The detected direction of the force onto a permanent magnet, caused by the displacement current in a wire gap, supports Weber electrodynamics

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Abstract

This article compares Maxwell's electrodynamics with the almost forgotten Weber electrodynamics as test theory by means of an easily reproducible and simple experiment. For this purpose, it is first theoretically inferred in two different ways that when charging a specially designed capacitor with a current source because of the displacement current a force should occur onto a permanent magnet between the plates, which is diametrically different in its direction in both theories. Subsequently, the experimental setup is described and, based on the results, it is determined that nature seems to follow Weber's law of force in this case. The result shows furthermore that Maxwell's magnetostatics can lead to false predictions under specific everyday conditions.

1. Introduction

Maxwell's equations have been very successful in describing electromagnetic waves for more than one hundred years. Their rise to the sole theory of electromagnetism begins with an article by James Clerk Maxwell in 1865 [1]. In this article he shows that from the complete set of Maxwell's equations, including the displacement current, a wave equation can be derived in which electromagnetic disturbances of the field propagate at the speed of light. His assumption was that light would be an electrical phenomenon. In 1886 Heinrich Hertz succeeded in experimentally generating and detecting electromagnetic waves for the first time [2]. Due to the success of Maxwell's equations, the electrodynamics of Wilhelm Weber and Carl Friedrich Gauss, which was widespread at this time, was increasingly forgotten because it was not compatible with the invariance of electric charge. Today it is largely unknown, although there have always been a few scientists who have studied it intensively [3] [4] [5].

The majority of scientists do not know Weber's electrodynamics and is probably firmly convinced that Maxwell's equations describe everyday physics perfectly due to their age. But old theories should be revised anew from time to time. To do that, a test theory is needed. Weber electrodynamics represents such a test theory, even though various experiments have apparently shown that the electric charge does not depend on the relative velocity, as the Weber force formula (4) would suggest (e.g. [6]). On the other hand, there are also a number of experiments that support the force law (21) between current elements which can only be derived from Weber electrodynamics (e.g. [7]).

This article belongs to the second type and examines the differences between the laws of force for current elements resulting from Maxwell's electrodynamics and Weber's electrodynamics. In particular, the question is investigated whether the magnetic force effect within a capacitor onto a permanent magnet differs in both theories and, if so, how this difference can be captured experimentally.

It has to be pointed out that a quantitative analysis of the magnetic field inside a capacitor is difficult and still subject of current research [8]. However, it is much easier not to measure the field strength electronically but only to detect the force effect and in particular its direction. The theory behind such an experiment, as well as the experiment itself, are described in this article. As far as the author knows, such an experiment has never been carried out before, probably because no one had previously assumed that there is a pendant for the displacement current in Weber electrodynamics as well.

2. Basics

2.1. The force between two uniformly moving point charges according to Maxwell and Weber

The following is about the electromagnetic force that a uniformly moving point charge q_s exerts onto another uniformly moving point charge q_d. The configuration is shown in figure 1. An approximation that neglects the magnetic component is the Coulomb law.

2.1.1. Weber force

An extension of Coulomb's law, which also contains the magnetic part, is from Wilhelm Weber in 1846 [9]:

$$\boldsymbol{F}_W = \left(1 - \frac{\dot{r}^2}{2\,c^2} + \frac{r\,\ddot{r}}{c^2}\right) \frac{q_s\,q_d}{4\,\pi\,\varepsilon_0}\,\frac{\boldsymbol{r}}{r^3}. \tag{1}$$

In this, $\dot{r}$ is the radial velocity and $\ddot{r}$ the radial acceleration.

Because of $\boldsymbol{v} := \dot{\boldsymbol{r}} = \boldsymbol{v}_d - \boldsymbol{v}_s$ and $\ddot{\boldsymbol{r}} = \dot{\boldsymbol{v}}_d - \dot{\boldsymbol{v}}_s = 0$, and by using the relations

$$\dot{r} = \frac{\mathrm{d}}{\mathrm{d}t}\sqrt{\boldsymbol{r}\cdot\boldsymbol{r}} = \frac{\boldsymbol{r}\cdot\dot{\boldsymbol{r}}}{r} = \frac{\boldsymbol{r}\cdot\boldsymbol{v}}{r} \tag{2}$$

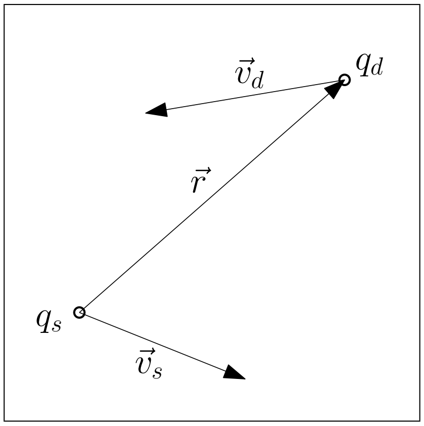

Figure 1: Configuration

and

$$\ddot{r} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\sqrt{\boldsymbol{r}\cdot\boldsymbol{r}} = \frac{\boldsymbol{r}\cdot\ddot{\boldsymbol{r}}}{r} - \frac{(\boldsymbol{r}\cdot\dot{\boldsymbol{r}})^2}{r^3} + \frac{\dot{\boldsymbol{r}}\cdot\dot{\boldsymbol{r}}}{r} = -\frac{(\boldsymbol{r}\cdot\boldsymbol{v})^2}{r^3} + \frac{v^2}{r} \quad (3)$$

the formula

$$\boldsymbol{F}_W(\boldsymbol{r},\boldsymbol{v}) = \left(1 + \frac{v^2}{c^2} - \frac{3}{2}\left(\frac{\boldsymbol{r}}{r}\,\frac{\boldsymbol{v}}{c}\right)^2\right)\frac{q_s\,q_d}{4\,\pi\,\varepsilon_0}\,\frac{\boldsymbol{r}}{r^3} \quad (4)$$

follows. Equation (4) is referred to in the following as *Weber force*.

2.1.2. *Maxwell force*

Also from Maxwell's equations a formula can be derived which gives the electromagnetic force between two uniformly moving point charges. To get to this, it is necessary to solve Maxwell's equations. For point charges, the special solution is called Liénard-Wiechert potentials [10, page 618].

