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Cold Plasma Oscillations According to Weber's Law. An Unphysical Result

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Applying Weber's law to particle interactions in a uniform infinite cold plasma, a dispersion relation for longitudinal plasma oscillations is found. The classical result of plasma frequency oscillations is recovered for small wavelengths. Depending on the wavelength, the phase and group velocities may be singular. This unphysical result is due to the absence of transverse interaction in Weber's law.

In the past century, prior to Maxwell's introduction of the displacement current concept, Wilhelm Weber formulated an electrodynamic theory based on the interacting force between two charged particles in motion (Weber, 1846). Such a force, known as Weber's law, can be expressed as

$$\mathbf{F}_{ji} = -q_i q_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^2} \left\{ 1 + \frac{|\dot{\mathbf{r}}_j - \dot{\mathbf{r}}_i|^2}{c^2} - \frac{3[(\dot{\mathbf{r}}_j - \dot{\mathbf{r}}_i) \cdot (\mathbf{r}_j - \mathbf{r}_i)]^2}{2c^2} + \frac{(\mathbf{r}_j - \mathbf{r}_i) \cdot (\ddot{\mathbf{r}}_j - \ddot{\mathbf{r}}_i)}{c^2} \right\} \tag{1}$$

where $q_{i,j}$, $r_{i,j}$, $\dot{\mathbf{r}}_{i,j}$, and $\ddot{\mathbf{r}}_{i,j}$ represent the charge, position, velocity, and acceleration of particles i and j, respectively. $\mathbf{F}_{ji} = -\mathbf{F}_{ij}$ represents the force due to particle j acting on particle i; clearly it satisfies the action and reaction principle and it can be shown to be conservative (Maxwell, 1954). The force (1) can be derived from Ampère's law for interaction between current elements (Ampère, 1825) and it has been successful in describing most magnetostatic phenomena and also the Faraday effect (Maxwell, 1954). However, being an action at a distance force, there is no place for

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electromagnetic radiation in a Weber scenario. Wesley (1987) tried to circumvent this failure by introducing retardation and fields, finding that an additional field to the electric and magnetic ones should be necessary and a new kind of wave, which he called Weber's wave, should exist together with the usual electromagnetic radiation. Anyway, we think that the convenience of a force formula relies just on the fact that fields can be omitted (insofar as radiation is not considered). Moreover, there is some similarity between Weber's law and the force arising from the Darwin Lagrangian for interacting charged particles (Darwin, 1920), which suggests that some kind of retardation should already be implicit in the force (1).

The fact that Weber's law is an action at a distance force and is completely relational stimulated some recent studies toward a demonstration of its validity (Rambaut and Vigier, 1990; Phipps, 1990; Clemente and Assis, 1991; Assis, 1989), in contraposition to the usual Lorentz force. None of these studies has been conclusive; on the contrary, a recent work by Assis and Caluzi (1991), on the motion of a charged particle in an infinite plane capacitor, showed an unphysical result. According to Weber's law, the velocity of the particle can become singular and complex in some cases.

Here, we will show another unphysical result arising from the application of Weber's law for describing cold plasma oscillations.

It is a known result that using only the Darwin Lagrangian for particle interactions (which includes lowest order time retardation effects) in a cold plasma, transverse waves do not propagate if the electromagnetic field is not included and no modification to longitudinal plasma oscillations at plasma frequency is found (Trubnikov, 1968; Kaufman and Rostler, 1971). Applying Weber's law, the same result for transverse waves is found, but for longitudinal oscillations a dispersion relation arises that is clearly unphysical, since singularities in phase and group velocities appear.

