

Measuring the Gravitational Constant

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ABSTRACT

By using a torsion pendulum, we determine the gravitational constant G to be $(9.46 \pm 0.11) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$, $\sim 43\%$ accuracy compared to the NIST value of $(6.674210 \pm 0.000010) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$.

1. Introduction

Newton in his *Principia Mathematica* outlined what is known as the inverse square law of gravity. He noticed that just as apples fell to the earth from a near on earth's surface, there was a certain force responsible and that it was the same force that keeps the moon revolving around the earth despite the moon being much farther away from the earth than an apple from a tree. From this Newton concluded that the gravitational force $\vec{F}$ depends both on the mass m_1 and m_2 of the two attracting bodies and the distance r between them:

$$F = -\frac{Gm_1m_2}{r^2}, \quad (1)$$

where G is known as the universal gravitational constant.

Mohr et al. (2007) find the NIST¹ value of G to be $(6.674210 \pm 0.000010) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$. They note that this measurement of G , basically using a torsion pendulum as we will, is valid on $0.01 - 1.0 \text{ m}$ scales. Hoyle et al. (2001) search for extra dimensions by measuring G on sub-millimeter scales. Our experiment, however, will measure G roughly on centimeter scales.

2. Theory and Experimental Setup

So how does one go about determining G , a small, imprecisely-known physical constant? We use a torsion pendulum, as shown in Figure 1, which is a part of the basic experimental setup shown in Figure 2.

A laser incident on a mirror attached to the fulcrum of the pendulum, where a thin tungsten wire coming out of the page of Figure 1 suspends it, reflects off the mirror onto a screen on which we attached a meter stick to measure the small angular displacements of the pendulum. We show in Figure 2 the view of the screen as seen by the mirror attached to the torsion pendulum.

Referring to Figure 1, we see that there are two fixed big masses each of mass M separated by a distance $2d$. These masses will exert a gravitational force predominately on the small mass m closest to each of big masses M . The small masses m are affixed to the ends of the torsion pendulum's beam, which is entirely enclosed in an evacuated chamber, hence the gravitational force between each of the small and big masses exerts a torque on the pendulum causing a displacement which we can measure via the reflected laser beam on the screen a distance L away. We then change the configuration of the stationary big masses such that it is not rotating counter-clockwise as shown in the figure but clockwise. Based on the amplitude of the oscillations and

¹<http://physics.nist.gov/cgi-bin/cuu/Value?bg>

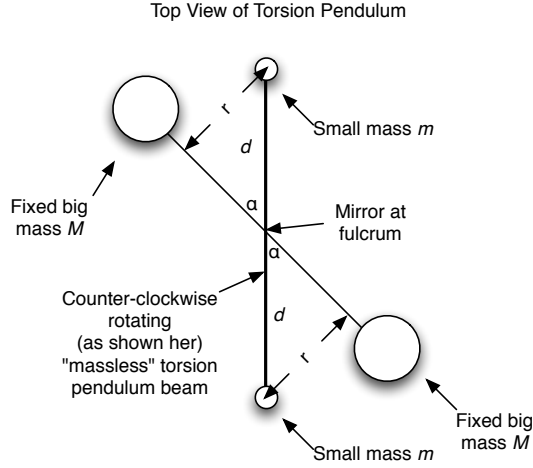


Fig. 1.— Top view of the torsion pendulum. The laser and screen of Figure 2.

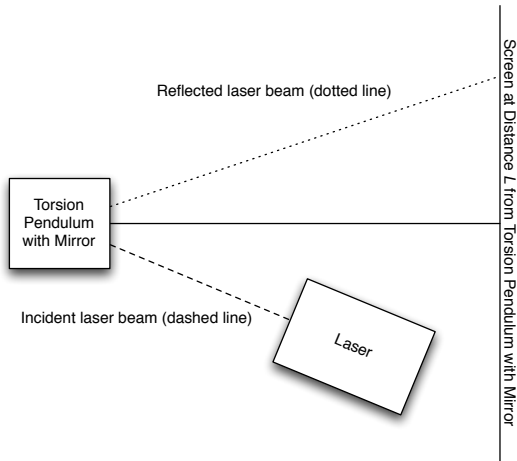


Fig. 2.— Basic experimental setup

the difference in the mean displacements of the clockwise and counter-clockwise configurations' oscillations, we can eliminate many non-gravitational effects to determine G .

