1941ApJ....94..232S

THE DYNAMICS OF THE INTERSTELLAR MEDIUM

II. RADIATION PRESSURE

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ABSTRACT

Radiation pressure will repel dust particles from a late-type star of high luminosity in the manner discussed by Schalén, while the atoms will be relatively unaffected by the encounter. Near a star of early type, however, the hydrogen is ionized, and the large interaction between dust and protons will force the interstellar medium to behave as a uniform homogeneous gas, provided that the velocity of the star is not much greater than 10 km/sec. The radiation pressure on dust near such a star will therefore tend to repel all the constituents of the medium, though the effect is considerably less than would appear if only dust were present. The pressure of radiation on atoms is less important; the radiation pressure per unit mass on interstellar atoms other than hydrogen is shown to be less, on the average, than the corresponding force per unit mass on dust. In the case of hydrogen the scattering of *La* radiation is so great that a quantum of *La* will diffuse slowly through space and will be absorbed by a dust particle before it has gone more than a fifth of a parsec; radiation pressure on hydrogen atoms is therefore negligible. It follows that for any one star the radiation pressure on the dust particles in a unit volume is much greater than that on all the atoms in the same volume.

The direct effects of the general galactic radiation field are small. The radiative force on dust particles in the galactic plane will produce a slight difference between the medium as a whole and the stars in the net attraction of each toward the galactic center, the difference in equilibrium rotational velocities amounting to not more than a few tenths of a kilometer per second in the neighborhood of the sun. If the short time scale is assumed, interaction between dust and atoms will prevent any sorting-out of the different constituents of the medium under the pressure of radiation either parallel or perpendicular to the galactic plane. The galactic radiation field is indirectly responsible for a force between two dust particles. The absorption of radiation by two particles will reduce the energy density between them; radiation pressure will therefore push the particles toward each other with a force which varies as the inverse square of the distance between them. For *Fe* particles with a radius of less than 10^{-5} cm. this force is more than two hundred times the mutual gravitational attraction of the two particles.

An investigation of the physical conditions of interstellar matter[1] shows that dust particles, in general, are negatively charged, that equipartition of kinetic energy exists between the dust and atoms of the medium, and that the medium behaves as a perfect fluid in its flow past stars. In this paper the effects of radiation pressure on interstellar matter are examined. A detailed quantitative investigation is not desirable at the present time, since there is so little observational material with which theoretical results may be compared. It is valuable, however, to investigate what phenomena in this connection may be important.

The effects produced by radiation pressure may be divided into two classes: (1) the interaction between interstellar matter and single bright stars and (2) the influence of the general galactic radiation field. These two topics are discussed in the first and second sections, respectively, of the present paper. Section 3 presents an analysis of the average force between dust particles which results from the presence of the general galactic radiation field.

1. SINGLE STARS

The radiation pressure on dust particles in the neighborhood of a bright star has been examined by several authors,[2] who have suggested that the high ratio of radiation pressure to gravity for the B and O stars would blow the lighter dust particles away from

[1] L. Spitzer, Jr., *Ap. J.*, **93**, 369, 1941. This paper will hereafter be referred to as paper I.

[2] E. Schoenberg and B. Jung, *A.N.*, **247**, 413, 1932; B. P. Gerasimovič, *Zs. f. Ap.*, **4**, 265, 1932; J. Greenstein, *Ap. J.*, **85**, 242, 1937.

the neighborhood of such stars. Schalén[3] has extended the analysis to show that radiation pressure sets a limit to the distance within which a dust particle may approach a luminous, moving star. For the early B and O stars this limiting distance, which we shall denote by r_1, may amount to 10 parsecs or more, but for other stars the effect is small.

These analyses neglect the fact that, if the abundances assumed in paper I are correct, the dust velocities during an encounter will be influenced by the atoms present. In fact, if the velocities and external forces are not too great, atoms and dust will behave as a uniform gas, with the dust diffusing only slightly through the atoms. For changes which occur sufficiently rapidly, however, the dust and atoms will act independently.

It is desirable to have some rough criterion as to whether the atoms and dust particles behave independently, or not, in any particular case. For this purpose we may consider a simplified one-dimensional problem in which dust and atoms, moving initially with a velocity v, stream directly toward the center of a motionless star. The dust particles will be acted upon by two forces. The first tends to reduce the flow of dust through the atoms; we shall call this an "interaction force." The second is the external force on dust arising from the gravitational attraction of a star or from the pressure of stellar radiation.

The interaction force may be computed from the analysis of equipartition of kinetic energy given by the author[4] and summarized in paper I. Let V be the velocity of a dust particle in the frame of reference in which the atoms have no net momentum. If V is much greater than the equilibrium root mean square velocity of dust particles, and if other forces are absent, the kinetic energy $\frac{1}{2}m_d V^2$ of the dust particles will, on the average, decrease to $1/e$ its initial value in a time t_e, where t_e is the appropriate value for the time of equipartition;[5] m_d is the mass of a dust particle. Since $\frac{1}{2}m_d$ is constant, it follows that V^2 equals its initial value multiplied by exp $(-t/t_e)$, and dV/dt is equal to $-V/2t_e$. This represents the decrease of the scalar velocity of the dust particle, which in the general case may differ from the acceleration in any particular direction, since deflections may be important. It is clear physically, however, that a heavy, rapidly moving particle will tend to be slowed down, rather than deflected, by encounters with lighter particles; in such a case the loss of momentum results primarily from the loss of kinetic energy. The acceleration of the particle toward the star is therefore equal to the scalar acceleration found above; and the interaction force, which is the product of mass and acceleration, becomes $-m_d V/2t_e$. The external force at a distance r from a star of mass M is simply $(\alpha - 1)GMm_d/r^2$, where α is the ratio of radiation pressure to gravity; we shall assume that α is greater than unity, since this is the case of primary interest.

