1:686)

Siegel’s explanation of Maxwell’s confidence is the discontinuity narrative: despite PLF’s talk of ‘imitation’ and ‘analogy’, Maxwell took the molecular vortices model as a *physical hypothesis*. On his view, Maxwell was certainly aware that other mechanical hypotheses were conceivable that could account for all the known electromagnetic facts. However, for Siegel Maxwell placed the hypothesis of molecular vortices above the rest as the result of its “simplicity” (1991:79) and its fulfilling various theoretical desiderata.²² Accordingly, by the time Maxwell had shown that all known electromagnetic phenomena could be deduced from his hypothesis, he could be sufficiently confident in it to infer the identity of the electromagnetic and optical media.

²⁰ Passages such as this lead [Siegel \(1991\)](#) to interpret the molecular vortices model as “a realistically intended theory of electromagnet[ism]” (83). On a double nature reading, instead, Maxwell needs his imaginary models to have at least some connection with real-world physical systems; it is for this reason that he cannot *assume* from the start that the vortices are spherical but has to regard it as an approximation. See also [Bokulich \(2015\)](#) on the ‘intermediary’ status of physical analogies – something in between pure geometry and physics.

²¹ The influence of [Siegel’s \(1991\)](#) reading is clear in other discontinuity theorists, e.g., [Harman \(1998\)](#), [Cat \(2001\)](#), [Hon and Goldstein \(2020\)](#). Below I will take Siegel’s view as the representative of this whole tradition.

²² Many thanks to an anonymous referee for indicating this solution on behalf of the discontinuity view to the problem (well-known to Maxwell; II:419) of under-determination of mechanical hypotheses by evidence.

The interpretative disagreement between Nersessian and Siegel exemplifies a general difficulty in reconciling Maxwell's texts with a credible epistemological story that can rationalize his scientific breakthroughs. Once the double nature of the molecular vortices analogy is recognized, however, the difficulty disappears: for it becomes possible to see how Maxwell could be legitimately confident in his identity claim (contra Nersessian) without having to in any way depart from the FLF methodology (contra Siegel). Unpacking this central claim of harmony between the text and the underlying epistemology is the task of the next subsection.

6.2. Harmony defended

In order to understand Maxwell's reasoning to the displacement current and the inferences he drew from it, we must start from a complete account of his problem situation. Nersessian (2008) offers a merely partial description of it when she writes that the “adaptation leading to [the postulation of elasticity, and thus to the displacement current,] arose from a constraint on the model as a mechanical system” (44). Siegel's (1991) reconstruction is, in this specific respect, more accurate (cf. also Cat, 2001:434). While conceding that the historical Maxwell might have first conceived of the solution by the study of his mechanical model, on Siegel's view Maxwell eventually came to recognize that the idea of the elasticity of the medium and the subsequent introduction of the displacement current were indicated by two separate directives, which we might call a broadly ‘empirical’ and a ‘theoretical’ one. Let's consider them in detail.

On the empirical side, Maxwell had two main pieces of data to work with. The first is that Ampere's law connecting currents with magnetic attractions and repulsions had been verified only with experiments on closed circuits; that there was an open question as to whether the law held more generally in open circuits as well had already been emphasized in FLF:

It is to be observed, that the currents with which Ampere worked were ... re-entering. All of his results are therefore deduced from experiments on closed currents (I:193)

The second piece of data was that, as shown in part I and II, the molecular vortices model could account for the phenomena of magnets and of induced currents; but electrostatics was left out:

If we can now explain the condition of a body with respect to the surrounding medium when it is said to be “charged” with electricity, ...we shall have established a connexion between all the principal phenomena of electrical science. (I:490)

The hypothesis that the electromagnetic medium had properties comparable to that of an elastic body must have struck Maxwell as a solution of impressive elegance; as he commented:

When we find electromotive force producing electric displacement in a dielectric [think of an insulator placed between two oppositely charged metal plates], and when we find the dielectric recovering from its state of electric displacement with an equal electromotive force, we cannot help but regarding the phenomena as those of an elastic body (I:492)

