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Physics in Perspective

Ludvig Lorenz and His Non-Maxwellian Electrical Theory of Light

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Maxwell's celebrated electromagnetic theory of light dates from 1865. Two years later, without appealing to the ether as a carrier of light waves, the Danish physicist Ludvig Lorenz (1829–1891) independently published another electrical theory of light based on optical equations and the novel idea of retarded potentials. In spite of resting on a very different conceptual foundation, Lorenz's theory led to almost the same results as Maxwell's. But whereas Maxwell's field theory heralded a revolution in physics, Lorenz's alternative was largely forgotten and soon relegated to a footnote in the history of physics. In part based on archival material and other sources in Danish, this paper offers a detailed contextual account of Lorentz's theory and its reception in the physics community. Moreover, it includes a brief introduction to other of Lorenz's scientific contributions and discusses the reasons why his electrical theory of light failed to attract serious interest.

Key words: Ludvig Lorenz; electromagnetism; optics; James Clerk Maxwell; retarded potentials; ether.

Introduction

Recognizing the vital role that light plays in science and our daily life, the United Nations proclaimed 2015 the International Year of Light. Among many other things, it was an opportunity to celebrate the historical development of the science of light and its numerous technological applications. Not by accident, the year marked the 150th anniversary of James Clerk Maxwell's momentous electromagnetic theory of light in which, to quote Albert Einstein, Maxwell "showed that the whole of what was then known about light and electromagnetic phenomena was expressed in his well-known double system of differential equations."[1] Einstein called Maxwell's introduction of the continuous electromagnetic field the most profound change "in the conception of reality … that has come to physics since Newton." Whereas Maxwell's theory belongs to the high points in the history of science, it is not generally known that just two years later a twenty-eight-year-old Danish physicist by the name Ludvig Lorenz published another electrical theory of light, which was not only independent of Maxwell's theory but also

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differed radically from it in both form and physical interpretation. Not only did Lorenz's theory lack electrical and magnetic fields, it also had no ether. And yet the two theories led to almost the same results. Lorenz's system of potential equations was physically equivalent to Maxwell's field equations. Moreover, Lorenz predicted from his theory, as Maxwell did from his, that transverse electrical waves propagate in free space with the velocity of light.

This paper offers a comprehensive and contextual analysis of Lorenz's 1867 theory, including its relationship to Maxwell's earlier theory. It takes into account not only Lorenz's scientific paper, which appeared in translation in *Annalen der Physik und Chemie* and *Philosophical Magazine*, but also a popular paper in Danish, which contains passages not found elsewhere. Although Maxwell was aware of Lorenz's alternative and responded to it early on, the theory did not arouse much attention either in England or on the Continent. By the time Lorenz died in 1891, it was nearly forgotten. In addition to analyzing the content and meaning of Lorenz's theory and comparing it with contemporary theories of light and electricity, this paper follows in detail how the theory was received by other physicists. It was characterized by two novelties that eventually attracted interest, one being the retarded potentials and the other being the condition or "Lorenz gauge" relating the scalar potential to the vector potential. Lorenz did not himself return to the theory, but a few physicists found it worthy of mention, even if that mention was often critical. By following the reception of Lorenz's theory, we follow parts of the history of electrodynamics in the late nineteenth century. Today the theory is not quite as unknown as it used to be and has found its limited place not only in the physics literature but also in works on the history of physics.[2]

Lorenz, Not Lorentz

Although not unknown in the history of science, the nineteenth-century Danish physicist Ludvig Valentin Lorenz (1829–1891) is generally regarded a secondary and somewhat peripheral figure (figure 1).[3] Born in Helsingør (Elsinore) to a German émigré, in 1848 he enrolled as a chemistry student at Copenhagen's Polytechnic College. He graduated as a polytechnic candidate four years later, but never worked as a chemical engineer nor expressed any interest in chemistry. What interested him greatly was physics and mathematics, fields in which he was largely an autodidact. Unable or unwilling to find a job as a chemist, for more than a decade Lorenz earned his living by grants and teaching at high schools and teachers' colleges. Under these circumstances he published several important works in optical theory with the result that in 1866 he was elected a member of the Royal Danish Academy of Sciences and Letters. The same year he was appointed physics teacher at the Royal Military High School in Copenhagen, a position he held for twenty-one years. In 1887 he accepted a generous offer from the Carlsberg Foundation to pay him as an independent researcher for the rest of his life. He died unexpectedly of a heart attack in 1891.

Fig. 1. Ludvig Valentin Lorenz (1829–1891). Source: Royal Library, Copenhagen, Picture Collection

As early as 1860, Lorenz introduced the hypothesis of transition layers in optics, meaning that in the case of refraction, for example, the two optical media are separated by a large number of very thin sheets with different optical properties.[4] At first the concept of transition layers was ignored or rejected, but by the 1890s it had become part of mainstream optical theory. Apart from his early theoretical works in optics and his electrodynamic theory, to be dealt with below, Lorenz is today best known for his contributions to three areas that are still part of textbook physics. In 1869, he derived from experiments and optical theory a relationship between the density d of a substance and its refractive index n, namely,

$$\frac{n^2 - 1}{n^2 + 2}\left(\frac{1}{d}\right) = \text{constant}.$$

Nine years later, Hendrik Antoon Lorentz independently derived this law on the basis of electromagnetic theory, for which reason it is known as either the Lorenz-Lorentz or the Lorentz-Lorenz law.[5] In papers of 1872 and 1881, Lorenz investigated the thermal and electrical conductivities of metals (κ and σ, respectively), concluding that the ratio κ/σ was proportional to the absolute temperature.[6] The result, $\kappa/\sigma = LT$, was an extension of the Wiedemann-Franz law of 1853 according to which the ratio was a constant. Today, the constant of proportionality L in the Wiedemann-Franz-Lorenz law is known as the Lorenz number, with present value of $L = 2.44 \times 10^{-8} WK^{-2}$.

