COMPLEMENTARITY OF ABSOLUTE AND RELATIVE MOTION

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The physical origin of inertial forces is investigated within the framework of general relativity. It is shown that the translational inertial force cannot be caused by the gravitational influence of distant masses. Minkowski's absolute spacetime must therefore be called upon in order to explain inertial forces. Nevertheless, only relative motion appears to have operational significance. This dichotomy is resolved in this paper by extending the principle of complementarity to include motion. Wave–particle duality is associated with the nature of a *particle*: its *motion* has corresponding complementary aspects in absolute and relative movements, respectively. This viewpoint is adopted in the description of motion in a gravitational field.

1. Introduction

It has been argued recently that the standard relativistic theory of gravitation provides a consistent description of wave properties only in the limit of vanishing wavelength of the radiation [1–3]. A tentative extension of the standard theory has been proposed based on the Fourier analysis of the measured components of the external radiation field [3]. The frequency and wave-vector content of the radiation field are then covariantly defined using the phase of each Fourier component in the standard manner. This procedure places a significant limitation on the magnitude of the wave vector of the radiation, namely, wavelengths longer than a certain characteristic radius of curvature of spacetime cannot be defined. This circumstance is in conflict with the fundamental postulate of wave–particle duality since no basic restriction is imposed on the momentum of a classical test particle in the framework of curved spacetime.

The purpose of this paper is to provide a basic resolution of this conflict: *Spacetime is flat. The gravitational field in flat spacetime can be interpreted in terms of an effective "curvature" of spacetime only in the eikonal limit.* It is important to recognize that this viewpoint is not in conflict with the theoretical as well as the observational basis of the general theory of relativity.

Absolute motion together with its significance for the explanation of inertial effects in newtonian physics is discussed in section 2. It is shown in section 3 that the problem of origin of inertial forces does not find a resolution in general relativity. That is, inertial effects cannot be gravitational in origin if the standard interpretation of general relativity is maintained. The situation can be summarized as follows: The absolute space and time of newtonian physics refer to the equivalence class of all inertial frames that are related to each other by galilean transformations (cf. Newton's *Principia*, book I, scholium to the definitions and the fifth corollary following the laws of motion; see also ref. [4]). The absolute spacetime of Minkowski is a generalization of this concept where galilean invariance is replaced by Lorentz invariance. Thus the laws of motion with respect to absolute spacetime must conform to the principle of relativity. Conversely, starting with the principle of relativity the laws of motion can be generalized to arbitrary systems of coordinates in curved spacetime according to the scheme of general relativity; however, this theory is inconsistent with relativity of arbitrary motion and the existence of absolute spacetime must be invoked in order to account for the origin of inertial effects. Classical mo-

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tion is either absolute or relative; however, these apparently contradictory classical features of movement are shown to be complementary in section 4. Finally, in section 5 the consistency of the data of experimental gravity with the existence of an underlying Minkowski spacetime is critically examined.