These empirical considerations were matched by equally pressing considerations on the theoretical side. Maxwell was, after all, seeking a physical analogy with a system that was not just consistent with the local action picture (as the FLF fluid analogy) but that had the feature of being mechanically conceivable. The idea of the molecular vortices that Maxwell borrowed from Thomson held promise as an account of Faraday's magneto-optic effect, which suggested that “some phenomenon of rotation is going on in the magnetic field” (LP, 1:670). Moreover, as Maxwell noted in his later ABS, the idea of vortices had strong appeal as a theory of matter:

In the vortex theory we have nothing arbitrary, no central forces or occult properties of any other kind. (II:223)

As Maxwell realized, if magnetic action is represented by molecular vortices, one of the few mechanically conceivable ways for the particles that stand for electricity to remain “in rolling contact” (I:489) with the vortices without loss of energy is for the medium to be in an elastic state. The analogy with the wave theory of light offered independent support for this idea:

The undulatory theory of light requires us to admit this kind of elasticity in the luminiferous medium ... We need not then be surprised if the magneto-electric medium possesses the same property. (I:489)

It is therefore likely that Maxwell regarded the solution of the elastic medium as a happy coincidence of the empirical and theoretical constraints that he had embraced at the very start of the PLF project. With the assumption of elasticity in the medium, he could define the accumulation of electrostatic charge by means of the mechanical model vortices, simply by taking electric charge to be equivalent to the excess density of the particle flux in the interstices of the magnetic vortices. In this way, practically all the known phenomena of electromagnetism could be accounted for. At the same time, the endowed elasticity also solved the problem of the mechanical conceivability of the system of vortices, showing that contribution of electric currents could be factorized into a *rotatory* (curl B) component and a *linear* component ($\Delta E/\delta t$) analogous to the contribution of other forces in familiar physical sciences.²³ As Siegel (1991) notes, these considerations generated in Maxwell the impression of “mutual reinforcement” (78), making the displacement current a non-arbitrary extension of the molecular vortex analogy.

An important difference with Siegel's (1991) reconstruction remains. On his view, the introduction of the displacement current takes the form of an auxiliary hypothesis that, together with the postulation of the molecular vortices as the material substratum of the electromagnetic medium, form a “consistent and realistically intended theory of electromagnetic phenomena” (83). This is consistent with Siegel's discontinuity narrative, which sees PLF as employing a hypothetical methodology as substitute for FLF's modest analogical approach. On the double nature account, instead, there is no need to see a discontinuity with the FLF method: the introduction of the displacement current takes the form of an extension of an imaginary model that Maxwell was partial to on independent grounds (e.g., its compatibility with “the law of conservation of energy”, as I:488 clarifies), and that had already been shown capable of naturally “imitating” the phenomena of magnetism and of induced currents. This way of extending an imaginary model to recover portions of the available experimental evidence, while refraining from invoking *ad hoc* hypotheses or “adding anything to that which has ... been proved by experiments” (I:159), is continuous with FLF's mode of physical investigation (see 4.1).

The status of the PLF model is accordingly different from the one that Siegel (1991) attributes. On the double nature view, by adding the term $+\Delta E/\delta t$ to Ampere's law in PLF, Maxwell did not mean to literally postulate a displacement current in the electromagnetic medium; rather, he was extending an independently plausible model to electrostatics

²³ Zapolski (1986) asks why Maxwell included *both* a curl-free and a divergence-free component of E in the definition of the displacement current $\Delta E/\delta t$ rather than just the curl-free component of E (which would have alone sufficed to restore charge conservation in Ampere's law). As Zapolski writes, this move was Maxwell's “true stroke of genius” (1140). It is plausible that Maxwell was motivated by the same considerations of the methodology of analogy via mathematical classes discussed in section five, which led him to suppose a linear component whenever a rotational component was postulated, and vice versa. See also Maxwell's point in Part IV of PLF: “the connection between electricity and magnetism has the same mathematical form as that between certain pairs of phenomena, of which one has *linear* and the other *rotatory* character” (I:503).

while taking the displacement current as a place-holder for whatever force is the ‘realizer’ of the term $+\Delta E/\delta t$. In a sense, there was no *need* for Maxwell to go beyond the physical analogy: once all the electromagnetic facts were shown to be appropriately “imitated” (I:488) by the analogy with an elastic medium composed of invisible molecular vortices, Maxwell could be content with the realization that the PLF molecular vortices analogy was (to put it in the language that he later developed) likely “correct”; and therefore that, whatever its exact features, the material substratum of electromagnetic phenomena was likely defined by quantities that at least belonged to the same “mathematical classes” as those instantiated by the quantities in the model.