Thus from Equation 1 we can see that the torque $\vec{\tau} \equiv \vec{d} \times \vec{F}$ on the small masses affixed to the pendulum would be

$$\sum \tau = 2d \frac{GM}{r^2}. \quad (2)$$

If the angle $\alpha \approx 0$ of Figure 1, then $r = d\alpha$. For small angular displacements, we know that a classical pendulum has an angular frequency $\omega = \sqrt{k/m}$, where k is a constant. Similarly, for the torsion pendulum case,

$$\omega = \sqrt{k/I}, \quad (3)$$

where $I = md^2$. Now we see that the period $T_0 = 2\pi/\omega = 2\pi\sqrt{I/k}$, from which we can solve for k :

$$k = \frac{2\pi^2 d^2 m}{T_0^2}. \quad (4)$$

Looking at Figure 2, we can see that if we assume $L \gg S$ —where L is the distance from the torsion pendulum to the screen and S , shown in Figure 2, is twice the amplitude of the displacement of the laser point on the screen—, then a corresponding angle θ relating L and S would be S/L . Now assuming that the total torque $\sum \tau \propto S$ with a constant of proportionality of k , we can solve for G :

$$\sum \tau = k\theta \quad (5)$$

$$= k \frac{S}{L} \quad (6)$$

$$= \frac{\pi^2 d^2 m S}{T_0^2 L}, \quad (7)$$

from which we get

$$G = \frac{\pi^2 d s r^2}{T_0^2 M L}. \quad (8)$$

We then used the following *Mathematica* code (see Appendix A) to find an optimal fit to an exponentially damped sinusoid for determining the S and T_0 of our measured time series.

3. Analysis and Results

Shown in Tables 1 and 2 are the setup parameters for each of our configurations with a big stationary ball and a small stationary ball, respectively. See the *Mathematica* code at the end of this document for the exactly fit angular frequencies and displacements.

$$\begin{aligned} M &= 1.465 \pm 0.001 \text{ kg} \\ L &= 4.26 \pm 0.08 \text{ m} \\ d &\equiv 0.05 \text{ m} \\ r &= 0.03 \pm 0.01 \text{ m} \\ G &= (9.46 \pm 0.11) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2 \end{aligned}$$

Table 1: Parameters for determining G using a big mass

$$\begin{aligned} M &= 0.500 \pm 0.001 \text{ kg} \\ L &= 4.26 \pm 0.08 \text{ m} \\ d &\equiv 0.05 \text{ m} \\ r &= 0.03 \pm 0.01 \text{ m} \\ G &= (3.08 \pm 0.68) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2 \end{aligned}$$

Table 2: Parameters for determining G using a small mass

Note that the errors in G are estimated because we did not run Monte Carlo simulations varying the parameters of our fit to see how much they change, for example, T_0 and S .

Shown in Figure 4 are the four plots of our time series. Forming a time series aided us in determining a precise displacement for the torsion pendulum in each of its clockwise and counterclockwise configurations by fitting these data with a damped sine wave function.

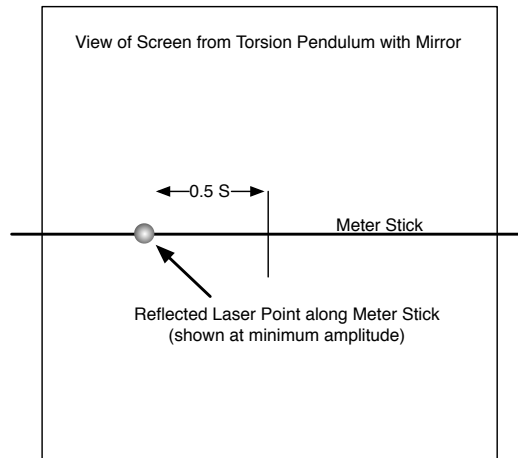
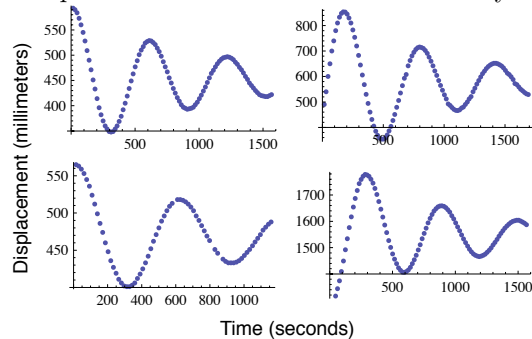