It is clear that dust and atoms will react independently with a star if during most of the encounter the external force on the dust particles is much greater than the interaction force, i.e., if for an appreciable range in r,

$$\frac{V}{2t_e} < \frac{(\alpha - 1)GM}{r^2} . \tag{1}$$

To apply this criterion to a particular encounter we first assume that the interaction forces are negligible and that each dust particle acts independently of the rest of the medium. The value of V as a function of r may then be calculated. If the V so computed satisfies inequality (1), for the relevant range in r, then the initial assumption that dust and atoms act independently has been verified.

[3] *Zs. f. Ap.*, **17**, 260, 1939.

[4] *M.N.*, **100**, 396, 1940.

[5] This result was proved initially on the assumption of Maxwellian velocity distributions for all particles. It is readily shown that this same result holds for the loss of energy by heavy particles of any velocity V, provided only that V is much less than the root mean square velocity of the lighter particles.

The quantity V is the difference between the velocity of the dust particles calculated from the usual energy equation and the velocity of the atoms relative to the star, and we must make some assumption about this latter velocity to test the validity of inequality (1). As we shall see below, the atoms are, on the average, less affected by an encounter with a star than are the dust particles, and to maximize V we shall assume that they are in fact completely unaffected. The velocity of the atoms relative to the star is therefore equal to v, the velocity of the medium relative to the star when r is very large.

The analysis is simplest when r is equal to r_1, the distance of closest approach if the interaction force is negligible. At this point the initial kinetic energy of a dust particle is converted wholly into potential energy, and we have

$$\tfrac{1}{2}v^2 = \frac{(\alpha - 1)GM}{r_1} . \tag{2}$$

At this point of closest approach the velocity of the dust is zero; but since the atoms are assumed to be moving with the initial velocity v relative to the star, the velocity V of the dust particles relative to the atoms will equal v. If we set r equal to r_1 and V equal to v in inequality (1), and use equation (2) to eliminate $(\alpha - 1)GM$, we find

$$r_1 < vt_e . \tag{3}$$

Since inequality (1) must hold for larger values of r as well, if the dust and atoms are to behave independently throughout most of the encounter, and since V does not decrease so rapidly as $1/r^2$, we conclude that r_1 must be considerably less than vt_e if the interaction forces are to be negligible.

The value of t_e to be used in inequality (3) depends on the state of ionization of hydrogen. Near late-type stars the hydrogen is neutral, and the value of t_e computed in paper I for such a case is 4×10^5 years. If v is 10 km/sec, vt_e equals 4 parsecs, which is greater than the values of r_1 for most such stars. We may infer that dust and atoms interact independently with these stars and that in this case the Schalén analysis holds.

For early-type stars, however, the values of s_0, the radius of an H II region of ionized hydrogen,[6] are greater for the most part than the values of r_1. Hence for t_e we must take 10^4 years, the value relevant to H II regions. If v is again 10 km/sec, vt_e is now a tenth of a parsec. The values of r_1 calculated by Schalén for these stars are all greater than this; it follows that for early-type stars the Schalén analysis is not applicable. For stars of sufficiently high velocity,—35 km/sec or more—dust and atoms will again become independent; the value of r_1 in such a case is only a tenth, or less, of the values given by Schalén, and the effect is therefore not important. On the other hand, the value of t_e used is an upper limit, since dust particles with a radius of less than 10^{-5} cm and with a density of less than 7.8 gm/cm^3 will come into equipartition with atoms more rapidly than has been assumed here.

Where the repulsive force of radiation is most important, therefore, the dust and atoms may be assumed to behave as a single gas. The dust will tend to diffuse slowly through the atoms, but the velocity V of diffusion will be small, in general, compared to the velocity v of the star relative to the medium.

To prove this we consider the factors determining V. This relative velocity will tend to increase to the point at which the interaction force nearly balances the external force. Hence V/t_e will be very nearly equal to $2(\alpha - 1)GM/r^2$; if we use equation (2) to eliminate $(\alpha - 1)GM$ as before, we see that V/v equals $vt_e r_1/r^2$. If vt_e/r_1 is small, V/v will therefore also be small, unless r is much less than r_1; it is physically obvious that when r is sufficiently small the atoms and dust will separate to a large extent. If diffusion is negli-

[6] B. Strömgren, *Ap. J.*, **89**, 526, 1939.

gible when r is equal to r_1, however, one may safely neglect diffusion in discussing the major effects involved in the flow of interstellar matter around stars. If the diffusion of dust through atoms is small, the diffusion of atoms of different kinds through one another will be even smaller, since the value of t_e for atoms is much less than for dust particles.