If the point charge q_s is at the coordinate origin at the time $t = 0$ and moves uniformly away from there with the velocity $\boldsymbol{v}_s$ the scalar potential is

$$\varphi = \frac{c\,q_s}{4\,\pi\,\varepsilon_0\sqrt{\left(c^2\,t - \boldsymbol{v}_s\,\boldsymbol{r}\right)^2 + (c^2 - v_s^2)\,(r^2 - c^2\,t^2)}}, \quad (5)$$

while the vector potential $\boldsymbol{A}$ is

$$\boldsymbol{A} = \frac{\boldsymbol{v}_s}{c^2}\,\varphi. \quad (6)$$

By using the equations [10, page 451]

$$\boldsymbol{E} = -\nabla\varphi - \frac{\partial\boldsymbol{A}}{\partial t} \quad (7)$$

and

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} \quad (8)$$

it is possible to obtain the electric field strength

$$\boldsymbol{E} = \frac{c\,q_s\left(c^2 - v_s^2\right)(\boldsymbol{r} - \boldsymbol{v}_s\,t)}{4\,\pi\,\varepsilon_0\left((r^2 - c^2\,t^2)\,(c^2 - v_s^2) + (c^2\,t - \boldsymbol{r}\,\boldsymbol{v}_s)^2\right)^{\frac{3}{2}}} \quad (9)$$

and the magnetic flux density

$$\boldsymbol{B} = \frac{\boldsymbol{v}_s}{c^2}\times\boldsymbol{E}. \quad (10)$$

Replacing $\boldsymbol{r}$ with the linear equation $\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}_s\,t$ in equation (9) shows that by doing so the time dependency disappears. This means that the electric field allways retains its shape regardless of the point in time. Because we are not interested in the temporal shift of the actually time-independent field of the source charge, we can set t to zero without loss of generality. It follows

$$\boldsymbol{E} = \frac{c\,q_s\left(c^2 - v_s^2\right)\boldsymbol{r}}{4\,\pi\,\varepsilon_0\left(r^2\,(c^2 - v_s^2) + (\boldsymbol{r}\,\boldsymbol{v}_s)^2\right)^{\frac{3}{2}}}. \quad (11)$$

The formula (10) remains unaffected by this.

However, the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ themselves are not actually measurable, but only their force effects on test charges. In order to calculate the force $\boldsymbol{F}$ onto another point charge q_d, additionally its velocity $\boldsymbol{v}_d$ and the formula of the Lorentz force $\boldsymbol{F} = q_d\,\boldsymbol{E} + q_d\,\boldsymbol{v}_d\times\boldsymbol{B}$ is needed. By inserting the equations (11) and (8) follows the relation

$$\boldsymbol{F}_M(\boldsymbol{r},\boldsymbol{v}_s,\boldsymbol{v}_d) = \frac{c\,\left(c^2 - v_s^2\right)\left(\boldsymbol{r} + \frac{1}{c^2}\,\boldsymbol{r}\times\boldsymbol{v}_s\times\boldsymbol{v}_d\right)}{\left((c^2 - v_s^2) + \left(\frac{\boldsymbol{r}}{r}\,\boldsymbol{v}_s\right)^2\right)^{\frac{3}{2}}}\,\frac{q_s\,q_d}{4\,\pi\,\varepsilon_0\,r^3} \quad (12)$$

for the force of a uniformly moving ideal point charge q_s onto another uniformly moving ideal point charge q_d. The equation (12) is referred to in the following as *Maxwell force*.

It should be noted that the 2-th order approximation of the Maxwell force with respect to the velocities $\boldsymbol{v}_s$ and $\boldsymbol{v}_d$ has the form

$$\boldsymbol{F}_M(\boldsymbol{r},\boldsymbol{v}_s,\boldsymbol{v}_d) \approx \boldsymbol{F}_W(\boldsymbol{r},\boldsymbol{v}_s) + \frac{1}{c^2}\left(\boldsymbol{r}\times\boldsymbol{v}_s\times\boldsymbol{v}_d - \frac{v_s^2\,\boldsymbol{r}}{2}\right)\frac{q_s\,q_d}{4\,\pi\,\varepsilon_0\,r^3}, \quad (13)$$

which corresponds to the Liénard-Schwarzschild force for small, negligible accelerations ([4, page 146], [3]).

It is obvious that the Maxwell force is not opposite equal, when source point charge and target point charge are permutated, because

$$\boldsymbol{F}_M(\boldsymbol{r}_d - \boldsymbol{r}_s,\boldsymbol{v}_s,\boldsymbol{v}_d) \neq -\boldsymbol{F}_M(\boldsymbol{r}_s - \boldsymbol{r}_d,\boldsymbol{v}_d,\boldsymbol{v}_s). \quad (14)$$

Therefore the third of Newton' s laws is violated for the Maxwell force. In Maxwell electrodynamics the energy conservation, the conservation of momentum and the conservation of angular momentum are only fulfilled if the emitted electromagnetic waves are included. The logical problem, however, is that this should also apply to the magnetostatic case, as will be explained later. For the Weber force (4) at the other hand

$$\boldsymbol{F}_W(\boldsymbol{r}_d - \boldsymbol{r}_s,\boldsymbol{v}_d - \boldsymbol{v}_s) = -\boldsymbol{F}_W(\boldsymbol{r}_s - \boldsymbol{r}_d,\boldsymbol{v}_s - \boldsymbol{v}_d) \quad (15)$$

applies.

Both formulas – the Weber force (4) and the Maxwell force (12) – therefore differ fundamentally, even in the first order. Therefore, both cannot be correct at the same time.

2.2. Forces between current elements

Measuring the electromagnetic force directly between two moving point charges is practically impossible. Much easier is to measure the force between currents. For this reason, this section analyzes two oppositely equal charges q_s and $-q_s$, which are at the coordinate origin at the time $t = 0$ and are moving at the speed $\boldsymbol{v}_s/2$ and $-\boldsymbol{v}_s/2$.

It is obvious that the both oppositely moving charges will no longer be in the same place after only a short time. However, if one imagines many such current elements arranged in a line, it becomes clear that there are always two oppositely equally sized charges at a location at any given time, since the neighbors repeatedly replace the outflowing charge carriers. Such a line represents a direct current, because electric current is defined as the number of charge carriers passing through a surface transverse to the direction of movement per unit of time.

The force $\boldsymbol{F}_{WC}$ of such a current element on a charge q_d with the velocity $\boldsymbol{v}_d$ at the location $\boldsymbol{r}$ is, when using the Weber force (4),

$$\begin{aligned} \boldsymbol{F}_{WC} &= \boldsymbol{F}_W(\boldsymbol{r}, \boldsymbol{v}_d - \boldsymbol{v}_s/2) - \boldsymbol{F}_W(\boldsymbol{r}, \boldsymbol{v}_d + \boldsymbol{v}_s/2) \quad (16) \\ &= \left(3\left(\frac{\boldsymbol{r}}{r}\,\boldsymbol{v}_s\right)\left(\frac{\boldsymbol{r}}{r}\,\boldsymbol{v}_d\right) - 2\,\boldsymbol{v}_s\,\boldsymbol{v}_d\right)\frac{q_s\,q_d}{4\,\pi\,\varepsilon_0\,c^2}\,\frac{\boldsymbol{r}}{r^3}. \quad (17) \end{aligned}$$