2. Absolute motion

In classical mechanics, absolute motion signifies motion with respect to an ensemble of inertial frames (i.e., cartesian systems of reference that are homogeneous and isotropic in space and time) related to each other by galilean transformations. *Absolute motion is not directly observable; only relative motion has any operational significance.* A penetrating analysis of these and related concepts has been given by Poincaré [5]. The difficulty is partially resolved by introducing observers which are approximately inertial in any given physical problem. All actual observers are, however, noninertial (i.e., accelerated with respect to absolute space). The newtonian laws of motion are expressed in terms of ideal inertial observers. To compare the theoretical predictions with observations, it is therefore necessary to determine the laws of motion according to noninertial observers. This is accomplished through a postulate of locality as follows [2]: Let the transformation law from the inertial system $(t, \boldsymbol{x})$ to the accelerated reference system $(t', \boldsymbol{x}')$ be $t=t'$, $\boldsymbol{x}=\boldsymbol{x}(t', \boldsymbol{x}')$; then, the lagrangian in the accelerated frame, L', is given by $L'(t', \boldsymbol{x}', \boldsymbol{v}')=L(t, \boldsymbol{x}, \boldsymbol{v})$. This postulate of locality is made manifest in newtonian mechanics by the fundamental assumption that the equations of motion of a body can be obtained from those of its constituents which are classical point particles. Thus the function of the postulate of locality is derived from the basic role played by the classical point test particle in newtonian mechanics. The noninertial observer is thus *locally* equivalent to a hypothetical inertial observer at the same event and moving with the same velocity; however, in a finite domain in space and time the movement of bodies in the rest frame of the noninertial observer is perturbed by inertial forces which arise due to acceleration of the observer with respect to absolute space. Thus an observer can decide whether or not it is inertial by measuring relative motions of physical systems in a finite region of space and time. These mechanical considerations remain essentially unchanged when the newtonian space and time are generalized and replaced by Minkowski spacetime.

3. Relative motion

Inertial forces must be explained without any reference to absolute space and time, provided only relative motion has observational significance. As a result of developments in electrodynamics, galilean invariance was generalized and replaced by Lorentz invariance and the absolute spacetime of Minkowski took the place of absolute space and time of newtonian physics. The laws of classical mechanics and electrodynamics could then be formulated in a Lorentz-invariant manner in Minkowski spacetime. Following the ideas of Mach [4], Einstein proposed to develop a relativistic field theory of gravitation which would be a generalization of newtonian theory except that inertial effects would result from the gravitational influence of distant masses in the universe. In this way, classical physics would only be concerned with measurable quantities and concepts such as absolute spacetime would be excluded. It will be shown in the rest of this section that Mach's conception of inertia is in fact neither explicitly included in the foundations of general relativity nor follows from it. This means that absolute spacetime is retained in Einstein's theory, despite appearances to the contrary. This conclusion would confirm previous indications of this fact as noted by various authors (cf. ref. [6]).

The steps leading to general relativity may be summarized as follows: The theory of relativity that is restricted to Lorentz invariance provides a description of physical phenomena according to *inertial observers.* The restrictions involved in this theory are twofold: (a) only cartesian coordinates are assigned to events by inertial observers, and (b) the only measurements of interest pertain to inertial observers. Restriction (a) is of a purely mathematical nature. It is removed once the laws of physics are expressed (by inertial observers) in terms of arbitrary coordinates assigned to events in Minkowski spacetime. In a given problem, for instance, inertial observers are free to choose any system of coordinates that is relevant to the symmetries or boundary conditions involved. This circumstance is a generalization to Minkowski spacetime of the common use of curvilinear coordinates in euclidean space. The observables are evidently independent of the choice of coordinates. This fact can be made explicit by means of general tensor calculus in accordance with Minkowski's geometric viewpoint. It is therefore possible to express the laws of physics in the same mathematical form irrespective of the coordinate system employed.

To every coordinate system corresponds a set of observers which are at rest in the system and are in general noninertial. The components of a tensor with respect to the noninertial observers would acquire physical significance once restriction (b) is removed. If a prescription for the definition of physical quantities according to noninertial observers is given, it should be possible to express the laws of physics in a *generally covariant* manner. The prescription that is adopted in the theory of relativity, namely, the *hypothesis of locality*, is a direct generalization of the postulate of locality in classical mechanics; i.e., the results of measurements performed by an accelerated observer at any point in spacetime are assumed to be identical with those of a hypothetical inertial observer at the same event and with the same velocity. It follows from this hypothesis that the results of measurements are scalar invariants regardless of the motion of the observer. The limitations that the hypothesis of locality places on the description of wave phenomena have been discussed previously [1,2].