When vt_e/r_1 is small, therefore, we may, for the most part, treat the interstellar medium by the usual methods of aerodynamics. The force on each volume element will be the resultant of the gravitational attraction and the radiation pressure, summing this over all types of particles. The flow of a compressible gas around a body which attracts or repels the gas with an inverse-square force has never been analyzed in detail, but certain general conclusions may be drawn from the magnitude of the forces involved.

The value of the gravitational attraction is readily computed; the radiation pressure on dust particles has already been analyzed in considerable detail. The radiation pressure on atoms, which has not been thoroughly investigated in this connection, may be shown to be unimportant. Let us compute the ratio of the radiative force per unit mass on an atom to the corresponding force per unit mass on a dust particle. The force per unit mass will be called a "specific force," and the ratio between the two specific radiative forces on atoms and on dust will be denoted by ξ. Let subscripts a and d refer to the atom and dust particle, respectively.

For a spherical dust particle of radius σ at a distance r from a star whose luminosity is $L(\nu)d\nu$ in the frequency interval $d\nu$, the force of radiation F_{rd} is given by

$$F_{rd} = \pi\sigma^2 \int_0^\infty \kappa Q(\nu) \frac{L(\nu)}{4\pi r^2 c} d\nu , \tag{4}$$

where c is the velocity of light. The efficiency factor $Q(\nu)$ is the ratio of the energy scattered or absorbed to that which is incident on the geometrical cross-section $\pi\sigma^2$; for a large opaque particle $Q(\nu)$ is unity. The quantity κ is the ratio of the momentum received by the particle to the momentum of the radiation scattered or absorbed. For absorbing particles κ is clearly unity, while for scattering particles κ depends on the phase function. Since κ probably does not vary widely, we shall set it equal to unity.

The mass m_d of a dust particle is $4\pi\sigma^3 d/3$, where d is the density of matter within the particle; from equation (4) we therefore find

$$\frac{F_{rd}}{m_d} = \frac{3L\bar{Q}}{16\pi\sigma d r^2 c} , \tag{5}$$

where L is the total luminosity of the star in ergs per second and

$$\bar{Q} = \frac{1}{L} \int_0^\infty Q(\nu) L(\nu) d\nu . \tag{6}$$

For an atom, on the other hand, the force of radiation is given by the relationship

$$F_{ra} = \int_0^\infty a_\nu \frac{L(\nu)}{4\pi r^2 c} d\nu , \tag{7}$$

where a_ν is the absorption coefficient per atom. In interstellar space only the ground state is populated, and, in general, only a few ultimate lines will contribute significantly to F_{ra}. We shall consider only the lowest ultimate line—that of longest wave length—and shall assume that this has an oscillator strength of unity; this maximizes F_{ra} in general. If $L(\nu)$ is assumed to change but slightly over the line, and a_ν is replaced by its integrated lue $\pi e^2/m_e c$, where m_e is the mass of the electron, the specific radiation force on an atom comes

$$\frac{F_{ra}}{m_a} = \frac{e^2 L(\nu)}{4 m_e m_a r^2 c^2} . \tag{8}$$

ally, if we divide equation (8) by equation (5) and use the expressions of Planck and an in the ratio $L(\nu)/L$, we have

$$\xi = \frac{20 e^2 \sigma d}{\pi^3 m_e m_a c \nu \bar{Q}} \left[\left(\frac{h\nu}{kT}\right)^4 \frac{1}{\exp \frac{h\nu}{kT} - 1} \right] . \tag{9}$$

The assumption that the energy distribution of stellar radiation corresponds to black-body radiation at the effective temperature of the star is not accurate in detail but should provide an adequate approximation. The neglect of stellar absorption lines again increases ξ.

To maximize ξ we may replace the bracketed expression in equation (9) by its maximum value, 4.78, and let d equal 7.8, its value for *Fe*. Then, if we express ν in terms of the corresponding energy in electron volts, which will be denoted by E_ν, and let m_a equal Am_0, where m_0 is the mass of unit atomic weight, we have, finally,

$$\xi < 5.1 \times 10^8 \frac{\sigma}{A E_\nu \bar{Q}} . \tag{10}$$

Schalén[7] has computed extensive tables of a quantity $\psi_\lambda(2\sigma)$, which in the present notation is $Q(\nu)/8\pi\sigma$. When σ is equal to or less than 10^{-5} cm it is evident from Schalén's graphs that $\psi_\lambda(2\sigma)$ is greater than 10^4 for wave lengths shorter than 4000 A. For stars of type A or earlier most of the radiation lies to the violet of λ 4000, and we may adopt 10^4 as a lower limit for $\psi_\lambda(2\sigma)$ in such cases. Equation (10) then becomes

$$\xi < \frac{2.0 \times 10^3}{A E_\nu} . \tag{11}$$

For particles of lesser σ or d, ξ will be even smaller than the value given by equation (11). Dust particles of much larger radius cannot be very abundant and are presumably of small importance.

For long wave lengths $\psi_\lambda(2\sigma)$ varies primarily as $1/\lambda$; hence for stars of even the latest spectral type the average value of ψ should not be less than 10^3. On the other hand, the bracketed expression in equation (9) is very small for these cool stars, since almost all ultimate lines lie relatively far in the ultraviolet. If, for instance, T is less than 4000° and ν corresponds to a wave length of less than 3500 A, this bracketed term will be less than one-tenth the maximum value which was used to derive equation (11). We may therefore assume that equation (11) is correct for all stars.