From the second order approximation (13) of the Maxwell force follows

$$\begin{aligned} \boldsymbol{F}_{MC} &= \boldsymbol{F}_M(\boldsymbol{r}, -\boldsymbol{v}_s/2, \boldsymbol{v}_d) - \boldsymbol{F}_M(\boldsymbol{r}, +\boldsymbol{v}_s/2, \boldsymbol{v}_d) \quad (18) \\ &= \frac{q_s\,q_d}{4\,\pi\,\varepsilon_0\,c^2\,r^3}\,\boldsymbol{r}\times\boldsymbol{v}_s\times\boldsymbol{v}_d. \quad (19) \end{aligned}$$

2.3. Magnetic forces in a wire gap

The force of a current element on another current element is the sum of the force of the current element on a positive point charge q_d at the location $\boldsymbol{r}$ with the velocity $\boldsymbol{v}_d/2$ and the force on an opposite equal point charge at the same location with the opposite velocity. In Maxwell's electrodynamics, the force of a current element on a point charge is given by the equation (19). The force of a current element onto another current element is therefore because of $q_d\,\boldsymbol{v}_d/2 + (-q_d)\,(-\boldsymbol{v}_d/2) = q_d\,\boldsymbol{v}_d$ and $1/\mu_0 = c^2\,\varepsilon_0$,

$$\boldsymbol{F}_M(\boldsymbol{r}) = \frac{\mu_0\,q_s\,q_d}{4\,\pi\;r^3}\,\boldsymbol{r}\times\boldsymbol{v}_s\times\boldsymbol{v}_d. \quad (20)$$

In Weber electrodynamics this force is given by the equation (17) and it applies

$$\boldsymbol{F}_W(\boldsymbol{r}) = \frac{\mu_0\,q_s\,q_d}{4\,\pi}\,\frac{\boldsymbol{r}}{r^3}\left(3\left(\frac{\boldsymbol{r}}{r}\,\boldsymbol{v}_s\right)\left(\frac{\boldsymbol{r}}{r}\,\boldsymbol{v}_d\right) - 2\,\boldsymbol{v}_s\,\boldsymbol{v}_d\right). \quad (21)$$

Both formulas (20) and (21) are regarded in the scientific literature as variants of Ampère's law, whereby it is still discussed whether both laws are equivalent [11] [12] [13]. Note that formula (20) is sometimes referred to as Biot-Savart or Grassmann force [4].

It is undisputed, however, that both equations lead to the same magnetic field for closed conductor loops and homogeneous current densities. For non-closed conductor loops, however, both equations are by no means equivalent. By using a *current source*, for example, it is possible to let a direct current flow into an electrical conductor *with a gap* for a comparatively long time. This will now be discussed.

To get the force that an entire wire exerts onto a current element, the line integral along the wire has to be calculated. Now two forces shall be calculated, namely

- the force of a wire which is located on the x-axis, begins at $-\infty$ and ends at the coordinate origin and
- the force of a wire which is located on the x-axis, begins at the coordinate origin and ends at $+\infty$.

Of course, no permanent current flow is possible in such wire segments, since the current flow changes the net charge over time. On short time scales, however, the current is sufficiently uniform so that the configuration can be considered quasi-stationary, especially if a current source is used instead of a voltage source.

With $q_s \to \lambda_s$ and $I_s = \lambda_s\,v_s\,\boldsymbol{e}_x$ for the Maxwell electrodynamics follows

$$\begin{aligned} \boldsymbol{F}_{IM}^{(\pm)}(\boldsymbol{r}, \boldsymbol{v}_d) &= \int_0^\infty \boldsymbol{F}_M(\boldsymbol{r} \pm s\,\boldsymbol{e}_x)\,\mathrm{d}s \quad (22) \\ &= \frac{\mu_0\,I_s\,q_d\,\boldsymbol{r}\times\boldsymbol{e}_x\times\boldsymbol{v}_d}{4\,\pi\,r\,(r \pm x)}. \quad (23) \end{aligned}$$

For Weber electrodynamics, on the other hand, the result is

$$\begin{aligned} \boldsymbol{F}_{IW}^{(\pm)}(\boldsymbol{r}, \boldsymbol{v}_d) &= \int_0^\infty \boldsymbol{F}_W(\boldsymbol{r} \pm s\,\boldsymbol{e}_x)\,\mathrm{d}s \quad (24) \\ &= \boldsymbol{F}_{IM}^{(\pm)}(\boldsymbol{r}, \boldsymbol{v}_d) \pm \frac{\mu_0\,I_s\,q_d\,(\boldsymbol{r}\cdot\boldsymbol{v}_d)\,\boldsymbol{r}}{4\,\pi\,r^3}, \quad (25) \end{aligned}$$

which is the same result as in Maxwell's electrodynamics plus an additional term. The plus sign is valid for the wire to the left of the y-z plane and the minus sign for the wire to the right.

Figure 2 shows in (A) the fields of this force onto right hand directed current elements for a wire stub that is just positively charging. At the top the field is shown which follows from Maxwell electrodynamics. Below, the corresponding field is shown, which is predicted by Weber electrodynamics. It can be noticed, that – as was to be expected – currents which flow in the same direction attract each other.

In figure 2 (B) the fields for current elements with flow direction upwards are shown. It is obvious that the fields in (A) as well in (B) become more and more similar to each

