so much feebler than the recurrent image that it produces no alteration in its tint.

In conclusion I would remark upon the apparent analogy between the phenomenon of recurrent vision and that of induced currents in electricity. A nerve-current in one kind of nerves appears to induce nerve-currents in the other kinds in a manner analogous to that by which a current of electricity in one conducting wire induces currents in parallel conducting wires.

Leeds Grammar School.
November 9, 1872.

LXIII. *On the Theory of Electrodynamics. By* M. Helmholtz*.

THE theory of electrodynamic actions, besides its immediate value for the understanding of this important and prolific branch of physics, is more universally interesting in its relation to the fundamental principles of general mechanics. All the other known actions at a distance can be easily and completely reduced to attractive and repulsive forces of points of masses, while the intensity of these forces depends only on the reciprocal distances of the points and not on their motion. Moreover the hitherto known actions between molecules can either be entirely referred to such forces, or at least are so similar in their whole manner of appearance to the effects produced by gravity that we find no difficulty in imagining them the effects of forces similar in character. But the electrodynamic forces constitute an exception. They form a class of distant actions produced only by the state of motion of the efficient agent, the electricity,—a state of motion which makes itself perceptible as such by a whole series of phenomena—by development of heat in solid conductors, chemical decomposition in liquid conductors, &c. The real laws of the manner of appearance of these forces are, in the main, well known, and have been reduced by F. E. Neumann, Sen., to a comparatively simple expression, which, however, gives not the action of mass-point upon mass-point, but of one linear element of a current upon the other. I have myself given to Neumann's expression of the potential a more general form†, in which it embraces also the differing expressions resulting from the theories of W. Weber and Maxwell for the potential of each two current-elements. For closed currents all these expressions give the same results; on the contrary, for open ones, the actions of which have, indeed, at present been little investigated, they

* Translated from the *Monatsbericht der Kön. Preuss. Akad. d. Wissenschaften zu Berlin* for April 1872.

† *Journal für reine und angewandte Mathematik*, vol. lxxii.

exhibit differences. The plan of my memoir was principally to seek out those differences which it might be possible to discover by practicable experiments. It must here be remarked that the various potential-expressions which I formed differ from one another only by a constant (in my memoir, denoted by k). We obtain Neumann's expression if we put $k=+1$, Maxwell's if $k=0$, W. Weber's if $k=-1$. The investigation showed that the expressions with k negative led to impossible consequences—namely, to an unstable equilibrium of the electricity in conductors, which, once disturbed, might give rise to infinitely great current-intensities and unlimited charges. On the other hand, the expressions with k positive, or with $k=0$, gave stable equilibrium, and, even for open currents, only such differences as, with our present experimental means, can hardly be detected; so that what is yet doubtful in the mathematical conception of the law, viz. the value of the constant k, appears to have no effect in the application of it to experiment.

These expressions for the potential of each two current-elements, however, are manifestly not elementary expressions of the last acting forces; for they lead, if we imagine each current-element as a solid body, to at least two forces for each, or to a force and a pair of forces; and the quantity and partly the direction of these forces depend not merely on the situation of the elements, but also on the velocity of the electric currents. The phenomena of induction are only indirectly derived from the electrodynamic potential, through the interposition of the law of the conservation of energy.

Among the further-penetrating hypotheses which seek to ascertain the elementary forces that lie at the base of electrodynamic phenomena, two especially must be mentioned. Mr. Clerk-Maxwell drops the assumption of action at a distance, and assumes that all magnetic, electrostatic, and electrodynamic actions are translated to a distance by the propagation of molecular motions and forces in an elastic medium which fills space. As the theory finally gives for this medium the capability of executing oscillations which are perfectly similar to those of light and have also the velocity of propagation of light, this medium must be identified with the luminiferous æther. It is true that, for the reciprocal action of neighbouring volume-elements of this medium, he assumes laws considerably different from those of the elastic bodies known to us; but he has shown that a kind of reciprocal action, such as he attributes to the æther, can indeed be produced by a mechanical combination of solid elastic bodies. For this purpose a system of cells with elastic walls and cylindrical cavities must be taken, in which elastic balls can rotate and be flattened out by the centrifugal force. In the walls of the cells there must be other balls, of invariable volume, as friction rollers. These would rotate freely; but their centres of gravity, in insulating media, would merely be displaced by elastic yielding of the cell-wall; in conducting media, on the contrary, at every displacement they must suffer a resistance similar to friction in a viscous liquid. The transference of motion between these balls would be effected only through the adhesion of their surfaces to one another. Displacement of the last-mentioned balls gives dielectric polarization of the medium; streaming of the same, an electric current; rotation of the elastic balls corresponds to the magnetizing of the medium, the axis of rotation being the direction of the magnetic force.