To describe physical phenomena according to an observer that is falling freely in a gravitational field, Einstein postulated the *local* equivalence of this observer with one that is accelerated with respect to absolute spacetime. This proposition is only pointwise valid in newtonian mechanics, provided the equivalence of the inertial and gravitational masses holds for all particles. Einstein's principle of equivalence and the hypothesis of locality together imply that each observer in a gravitational field is locally identical to an inertial observer. The spacetime is therefore flat in the immmediate neighborhood of each event; the totality of events represents a lorentzian manifold whose curvature can be identified with the gravitational field. The laws of mechanics as well as electrodynamics in arbitrary frames of references are then generalized in a natural way to take into account the presence of a gravitational field. Finally, the gravitational field equations can be established as relativistic generalizations of Poisson's equation for the newtonian gravitational potential.

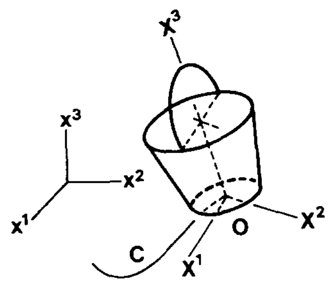

Fig. 1. A bucket of water that is subject to translational as well as rotational acceleration relative to an inertial frame of reference.

It must be clear from this description of the physical basis of general relativity that this theory – despite its name – does not include a generalization of the principle of relativity. To show furthermore that relativity of non-uniform motion does not follow from the theory, it proves interesting to discuss anew Newton's well-known experiment with the rotating vessel of water. The general argument that follows was first briefly presented in ref. [7]. Imagine a bucket of water that is accelerated along a curve $C(t)$ and rotated with an instantaneous angular velocity $\boldsymbol{\Omega}(t)$ with respect to the inertial axes as in fig. 1. Let X represent the position of a fluid element with respect to the frame of the bucket and let $\boldsymbol{x}$ denote the position of the fluid particle with respect to the inertial frame, then

$$x^i = c^i(t) + R_{ij}(t)\, X^j\,, \tag{1}$$

where $\boldsymbol{c}(t)$ denotes the position of O with respect to the inertial frame and R is the rotation matrix connecting the local frame of the bucket to the inertial axes with origin at O. R and $\boldsymbol{\Omega}$ are related by

$$(R^{-1}\, \mathrm{d}R/\mathrm{d}t)_{ij} = -\epsilon_{ijk}\Omega^k\,, \tag{2}$$

where ϵ designates the alternating symbol. Latin indices run from 1 to 3 throughout this paper. The newtonian law of motion for the fluid element is

$$\mathrm{d}^2\boldsymbol{x}/\mathrm{d}t^2 = \boldsymbol{a}\,, \tag{3}$$

where $\boldsymbol{a}$ is the sum of the acceleration of gravity (due to the Earth and all other masses in the universe) and the force per unit mass due to the influence of other fluid elements, etc., brought about by the electromagnetic interaction. The equation of motion with respect to an observer at rest with the bucket is

$$\frac{d^2X^i}{dt^2}+R_{ji}(t)\frac{d^2c^j}{dt^2}+\left(2\boldsymbol{\Omega}\times\frac{d\boldsymbol{X}}{dt}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{X})+\frac{d\boldsymbol{\Omega}}{dt}\times\boldsymbol{X}\right)^i=\mathscr{A}^i(t,\boldsymbol{X})\,, \tag{4}$$

where $\mathscr{A}^i=R_{ji}a^j$, $i=1,2,3$, are the components of the net force per unit mass of the fluid element with respect to the frame attached to the bucket. In the general relativistic description of the motion of the fluid with respect to the bucket, the acceleration and rotation of the bucket occur presumably with respect to distant masses in the universe; however, the equation of motion is expected to reduce to eq. (4) in the newtonian limit [8]. It must be emphasized that the time-scale of the phenomena envisaged in the motion of the bucket and the fluid are expected to be so short in practice compared to the main periods connected with Earth motion that the rest frame of the Earth can be considered approximately inertial and may therefore be identified in this sense with the (x^1, x^2, x^3) system.