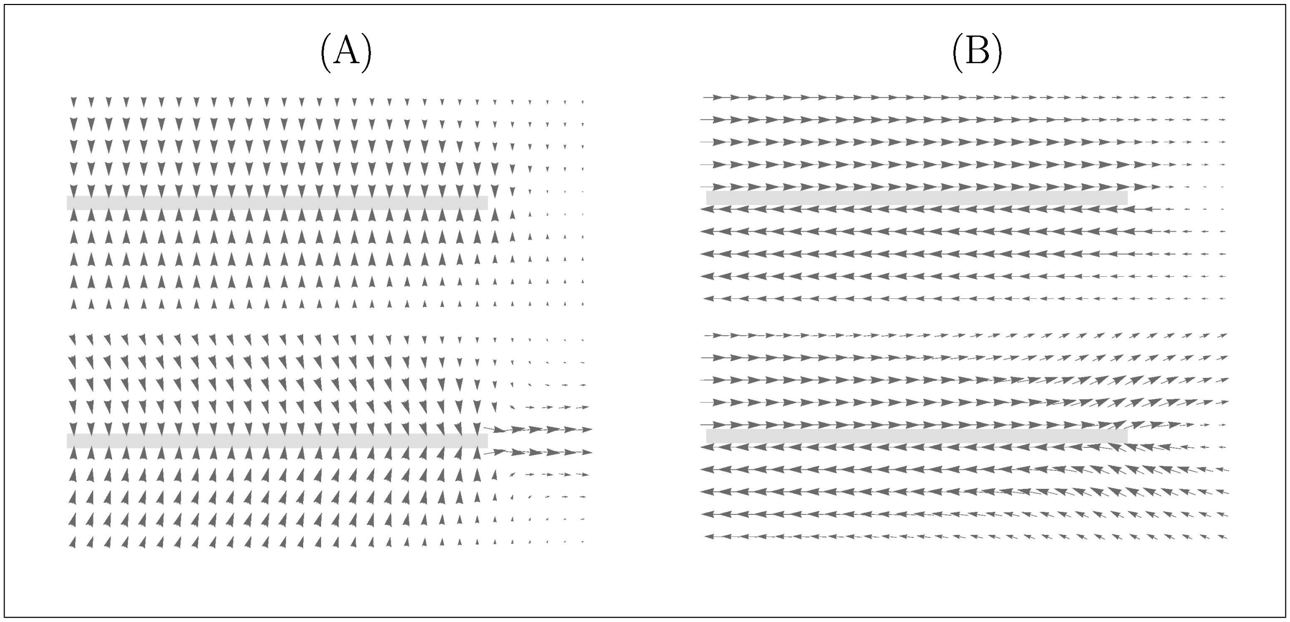

Figure 2: The fields of the magnetic force (not flux densities) for Weber (bottom) and Maxwell (top) onto current elements with flow direction to the right (A) and flow direction upwards (B).

other on the left. This is also the reason why it is so difficult to distinguish between Maxwell and Weber electrodynamics simply by measuring the forces around direct currents. Only where the wire ends the fields differ. It is obvious that exactly this should be exploited in order to decide experimentally between the two force laws.

It should be noted that for Maxwell's electrodynamics, the field of a wire stub cannot only be determined by integration, but also by solving the Maxwell equations directly. In the next section this calculation is performed and it is shown that it is actually not necessary to use the Liénard-Wiechert potentials.

But back to the experiment: The formulas (23) and (25) represent the force which the wire stub asserts onto current elements. For experiments, however, it is much more interesting to know which force and which torque acts on a small permanent magnet, since such a magnet is not influenced by an electric field. Both will be calculated in the following. For this purpose it is assumed that at $\boldsymbol{r}$ a very small conductor loop with the magnetic dipole moment $\boldsymbol{\mu}$ is located (equivalent to a small permanent magnet). The figure 3 shows the configuration in form of a sketch.

Let $\boldsymbol{e}_a$ and $\boldsymbol{e}_b$ be two mutually perpendicular unit vectors for which the equation

$$\boldsymbol{e}_a \times \boldsymbol{e}_b = \boldsymbol{\mu}/\mu \tag{26}$$

applies. For the net force $\boldsymbol{F}_L$ on a conductor loop with the radius R, as one can see from the sketch 3, follows the equation

$$\boldsymbol{F}_L = R \int_0^{2\pi} \boldsymbol{F}_I \left(\boldsymbol{r} + R\, \boldsymbol{e}_\phi(\phi)\, , \frac{I_d}{\lambda_d}\, \boldsymbol{e}_\phi(\phi + \pi/2) \right) \mathrm{d}\phi \tag{27}$$

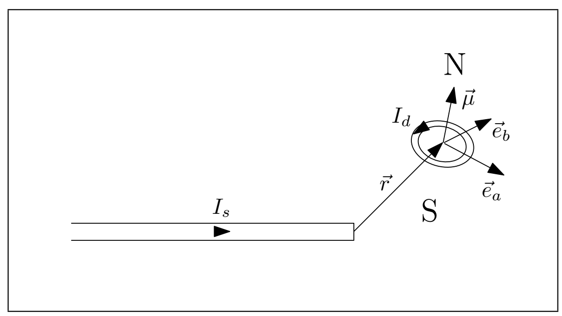

Figure 3: Small conductor loop as model of a permanent magnet in the vicinity of the wire stub

with $q_d \to \lambda_d$,

$$\boldsymbol{e}_\phi(\phi) := \boldsymbol{e}_a \cos(\phi) + \boldsymbol{e}_b \sin(\phi) \tag{28}$$

and

$$\mu = I_d\, \pi\, R^2. \tag{29}$$

For the torque $\boldsymbol{M}_L$ follows quite equivalent according to the sketch the curve integral

$$\begin{aligned} \boldsymbol{M}_L = R^2 \int_0^{2\pi} & \boldsymbol{e}_\phi(\phi) \times \\ & \boldsymbol{F}_I \left(\boldsymbol{r} + R\, \boldsymbol{e}_\phi(\phi)\, , \frac{I_d}{\lambda_d}\, \boldsymbol{e}_\phi(\phi + \pi/2) \right) \mathrm{d}\phi. \end{aligned} \tag{30}$$

In order to solve the complicated integrals for the forces (23) and (25) it is useful to take advantage of the fact that the radius R of the conductor loop is very small, which allows a Taylor approximation of first order of $\boldsymbol{F}_I$ at zero.

The resulting approximation can then be used to obtain the force

$$\boldsymbol{F}_{LM}^{(\pm)}(\boldsymbol{r}) = \frac{\mu_0\, I_s}{4\,\pi\, r^3}\left(\frac{r^2}{r \pm x}\boldsymbol{\mu}\times\boldsymbol{e}_x + \frac{(2\,r \pm x)\,\boldsymbol{r} \pm r^2\,\boldsymbol{e}_x}{(r \pm x)^2}(\boldsymbol{\mu}\times\boldsymbol{r})\cdot\boldsymbol{e}_x\right) \tag{31}$$

for Maxwell electrodynamics by using the equations (26) and (29). Weber electrodynamics, on the other hand, gives the force

$$\boldsymbol{F}_{LW}^{(\pm)}(\boldsymbol{r}) = \boldsymbol{F}_{LM}^{(\pm)}(\boldsymbol{r}) \pm \frac{\mu_0\, I_s\, \boldsymbol{r}\times\boldsymbol{\mu}}{4\,\pi\, r^3}. \tag{32}$$

For the torque, Maxwell electrodynamics provides after calculation of the integral (30) and simplification of the terms the solution

$$\boldsymbol{M}_{LM}^{(\pm)}(\boldsymbol{r}) = \frac{\mu_0\, I_s\, \boldsymbol{r}\times\boldsymbol{e}_x\times\boldsymbol{\mu}}{4\,\pi\, r\,(r \pm x)}. \tag{33}$$

For electrodynamics according to Weber follows

$$\boldsymbol{M}_{LW}^{(\pm)}(\boldsymbol{r}) = \boldsymbol{M}_{LM}^{(\pm)}(\boldsymbol{r}) \pm \frac{\mu_0\, I_s\, \boldsymbol{r}\times\boldsymbol{\mu}\times\boldsymbol{r}}{4\,\pi\, r^3}. \tag{34}$$

The figures 4 and 5 show the field of the two — slightly separated – wires on a permanent magnet with the north pole aligned into the drawing plane. Both wires together form a capacitor of low capacitance.