Let us consider an infinite uniform cold plasma, with singly charged ions and electrons, without any macroscopic field. If we assume infinite inertia for the ions and perturb such an equilibrium state, approximating the discrete ions as a fixed background of constant density n_0 and the discrete electrons as a continuum of density $n(\mathbf{r}, t) = n_0 + \delta n(\mathbf{r}, t)$ and velocity $\delta\mathbf{v}(\mathbf{r}, t)$, it is possible to obtain to lowest order in δn and $\delta\mathbf{v}$, according to Weber's law, the following equation of motion for the electrons:

$$\frac{\partial \delta\mathbf{v}(\mathbf{r}, t)}{\partial t} = -\frac{e^2}{m_e} \int d\mathbf{r}' \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3} \left[\delta n(\mathbf{r}', t) + \frac{n_0}{c^2} (\mathbf{r}' - \mathbf{r}) \cdot \frac{\partial \delta\mathbf{v}(\mathbf{r}', t)}{\partial t} \right] \qquad (2)$$

If we Fourier analyze this equation assuming $\delta n(\mathbf{r}, t) = \delta n \exp i(\mathbf{kr} - \omega t)$ and $\delta\mathbf{v}(\mathbf{r}, t) = \delta\mathbf{v} \exp i(\mathbf{kr} - \omega t)$ with δn and $\delta\mathbf{v}$ constants, and use the lowest order continuity equation for the electrons ($\omega\,\delta n = \mathbf{k}\cdot\delta\mathbf{v}n_0$), we find the following equation for $\delta\mathbf{v}$:

$$\delta\mathbf{v} = \frac{\mathbf{k}\;\delta\mathbf{v}\cdot\mathbf{k}}{k^2}\left(\frac{\omega_p^2}{c^2k^2} + \frac{\omega_p^2}{\omega^2}\right) - \frac{\omega_p^2}{c^2k^4}\,\mathbf{k}\times(\delta\mathbf{v}\times\mathbf{k}) \tag{3}$$

where $k = |\mathbf{k}|$ and $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the classical plasma frequency.

Equation (3) has two solutions; when $\delta\mathbf{v}\cdot\mathbf{k} = 0$ it follows that

$$\omega_p^2 = -c^2k^2 \tag{4}$$

which implies no propagation of transverse waves, the same result which arises from the use of the Darwin Lagrangian (Trubnikov, 1968; Kaufman and Rostler, 1971). When $\delta\mathbf{v}\times\mathbf{k} = 0$ it results that

$$\frac{\omega^2}{\omega_p^2} = \frac{c^2k^2}{c^2k^2 - \omega_p^2} \tag{5}$$

which shows a dispersion relation for longitudinal plasma oscillations. For short wavelengths ($c^2k^2 \gg \omega_p^2$) classical plasma oscillations at the plasma frequency are recovered, but at longer wavelengths propagation is possible and in principle the phase velocity ω/k becomes singular together with the group velocity when $c^2k^2 = \omega_p^2$. Such behavior, apart from being unphysical, has never been observed in any experiment. The Bohm–Gross dispersion relation for electrostatic oscillations in plasmas (Bohm and Gross, 1949),

$$\omega^2 = \omega_p^2 + 3KT_e k^2/m_e \tag{6}$$

where T_e is the equilibrium electron temperature and K the Boltzmann constant, has been quite well verified in experiments (Looney and Brown, 1954; Barret *et al.*, 1968), and the result of almost no propagation for cold plasmas is a reality. Therefore we can conclude that the use of Weber's law in cold plasmas leads to wrong results and the law itself cannot be considered completely true.

It is interesting to point out that comparing expression (2) with the linearized expression arising from the use of the Darwin Lagrangian (Trubnikov, 1968) shows that a term due to transverse interaction is missing:

$$-\frac{e^2n_0}{2m_ec^2}\int d\mathbf{r}'\,\frac{\mathbf{r}'-\mathbf{r}}{|\mathbf{r}'-\mathbf{r}|^3}\times\left[\frac{\partial\delta\mathbf{v}}{\partial t}\times(\mathbf{r}'-\mathbf{r})\right] \tag{7}$$

Adding this term to the right member of expression (2), one can recover $\omega^2 = \omega_p^2$ for longitudinal oscillations. The fact that Weber's law satisfies action and reaction in a strict sense seems to be its worst defect and one has to admit the reality of fields in the description of our universe.

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