From equation (11) we may evaluate the relative importance of radiation pressure on atoms other than hydrogen. The assumption will be made that the total mass abundance of atoms other than hydrogen in interstellar space is not greater than the mass abundance of dust. Let $\bar{\xi}$ denote an average value of ξ over all such atoms, each atom being weighted in accordance with its mass abundance.

[7] *Uppsala Ann.*, **1**, No. 2, 1939; Schalén and Wernberg, *Arkiv för Math., Astr., och Fysik*, **27A**, No. 26, 1941. (Uppsala Medd., No. 83.)

The value of ξ depends on whether or not the hydrogen is ionized. In H II regions atoms will be highly ionized, and the energy of even the lowest ultimate lines will be e.v. for the most part. The atomic weight A is also greater than 10 for all but the light atoms, and $E_\nu A$ should therefore much exceed 10^2 for all but occasional neutral or sin ionized elements. Hence the value of ξ averaged over all atoms other than hydrogen certainly be less than 10.

In H I regions the lighter elements of ionization potential greater than 13.5 volts O, F, and Ne—will be neutral, which will decrease E_ν for these atoms and tend to incre somewhat. But the elements of lesser ionization potential will become more highly ized, owing to the decrease of the number of free electrons per cm^3 when H is neu This should largely offset any increase in $\bar{\xi}$. It follows that $\bar{\xi}$ in any case is less than

Most of the approximations made in evaluating $\bar{\xi}$ have maximized this quantity. particular, the bracketed expression in equation (9) will usually be much less than t value assumed. This quantity is obviously a maximum when E_ν for the ultimate lin is approximately the energy $E_{\max}$ at which the radiation per unit frequency from the star is a maximum. But the electron density of interstellar space is so low that most atoms will be ionized to the point at which their next ionization potential, E_i, is very much greater than $E_{\max}$. Since the energy absorbed in the lowest ultimate line is never less than $E_i/5$ and is usually greater, E_ν will also be considerably greater than $E_{\max}$, on the average. For many elements ξ will therefore be very small, particularly for matter which is relatively close to a star.

The most serious approximation here is perhaps the neglect of a possible excess of ultraviolet radiation, of the type observed for the sun. Such a deviation from black-body radiation will not affect these results, however, unless a very considerable proportion of the energy of the star appears in the ultraviolet. The bracketed term in equation (9) has been substituted for the dimensionless quantity $\nu L(\nu)/L$; when ν is much greater than $\nu_{\max}$, this quantity will be small unless an appreciable fraction of the total luminosity L appears in radiation of frequency much greater than $\nu_{\max}$. Although the flux of solar radiation in the far ultraviolet is enormously greater than one would expect from a black body at the solar effective temperature, it is relatively small compared to the total solar luminosity. If this result is valid for other stars as well, the presence in stellar spectra of marked deviations from the Planck formula should not affect appreciably the foregoing results. We may therefore conclude that $\bar{\xi}$ is actually not greater than unity, at most, and is probably considerably less.

Hence it is evident, if we remember the definition of $\bar{\xi}$, that the total force of radiation on a gram of atoms (other than hydrogen) is at most equal to the corresponding force on a gram of dust particles. Since the interstellar density of all such atoms has been assumed to be somewhat less than that of dust, it follows that the total force of radiation on all such atoms in a unit volume of interstellar space is no greater than the corresponding force on the dust particles in the same volume.

To complete the analysis, the force on hydrogen must also be considered. For free protons and electrons the influence of radiation pressure is clearly small. In H II regions the relative ionization is so high, on the average, that any radiation forces on the hydrogen in such regions may be neglected.

In regions of neutral H, however, a separate analysis is necessary, since from equation (11) we see that ξ for H atoms may be as great as 10^2. In addition, the intensity of $L\alpha$ radiation is much increased by the absorption of ultraviolet radiation. As in the planetary nebulae, every quantum of radiation beyond the Lyman limit emitted by a star will produce one quantum of $L\alpha$, together with other quanta of the higher series.

The very large radiation pressure which one might expect from this source in all H I regions will be eliminated by the presence of absorbing dust particles. The optical depth in $L\alpha$ in a single parsec is roughly 6.9×10^4 if one H atom per cm^3 is assumed, together with a flat-topped profile corresponding to a Doppler broadening of ± 30 km/sec. A quantum of $L\alpha$ will therefore diffuse very slowly through the interstellar medium and will be absorbed by a metallic particle before it has gone very far.