The translational and rotational inertial forces whose existence can be deduced from eq. (4) are all proportional to the inertial mass. The equivalence of inertial and gravitational masses raises the possibility that these apparent forces could in fact be true gravitational forces. To investigate this possibility, imagine that all bodies undergo acceleration relative to the bucket in such a way as to generate the same relative motion. That is, consider the description of phenomena from the standpoint of an observer at rest with respect to the bucket taking into account the gravitational influence of all the matter in the universe in accordance with general relativity. The resulting gravitational field would affect the motion of each particle of the vessel as well as of the fluid in much the same way as a consequence of the *universality* of the gravitational interaction. The relative gravitational accleration of any two neighboring particles is therefore expected to be proportional to their relative distance. This suffices to show that the translational inertial force, which is independent of the *position* of a fluid element relative to the vessel, cannot be of gravitational origin. This conclusion is independent of how the rest of the universe moves relative to the bucket of water; therefore, it is unimportant for this demonstration how it is ascertained that the *same* relative motion is generated. To present this argument in detail, it is necessary to make use of the theory of motion of a body in a gravitational field within the framework of general relativity [8]. The discussion is simplified if the self-gravity of the vessel as well as of the fluid is neglected. An observer comoving with the bucket can establish, in principle, a Fermi frame along the wordline of the point mass O of the bucket. Let τ be the proper time along this path. The rest frame of the bucket would, in general, rotate at a rate $\boldsymbol{\Omega}^*(\tau)$ with respect to the gyro axes of the Fermi frame carried along O. The equation of motion of a fluid element with coordinates X^i, $i=1,2,3$, with respect to the rest frame of the bucket can be written approximately as

$$\frac{d^2X^i}{d\tau^2}+\left(2\boldsymbol{\Omega}^*\times\frac{d\boldsymbol{X}}{d\tau}+\boldsymbol{\Omega}^*\times(\boldsymbol{\Omega}^*\times\boldsymbol{X})+\frac{d\boldsymbol{\Omega}^*}{d\tau}\times\boldsymbol{X}\right)^i+k^i{}_j(\tau)\,X^j=A^{*i}(\tau,\boldsymbol{X})-A^{*i}(\tau,\boldsymbol{0})\,, \tag{5}$$

where $k(\tau)$ is the tidal matrix, i.e., its elements are the "electric" components of the Riemann curvature tensor along O, and $\boldsymbol{A}^*(\tau,\boldsymbol{X})$ is the non-gravitational acceleration that a point mass at $(\tau,\boldsymbol{X})$ experiences as measured in the rest frame of the bucket (cf. appendix A of ref. [8]). In comparing eqs. (4) and (5), it should be noted that a relationship can be established between t and τ once eq. (4) is thought of as a limiting form of a general relativistic treatment. This difference cannot, however, account for the main discrepancy between eqs. (4) and (5), namely, the absence of the translational inertial acceleration among the gravitational effects in eq. (5).

The presence of $\boldsymbol{\Omega}^*$ in eq. (5) is in part due to the fact that the rotation of the rest of the matter in the universe about the bucket generates, in addition to the usual gravitational "electric" field, a gravitational "magnetic" field which can be more accurately described by the dragging frequency of the inertial frames [7]. A comparison of eqs. (4) and (5) reveals that the Coriolis effect is the only one of the rotational inertial effects that may be considered machian in appearance. Indeed, the usual "demonstrations" of Mach's principle are based on the Coriolis effect either for the motion of particles (e.g., Foucault pendulum) or for light rays (e.g., Sagnac effect).

In summary, relativity of arbitrary motion is not contained in the general theory of relativity which is supported by the experimental data available at present. Crudely, when a bucket of water is accelerated with respect to absolute spacetime, the water inside the bucket is free of the acceleration so that its motion relative to the bucket reflects the existence of the inertial force. On the other hand, when the rest of the matter in the universe is accelerated relative to the bucket of water the resulting gravitational field acts both on the water and the container so that the relative motion is due to the difference of gravitational force. Hence the origin of inertial forces in general relativity must be essentially the same as in Newton's theory, namely, acceleration with respect to absolute spacetime.