Let d be the width of the air gap. The net force is then given by the equation

$$\boldsymbol{F}_T(\boldsymbol{r}) = \boldsymbol{F}_L^{(+)}(\boldsymbol{r} - d/2\,\boldsymbol{e}_x) + \boldsymbol{F}_L^{(-)}(\boldsymbol{r} + d/2\,\boldsymbol{e}_x). \tag{35}$$

It is obvious that in the middle of the wire gap forces occur that differ in sign. For Maxwell electrodynamics, $\boldsymbol{\mu} = \mu\,\boldsymbol{e}_y$ (north pole points into the drawing plane) applies

$$\boldsymbol{F}_{TM}(\boldsymbol{0}) = -\frac{\mu_0\, I_s\, \mu}{\pi\, d^2}\,\boldsymbol{e}_z, \tag{36}$$

while Weber electrodynamics predicts force

$$\boldsymbol{F}_{TW}(\boldsymbol{0}) = +\frac{\mu_0\, I_s\, \mu}{\pi\, d^2}\,\boldsymbol{e}_z. \tag{37}$$

Both theories lead here to *opposite statements*, which makes a detection of this force direction almost predestined for an experiment.

By the way, there is also something about torque that is worth mentioning. From equation (33) it is easy to deduce that $\boldsymbol{M}_{LM}^{(\pm)}(\boldsymbol{r})\cdot\boldsymbol{\mu} = 0$ applies, i.e. in Maxwell's electrodynamics is nowhere a torque which is aligned parallel to the magnetic moment $\boldsymbol{\mu}$. This is not the case in Weber electrodynamics! For example, when the magnet is positioned directly at the end of the wire stub. This is quite remarkable as it goes beyond the frame given by the magnetic field line concept. On the other hand, such a torque is necessary to ensure the conservation of the total angular momentum, since the magnet generates an angular momentum on the wire as well. A similar statement applies to the magnetic force in the air gap, since only in Weber electrodynamics the conservation of momentum is fulfilled.

2.4. Solution of Maxwell's equations for a wire stub

In this section, the field which was calculated in the previous section is determined again directly based on Maxwell's equations. This serves above all to show that the statement of Maxwell's electrodynamics regarding the previously calculated force effects does not depend on the solution path.

In this section the full set of Maxwell equations in vacuum is used:

$$\nabla\cdot\boldsymbol{E} = \frac{\rho}{\varepsilon_0} \tag{38}$$

$$\nabla\cdot\boldsymbol{B} = 0 \tag{39}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \tag{40}$$

$$\nabla\times\boldsymbol{B} = \mu_0\,\boldsymbol{j} + \mu_0\,\varepsilon_0\,\frac{\partial\boldsymbol{E}}{\partial t}. \tag{41}$$

In the following, the electromagnetic field of a direct current-carrying wire is calculated, which lies exactly on the x-axis, begins somewhere far to the left and ends exactly at the coordinate origin. It is obvious that due to the current flow and the discontinuity, the wire cannot remain electrically neutral, but must charge or discharge over time. This results in a time-varying electric field, which in turn must be taken into account in Ampère's circuital law (41).

It is again noted that even in a non-closed conductor loop a direct current can be maintained for a certain time, i.e. when the wire segment is charged via a current source. The current can flow hereby until the amount of charge in the wire segment has become so large that the technical limits of the current source are exceeded. As model for the current density, therefore the time independent ansatz

$$\boldsymbol{j} = \frac{I_s}{2}\left(1 - \mathrm{erf}\left(\frac{x}{\sqrt{2\,\nu}}\right)\right) g(y,\nu)\, g(z,\nu)\,\boldsymbol{e}_x \tag{42}$$

can be used. I_s represents the current and g the Gaussian function:

$$g(u,\nu) := \frac{1}{\sqrt{2\,\pi\,\nu}}\,\exp\left(-\frac{u^2}{2\,\nu}\right). \tag{43}$$

The variance ν can be very small for a real wire. For $\nu \to 0$, equation (42) turns into the distribution (52) and it becomes obvious that the current density ansatz (42) is indeed a smoothed modell for the already discussed current-carrying wire which lies exactly on the x-axis, begins somewhere far to the left and ends exactly at the coordinate origin. The reason for the smoothing is that it eliminates singularities and makes the function continuously differentiable, which is a requirement for using Maxwell's equations.

Inserting of equation (42) into the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = 0 \tag{44}$$

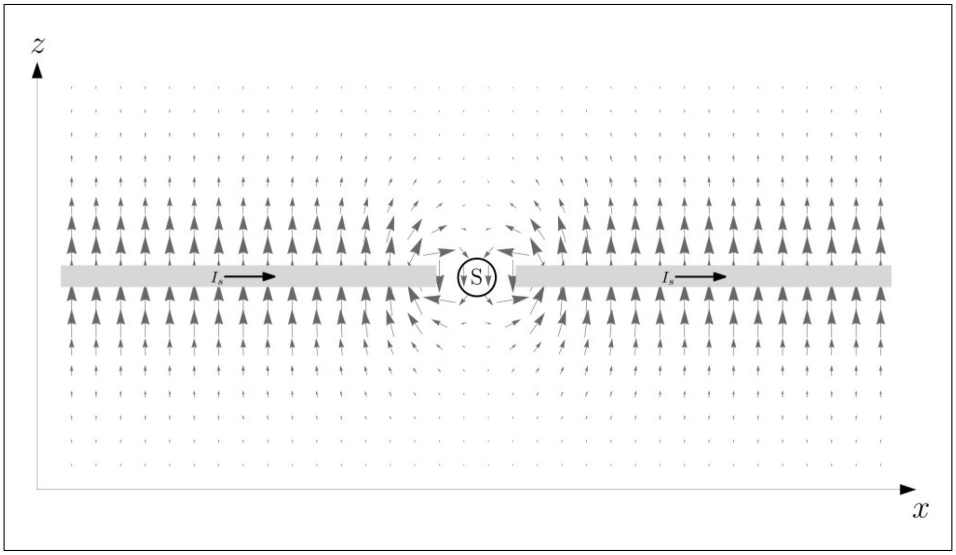

Figure 4: The force effect of a direct current according to Maxwell onto a permanent magnet with north pole pointing into the drawing plane. The current can be driven by a current source.

yields the charge density

$$\rho = -\int \nabla \cdot \boldsymbol{j}\, \mathrm{d}t = \frac{I_s\, t}{(2\,\pi\,\nu)^{\frac{3}{2}}}\, e^{-\frac{r^2}{2\nu}} \tag{45}$$

with $r^2 = x^2 + y^2 + z^2$, assuming that the wire is electrically neutral at $t = 0$. The calculated charge density shows that the wire becomes electrically charged where it ends, since the current cannot flow on here.