Let us consider more quantitatively the flow of radiation outward from a spherical source of radius s_0. The source is simply the H II region surrounding the star; the total luminosity of the source in $L\alpha$ radiation may be taken as given. Scattering by neutral atoms in H I will give rise to a large mass scattering coefficient l_ν; a small absorption coefficient k_ν will arise from the presence of dust particles. If the relative change of r—the distance from the star—with increasing optical depth in the line is small, the Eddington approximation is legitimate, and the equation of transfer in a spherical system gives[8] in the usual notation

$$\frac{1}{r^2}\frac{d}{d\tau_\nu}\left(r^2\frac{dJ_\nu}{d\tau_\nu}\right) = q^2(J_\nu - B_\nu) , \tag{12}$$

where

$$d\tau_\nu = k_\nu \rho dr ; \tag{13}$$

ρ is the density of interstellar matter, and, since ϵ is negligible,

$$q^2 = 3\left(1 + \frac{l_\nu}{k_\nu}\right) . \tag{14}$$

The quantity τ_ν is not the optical depth in the line but only that very small fraction of the optical depth which arises from true absorption. For convenience we shall take τ_ν equal to zero at the boundary of the H II region, when r equals s_0. Contrary to the usual convention, τ_ν here increases with increasing r.

In the present case the re-emission from the dust particle corresponds to a temperature of a few degrees absolute, and B_ν may be neglected. If we also assume that l_ν, k_ν, and ρ are constant, equation (12) has the solution

$$J_\nu = \frac{Ae^{-q\tau_\nu}}{r} , \tag{15}$$

where A is a constant.

To determine A we have the condition that when r equals s_0, the radius of the H II region, the total flux $4\pi r^2 F_\nu$ must approach $L(\nu)$, the luminosity of the source in the frequency ν. We find that F_ν is given by

$$F_\nu = \frac{L(\nu)}{4\pi r^2} e^{-q\tau_\nu}\left(\frac{1 + qk_2\rho r}{1 + qk_2\rho s_0}\right) . \tag{16}$$

We may define the total optical depth t_ν in the line by the relationship

$$dt_\nu = \rho(l_\nu + k_\nu)dr . \tag{17}$$

If we assume again that l_ν, k_ν, and ρ are constant and that both optical depths are zero when r equals s_0, then we have from equations (13) and (14),

$$q\tau_\nu = (3\tau_\nu t_\nu)^{1/2} . \tag{18}$$

In the present case k_ν is very much less than l_ν and may be neglected in equation (17).

[8] S. Chandrasekhar, *M.N.*, **94**, 444, 1934, eq. (83).

The value of F_ν at a distance of 1 parsec from the boundary of the H II region may be determined directly from equations (16) and (18). The optical depth t_ν corresponding to this distance will be 6.9×10^4 as before. The corresponding optical depth for extinction (scattering and absorption) by interstellar dust will be roughly 10^{-3}, giving a moderate absorption of 1 mag/kpc. The albedo of interstellar particles—the ratio of pure scattering to the total extinction—is somewhat uncertain,[9] but the most probable value is[10] about 0.5, giving equal amounts of scattering and absorption. At a distance of 1 parsec from the region of ionized hydrogen the value of τ_ν is therefore 5×10^{-4}, and from equation (18) we see that the corresponding value of $q\tau_\nu$ equals 10. At this distance the value of F_ν found from equation (16) is quite negligible. Considerable variation in the density of dust is possible without affecting this result. A more complete analysis, taking into account the decrease of $L(\nu)$ through absorption of radiation by dust particles within the H II region and the increase of $L(\nu)$ by fluorescent processes, leads to an even smaller value of F_ν for a given r. It is evident that the effect of radiation pressure on hydrogen, neutral or ionized, may be neglected.

From the foregoing analyses it follows that the radiation pressure on dust particles is clearly more important than that on atoms. Even if ξ is as great as unity for atoms other than hydrogen, the assumed mass abundance of hydrogen is sufficiently great so that the final value of the specific radiation pressure for all interstellar atoms is considerably less than the corresponding quantity for dust particles. This varifies the assumption, which was made above, that, when the dust and atoms act separately, the atoms will be less influenced by the encounter than will the dust.

Hence, late-type supergiant stars will repel dust particles without affecting the distribution of atoms appreciably. Some distance from such a star the dust particles which have been repelled from the star will be slowed down by the atoms, giving up their momentum to the medium. To this extent the conclusion at the end of paper I must be modified, since if a star is surrounded by an H I region, the interaction of dust and atoms will not appreciably affect the flow of atoms around the star. A supergiant M star will therefore impart momentum to the medium even if its velocity through the medium is less than 10 km/sec. If its velocity is as low as 3 or 4 km/sec, however, vt_e/r_1 will be small, since this quantity varies as v^3; in such a case the dust and atoms will not separate appreciably and the medium will receive no net momentum.

For early-type stars, on the other hand, the dust and atoms will behave as a homogeneous gas. The repulsive force on the dust particles will tend to repel the entire interstellar medium from stars whose luminosity is sufficiently high. Both the high random atomic motions and the mass abundance of hydrogen—presumably somewhat greater than that of dust—will tend to reduce the effective mean distance of closest approach for the medium as a whole to a value below r_1, the distance of closest approach if dust alone were present. Observational evidence on the distribution of interstellar matter near bright stars would make possible an estimate of the relative abundances of dust and atoms.

2. THE GALACTIC RADIATION FIELD

The distant stars of the Milky Way produce a moderately smooth radiation field which exerts an appreciable pressure on interstellar particles. This galactic radiation field has been treated in detail by Greenstein,[11] who finds that it is much more nearly homogeneous in galactic longitude than one would expect from the assumed distribution of mass in the Galaxy. This effect results from heavy absorption in the direction of the galactic center; this absorption reduces the resultant force of radiation on dust particles

[9] O. Struve, *Ap. J.*, **85**, 194, 1937; L. G. Henyey, *Ap. J.*, **85**, 255, 1937; J. L. Greenstein and L. G. Henyey, *Ap. J.*, **89**, 647, 1939; *ibid*, **93**, 70, 1941.