The third work for which Lorenz is remembered was also his last contribution to physics, a detailed mathematical analysis of how a plane light wave traverses a collection of homogeneous, transparent, and isotropic spheres. In contrast to most of his other papers, this one was published in Danish only, in the transactions of the Royal Danish Academy of Science, and consequently it remained unknown to the international community of physicists. Lorenz's scattering theory was eventually translated into French in two volumes containing his collected papers in physics and mathematics.[7] However, the publication was little read and rarely referred to in the literature. But much later, the 1890 paper was recognized as "truly one of the most remarkable memoirs to be published in the 19th century."[8] Today, Lorenz's memoir is seen as a non-electromagnetic version of the seminal paper that Gustav Mie published in 1908 on electromagnetic scattering of dielectric spheres. Modern physicists increasingly refer to this foundational theory as the Mie-Lorenz or Lorenz-Mie theory.[9]

Despite his important work in several areas of physics, Lorenz remains a peripheral figure whose name, when it occurs in a physics context, is often thought to be a misspelling of the surname of H. A. Lorentz. It was Lorenz's destiny to have a name that could too easily be confused with that of a towering figure whose name is known to all physicists and historians of science. The two physicists sometimes worked in the same branches of science, with the result that Lorenz's priority has not always been recognized. This is the case with the Lorenz-Lorentz formula and the Lorenz gauge in electrodynamics (see below), and Lorentz's name may even replace Lorenz's in areas in which H. A. Lorentz had no share. Thus, there are many references in the physics literature to "Lorentz-Mie theory," "Wiedemann-Franz-Lorentz law" and "Lorentz number." It does not reduce the confusion that the surname of H. A. Lorentz is not infrequently misspelled as Lorenz.

Light as Electrical Vibrations

In his works on optics from the early 1860s, Lorenz subscribed to the then-standard view that light consists of transverse waves made up of deformations in an incompressible ethereal medium. Thanks to a government grant, he spent 1859 in Paris, where he followed lectures by illustrious French scientist such as Joseph Liouville, Gabriel Lamé, and Henri Regnault. During this fruitful stay, Lorenz learned that the ether was necessary to understand optics in particular and to unify the diversity of physical phenomena in general. He eagerly studied Lamé's celebrated textbook on the mathematical theory of elasticity, in which the author declared that "it is impossible to arrive at a rational and complete explanation of physical nature without interposing this agent [the ether], whose presence is inevitable."[10] However, Lorenz soon came to regard the luminiferous ether as a medium with no physical characteristics. Although the concept figured in his works from 1862 to 1865, by then it was merely a name, a somewhat unnecessary substitute for the medium of light propagation.

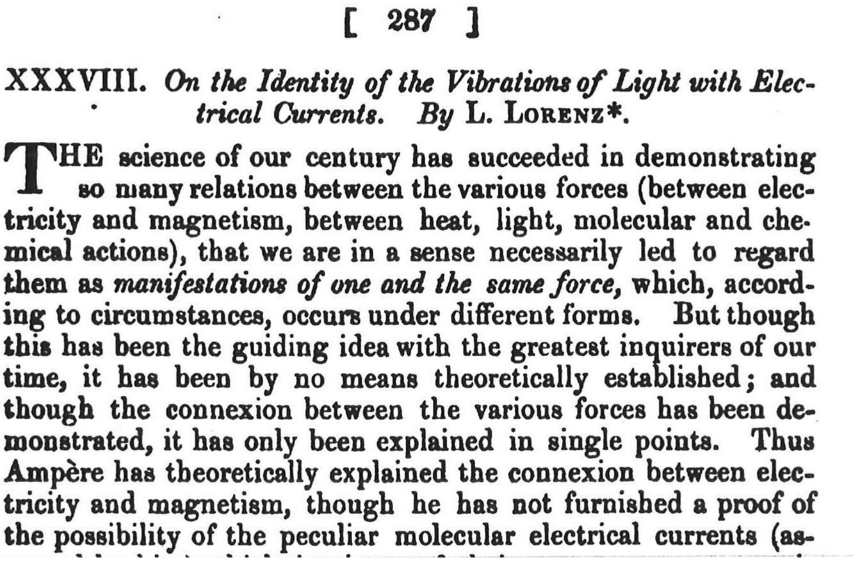
It was only in two papers of 1867, in which he introduced his new electrodynamic theory of light, that he unequivocally rejected the ether as a concept flawed from a methodological point of view. Three years earlier, he had characterized certain physical hypotheses with Francis Bacon's term *ignes fatui*, meaning idols that hide the true nature of things and delude rather than reveal the truth.[11] He now concluded that the ether was nothing but a superfluous *ignes fatuus*:

> The assumption of an ether would be unreasonable; because, it is a new non-substantial medium which has been thought of only because light was conceived in the same manner as sound and it hence had to be a medium of exceedingly large elasticity and small density in order to explain the large velocity of light.... If we need to assume some medium for light between the celestial globes, we do not have to conceive it as different from the known gases. On the whole it is most unscientific to fabricate a new substance when its existence is not revealed in a much more definite way.[12]

Without referring to Maxwell and most likely without thinking of him, Lorenz implicitly suggested that the very basis of Maxwell's electromagnetic theory was "unscientific." For, according to the Scottish physicist, it was imperative that light propagated in what he called an "aethereal medium filling space and permeating bodies."[13] Not only light but all electromagnetic phenomena were effects of the dynamics of this medium without which energy could not be transmitted.