That the PLF model was set apart as a “correct” analogy arguably explains why, when Maxwell realized that the velocity of propagation of waves in the imaginary medium is the same as the measured velocity of light, he could “scarcely avoid the inference” (I:500) about the identity of the electromagnetic and optical media. On the double nature reading, this is because the analogical inferences drawn from his molecular vortices model come already equipped with a degree of inductive support that the predictions of rival models do not possess. This is why, if an arbitrary rival model that accommodated all the known facts about electromagnetism had returned a number for one of its parameters which was close to the measured velocity of light, we would have had all the reasons to suspect that the coincidence was “purely numerical” (LP, 1, 686). However, when a derivation from the molecular vortices model returns such a surprisingly close number, the numerical similarity becomes *evidence* that there exists a real connection. Once we grasp the modelling process whereby the molecular vortices analogy (in light of considerations of both the theoretical and of the experimental kind) gradually emerges as the favored physical analogy, it becomes clear that Maxwell’s inference to the identity of the media is a legitimate result of the method of physical analogy that he had defended elsewhere.

The double nature reading is in a privileged position to explain why, in announcing his groundbreaking PLF results in correspondence with Faraday, Maxwell wrote that:

I think we have now strong reason to believe, *whether my theory is a fact or not*, that the luminiferous and electromagnetic medium are one. (LP, 1:686, our emphasis)

Maxwell’s expression “whether my theory is a fact or not” signals that, *pace Siegel (1991)*, the credibility of the result about the velocity of the transverse waves was independent of any commitment as to the realistic status of the molecular vortices. Indeed, so long as considerations of the empirical and theoretical kind were telling in favor of the physical analogy, that velocity had a special claim to be the analogue of some actually existing quantity in the electromagnetic target. That the method of PLF was analogical rather than hypothetical therefore explains why novel inferences about electromagnetism could be drawn from the molecular vortices analogy without thereby making any epistemic commitment as to the realistic status of the model.

The naturalness with which the double nature reading accounts for Maxwell’s comments in (and about) PLF puts pressure on the claim that “he appealed to a hypothetical ontology that supports causal argumentation with the goal of explaining the phenomena” (Hon & Goldstein, 2020: 99). Even though the goal has shifted to providing a micro-level mechanical perspective on electromagnetism, the epistemological problem of “being carried beyond the truth by a favorite hypothesis” (I:156) that Maxwell had denounced in FLF remains. It is therefore far from clear that, when he turned to PLF’s project, Maxwell thereby saw the need for a change in approach. If anything, the logic whereby shifting to a mechanical perspective requires a shift to hypothetico-deductivism may well obscure the main methodological lesson that Maxwell wanted to emerge from PLF: that the method of analogy “borrow[ed] for a season” (LP, 1:306) from Thomson in FLF offered a much safer route to micro-level mechanical theories than the bold hypothetical methodology that Thomson had turned to in his later years. Recognizing the double

nature reading as a serious interpretative hypothesis may therefore be crucial to rescuing the originality of Maxwell’s position in the methodological debate of which he was part.