To determine the electric field of this charge, the first Maxwell equation in integral form

$$\varepsilon_0 \oiint_{\partial V} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{A} = \iiint_V \rho\, \mathrm{d}V \tag{46}$$

is used. The integral on the right corresponds to the charge Q enclosed in the volume. If a sphere with radius R is selected as volume than

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho\, r^2\, \sin(\theta)\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}\phi, \tag{47}$$

i.e.

$$Q = I_s\, t \left(\operatorname{erf}\left(\frac{R}{\sqrt{2\,\nu}}\right) - \sqrt{\frac{2}{\pi\,\nu}}\, R\, e^{-\frac{R^2}{2\nu}}\right). \tag{48}$$

Because of the spherical symmetry of the charge distribution ρ, it can be concluded that the electric field $\boldsymbol{E}$ is also radially symmetric and that the field lines are always perpendicular to the surface of the sphere. The integral on the right side of the equation (46) can therefore be solved immediately and the result is

$$\varepsilon_0 \oiint_{\partial V} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{A} = \varepsilon_0\, E\, 4\,\pi\, R^2. \tag{49}$$

From the results (48) and (49) and the symmetry follows then

$$\boldsymbol{E} = \frac{I_s\, t\, \boldsymbol{r}}{4\,\pi\,\varepsilon_0\, r^3} \left(\operatorname{erf}\left(\frac{r}{\sqrt{2\,\nu}}\right) - \sqrt{\frac{2}{\pi\,\nu}}\, r\, e^{-\frac{r^2}{2\nu}}\right). \tag{50}$$

Next, the magnetic flux density $\boldsymbol{B}$ can be determined by using the fourth of Maxwell's equations (41). For this purpose the current density (42) and the electric field (50) are used. Afterwards, Poincarés-Lemma can be applied and one gets to

$$\boldsymbol{B} = \frac{I_s\, \mu_0 \left(1 - \frac{x}{r}\operatorname{erf}\left(\frac{r}{\sqrt{2\,\nu}}\right) - e^{-\frac{r^2-x^2}{2\nu}}\left(1 - \operatorname{erf}\left(\frac{x}{\sqrt{2\,\nu}}\right)\right)\right) \boldsymbol{e}_x \times \boldsymbol{r}}{4\,\pi\,(r^2 - x^2)}. \tag{51}$$

Inserting the equations (42), (50) and (51) in the four Maxwell equations (38) to (41) shows that all conditions are fulfilled. The equations (50) and (51) therefore describe the fields which, according to Maxwell's electrodynamics, result from the charging process of the wire stub. It should be noted that the magnetic flux density $\boldsymbol{B}$ is time-independent. This means that this is a special case of *magnetostatics*.

Finally, the results can be simplified by calculating the limit $\nu \to 0$. For the current density follows

$$\boldsymbol{j} = I_s\, (1 - \Theta\,(x))\, \delta(y)\, \delta(z)\, \boldsymbol{e}_x, \tag{52}$$

with Θ being the Heaviside step function and δ the Dirac delta function. The electric field strength becomes

$$\boldsymbol{E} = \frac{I_s\, t\, \boldsymbol{r}}{4\,\pi\,\varepsilon_0\, r^3}, \tag{53}$$

and the magnetic flux density

$$\boldsymbol{B} = \frac{I_s\, \mu_0}{4\,\pi\,(r^2 - x^2)} \left(1 - \frac{x}{r}\right) \boldsymbol{e}_x \times \boldsymbol{r} = \frac{I_s\, \mu_0\, \boldsymbol{e}_x \times \boldsymbol{r}}{4\,\pi\, r\,(r + x)}. \tag{54}$$

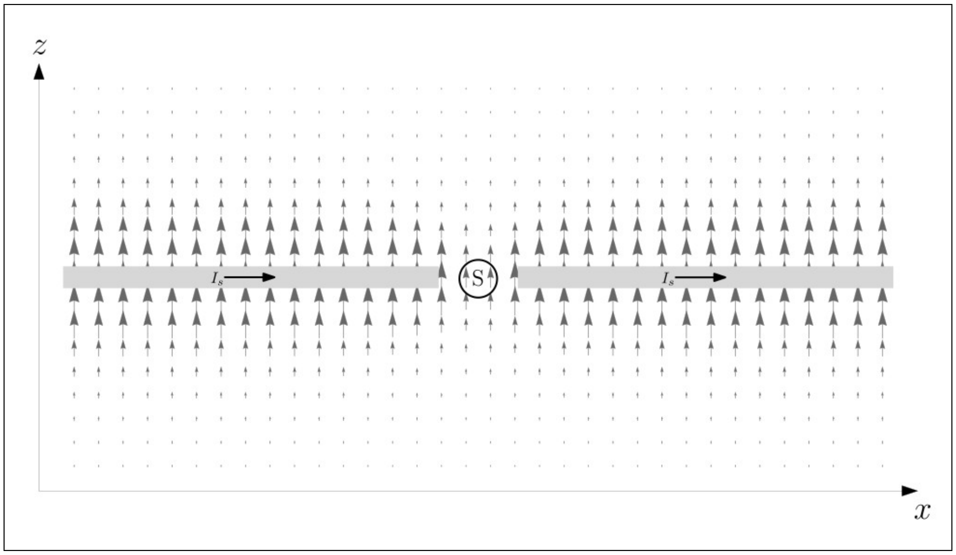

Figure 5: The force effect of a direct current according to Weber onto a permanent magnet with north pole pointing into the drawing plane. The current can be driven by a current source.

Because of

$$\lim_{x\to-\infty} r(r+x) = \frac{1}{2}(y^2+z^2) \tag{55}$$

and

$$\lim_{x\to-\infty} \boldsymbol{e}_x \times \boldsymbol{r} = \boldsymbol{e}_x \times \boldsymbol{r} \tag{56}$$

this magnetic field to the left of the y-z plane changes into the usual field of a line current. On the right, however, the strength of the field decreases with the square of the distance to the y-z plane, but otherwise retains its shape and orientation unchanged, just as if the current were still flowing slightly beyond the y-z plane.

With the help of the Lorentz force

$$\boldsymbol{F} = q_d\,\boldsymbol{E} + q_d\,\boldsymbol{v}_d \times \boldsymbol{B}, \tag{57}$$

and by inserting the equations (53) and (54) one gets finally the force

$$\boldsymbol{F} = \frac{q_d\,I_s\,t\,\boldsymbol{r}}{4\,\pi\,\varepsilon_0\,r^3} + \frac{q_d\,I_s\,\mu_0\,\boldsymbol{r}\times\boldsymbol{e}_x\times\boldsymbol{v}_d}{4\,\pi\,r\,(r+x)}, \tag{58}$$

which the wire stub exerts on a point charge q_d with the velocity $\boldsymbol{v}_d$ according to Maxwell's equations. A comparison shows that the magnetic part of the force corresponds exactly to formula (23).