The context of Lorenz's dismissal of the imponderable ether was a new theory in which the vibrations of light were considered to be electrical currents. He presented the theory to the Royal Danish Academy at a meeting of January 25, 1867, and later in the year published two papers on it, one in a new popular Danish science journal and the other in the proceedings of the Academy. Both were in Danish. Whereas the translated title of the popular paper was simply "On Light," the full technical paper was titled "On the Identity of the Vibrations of Light with Electrical Currents." To make his work accessible to an international audience of physicists, Lorenz sent a copy of his paper to Johann Christian Poggendorff, the editor of *Annalen der Physik und Chemie*, where it appeared in German translation later the same year. The German version served as the basis for an English translation in *Philosophical Magazine*, also in 1867 (figure 2).

What Lorenz had written about the ether in his popular Danish paper he repeated in different words in his scientific paper. Although "the present general opinion regards light as consisting of backward and forward motions of particles of aether," according to Lorenz, "light cannot ... consist of vibrations of the kind hitherto assumed." In his new electrical theory, there was "scarcely any reason for adhering to the hypothesis of an aether, for it may well be assumed that in the so-called vacuum there is sufficient matter to form an adequate substratum for the



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XXXVIII. *On the Identity of the Vibrations of Light with Electrical Currents.* By L. Lorenz*.

THE science of our century has succeeded in demonstrating so many relations between the various forces (between electricity and magnetism, between heat, light, molecular and chemical actions), that we are in a sense necessarily led to regard them as *manifestations of one and the same force*, which, according to circumstances, occurs under different forms. But though this has been the guiding idea with the greatest inquirers of our time, it has been by no means theoretically established; and though the connexion between the various forces has been demonstrated, it has only been explained in single points. Thus Ampère has theoretically explained the connexion between electricity and magnetism, though he has not furnished a proof of the possibility of the peculiar molecular electrical currents (as-

Fig. 2. Lorenz's paper on the electrodynamic theory of light as translated from the German version in *Philosophical Magazine*

motion."[14] We shall return to the question of the position of the ether in Lorenz's theory.

What may be called Lorenz's phenomenological attitude to the problems of light and electricity differed from that of most German and French physicists by incorporating aspects of the dynamical thinking that characterized such natural philosophers as Michael Faraday and Hans Christian Ørsted. In most respects the mathematically inclined Lorenz differed a great deal from these two thinkers, but he was probably influenced by Ørsted in his search for analogies that might reveal the underlying unity of the natural forces. While a student at the Polytechnic College, he followed Ørsted's lectures on chemical physics and the diary he kept at the time strongly suggests that he was sympathetic to his teacher's philosophy of nature.[15] As early as 1816, Ørsted thought of light as "a series of immeasurably small electrical sparks," an idea which appealed to him because "it does not presuppose any force or matter whose existence has not been experimentally proved."[16] The influence from Ørsted was instrumental in Lorenz's 1867 identification of light with electric disturbances. Almost paraphrasing Ørsted, he explained his guiding principle as follows: "The endeavors to search for connections between the various forces have been a significant reason for the progress of recent science; the idea that the various forces in nature are merely different manifestations of the one and same force has proved itself more fertile than all physical theories. It turned out that only one further step along the already established road had to be made, and this step leads to the remarkable result that *the vibrations of light are electrical currents*."[17] In Lorenz's scientific paper of 1867, he referred specifically to Ørsted's discovery of electromagnetism as a partial confirmation of the unity of forces. With regard to his own electrical theory of light, he said that "[it] manifestly lead[s] us a step further towards developing the idea of the unity of forces, and opens up a fresh field for future inquiries."[18]

Lorenz introduced his paper with critical comments of a methodological nature. Although acknowledging the idea of the unity of force as a valuable guiding theme, he objected that it had not been proved experimentally and lacked a theoretical foundation. With regard to the theories of electricity based on electrical fluids and those of optics based on the ether, he wrote: "Yet these physical hypotheses are scarcely reconcilable with the idea of the unity of force, and … have only been useful inasmuch as they furnish a basis for our imagination." His alternative was to adopt a phenomenological or positivistic strategy: "Hence it would probably be best to admit that in the present state of science we can form no conception of the physical reason of forces and of their working in the interior of bodies; and therefore (at present, at all events) we must choose another way, free from all physical hypotheses, in order, if possible, to develop theory step by step in such a manner that the further progress of a future time will not nullify the results obtained." As he saw it, the electrical theory of light demonstrated the usefulness of the chosen strategy, which he had earlier advocated in connection with his phenomenological theory of light. Indeed, from a methodological and formal point of view, the electrical light theory rested on the same principles that characterized his elastic theory of light as presented between 1862 and 1865. A leading historian of physics has even called Lorenz's wave equation from that period "a legitimate anticipation of the electromagnetic equation."[19]

But of course the new theory was more than just a continuation of the older optical theory. As far as electricity was concerned, it relied directly on an earlier theory by Gustav Robert Kirchhoff, who in 1857 had published two important papers on the propagation of electricity in conducting media (figure 3).[20] Kirchhoff's theory was based on a set of local equations comprising Ohm's law and the law of induction, which in modern notation and slightly anachronistically can be summarized in the vector equation:

$$\mathbf{j} = -\sigma\left(\nabla\varphi + \frac{\partial \mathbf{A}}{\partial t}\right).$$

Here, $\mathbf{j}$ denotes the current density vector, σ the specific electric conductivity, φ the scalar potential and $\mathbf{A}$ the vector potential. In Kirchhoff's theory, the two potentials were expressed in terms of the density of free electricity ρ and the current density $\mathbf{j}$, namely as