4. Complementarity

It has been shown that in order to account for the origin of inertial forces recourse to the concept of absolute spacetime is necessary. On the other hand, this concept contains the idea of relativity insofar as it refers to the ensemble of all inertial frames related to each other by Lorentz transformations. The problem of physical reality of absolute spacetime remains still unresolved, however, since motion of a classical particle purely with respect to absolute spacetime is not directly measurable. To overcome this difficulty, the concept of relative motion must be examined more closely.

Relativity necessitates, in principle, the ability to shift the standpoint for the purpose of observation. Description of relative motion therefore brings to mind images of finite bodies such that an observer can be comoving with each body in turn. Relativity of motion therefore properly belongs to the mechanics of *classical particles*. On the other hand, it is natural to associate the idea of absolute motion with classical electromagnetic waves. This possibility was pointed out by Lorentz [9]. It is evidently connected with the historical development of ideas concerning the way spatially separated bodies can influence each other.

Radiation of electromagnetic waves is caused when electric charges accelerate with respect to absolute spacetime. Furthermore, it has been suggested that an observer can never be at rest with respect to an electromagnetic wave [1]. This assertion, which goes beyond the theory of relativity, is a natural extension of the well-known result for inertial observers to arbitrary accelerated observers. It is related to the quantum invariance condition [2] which postulates the independence of the number of quanta from the motion of the observer. Imagine, e.g., an observer that starts from rest and accelerates very slowly; the *adiabatic* variation of the observer's velocity is not expected to lead to any change in the *number* of quanta observed. The quantum invariance condition is a generalization of this result to arbitrary acceleration. Thus the motion of a classical particle (or wave) with respect to an electromagnetic wave cannot be described as *relative* since it is impossible to imagine an observer comoving with the wave. It is natural to generalize this association of absolute motion with classical electromagnetic waves to include any fundamental wave field.

It is now possible to put forward a generalization of the principle of complementarity to include *motion*. True (i.e., quantum) motion has complementary classical manifestations in absolute and relative movements in complete correspondence with the duality of classical waves and particles. Space and time are *classical* modes of description of extension and motion of the objects of perception. From this point of view, the motion of a massive particle relative to absolute spacetime cannot be observed since the particle must be represented in the spacetime picture by a wave field which is never static; i.e., there is no frame of reference in which the particle is at rest (or the field is static), otherwise the uncertainty principle would be violated.

The integration of classical notions of particle and wave in the quantum concept of *particle* can therefore be extended to the *motion* of the particle as well.

In this sense motion is the union of mutually exclusive yet complementary classical notions of absolute and relative movements.

5. Motion in a gravitational field

The viewpoint regarding motion developed in this paper is based upon the Broglie's hypothesis of wave–particle duality as incorporated into the framework of quantum theory. Thus motion in a gravitational field must also be examined from the standpoint of complementarity. The general picture that emerges is that gravity must be treated as a field on Minkowski spacetime; however, this field can be interpreted as an *effective curvature* of spacetime in the eikonal limit.

Imagine the propagation of *test* electromagnetic radiation in a time-independent gravitational field. For simplicity, let this be the field exterior to an isolated mass. Invariance under an arbitrary translation in time implies that the frequency of electromagnetic radiation (determined by static observers) does not change as the wave propagates through the gravitational field. That is, a photon does not gain or lose energy when it propagates in a time-independent gravitational field. This is in conformity with wave–particle duality for the total energy of a classical test particle in such a gravitational field is conserved in newtonian mechanics. These observations may be combined in a *tentative* hypothesis: The potential energy of a system of *rest* mass m in a gravitational field with newtonian potential $\phi(\boldsymbol{x})$ is given by $m\phi$, where the principle of equivalence of inertial and gravitational masses has been assumed. This assertion is consistent with all observations regarding nonrelativistic motion of massive particles in a gravitational field, and especially with the data concerning the propagation and interference of thermal neutrons [10]; moreover, it is consistent with constant energy propagation of a photon since its rest mass is zero. It is important to recognize that a completely random collection of photons, such as in blackbody radiation, must be treated in effect as a body with rest mass E/c^2, where E is the total energy of the photons in a frame in which their net momentum vanishes. This leads directly to the concept of *weight* of radiation which is meaningful only in a classical context.

