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THE DEFINITION OF $\mathbf{B}^{(3)} = \mathbf{B}^{(0)}\mathbf{k}$: A REPLY TO E. COMAY

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The recent claim by Comay that the $\mathbf{B}^{(3)}$ field has non-zero divergence for dipole radiation is shown to be incorrect.

Key words: $\mathbf{B}^{(3)}$ theory, multipole radiation, reply to Comay.

1. INTRODUCTION

Recently [1] Comay has claimed that the B cyclic relations [2–10] result in non-zero divergence for the $\mathbf{B}^{(3)}$ field for dipole radiation. In this note the claim is shown to be erroneous, the correct way of defining $B^{(0)}$, the magnitude of $\mathbf{B}^{(3)}$, is by way of integration over the surface of a sphere in spherical polar coordinates.

2. DIPOLE RADIATION AND $\mathbf{B}^{(3)}$

The textbook [11–14] definition of the far-zone magnetic flux density due to an oscillating electric dipole is given in S.I. units by [11]

$$B_\alpha = \frac{\mu_0\omega^2}{4\pi rc}\,\epsilon_{\alpha\beta\gamma}n_\beta\mu_\gamma^{(0)}\,e^{i(\kappa r-\omega t)}\ , \tag{1}$$

where μ_0 is the vacuum permeability, ω the angular frequency, r the radial coordinate of the spherical polar system, c the conventional speed of light, $\epsilon_{\alpha\beta\gamma}$ the Levi-Civita symbol, n_β a unit vector, $\mu_\gamma^{(0)}$

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the electric dipole moment, κ the wavevector, and where the electromagnetic phase is $\kappa r - \omega t$ at instant t. This has the characteristic inverse r dependence in the far zone [11–14].

In the B cyclic theorem, however [2–10], $B^{(0)}$, the magnitude of $\mathbf{B}^{(3)}$, is defined as an amplitude constant in space and time, (e.g. Ref. [13], p. 204).

In order to derive $B^{(0)}$ from Eq. (1) we first find the total radiated power by integrating the square amplitude of B_α over a spherical surface. The square amplitude is

$$B^2 = \frac{\mu_0^2 \omega^4 \mu^{(0)2}}{16\pi^2 \mathrm{r}^2 \mathrm{c}^2}, \tag{2}$$

and the integral is [12]

$$W = \frac{\mathrm{c}}{\mu_0} \int_0^{2\pi} \int_0^{\pi} B^2 r^2 \sin\phi \, d\phi \, d\theta , \tag{3}$$

where W is the total radiated power in watts. Thus

$$W = \left(\frac{\mu_0 \mu^{(0)2}}{6\pi \mathrm{c}} \right) \omega^4 \tag{4}$$

and has the characteristic ω^4 dependence of classical radiation. Analytical dependence on r has disappeared through the integration procedure (3). The integrated radiation power density I (watts per square metre) is defined with respect to a cross section of the radiation, a beam area A independent of r, by

$$I = \frac{\mathrm{W}}{\mathrm{A}} . \tag{5}$$

Finally [2–10],

$$I = \frac{\mathrm{c}}{\mu_0} B^{(0)2} , \tag{6}$$

and, from Eq. (6), $B^{(0)}$ has no dependence on r.

3. DISCUSSION

Comay has used the unintegrated B_α of Eq. (1) and has asserted in an arbitrary manner that $\mathbf{B}^{(3)}$ is proportional to $1/r$, so that its divergence is non-zero *by construction.* The $B^{(0)}$ factor of the B cyclic theorem is that given by Jackson [13], and by Eq. (6). The $1/r$ dependence of the unintegrated B_α is a consequence of the fact that radiation takes place over the sphere, and of the fact that the unintegrated Poynting vector must decrease as $1/r^2$. The total radiated power is however independent of r and so are I and $B^{(0)}$. Thus $\mathbf{B}^{(3)} = B^{(0)}\mathbf{k}$ is an intrinsic property of radiation, in the same sense as the electron has a half integral spin. Thus $\mathbf{B}^{(3)}$, if properly defined, is independent of r and its divergence is zero [2–10]. The same is true for all multipoles of radiation [11–14].

The energy of one photon is $\hbar\omega$, where $\hbar$ is Dirac's constant, and is independent of r. The fact that the intensity of light decreases as $1/r^2$ is due to the fact that it is being radiated over a sphere, the energy and power ($\hbar\omega^2$) of one photon do not decrease as $1/r^2$. Therefore in order to achieve consistency between the fact that $B^{(0)}$ is independent of r, but that dipole radiation gives a field dependent on $1/r$, integration must be used as in Sec. 2. The energy and power of a light beam made up of one photon are both independent of r, because the photon is radiated in one axis, not over a sphere.

Examination of Comay's (6) shows that it has the incorrect units of Cs^{-2} for magnetic flux density. The correct units are tesla $= JsC^{-1}m^{-2}$. The square modulus of Comay's $\mathbf{B_T}$ is

$$| \mathbf{B_T} \times \mathbf{B_T^*} |= \frac{\mathrm{A_T^2}}{\mathrm{r^2}} \, . \tag{7}$$

A consistent form of the B cyclic theorem can easily be constructed from Comay's own definition (6) by multiplying the numerator and denominator by an area (A), independent of the radial coordinate r. This area can be a spherical surface or a cross section of the radiation. We then find the correct, divergentless $\mathbf{B}^{(3)}$ field through the equation

$$| \mathbf{B} | := \frac{\mathrm{A_T}}{\mathrm{A^{1/2}}} \, , \quad A_T = 2Q\omega^2 R\left(\frac{\mu_0}{\mathrm{c}}\right) \, , \tag{8}$$

in which $A^{1/2}$ is r independent. The scalar magnitude $B^{(0)}$ is defined by Comay's A_T divided by $A^{1/2}$, and adjusting Comay's incorrect units to the correct S.I. units of tesla through multiplication by μ_0/c of his factor A_T. We find

$$\mathbf{B} \times \mathbf{B}^* = iB^{(0)}\mathbf{B}^{(3)*}, \tag{9}$$

and $\mathbf{B}^{(3)}$ is divergenceless.

It is concluded that Comay's comment is another trivially erroneous assertion which demonstrates on the part of the author a complete lack of understanding of $\mathbf{B}^{(3)}$ theory. The author has chosen to ignore the more recent literature on $\mathbf{B}^{(3)}$, which shows approximately eight independent corroborations [2–10]. He has also chosen to ignore the fact that the reply to Rikken's paper [16] has been available for some time [17]. Comay again ignores voluminous work on $\mathbf{B}^{(3)}$ by Evans in co-authorship with others [2,3], and the work of all corroborating authors and groups [2–10]. This paper by Comay is similar to, and in parts almost identical with, another by Comay published in Ref. 18 and already replied to in Ref. 19. This paper was submitted simultaneously without the criticised author's knowledge. The criticised author was not offered the right of reply. This repeats a similar pattern observed over the past few years, i.e., the appearance of trivially incorrect, or otherwise misplaced, criticism of $\mathbf{B}^{(3)}$ theory, criticism made without right of reply. This process is highly detrimental to the progress of science, a progress which depends on free discussion between schools of thought. The present author managed to procure the right of reply through other journals or books, and these replies are now available in the scientific literature [2,17,19,20].

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I was not advised by Comay or anyone else that a paper critical of my work would be forthcoming in *Physica*, and neither did the responsible editor of this journal offer me the opportunity to reply, as is customary.

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