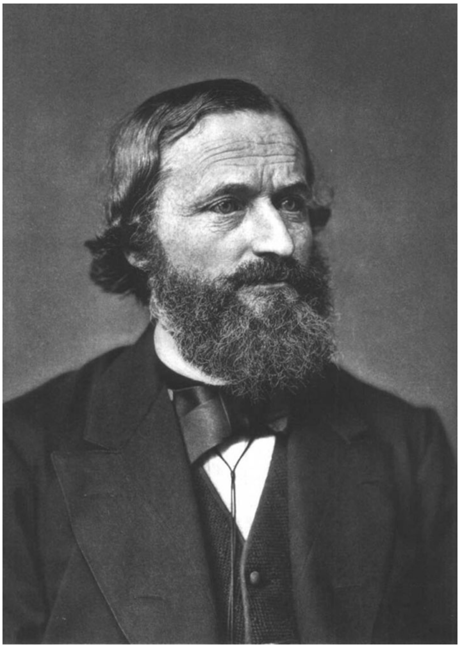

Fig. 3. G. Robert Kirchhoff (1824–1887). https://upload.wikimedia.org/wikipedia/commons/f/fe/Gustav_Robert_Kirchhoff.jpg

$$\varphi(\mathbf{x},t) = \frac{\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}'$$

and

$$\mathbf{A}(\mathbf{x},t) = \frac{\mathbf{j}(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}'.$$

Kirchhoff's potentials satisfied the continuity equation or equation of charge conservation, a result which he stated in a form corresponding to

$$\frac{1}{2}\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0.$$

The factor of ½ reflected the contemporary view that electrical currents are composed of negative as well as positive charges moving in opposite directions. Kirchhoff's paper was also important to Lorenz because it drew attention to a recent work of Wilhelm Weber and Rudolf Kohlrausch in which they had established a peculiar relation between, on the one hand, the ratio between an electrical charge measured in electromagnetic and electrostatic units (c_{w}) and, on the other, the velocity of light in air (c_0).[21] The relation was

$$c_w \cong \sqrt{2}c_0$$

According to the 1856 experiment of Weber and Kohlrausch, $c_w = 4.4 \times 10^8 \text{m s}^{-1}$ or $c_w/\sqrt{2} = 3.1 \times 10^8 \text{m s}^{-1}$, whereas the measured value of the speed of light was close to $3 \times 10^8 \text{m s}^{-1}$.

Kirchhoff not only concluded that the velocity of propagation of currents was independent of the nature of the conductor but also that it was approximately equal to the velocity of light. But although he considered the Weber-Kohlrausch relationship to be more than just a curious numerical coincidence, he refrained from drawing conclusions with respect to the nature of light. Weber's attitude was bolder and more in agreement with Lorenz's view. In 1864 Weber commented, "If this close agreement of the propagation velocity of electric waves with the velocity of light could be regarded as an indication of a deep connection between the two sciences, then this agreement would captivate our attention, considering the high importance of the search for such a connection." And yet Weber drew back from identifying light and electrical disturbances: "It is obvious that the true meaning of this velocity [Kirchhoff's] with respect to electricity must be considered, and this meaning is not of a kind that would allow great expectations."[22] Lorenz's expectations differed from Weber's.

An Electrodynamic Theory of Light

The 1867 theory grew out of an innovative combination of Lorenz's earlier optical theory and Kirchhoff's theory of electrical conduction in metallic wires. From this combination, Lorenz derived a consistent electro-optical theory and several novel results, which I discuss in four subsections.

The General Framework

In a nutshell, Lorenz argued in his 1867 paper that the electrical action in free space propagates in time and that the wave equation based on this idea justified the claim that light is nothing but periodically varying currents. Although he did not deduce the value of the velocity with which the electrical waves propagate, "yet it must be very great, of the same order as c [that is, c_w]."[23] Citing the results of Weber and Kirchhoff, he suggested that they were valid also for the propagation of electrical waves in free space. "We have therefore some reason for taking $a = c/\sqrt{2}$," he cautiously wrote. He was pleased to note that with this value his electrodynamic equations "assume now a very simple form, and lead to exactly the same differential equations as those which I formerly deduced for the vibrations of light." In his seminal paper of 1861–1862, "On Physical Lines of Force," Maxwell had reached what was essentially the same conclusion. His argument was this: "The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of MM. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."[24] However, Maxwell's "inference" was in reality a claim without proper justification. He was not as yet in possession of an electromagnetic theory of light from which the claim could be derived, or at least draw support.

Although Lorenz's reasoning was not generally guided by physical models or analogies, it was guided by formal analogies. Since he had derived electrical equations identical to those he had earlier found for light, he inferred that light was made up of electrical oscillations. In his 1867 paper in the popular Danish journal, the analogical form of reasoning appeared explicitly: "Where, then, should we look for these periodic currents, propagating more easily the more poorly they are conducted in the body and only in a direction perpendicular to the current, if not in *the ray of light*? After all, the vibrations of light are periodical and perpendicular to the direction of light; moreover, light can only pass through extremely poor conductors."[25] Such an inference from analogy to identity is not logically justified but rather amounts to a new hypothesis in disagreement to Lorenz's expressed desire to avoid physical hypotheses. He seems not to have noticed or admitted the inherent conflict between his theory and his professed methodology. On the contrary, he claimed that the identity of light and electrical oscillations "indubitably follows" from the analogous forms of the involved equations.

Lorenz furthermore argued that, since propagation of electrical currents in space followed from the equations and hence was possible, it had to correspond to a real phenomenon in nature. He thus applied what in the history of ideas is known as the principle of plenitude,[26] formulating his own version of it as: "What turns out to be possible in calculations deduced from really existing laws and conditions will always turn out to correspond to reality." In his popular paper, Lorenz briefly discussed whether the claimed identity of light and electrical vibrations could be confirmed by means of experiment. Given that "the electrical currents in a ray of light vibrate billions of times per second," a direct confirmation would scarcely be possible. On the other hand, for possible indirect confirmation he vaguely referred to "known facts indicating that vibrations of light might be transformed to electrical currents, namely when light hits upon the interface between two different metals."[27] He might have thought of the photovoltaic effect Alexandre-Edmond Becquerel discovered in 1839.