3. Experiment

3.1. Experimental setup

In section 2.3 a way was identified to experimentally determine whether magnetostatics is correctly described by Maxwell's electrodynamics or by Webers approach. It was found that the simplest way is to measure the magnetic force on a permanent magnet within a wire gap. Both force laws predict the same magnitude for the force, but the prediction differs diametrically in terms of direction. This predestines this effect for an experiment, since the direction of a force can be detected much more easily than to measure its magnitude.

In figure 6 the basic experimental setup is shown. It consists of an upside-down U-shaped capacitor with an air gap. Exactly inside the gap is a rod-shaped permanent magnet hanging from a shielded piezoelectric cantilever force sensor [14] (sensor: EKULIT EPZ-27MS44W). By closing a relay, the capacitor can be charged from 0 to 7kV via a series resistor. During this, a charging current flows for a short time in the plates and a displacement current within the wire gap. The permanent magnet, which is aligned with the north pole towards the observer, experiences during this charging event a magnetic force, which z-component can be detected as a voltage change of the piezoelectric sensor. The permanent magnet itself consists of 28 neodymium magnets with a diameter of 8mm and a total length of 86mm. To avoid dielectric breakdowns, the magnet was coated with an insulating coating.

As it becomes clear, the experiment is simple and direct. The only challenge is to reliably detect the small force that only acts for a short time. Furthermore, an electromagnetic pulse is generated during the charging event, which influences the experiment. Therefore, great importance must be attached to shielding. For this reason, the cantilever force sensor, on which the permanent magnet is suspended, was wrapped with electrically insulating foil and then with aluminium foil. The aluminium foil was then connected to the shield of a stereo audio cable in order to completely protect the two differential lines of the piezo up to the likewise shielded instrumentation amplifier.

The schematic of the charging circuit is shown in fig-

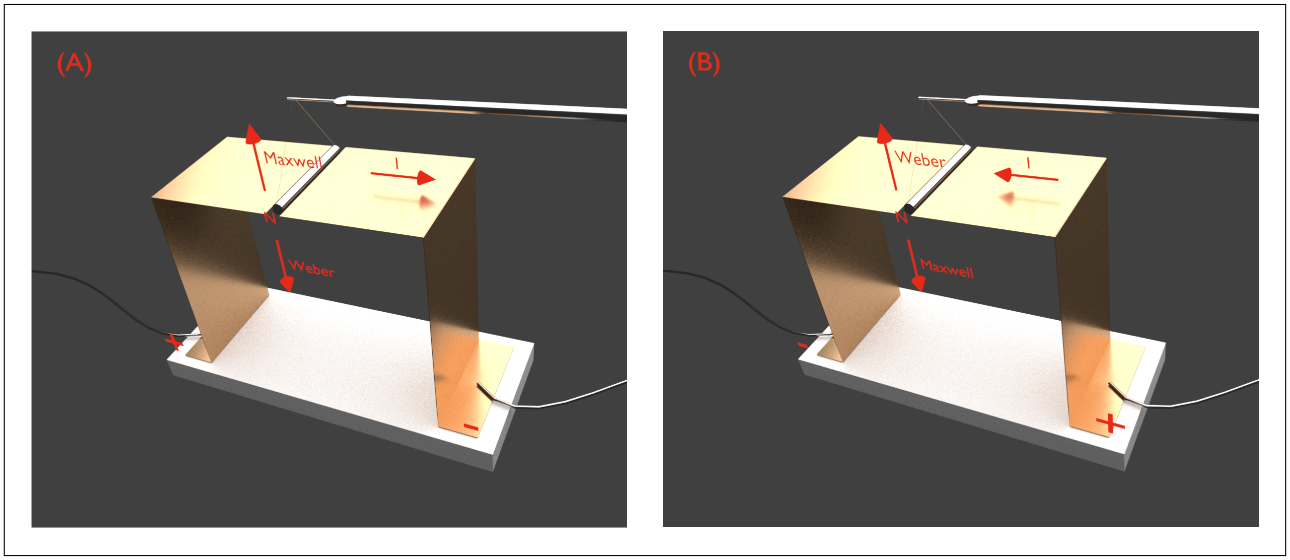

Figure 6: Experimental setup: capacitor and permanent magnet (north pole directed towards the observer)

ure 7. Here C_M is the measuring capacitor shown in figure 6. A measurement gave a capacity of about 8pF, of which only about one percent is due to the air gap. Since the high voltage source has a very large internal resistance R_i, there is a charging capacitor C_S parallel to the measuring capacitor, which ensures that the voltage of 7kV does not collapse when the relay S_1 is closed and that there is enough charge available to charge the measuring capacitor.

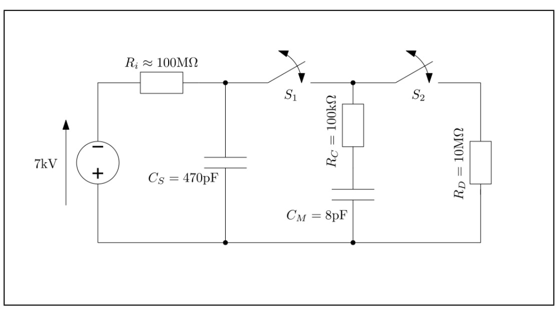

Figure 7: charging circuit

As can be seen from the circuit, the measuring capacitor needs about 2.4μs to be charged to 95 percent after closing the relay S_1, because the time constant τ has the value $R_C C_M = 0.8\mu$s. This time constant specifies the pattern that can be expected from the piezo. The relay S_2 and the resistor R_D are used to discharge the measuring capacitor and to establish a defined initial state.

Figure 8 shows the schematic of the amplifier. The INA111 is an integrated instrumentation amplifier, i.e. a differential amplifier with very high common mode rejection. Like the measuring cable to the sensor, the amplifier was completely shielded by installation in a metal housing.

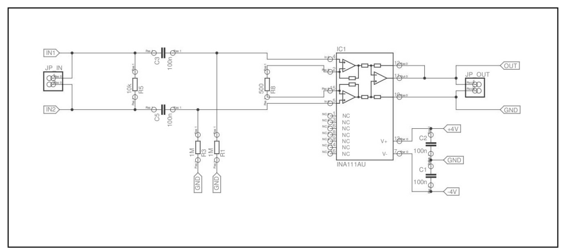

Figure 8: measuring amplifier

3.2. Results and evaluation

The forces generated at the piezoelectric cantilever force sensor are comparatively small. The magnetic moment μ of the used neodymium magnet can be roughly estimated to the value 3.8Am^2 with the help of the formula $\mu \approx \text{volume} \cdot 875000\text{A/m}$. With a gap width of 12mm and an initial charge current of 70mA, the formula (36) respectively (37) gives a force of at most $\pm 738\mu$N, which corresponds to the weight force acting on an object with a mass of 75mg. In reality, this force will be much smaller, since only a fraction of the current flowing into the measuring capacitor is effective as displacement current in the air gap.