Now, although the idea of such a molecular structure of the space-filling æther may be repugnant to our imagination as too artificial, yet the hypothesis of Maxwell appears to me very important on this account—because it proves that there is nothing in electrodynamic phenomena to compel us to attribute them to an entirely anomalous sort of natural forces, to forces depending not merely on the situation of the masses in question, but also on their motion. Indeed, out of the assumption of those reactions of the volume-elements of the æther upon each other which Mr. Maxwell has assumed, a complete and mathematically very elegant theory of all electric phenomena (magnetic, electrodynamic, and induction) can be developed; and the same theory also gives an account of the phenomena of light.

On the other hand, M. Weber's theory derives the explanation of electrodynamic actions from distant forces of a peculiar kind, acting between the points of the electrical masses, and depending simultaneously on the distances and the relative motions and accelerations of each pair of points. It gives comparatively simple explanations of electrodynamic attractions and of the induction-effects in linear conductors; and its analytic deductions accord perfectly, for all the phenomena to be observed in closed linear currents, with the consequences of the potential-law derived by F. E. Neumann from the phenomena. On this account, Weber's theory (which preceded Maxwell's) was very favourably received, especially by the German physicists. It had, and moreover retains, decidedly the merit of every acute and original thought which endeavours to strike out new paths in science when the old ones appear to lead into an inextricable thicket. I hardly need here remark that the value of such an attempt, if it was sufficient for the state of knowledge at the time, is not diminished when, after twenty-five years' progress of science, the impossibility is shown of carrying it out. Even then such an attempt has not been fruitless. A reconnaissance of unknown ground lying beside the road hitherto kept, if carried out carefully and intelligently, retains its value even if it should only teach that no way exists except the high road.

It was through Weber's hypothesis that a question of the highest significance for the principles of natural science was for the first time tested in the problems of facts, viz. whether elementary forces, incapable of further analysis, must be assumed dependent not merely on the position, but also on the motion of the acting points. In my work 'On the Conservation of Force,' I had stated that forces which depend only on the distance and the velocities, and therefore only on the coordinates of the points, and on their first differential quotient, must necessarily infringe the universal natural law of the conservation of energy, which law proves everywhere true also in electrodynamic phenomena. At that time, however, I had not considered this still more complicated case set up by the Weberian law, in which the forces depend on the coordinates and on the first and second differential quotients; and this case is certainly compatible with a somewhat extended form of the law of the conservation of energy. If we, as has always hitherto been done, name *vis viva* or *actual energy* the sum of the moved inert masses multiplied each by half the square of its velocity, then, in the usual form of the law, the quantity which I have called *quantity of tension-force,* and the English physicists *potential energy,* is a function of the coordinates of the moved points only; and the law of the conservation of energy affirms that the sum of the actual and potential energy remains constant in every motion of a mass-system not influenced from without.

If, however, under the action of external forces a self-repeating cyclical process takes place, at the end of which all the points of the system have exactly the same position, and the whole the same *vis viva,* as at the beginning, the sum of the work received from without and the work given out must be equal to zero, so that by no repetition of the process can work be permanently gained or destroyed. If the former were the case, there would be possible a perpetually continuous gain of work without a progressive alteration of the mass-system, and a *perpetuum mobile* might be constructed.

Weber's extension of the law of energy makes also the value of the *potential energy* a function not merely of the position, but also of the velocities of the mass-points. Under this assumption also, by no cyclical process which brings back not merely all the masses of the system to their initial positions, but also each one to its initial velocity, can more work be given out than is received from without, because those quantities of actual and potential energy which constitute the measure of the work are the same at the end of every such cyclical process as at the beginning.

Under these circumstances, however, the values of the forces must necessarily contain second differential quotients of the coordinates, because the sum of the force-components corresponding to the individual points and axes of coordinates, each multiplied by the corresponding component of the velocity, must be equal to the differential quotient of the potential energy, taken according to the time; and the latter, under the condition presupposed, necessarily contains also the second differential quotients of the coordinates according to the time.

In relation to complete cyclical processes, M. W. Weber* has proved that his assumption concerning the value of the electric forces admits no production of work without a corresponding expenditure of forces capable of producing it.