As for the physical mode of action responsible for the generation of light, Lorenz concluded that it could be expressed in several ways depending on the choice of hypotheses. Since he could see no good reason to prefer one physical hypothesis over another, he dismissed the question as unimportant. He wrote, "After careful investigation of this point, I have completely given up the idea of getting any good from physical hypotheses." Nevertheless, his paper did include such a physical hypothesis, namely that the mode of motion of light might be rotational rather than vibrational. This is what he had suggested four years earlier in the context of his phenomenological theory of light,[28] and he now returned to it:

> If we suppose light to consist of *rotating* vibrations in the interior of bodies, about axes which, according to the theory of electricity, we regard as directions of vibration, the electrical current is no translatory motion, but a rotation continued in one direction, and the axis of rotation becomes then the direction of the current. This rotation will only be continuous in good conductors, and the motion travel there in the direction of the axis, whereas it becomes periodical in bad conductors, and is propagated by what in electricity we call induction, in a direction at right angles to the axis of rotation.

That is, Lorenz conceived a steady current to be a steady rotation continued along its own axis, whereas oscillatory currents would be propagated by electromagnetic induction perpendicular to the axis of the current and in this way constitute light.

Potentials and Wave Equations

Lorenz noted that Kirchhoff's equations were "deduced in a purely empirical manner" and thus known to be true only to a degree corresponding to the accuracy of ordinary experiments. Hence it would be admissible to introduce very small terms the effect of which would not turn up experimentally. Moreover, such a modification was preferable for methodological reasons because it would lead to a more general theory of non-instantaneous interaction. Lorenz argued that the most general assumption would be to assume a finite velocity for the propagation of the electric action in the form of retarded potentials. He had introduced retarded action already in a paper of 1861 on elasticity theory published in what was commonly known as *Crelles Journal*, but at the time in a mathematical and abstract form only.[29] Six years later he adopted the notion to the context of electrodynamics, writing the scalar potential as

$$\varphi(\mathbf{x}, t) = \frac{\rho(\mathbf{x}', t - r/a)}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'$$

and the vector potential as

$$\mathbf{A}(\mathbf{x}, t) = \frac{\mathbf{j}(\mathbf{x}', t - r/a)}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'.$$

Thus, "the entire action between the free electricity and the electrical currents *requires time to propagate itself*.... The action in the point xyz at the moment t does not depend on the simultaneous condition $x'y'z'$, but on the condition in which it was at the moment $t - r/a$; that is, so much time in advance as is required to traverse the distance r with the constant velocity a." Lorenz proved by a series expansion that the retarded potentials give the instantaneous potentials in the limit $r/a \rightarrow 0$. He clearly understood that the physical interpretation of the new potentials φ and $\mathbf{A}$ is that at a given point $\mathbf{x}$ and a given time t, the potentials are determined by the charge and current that existed at other points in space $\mathbf{x}'$ at an earlier time.

Lorenz did not actually refer to φ and $\mathbf{A}$ as potentials, a name that does not appear in his paper. In agreement with Kirchhoff, he called the two functions "two components of electromotive force—one arising from the inducing action of free electricity, the other from the inducing action of the variable intensities of the current."[30] In contrast to Maxwell, neither Kirchhoff nor Lorenz associated the vector potential with magnetism. They considered both potentials to be electrical quantities. For this reason, the symbol for the vector potential in Lorenz's theory does not stand for quite the same as the vector potential appearing in Maxwell's equations.[31]

To deduce the physical consequences of his new approach, Lorenz adapted his optical results obtained in the early 1860s to the electrical case. With Δ being the Laplace operator, he stated the wave equation in the general form

$$\left(\Delta - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}\right)\frac{\rho(\mathbf{x}', t - r/a)}{|\mathbf{x} - \mathbf{x}'|}d\mathbf{x}' = -4\pi\varphi(\mathbf{x}, t).$$

Instead of providing a proof of the equation, Lorenz referred to his 1861 paper in *Crelles Journal*, where the same equation appeared in a non-electrical context and a proof was given.

After some further calculations Lorenz obtained three partial differential equations for the variation in space and time of the current density $\mathbf{j}$. "These equations for the components of the electrical current," he wrote, "agree fully with those I have already found for the components of light up to the last member, into which the electrical conductivity [σ] enters." The wave equation for the variation of $\mathbf{j}$ can in condensed form be expressed as

$$-\nabla \times (\nabla \times \mathbf{j}) = \frac{1}{a^2}\frac{\partial^2 \mathbf{j}}{\partial t^2} + \frac{16\pi}{a^2}\sigma\frac{\partial \mathbf{j}}{\partial t}.$$

Lorenz identified the last term with the absorption of light which would increase with the electrical conductivity. In agreement with experiments, it showed that all good conductors absorb light to a great extent and that the opposite is the case for poor conductors. For example, it was known that carbon in the form of the opaque graphite is a good conductor, whereas a transparent diamond is a non-conducting form of carbon.