[10] I am indebted to Dr. Greenstein for this value.

[11] *Harvard Circ.*, No. 422, 1937.

in the galactic plane to about 1 per cent, at most, of the gravitational attraction toward the center of the Galaxy. Moreover this radiative force is not directed toward the anticenter but deviates by about 50°.

The gravitational acceleration toward the galactic center may be set equal to v^2/r, where v is the circular velocity of 300 km/sec in the neighborhood of the sun, and r is the distance of 10,000 parsecs to the galactic center. The net force of radiation in the galactic plane would therefore produce an acceleration of an isolated dust particle roughly equal to 3×10^{-10} cm/sec^2. If dust particles alone were present they would rotate about the galactic center with a circular velocity less than that of the stars by about 1.5 km/sec.

The presence of atoms will modify the situation in the same way as before. We have seen that the radiative force per unit mass for atoms is less than the corresponding quantity for dust particles. When we discuss the general radiation field of a galaxy, we must average ξ not only over different types of atoms but over all different sources of radiation as well. When the bracketed expression in equation (9) is averaged over all values of T, weighting each value by the total apparent luminosity of all stars whose effective temperature is T, $\bar{\xi}$ will be decreased still farther from the upper limit found in section 1. It may be concluded that the net effect of galactic radiation pressure on atoms, like the pressure from individual stars, is, in all probability, considerably less important than the corresponding force on dust particles.

If the specific force of radiation on atoms were equal to that on dust particles and if the distribution of interstellar matter in the galactic plane were similar to that of the stars, the entire medium would show a velocity of rotation less than that of the stars by 1.5 km/sec. If, as seems more likely, the average value of ξ for all atoms is considerably less than unity, then the medium will still tend to rotate somewhat more slowly than the stars, but the effect will be reduced by a factor equal to the relative amount of interstellar matter which is in the form of small dust particles. Thus, if dust and atoms are equally abundant, the equilibrium circular velocity of galactic rotation for interstellar matter should be less than that of the stars by 0.75 km/sec. If hydrogen is ten times more abundant than dust, this difference of velocity is reduced to 0.15 km/sec. This difference between the stars and the interstellar medium in their net attraction toward the galactic center could result, of course, either in an actual difference of mean rotational velocities or in a difference between the pressure gradients along the galactic plane. This latter case would lead to a concentration of interstellar matter toward the galactic center somewhat less than that of the stars, although this tendency might be reversed by the difference of mean square velocities between the stars and the interstellar particles. In any case, however, the effects are probably too small to be observed.

When the medium as a whole is in equilibrium, the individual dust particles will drift through the atoms with a relative velocity such that the external and interaction forces are equal and opposite, where these terms have the meanings given them in section 1. The interaction force is, as we have seen, $-m_d V/2t_e$; the external force is $3\times 10^{-10} m_d$, if we use the acceleration of an isolated dust particle which was found above. Equating these two forces, we find

$$V = 6\times 10^{-10}\, t_e\,. \tag{19}$$

This assumes that the dust is less abundant than atoms and that therefore the velocity of the medium as a whole is not greatly affected by the forces on the dust particles. Since 4×10^5 years is an upper limit for t_e, we see that V is at most 8×10^3 cm/sec in the neighborhood of the sun. This is greater than the equilibrium random velocities computed in section 2 but is still small, and would produce a negligible separation of dust and atoms in the galactic plane during 10^9 years.

Although the force of radiation on dust particles in the galactic plane is small compared to the force of gravity, this is not true for the forces perpendicular to the plane. In fact, for particles of radius 10^{-5} cm Greenstein finds that the repulsive force of radiation a short distance away from the plane may exceed slightly the force of gravity toward the plane. For greater distances from the plane the concentration of the more luminous stars becomes relatively less, the luminosity per unit mass decreases, and the force of gravity becomes great enough to overcome the radiation pressure.

One might gather from Greenstein's analysis that particles of radius within a certain range would tend to gather in parallel planes some 50 parsecs or so on each side of the galactic plane. But the time required for dust to reach an equilibrium of this sort, even if no atoms were present, would be roughly one-fourth the time required for stars to perform a complete oscillation through the plane and back, or 10^7 years.

Since the level of ionization will increase with decreasing density and hence with increasing distance from the galactic plane, the average value of ξ for all atoms at a moderate distance from the galactic plane should be even less than its value in the plane. The radiative force perpendicular to the galactic plane is therefore much less on atoms than on dust; an atom will show little tendency to be repelled from the plane, on the average, but the dust will tend to diffuse outward through the atoms. The time required for dust to diffuse some 50 parsecs from the galactic plane is well over 10^9 years. Even if the adopted time scale were sufficiently long, the concentration of dust into clouds and the change of radii with time would probably act to prevent the attainment of such an equilibrium. It is evident that we may neglect the direct effects produced by the radiation pressure of the galactic field on both dust and atoms.