Fig. 3.— View of the screen from the torsion pendulum

Fig. 4.— Times series of displacements of the laser beam on the screen. *Clockwise from top left:* big ball clockwise rotation, big ball counter-clockwise rotation, small ball counter-clockwise rotation, and small ball clockwise rotation. The fit exponentially-damped sinusoids are not shown for clarity.



4. Conclusion

Our most accurately determined G was found when using the more massive stationary balls: $G = (9.46 \pm 0.11) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$ with $\sim 43\%$ accuracy compared to the NIST² value of $(6.674210 \pm 0.000010) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$ (Mohr et al. 2007). We could have improved the accuracy by measuring r more precisely than 1 mm because this was the largest source of error, due to the fact we could not take apart the evacuated chamber within which the torsion pendulum oscillated. That we neglected the gravity of the big mass on the small mass farther away ($\approx 2d$ away) is a more minor effect due to gravity's strong inverse dependence on distance.

5. Acknowledgements

I would like to thank my lab partner Cole Valencius for his assistance, our teaching assistants (TAs) Eliot and Will for their patience and Dr. Sandhu for his guidance in this fascinating lab project.

REFERENCES

- Hoyle, C. D., Schmidt, U., Heckel, B. R., Adelberger, E. G., Gundlach, J. H., Kapner, D. J., & Swanson, H. E. 2001, Physical Review Letters, 86, 1418
- Mohr, P. J., Taylor, B. N., & Newell, D. B. 2007, ArXiv e-prints, 801, arXiv:0801.0028

²<http://physics.nist.gov/cgi-bin/cuu/Value?bg>

This 2-column preprint was prepared with the AAS L^AT_EX macros v5.2.

A. *Mathematica* fitting commands

```

cw1data=Import["cw_1.txt","Table"];
ccw1data=Import["ccw_1.txt","Table"];
cw2data=Import["cw_2.txt","Table"];
ccw2data=Import["ccw_2.txt","Table"];
fitfunc = c+d Exp[t e] Sin[g+f t];
cw1fit = FindFit[cw1data,fitfunc,{c,450.},{d,150.},{e,1/2000.},{f,2 Pi/645.},{g,Pi/2.},t]
{c -> 451.68, d -> 141.134, e -> -0.000954096, f -> 0.010418, g -> 1.3649}
ccw1fit = FindFit[ccw1data,fitfunc,{c,600.},{d,250.},{e,1/2000.},{f,2 Pi/600.},{g,Pi/4.},t]
{c -> 568.309, d -> 351.253, e -> -0.00106222, f -> 0.0101228, g -> -0.359714}
G=(Pi^2*(0.03)^2*0.05*Abs[568.309-468.769]/100.)/(2*1.465*(2*Pi/((0.010418+0.0101228)/2))^2*4.26)
9.46341*10^(-11)
cw2fit = FindFit[cw2data,fitfunc,{c,475.},{d,150.},{e,1/2000.},{f,2 Pi/600.},{g,Pi/2.},t]
{c -> 468.769, d -> 96.8258, e -> -0.00106601, f -> 0.0103371, g -> 1.31574}
ccw2fit = FindFit[ccw2data,fitfunc,{c,1525.},{d,250.},{e,1/2000.},{f,2 Pi/600.},{g,-Pi/2.},t]
{c -> 1553.18, d -> 303.663, e -> -0.00114088, f -> 0.0103936, g -> -1.53367}
G=(Pi^2*(0.03)^2*0.05*Abs[468.769-1553.18]/100.)/(2*0.5*(2*Pi/((0.0103371+0.0103936)/2))^2*4.26)
3.07684*10^(-9)
Export["plots.eps",Show[ListPlot[cw1data],ListPlot[ccw1data],ListPlot[cw2data],ListPlot[ccw2data]]];

```