The Lorenz Gauge

Given the similarity of his optical wave equation and the one governing electrical currents, Lorenz could transfer most of his old results to the new electrodynamic theory of light. In the course of deriving his equations, Lorenz argued that the retarded potentials had to satisfy a certain condition. Largely following Kirchhoff's notation, he symbolized the scalar potential by $\bar{\Omega}$ and the vector potential by (α, β, γ). With this notation he wrote the constraint as

$$\frac{\partial \bar{\Omega}}{\partial t} = -2\left(\frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial x} + \frac{\partial \gamma}{\partial x}\right).$$

In modern notation and Gaussian units, the factor 2 disappears and the Lorenz constraint becomes

$$\nabla \cdot \mathbf{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0.$$

Without emphasizing the significance of this formal innovation, a particular fixing of the divergence of **A**, he thus introduced the constraint of the vector potential known today as the Lorenz gauge condition.[32]

In ordinary Maxwellian electrodynamics, the fields and potentials are related through the equations

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t} \text{ and } \mathbf{B} = \nabla \times \mathbf{A}.$$

Whereas the field quantities **E** and **B** are unique, this is not the case for the potentials φ and **A**. If they are replaced by

$$\varphi' = \varphi + \frac{\partial \beta}{\partial t} \quad \text{and} \quad \mathbf{A}' = \mathbf{A} - \nabla \beta,$$

where β is an arbitrary differentiable scalar, it leaves the **E** and **B** fields unchanged. With the full development of the Maxwell equations it was understood that the potentials have to be defined and that this can be done in different ways without affecting the physical meaning of the fields. The freedom corresponds to different "gauges." The constraint stated by Lorenz was and is still often attributed to the famous H. A. Lorentz, who used it in his important work on electromagnetism in the early twentieth century and spelled out in clear language the idea of the arbitrariness of the potentials. But paternity of the constraint used by Lorentz belongs to his Danish near-namesake. As far as priority is concerned, it is one of many cases where a discovery made by some scientist is or has been named after another, better-known scientist.[33]

Maxwell generally worked in what is known as the Coulomb or radiation gauge, according to which $\nabla \cdot \mathbf{A} = 0$. Despite its name, it owes nothing to the eighteenth-century physicist Charles Augustin Coulomb but was first stated by Maxwell in 1861, using Faraday's name "electrotonic state" for the vector potential.[34] Maxwell insisted that $\nabla \cdot \mathbf{A} = 0$ was not an arbitrary choice but that the condition was demanded for physical and not merely mathematical reasons. However, this gauge is not suited for dynamic fields where it causes the scalar field to adjust instantaneously to changes in electrical charges, formally corresponding to an infinite speed of propagation and implying problems of causality.[35] In terms of potentials, Maxwell formulated his equations in free space as

$$\frac{\partial^2 \mathbf{J}}{\partial t^2} + \frac{\partial \Psi}{\partial t} = 0.$$

He commented that "$\mathbf{J}$ [$= \nabla \cdot \mathbf{A}$] must be a linear function of t, or a constant, or zero, and we may therefore leave $\mathbf{J}$ and Ψ [$= \varphi$] out of account for periodic disturbances." As was discussed later in the century, the Coulomb gauge has the consequence that $\nabla^2 \varphi$ is independent of time and hence, apparently, that the scalar potential cannot be propagated in time but must work as a kind of action at a distance.[36] The problem worried the Irish physicist Gerald Fitzgerald, who by 1890 at the latest reached the conclusion that Maxwell's quantities $\mathbf{J}$ and Ψ were not independent but related in the form of the Lorenz gauge, meaning $\mathbf{J} = -\partial \Psi / \partial t$. He was at the time unaware of Lorenz's paper.[37]

Although first stated many years before the advent of relativity theory, the Lorenz gauge condition is, contrary to the Coulomb gauge, manifestly Lorentz invariant. In terms of the four-potential $\mathbf{A}^{\mu} = (\varphi / c, \mathbf{A})$ it can be written as $\partial_{\mu} \mathbf{A}^{\mu} = 0$. The first example of a particular gauge, a relation between the two electromagnetic potentials, can retrospectively be found hidden in Kirchhoff's 1857 paper, where it appears in a form corresponding to

$$\nabla \cdot \mathbf{A} - \frac{1}{c} \frac{\partial \varphi}{\partial t} = 0.$$

The little-used "Kirchhoff gauge" differs from Lorenz's condition only by a change in sign. Of more interest, in a lecture delivered in Göttingen in 1861 Bernhard Riemann interpreted the electrostatic scalar potential φ as the density of the ether and the electrodynamic (vector) potential $\mathbf{A}$ as the ether's current intensity.[38] He showed that the latter quantity could be chosen to satisfy a relation to the first quantity which in modern symbols can be expressed as $\nabla \cdot \mathbf{A} - \partial \varphi / \partial t = 0$. Although Riemann's lectures were only published in 1875, in a sense he anticipated the Lorenz gauge.

More on Retarded Potentials

In the conclusion of his 1867 paper, Lorenz summarized the new theory and pointed out its incompatibility with the class of action-at-a-distance theories favored by Weber and most other German physicists: "Electrical forces require time to travel, and … every action of electricity and electrical currents does in fact only depend on the electrical conditions of the *immediately surrounding* elements." He claimed no priority for this qualitative insight, for "This is well known to be an idea indicated by Ampère, and which several physicists, more particularly Faraday, have defended." Lorenz thought that retarded action might be valid for any kind of physical interaction, not only for electricity. In his popular paper, he expressed the belief as follows: "Among the certain results of this investigation is that it takes time for the actions of electricity to propagate from one place to another. What Rømer taught us about light 200 years ago is valid also for the electrical forces. We might well add that it is valid also for other forces such as the gravitational attraction and assume that in general no action can propagate instantaneously and thus be all over in space at the same time." With respect to the novel feature of retarded potentials in Lorenz's theory, it should be pointed out that he was preceded by Riemann, who introduced the idea in a paper submitted to the Göttingen Academy of Science on February 10, 1858 (figure 4). In fact, in a letter to Weber of 1845, Gauss had suggested that the potential at some point and at some time was due to the distribution of electricity elsewhere not at that instant but at an earlier time depending on the distance. He thus anticipated that the propagation of electrical action might take time, but neither Gauss nor Weber developed the idea into a scientific theory.[39]

Although Riemann was convinced that he had found the long-sought connection between electricity and light, he retracted the paper shortly after its submission. The reason for his retraction may have been a rather elementary mathematical error, such as pointed out by Rudolf Clausius in 1868, but it may also have been Riemann's failure to derive Weber's force law between two electrical

Fig. 4. G. F. Bernhard Riemann. Credit: Wikimedia Commons

particles.[40] In any case, the paper was only published posthumously nine years later and, as it happened, in the very same issue of *Annalen* containing Lorenz's paper, which followed immediately after Riemann's. One can reasonably assume that Poggendorff deliberately arranged the papers consecutively.