In summary, the attribution of an inductive function to the molecular vortices analogy in PLF offers a remarkably coherent reconstruction of Maxwell’s reasoning to the displacement current and the subsequent reasoning to the identity of the optical and electromagnetic media, one that weaves together and explains some otherwise puzzling remarks by Maxwell on PLF’s aims and achievements. Looking at PLF through the lens of a double nature interpretation vindicates some important insights behind Nersessian’s (1984, 2008) and Siegel’s (1991) accounts. In particular, the proposed reconstruction agrees with Nersessian’s point that PLF is methodologically continuous with FLF; and it agrees with Siegel’s claim that Maxwell took the PLF derivations as reasons to believe in the identity of the electromagnetic and optical media. At the same time, the historical picture of Maxwell that emerges from the double nature reconstruction is one of a substantially more coherent author, both synchronically and diachronically, than the rival accounts by Nersessian or Siegel. The next section will return to this idea of underlying coherence, proposing it as a guiding principle for a new wave of historical work on Maxwell.

7. Conclusion

A large portion of Maxwell scholarship has insisted that some of Maxwell’s major scientific breakthroughs were not reached by the method of physical analogy. The discontinuity narrative whereby PLF and DTG employ the hypothetical methodology criticized in FLF is only a recent example of this trend. It supersedes another long tradition, which traces back to Duhem (1902), aiming to show that Maxwell’s main electromagnetic results had been obtained through an adventurous use of the “mathematical method”, also criticized in FLF. On the Hesse-inspired view defended in this paper, however, it is a mistake to reduce Maxwell’s analogical approach to a heuristic method that would satisfy philosophical preconceptions. To grasp the true nature of Maxwell’s physical analogies, one must recognize that they possess a distinctive inductive function. The most striking feature of this reading is the naturalness with which it shows that Maxwell’s use of physical analogy in practice is consistent with his methodological remarks. This fact alone puts the proposed reconstruction, even in the broad outline offered here, above those noticeably more technical discussions of Maxwell’s work that, imbued with preconceptions about the role of analogy in physical inquiry, have often raised more questions than they solve.

Of course, to say that FLF, DTG, PLF and DTE employ a single method of physical analogy is not to say that they share some precisely describable, algorithmic procedure of reasoning. Except for the fact that it exploits resemblances with another domain and that it avoids the use of physical hypotheses, Maxwell’s analogical method remains a process more akin to the implementation of an artistic technique, whose mastery depends upon grasping its use in specific examples. In particular, the textual evidence shows that Maxwell intended his method to be applicable at very many levels of physical inquiry: at the macro-level, as in FLF’s geometrical representation of the lines of force and in DTE’s conception of the dynamical evolution of the electromagnetic field, as well as at the micro-level, as in PLF’s and DTG’s mechanical outlooks.²⁴ The physical analogies in each paper differed significantly and provided interestingly different perspectives on the phenomena under investigation. However, the multiplicity of perspectives does not affect the method’s underlying unity and generality. Indeed, as Maxwell notes, again emphasizing the credentials of the method for producing knowledge:

²⁴ See also Maxwell’s distinction between a “statical”, a “mechanical” and a “dynamical” perspective on physical phenomena in PLF (I:472). The distinction resurfaces in later works and notes (e.g., II:326-8).

When examples of this method of physical speculation have been properly set forth and explained, we shall hear fewer complaints of the looseness of the reasoning of men of science, and *the method of inductive philosophy* will no longer be derided as mere guess-work (II:420).

Viewing the historical Maxwell as engaged in a long-term project of developing a method alternative to the hypothetico-deductive orthodoxy makes him a far more original and systematic methodologist than predominant reconstructions have so far allowed. It can also be useful to guide new historical investigations into specific aspects his work. An especially important question in this regard concerns the relation between Maxwell's methodology and his physics. One of the "perennial issues in Maxwell scholarship", as Siegel (1991) writes, concerns "the unity and coherence of Maxwell's mechanical and mathematical formalisms" (168). In the case of Maxwell's work in electromagnetism, the discontinuity narrative is typically accompanied by the claim that the development of Maxwell's field theory "involved the formulation of a series of differing versions of the theory, each internally coherent" (Siegel, 1991:170) but ultimately inconsistent with one another. According to this traditional reconstruction, the differing "versions" of the electromagnetic theory replaced one another in both content and methodology until the definitive formulation of the *Treatise*. Will this account survive historical scrutiny once the accompanying discontinuity narrative is put into question? Further historical research into this and other directions may reveal a new picture of Maxwell's work and accomplishments.

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