In another place, applications which I endeavoured to make of the differential equations deduced by Kirchhoff from Weber's assumption had led me to the discovery that they corresponded to a state of unstable equilibrium of the electricity in conductors, and that, according to them, currents might be developed which would lead to infinite current-intensities and infinite electric densities.

Replies by MM. W. Weber and C. Neumann have induced me to resume and generalize these investigations, the results of which I will here briefly lay before the Academy†.

If we have any number, however great, of mass-points, the inert mass of which may be denoted by μ_n, and all or some of which contain *quanta* of electricity, which, measured according to electrostatic measure, may be denoted by e_n—if, further, r_{nm} is the distance between the points n and m, and q_n the resulting velocity of the point n, ϑ_{nm} the angle which it makes with the direction of the line r_{nm} prolonged beyond n, then the value

(1) of the electrostatic potential

$$P = \tfrac{1}{2}\Sigma\Sigma\left[\frac{e_n \cdot e_m}{r_{nm}}\right];$$

(2) of the electrodynamic potential

$$Q = \frac{1}{cc}\Sigma\Sigma\left[\frac{e_n \cdot e_m}{r_{nm}} \cdot q_n \cdot q_m \cdot \cos(\vartheta_{nm}) \cdot \cos(\vartheta_{mn})\right].$$

We put, further, the quantity

$$p_n = \frac{1}{cc}\Sigma\left[\frac{e_m}{r_{nm}}\cos^2(\vartheta_{mn})\right],$$

* "Electrodynamische Maassbestimmungen, insbesondere über das Princip der Erhaltung der Energie," *Abh. d. math.-phys. Classe der Königl. Sächsischen Ges. der Wissenschaften*, 1871. The value of the potential was given by the same author in Pogg. *Ann.* 1848, vol. lxxiii. p. 229.

† The investigation will be published entire in the *Journal für reine und angewandte Mathematik.*

and let V denote the potential energy of the remaining forces which act upon the inert masses; then the equation which, in Weber's sense, expresses the conservation of the force becomes

$$\tfrac{1}{2}\Sigma[(\mu_n - p_n e_n)q_n^2] + \mathrm{P} + \mathrm{V} - \mathrm{Q} = \text{const.}$$

The sum here occurring, which occupies the place of the *vis viva*, and which shall be denoted by L, differs from the ordinary form of this expression by the necessarily positive squares of q_n being not merely multiplied by the necessarily positive inert masses μ_n, but, instead of the latter, the differences $(\mu_n - e_n p_n)$ entering as coefficients of the squares. These differences, however, may become negative, since μ_n can at all events be reduced to what even Weber and C. Neumann regarded as an extraordinarily little inert mass of the electrical quantum e_n, while the quantity p_n, a function formed after the manner of potential functions, may proceed from as great electrical masses as we please. If, now, $e_n p_n > \mu_n$, the point e_n would possess a *quasi* negative mass. *Acceleration of its motion would correspond to a diminution of the vis viva.* If the *vis viva* L consisted of a number of positive and negative terms of this kind, it might conserve an unchanged final value while its negative and its positive terms alike augmented *ad infinitum.*

These relations are represented most simply when only one of the masses μ is supposed to be in motion, and the rest spread over and adherent to a spherical surface surrounding the mass μ (perhaps the surface of an insulator). Then p and P become constants independent of the position of the point μ in the sphere; further, Q=0; and the equation reduces itself to

$$\tfrac{1}{2}(\mu - ep)q^2 + \mathrm{V} = \text{const.}$$

If, now, the quantum of the electricity on the sphere is great enough, so that $ep > \mu$, then q^2 and V must increase and diminish together. If μ moves in a direction *opposite* to the force represented by V, V augments and the velocity q must *increase*. *If, on the contrary, μ moves in the direction of the force, the velocity diminishes.* If μ moves in a prescribed path against a force which constantly resists it (for example, against friction), its velocity must increase continually and *ad infinitum*, with which production of heat *ad infinitum* would be connected. If in its course the mass impinges again and again continually against a greater inert elastic mass, it will drive this onward, and at each impact increase its own velocity, so as to make the next collision more forcible. This would evidently give a *perpetuum mobile.*

It may here be remarked that, if the linear dimensions of the spherical electric layer be increased n-fold, but the density be preserved unaltered, the quantity p will be augmented to n times as much; so that we can make it as great as we please, in spite of continually increasing distance of the acting mass. We have here, therefore, by no means to do with actions at molecular distances, but with distant actions of the Weberian forces.