Much like Lorenz, Riemann based his work on "the assumption that the action of one electrical mass on the rest of them is not instantaneous, but is propagated to them with a constant velocity which, within the limits of error of observation, is equal to that of light."[41] The assumption implied that the electrical charge $q(t)$ in the Coulomb force had to be replaced by $q(t - r/c)$. Riemann found that the differential equation for the propagation of electricity was the same as the equation governing the propagation of light and possibly also of radiant heat. Also like Lorenz, he replaced Poisson's equation for the electrostatic potential with an equation containing the second-order time derivative of the potential. On the other hand, in contrast to Lorenz, the German mathematical physicist considered only the retarded form of the scalar potential φ and not of the vector potential $\mathbf{A}$. Moreover, he did not state the retarded scalar potential for a charge distribution in integral form, but only for a point charge. From 1858 until his death eight years later, Riemann often lectured on electricity and magnetism, but without returning to the connection to the propagation of light.

Despite its somewhat sketchy character, Riemann's posthumous paper was well known and much discussed. It influenced Maxwell's thinking and attracted the critical attention of, for example, Enrico Betti in Italy and Clausius and Carl Neumann in Germany.[42] Neumann took over from Riemann the idea of retarded potentials, but without drawing the conclusion that light was electrical in nature. According to Maxwell, "those who supposed that Neumanns [*sic*] potential travelled like light were greatly mistaken."[43] Neumann tended to believe that the analogy between optics and electrodynamics was superficial and not of great physical interest. To Clausius, the analogy was not only superficial but non-existant. In a paper of 1868, he denied any connection between Neumann's theory and the propagation of light.[44]

In what some authors call the Riemann-Lorenz formulation of classical electrodynamics, the scalar and vector potentials are the basic quantities from which all experimental facts can be deduced.[45] This is contrary to the usual formulation of Maxwell's equations, where the electric and magnetic fields **E** and **B** are the basic quantities. This formulation was not Maxwell's but due to later physicists such as Heinrich Hertz and Oliver Heaviside who realized that whereas the field strengths have direct physical effects, this is not the case for the potentials. They consequently regarded the potentials as unnecessary mathematical constructs, which could and should be avoided. In works from the turn of the century, the Riemann-Lorenz approach was developed into the so-called Liénard-Wiechert retarded potentials found independently by Alfred-Marie Liénard in 1898 and

Emil Wiechert in 1900. These potentials describe the Maxwell equations of arbitrarily moving point charges (electrons) in the Lorenz gauge. Wiechert referred in his derivation to Riemann, but not to Lorenz; Liénard did not refer to either Riemann or Lorenz.[46]

Responses to Lorenz's 1867 Theory

Lorenz may reasonably have hoped that the translation of his paper on the electrical theory of light into German and English would arouse wide interest in his theory. If so he must have been disappointed. His theory was presumably well known to contemporary physicists, but only very few of them actually referred to it and no one—including himself—sought to develop it. Remarkably, neither Clausius nor Neumann, nor (for that matter) Betti, mentioned Lorenz's work in their papers of 1868, though they almost certainly had read it. Neumann undoubtedly had, for in a long letter to Lorenz he explained his own view concerning what he called the "progressive propagation of the potentials."[47] Nor did Hermann von Helmholtz, in his important series of writings on electrodynamics, pay explicit attention to the Danish physicist.[48] Today Lorenz's theory is often thought to have been unjustifiably ignored. According to one physicist, "The greatness of Lorenz's electrodynamic theory was not realized when it appeared, nor during his lifetime, and apparently it was forgotten soon after 1867."[49] Although there is some truth in this evaluation, a closer study indicates that the theory was fairly well known during the last third of the nineteenth century. Contemporary as well as later responses constitute an important part of the theory's history and for this reason I shall cover the subject as comprehensively as possible.

Maxwell Versus Lorenz

With some delay, Lorenz's paper was carefully reviewed in the leading abstract journal, *Fortschritte der Physik*, established in 1845 by the German Physical Society. The reviewer, an "extraordinary" professor of physics and meteorology at the University of Bonn by the name Gustav Radicke, provided a fair and comprehensive summary of the new electrical theory. Nevertheless, Radicke was not convinced of Lorenz's conclusion, which he thought built on a somewhat shaky foundation: "It appears to him [Lorenz] that he has *proved* (!) that the oscillations of light and electric currents are *identical*; and moreover, that the result has been achieved without assuming any physical hypothesis."[50] However, the very first critical reference to Lorenz's theory did not come from the relatively unknown Radicke in Bonn but from none other than Maxwell in Cambridge (figure 5).