Fortunately, in this experiment it is not necessary to measure the magnitude of the force, since only its direction is of interest. For this reason, it is sufficient to clearly identify the force effect and to distinguish it from existing interference influences. Figure 9 shows in (A) the measured voltages of six different individual measurements on the piezoelectric cantilever force sensor immediately after switching on the relay S_1, exactly for the case shown in Figure 6. The current flows here from left to right and the north pole points in the direction of the beholder. (B) shows the voltages measured when the current flows from right to

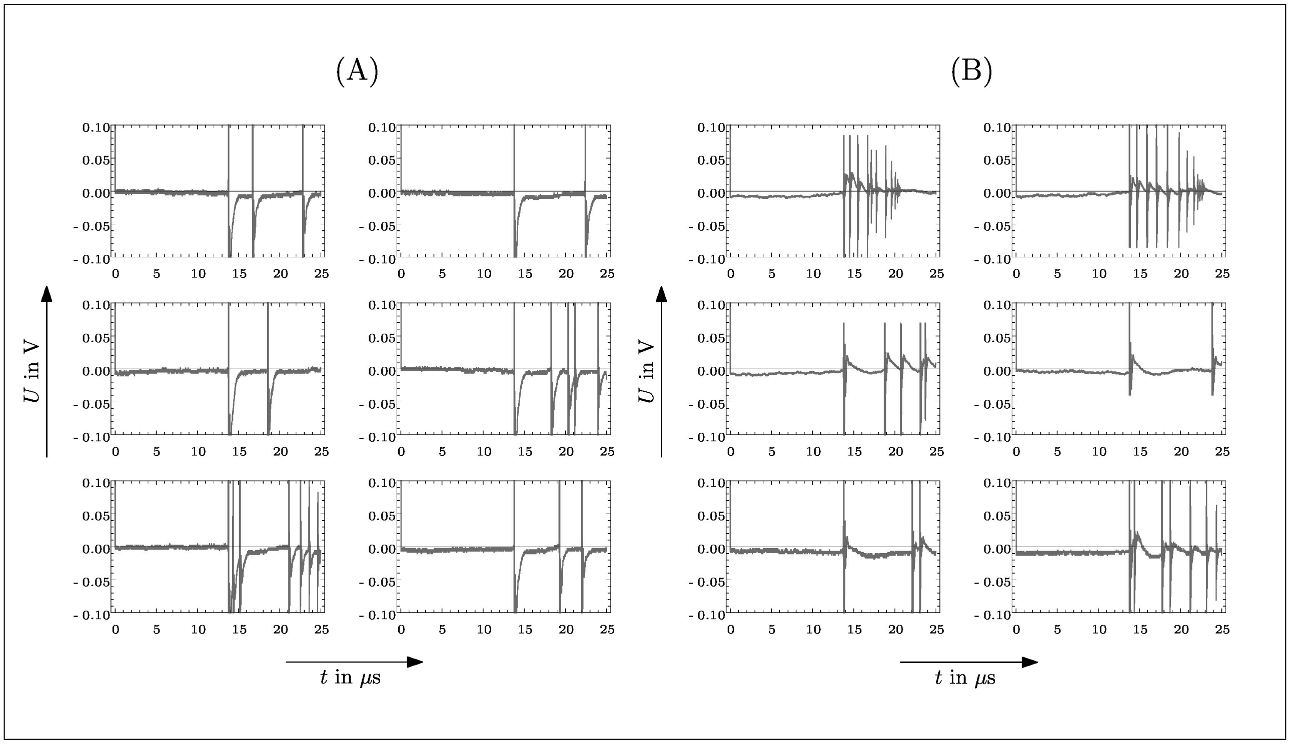

Figure 9: Results: In (A) is Plus on the left and Minus on the right. For (B) the capacitor is rotated, but the north pole is still pointing towards the beholder.

left.

What is noticeable at first when looking at the figure 9 are the seemingly random peaks, where the first peak always occurs at about $t = 14\mu$s. The reason for this delay is that the relay needs some time between triggering and closing. The peak itself is an effect of the electromagnetic pulse in the oscilloscope itself and occurs even when the magnet is disconnected from the force sensor or the amplifier is turned off. The peaks can therefore be regarded as disturbances. The voltage curves following a peak, however, are important for the experiment.

As can be seen, the voltage in figure 9 (A) approaches the x-axis after the first peak coming from below, while the opposite is true for (B). The shape of the voltage curve is thereby exponential, where the time constant is about 2μs. This is in good agreement with the time constant previously derived from the circuit in Figure 7. It is therefore possible to conclude that the voltage curves observed are each due to the magnetic force, which is proportional to the charging current in the capacitor.

The randomly distributed peaks following the first peak show the significant pattern as well. The randomness of the occurrence can be explained by the bouncing of the relay, which opens and closes several times for a short time after triggering.

The bottom line is that if the current flows from left to right, the cantilever force sensor outputs a voltage change downwards, while if the current flows from right to left, the voltage rises. By tapping the force sensor very lightly, it could be determined that a voltage drop corresponds to a downward pulling force. A voltage rise, on the other hand, corresponds to a force that pushes from bottom to top.

This finally makes it evident that a charging current flowing from right to left results in a downward force, provided that the north pole of the magnet points in the direction of the beholder. However, a current flowing from right to left pushes the magnet upwards. This *corresponds to the predictions of Weber electrodynamics*, as can be seen in the figures 4, 5 and 6, and shows at the same time that Maxwell electrodynamics is in contradiction with the experiment.

4. Summary and conclusion

This article showed that Weber and Maxwell electrodynamics can be experimentally distinguished by detecting the magnetic force directions within capacitors that are being charged or discharged. Subsequently, such an experiment was carried out and it was determined on the basis of the results that the detected force direction does not correspond to the predictions of Maxwell's electrodynamics in combination with the Lorentz force and that nature seems here to follow Weber's law of force.

The consequences of this statement are far-reaching, because this indicates that Maxwell electrodynamics seems to provide false predictions under specific everyday conditions. Should this be the case, it would be a veritable problem for modern physics, since practically everything that was added to physics in the twentieth century is based on Maxwell's equations and is thus indirectly affected. Furthermore, it is a disadvantage for engineering if the predicted forces in simulations and calculations for displacement currents are incorrect. This probably means that some technical applications have not yet been developed.

For this reason, the author feels compelled to urge the scientific community to repeat and analyze theory and experiment of this article as soon and carefully as possible. Furthermore, a way must be found to improve the existing theory of electrodynamics so that the magnetic forces can be predicted correctly in the case of spatially inhomogeneous current densities.

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