3. RADIATIVE ATTRACTION BETWEEN DUST PARTICLES

There is another effect of radiation pressure which has apparently not been considered. Two particles in an isotropic radiation field will be forced toward each other by radiation pressure, since the shadow of each on the other will produce an uncompensated force along the line joining the two particles. If the radiation field is not isotropic, the force on the first particle may be resolved into two forces, F_0, the force arising from the radiation pressure in the absence of the other particle, and F_d, the increment of force arising from the fact that the other dust particle cuts off some of the radiation. The average force on the first particle is then equal to $\overline{F}_0 + \overline{F}_d$, the sum of the averages of F_0 and F_d.

We have seen that the average value of F_0 for particles in the galactic plane produces no important effects. The value of F_d for any two particles will vanish except for those intervals when the second particle comes between the first one and either a star or a bright nebula. The force F_d will always be directed between the two particles, however, and one may take an average over time as the line between the two particles slowly changes its direction. This average will not vary by more than a factor of two as the line between the particles points toward different galactic longitudes, and to a first approximation F_d may be calculated as though the radiation field were rigorously homogeneous and isotropic.

For any two particles $\overline{F}_d$ is clearly is very much less than the minor statistical fluctuations in $\overline{F}_0$, but for large numbers of absorbing particles the effect is statistically significant. In other words, a dust cloud will tend to be blown about by the radiation from passing stars and by the general radiation field, but the net effect will be to force the cloud together. Hence F_d, though small, may produce a cumulative effect which is much more important than any result arising from $\overline{F}_0$.

This radiative force $\overline{F}_d$ between two dust particles will be denoted hereafter by f_r. Let us calculate f_r in a radiation field of energy density $U(\nu)$ ergs per cm^3 per frequency interval. If the dust particles are not separated by more than a few hundred parsecs, the radiation arising from stars between the two particles will be much less than that from beyond and may be neglected.

If the second particle has a geometrical cross-section $\pi\sigma_2^2$, it will subtend a solid angle $\pi\sigma_2^2/r^2$ at the first particle, where r is the distance between them. If the absorption efficiency of the second particle is unity, no radiation will reach the first particle in this solid angle, and the energy density of frequency ν at the first particle will be reduced by an amount $\pi\sigma_2^2 U(\nu)/4\pi r^2$. Since the absorption efficiency is not unity, in general, we must multiply this computed reduction in energy density by $(1 - \gamma_{2\nu})Q_2(\nu)$, the absorption efficiency of the second particle for radiation of frequency ν. As in section 1, $Q(\nu)$ is the ratio of the total radiation scattered or absorbed to the radiation incident on the geometrical cross-section $\pi\sigma^2$; $\gamma_{2\nu}$ is the albedo of particle 2 in the frequence ν, and $1 - \gamma_{2\nu}$ is, therefore, the fraction of the extinguished radiation which is absorbed rather than scattered. The scattering of radiation will not affect the energy density between the particles if the radiation field is isotropic.

If r is much greater than σ, all the intercepted radiation may be assumed parallel to r; the total force of radiation on the first particle may then be found by multiplying the change in energy density by $\pi\sigma_1^2\kappa_1 Q_1(\nu)$, which is the extinction cross-section for the first particle multiplied by κ_1, the ratio of the momentum gained by the particle to the momentum of the radiation scattered or absorbed. As in section 1, we shall set κ_1 equal to unity. The total force f_r on the first particle is found finally by an integration over all frequencies, which yields

$$f_r = \frac{\pi\sigma_1^2\sigma_2^2}{4r^2}\int_0^\infty (1 - \gamma_{2\nu})Q_1(\nu)Q_2(\nu)U(\nu)d\nu \,. \tag{20}$$

The gravitational attraction between the two particles may be denoted by f_g. The two particles will be assumed identical, the subscripts 1 and 2 omitted, and $4\pi\sigma^3 d/3$ substituted for the mass of each particle. If we follow Schalén's notation[7] and substitute $8\pi\sigma\psi_\lambda(2\sigma)$ for $Q(\nu)$, then the ratio of f_r to f_g becomes

$$\frac{f_r}{f_g} = \frac{9\pi UK}{Gd^2}\,, \tag{21}$$

where G is the gravitational constant, U is the integrated energy density of radiation, and

$$K = \frac{1}{U}\int_0^\infty (1 - \gamma_\nu)\psi_\lambda^2(2\sigma)U(\nu)d\nu \,. \tag{22}$$

When the radius σ of a metallic particle is much less than $\lambda/2\pi$, the scattering becomes very small and γ_ν vanishes; in addition, $\psi_\lambda(2\sigma)$ becomes[11] equal to $F(\lambda)/\lambda$, where $F(\lambda)$ is independent of σ and depends only on the optical constants of the metal for the wave length λ. For *Fe* particles, for instance, $F(\lambda)$ increases slightly from 0.55 to 0.75 as λ decreases from 5800 A to 3000 A. It is evident that as σ decreases, f_r/f_g approaches a limiting value which is independent of σ. With Dunham's value[12] of 5.2×10^{-13} ergs/cm^3 for U, this limit becomes

$$\frac{f_r}{f_g} = 2.2\times 10^{-4}\,\frac{1}{d^2}\,\overline{\left(\frac{F(\lambda)}{\lambda}\right)^2}\,, \tag{23}$$

where $(F(\lambda)/\lambda)^2$ is averaged over $U(\nu)$.