The case I previously indicated, in which the mass μ attains infinite velocity, rests on the fact that this must always happen as often as, under the action of an accelerating force, it arrives at any place where the coefficient $(\mu - pe)$ representing the mass becomes $=0$, because the mass zero receives infinite acceleration from a finite force. Besides, in the present memoir, I have shown that neither is it necessarily at molecular distances that this takes place, nor does it require an infinite initial velocity, if only sufficiently large electrical masses are chosen, and if upon the whole path of the two masses an exterior force acts which impels them towards each other and is powerful enough to overcome their electrostatic repulsion.

The objections raised by W. Weber against one of the physically impossible consequences which, in my earlier memoir, are deduced from his theory are thus removed.

In his most recent electrodynamic researches, M. C. Neumann has expressed his concurrence in Weber's objections, and, for his own part, has endeavoured to remove from the theory the deficiencies pointed out by me, in that he has introduced an alteration into Weber's expression for very small distances. From what has just been said it is evident that such an alteration cannot obviate the physical impossibilities mentioned.

Also, for electric currents, no introduction of molecular processes, motions, or forces can get rid of the unstable equilibrium, because when the dimensions are increased n-fold and the electric densities unchanged the work-equivalent of the molecular processes increases only as n^3, but that of the potentials as n^4 or n^5, according to whether they proceed from surfaces or spaces; so that the latter, if they represent a quantity of work which is smaller than in the resting equilibrium of electricity, always obtain the preponderance with a sufficient augmentation. When everywhere equal quanta of positive and negative electricity move in opposite directions, the quantities p_n vanish, but the electrodynamic potential $(-Q)$ may become less than zero. That such a distribution of electric densities and currents may occur has been shown in my earlier memoir, quite independently of the differential equations which regulate the course of the currents.

Given a current-distribution which represents a less quantity of work than that of electric equilibrium, such a flow can only by the application of exterior work be brought to rest, and must otherwise, by withdrawal of work, such as takes place by the development of heat in conductors, be augmented *ad infinitum.*

In this manner an example makes clear how important it is that the analytical expression of the *vis viva* should contain only positive terms; and that this condition is not fulfilled by the action at a distanee of Weber's law is here exhibited as the last cause of the physically impossible consequences to which it leads. These, at all events, cannot be removed without very bold new auxiliary hypotheses, which must not only vary the actions at molecular, but also those at greater distances.

In conclusion I have, in the present memoir, endeavoured to clear up the doubts expressed by M. J. Bertrand* respecting the structure of the differential equations of the motion of electricity.

LXIV. *Proceedings of Learned Societies.*

ROYAL SOCIETY.

[Continued from p. 473.]

June 20, 1872.—Sir James Paget, Bart., D.C.L., Vice-President, in the Chair.

THE following communication was read :—

"On the Spectrum of Nitrogen." By Arthur Schuster, Student at the Physical Laboratory of Owens College.

1. *Introductory.*—The formation of the different spectra which one gas is said to exhibit, when examined under different conditions, still remains one of the most obscure points of spectrum analysis. In 1864, when Plücker and Hittorf published their researches "On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance"†, they drew attention to the close resemblance in character of the band-spectra which certain metals yield at a comparatively low temperature to the band-spectrum of nitrogen and sulphur. Roscoe and Clifton, in their paper "On the effect of increased Temperature upon the nature of the Light emitted by the Vapour of certain Metals or Metallic Compounds"‡, rendered it probable that the band-spectra of the metals belonged really to the oxides. The two spectra of nitrogen were not, however, examined from that point of view, but, on the contrary, they were made the starting-point of new investigations by Wüllner, who came to the conclusion that certain gases may give even more than two different spectra. Ångström §, expressing his doubts about the trustworthiness of Wüllner's experiments, says in a note: "As regards the spectra which are usually attributed to nitrogen, I mention here, as a general fact, that it is my conviction that the fluted bands which are so characteristic of the oxides of metals are never found in spectra of elementary gases."

I propose to show, in the present communication, (1) that pure

* *Comptes Rendus de l'Acad. des Sciences,* vol. lxxiii. p. 968.

† Philosophical Transactions, vol. clv. p. 1.

‡ Chemical News, vol. v. p. 233. § Comptes Rendus, Aug. 1871.