Consider now a system S_0 which is at rest far from the source of the gravitational field ($\phi \to 0$). In a transition of the system S_0 from an excited state of energy E_2 to the ground state of energy E_1 a photon of frequency $\omega_0 = (E_2 - E_1)/\hbar$ would be emitted. If a system S that is identical with S_0 is quasi-statically placed in the gravitational field at $\boldsymbol{x}$, $\phi(\boldsymbol{x}) < 0$, the energy levels would be shifted to $E_2(1+\phi/c^2)$ and $E_1(1+\phi/c^2)$, and the emitted photon in te corresponding transition would have frequency $\omega = \omega_0(1+\phi/c^2) < \omega_0$. As the photon propagates away from the source of the field, ω remains constant so that an observer at rest with S_0 would conclude that the radiation spectrum originating at S has shifted toward the red in agreement with experimental data [11]. In this analysis the redshift is solely determined by the difference in gravitational potential; however, in a more general and extensive treatment the ratios between the wavelength of the radiation, the size of the system S, and the length scale of the variation of the field are also expected to play significant roles. In general relativity, the radiation is considered at the limit of vanishing wavelength and all the (non-gravitational) energies at any given point in space scale with the same factor so that an observer at any point measures the same relative quantities as in the absence of the field. In this way, each observer has a local (i.e., pointwise) inertial frame and the gravitational redshift is then explained by the relative clock rate at the locations of S and S_0.

The above considerations have been based on the constancy of the total energy (and hence the frequency) of a particle in a time-independent gravitational field. The particle's momentum (or, equivalently, its wave vector) does not remain constant, however, since the gravitational potential is position-dependent. This circumstance extends to a photon as well by continuity inasmuch as the motion of a particle in a gravitational field is independent of its mass (no matter how small). The resulting classical bending of light in a gravitational field has led to the development of a useful analogy with the refraction of light in a material medium with time-independent constitutive properties. In general relativity, Maxwell's equations in a gravitational field can be reformulated as the electromagnetic field equations in flat spacetime but in a "medium" with certain linear constitutive relations that include equal dielectric and permeability tensors [12,13].

The electromagnetic radiation propagating in a material medium is the superposition of many wavelets scattered from different parts of the medium. Constructive interference of the waves leads – in the eikonal approximation – to rays of radiation which exhibit the phenomena of refraction, reflection, etc. One may speculate, by analogy, that the scattering of electromagnetic radiation by a gravitational field would give rise to phenomena as yet unknown which, however, in the eikonal limit would correspond to rays propagating along null geodesics of a lorentzian manifold in accordance with general relativity. The development of a complete theory in agreement with the ideas presented in this paper remains a task for the future.

Finally, the observed large scale structure of the universe must be reconciled with an underlying Minkowski spacetime. The correspondence between the Friedmann–Lemaître–Robertson–Walker cosmological models and newtonian cosmology [14] together with detailed considerations regarding the distribution of cosmic electromagnetic background radiation [12] ensures that the basic known facts of modern observational cosmology are consistent with the existence of an underlying Minkowski spacetime. The event at which the Hubble expansion originally began is not expected to have any *a priori* significance in Minkowski spacetime; therefore, one may speculate on the possibility that the universe might contain other worlds of which Earth-based observations have not yet provided any clue.

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