To find a lower limit for this average we may replace $F(\lambda)$ by 0.25. The average of $1/\lambda^2$ over the energy distribution for interstellar space computed by Dunham for no se-

[12] *Proc. Amer. Phil. Soc.*, **81**, 277, 1939.

lective absorption is roughly 3×10^9 cm^{-2} but is decreased to 1×10^9 cm^{-2} if the values of $U(\nu)$ for an assumed selective absorption are assumed. To obtain a final lower limit for f_r/f_g in equation (23) we replace $1/\lambda^2$ by 10^9 and d by 7.8, which gives f_r/f_g equal to 2.1×10^2. If d is unity, however, the ratio increases to 1.4×10^4.

In the more general case where σ may have any value, we must replace $F(\lambda)/\lambda$ in equation (23) by $\psi_\lambda(2\sigma)$. It is evident from the graphs given by Schalén that an increase in σ increases $\psi_\lambda(2\sigma)$ until σ becomes approximately equal to $\lambda/2\pi$, after which $\psi_\lambda(2\sigma)$ decreases as $1/\sigma$. Since roughly half the stellar radiation, as computed by Dunham, is of wave length longer than 4000 A, an increase of σ from zero to 10^{-5} cm will not decrease f_r/f_g appreciably. Particles of much larger radius may again be neglected, owing to their low abundance.

An increase in σ will, however, increase the albedo γ_ν. In addition, all nonmetallic particles have an albedo which is nearly unity. Although scattering particles will not be forced together in a purely isotropic radiation field, the intensity of radiation in the galactic system varies so greatly with galactic latitude that a force between such particles will actually appear. This force will be attractive if the line between the two particles lies in the galactic plane and repulsive if this line is perpendicular to the plane. The force between scattering particles will always vanish on the average, and we shall not consider it further.

Since scattering is to be neglected, a lower limit on f_r/f_g for particles of all relevant sizes and composition may be found if we multiply the value of 210 found above by the average value of $(1 - \gamma_\nu)$ for interstellar dust particles. Since the most probable value of the mean albedo is[10] about 0.5, as we have seen above, it follows that the average force of attraction between any two dust particles is at least a hundred times as great as the attraction of gravity between them. Since the galactic radiation is greatly concentrated to the galactic plane, the actual forces between particles will be greater than this lower limit if the line joining them is parallel to the galactic plane, and less if it is perpendicular. Electrostatic forces between the particles will not affect any of these conclusions, since each dust particle will be accompanied by a sufficient number of free positive ions to neutralize the effect of its charge a few meters away.

If the dust and atoms are assumed to behave as a homogeneous gas, the effective value of f_r/f_g for interstellar matter will be reduced. If each gram of dust is accompanied by 10 gm of atoms, for instance, the gravitational force between two unit cubes of interstellar matter will be 121 times its value in the absence of atoms, and f_r/f_g will be decreased by more than a hundred. The ratio of atoms to dust is probably not quite so high, however, and between any two volume elements of interstellar matter the radiative attraction should be considerably greater than the corresponding gravitational force.

Within large clouds of dust the energy density will be much reduced by absorption, and radiation pressure will yield a force wholly on the surface of the cloud. For clouds of small optical depth, however, this force among dust particles behaves mathematically in the same way as the gravitational attraction between them. Such a force is clearly of great importance in the formation and equilibrium of condensations within the medium.

The similar force on an atom in the neighborhood of a dust particle cannot be of much importance in the distribution of interstellar matter. Let us consider the extreme case in which all the line radiation has been absorbed by a cloud and, as a result, atoms have been attracted to and concentrated in the cloud. The excess pressure of atoms within the cloud is given by $(1 + Z)nkT$, where n is the number of atoms per cm^3 within the cloud minus the corresponding number outside, Z is the average number of free electrons per atom, and k is the usual Boltzmann constant. If these atoms are held within the cloud by radiation pressure, P_r, we must have

$$P_r = \tfrac{1}{3}\zeta U = (1 + Z)nkT\,, \tag{24}$$

where ζ is the fraction of the total radiation which the atoms are capable of absorbing in all stages of ionization and P_r is the pressure on the atoms arising from this source. Since ζ is less than 10^{-3}, we have

$$n < \frac{1.3}{(1 + Z)T}, \tag{25}$$

provided that U, the energy density of radiation, is again set equal to 5.2×10^{-13} ergs/cm^3. Since T is presumably at least $1000°$, the increase in the concentration of singly ionized atoms from this source amounts, at most, to 7×10^{-4} atoms per cm^3. This is negligible compared to an abundance of one hydrogen atom per cm^3 present normally in interstellar space. Any radiative force on atoms is clearly negligible in this connection.

For a cloud composed entirely of dust the upper limit on the density found from equation (24) is so great that a large cloud of such a density would be held together primarily by gravitational attraction; the limit on the density in such a case is therefore of no importance. If, on the other hand, atoms are diffusing out of a cloud but are still forming an appreciable fraction of the mass of the cloud, the maximum density will again be small. Whether dust clouds can actually form in less than 10^9 years from an initially homogeneous distribution of dust and atoms is a question which will be postponed to a later paper.

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April 1941