Briefly commenting on Lorenz's theory in a paper of 1868, Maxwell wrote that "The propagation of attraction through space forms part of this hypothesis also, "though the medium is not explicitly recognised." Claiming that the continental theories were inconsistent with energy conservation and Newton's third law, he criticized Riemann and Lorenz for ignoring the action of the medium through which the electric disturbances propagated: "From the assumptions of both these papers we may draw the conclusion, first, that action and reaction are not always equal and opposite, and second, that apparatus may be constructed to generate any amount of work from its resources."[51] In a letter of March 12, 1868, to his friend, the mathematical physicist Peter Guthrie Tait, he phrased the critique in different wording. The theories of Riemann and Lorenz, Maxwell wrote, would lead to the absurd consequence of "a locomotive engine fit to carry you through space with continually increasing velocity." To stress the absurdity, he compared the consequence with "Gulliver's Travels in Laputa," a reference to a flying and highly imaginary island appearing in Jonathan Swift's *Gulliver's Travels*.[52]

At the 1870 meeting of the British Association for the Advancement of Science in Liverpool, Maxwell again commented on the theories of electricity favored by Weber and other physicists in German-speaking Europe. Referring to the force between electrical particles, he said, "according to a theory hinted at by Gauss, and developed by Riemann, Lorenz, and Neumann, [it] acts not instantaneously, but after a time depending on the distance." Although Maxwell found the theory of "these eminent men" to be valuable and deserving careful study, he much preferred his own field theory based on disturbances in the ethereal medium. Considering the two classes of theory to be temporarily equivalent, he philosophized:

> Both these theories are found to explain not only the phenomena by the aid of which they were originally constructed, but other phenomena, which were not

Fig. 5. James Clerk Maxwell (1831–1879). Source: Front page, Maxwell (1965), first edition 1890

> thought of or perhaps not known at the time; and both have independently arrived at the same numerical result, which gives the absolute velocity of light in terms of electrical quantities. That theories apparently so fundamentally opposed should have so large a field of truth common to both is a fact the philosophical importance of which we cannot fully appreciate till we have reached a scientific altitude from which the true relation between hypotheses so different can be seen.[53]

Maxwell clearly realized that the situation in electrodynamics was underdetermined by the available empirical evidence. Two very different theories might both agree with known phenomena within their common domain and also predict new phenomena, and yet they cannot be distinguished by these empirical factors alone. This is indeed a matter of "philosophical importance."[54] Three years later, in his classic *Treatise on Electricity and Magnetism*, Maxwell returned to Lorenz, who had found "from Kirchhoff's equations of electric currents, by the addition of certain terms which do not affect any experimental result, a new set of equations, indicating that the distribution of force in the electromagnetic field may be conceived as arising from the mutual action of contiguous elements, and that waves, consisting of transverse electric currents, may be propagated, with a velocity comparable to that of light, in non-conducting media. He therefore regards the disturbances which constitute light as identical with these electric currents."[55] In a sense, Maxwell's successor in electromagnetic theory, H. A. Lorentz was aware of the work of his Danish near-namesake at an early date. In his doctoral dissertation from 1875, a detailed investigation of physical optics based on the electromagnetic theories of Maxwell and Helmholtz, the twenty-two-year-old Dutch physicist referred on the very first page to these two famous physicists. He also pointed out with reference to the 1867 *Annalen* paper that "independently of Maxwell but following a different route the same results have been obtained by Lorenz."[56]

Much later, in his Nobel lecture delivered on December 11, 1902, Lorentz again referred to the Danish physicist. Perhaps to please his audience in Stockholm, he dealt at some length with the nineteenth-century Swedish physicist and meteorologist Erik Edlund, who in the 1870s proposed an electrical theory based on the hypothesis of a single ethereal fluid consisting of what he called "molecules of ether."[57] According to Lorentz: "Edlund went as far as to identify the electric fluid with the ether, ascribing to a positively charged body an excess of ether and to a negatively charged one a deficiency of ether.… The way pioneered by Edlund, in which the distinction between ether and electricity was completely swept aside, was incapable of leading to a satisfactory synthesis of optical and electrical phenomena. Lorenz at Copenhagen came nearer to the goal. You know, however, that the true founders of our present views on this subject were Clerk Maxwell and Hertz."[58] As to Edlund's rather speculative theory, it differed in most respects drastically from Lorenz's. Not only did it presuppose the ether as a real substance, it also likened the free ether to a perfect electrical conductor. On the other hand, Edlund's theory shared with Lorenz's the general idea of retarded electrical actions, something which Lorentz highlighted in his Nobel lecture, albeit without acknowledging Lorenz's priority.

Other Contemporary Responses

Generally speaking, Lorenz's theory aroused little interest in the physics community. It was known in the 1870s and 1880s but without attracting much attention compared to other theories such as those of Maxwell, Neumann, and Helmholtz. When Josiah Willard Gibbs during his tour to Europe 1866–1869 first encountered advanced electromagnetic theory, he studied the works of Kirchhoff, Riemann, and Lorenz, but not those of Maxwell. According to Ole Knudsen, a historian of physics, "Gibbs' first encounter with the electromagnetic theory of light was almost certainly through Lorenz's paper.… It is tempting to infer that Lorenz made a lasting impression on him."[59] Perhaps so, but the impression cannot have been very deep, since Gibbs never referred to the Danish physicist in any of his scientific works.

As an illustration of the lack of impact of Lorenz's theory, consider the influential and systematic report on electromagnetic theories that Joseph John Thomson gave in 1885 to the British Association for the Advancement of Science.[60] Thomson covered in some detail the theories of, for example, Weber, Riemann, Clausius, and Neumann, and also those of less well-known scientists such as Hermann Grassmann, Josef Stefan, and Diederik Korteweg. He paid particular attention to Helmholtz's theory and its relation to Maxwell's. But although his report was aimed to be comprehensive, for some reason he did not refer to Lorenz's memoir of 1867. At the time of Thomson's report Lorenz was well known to British physicists for his work on the determination of the unit of electrical resistance,[61] but not, apparently, for his earlier contribution to electrodynamics. The report probably had the effect of further marginalizing his electrical theory of light in the British physics community.

Lorenz's paternity of the retarded potentials was not generally recognized and soon forgotten. All that the prominent electron theorist Max Abraham had to say about the matter in his influential textbook *Theorie der Elektrizität* from 1905 was that the retarded potentials "have been employed by H. Poincaré, E. Beltrami, V. Volterra, H. A. Lorentz and others."[62] And Heaviside, obviously unaware of the continental tradition, ascribed what he called the "progressive potentials" to FitzGerald.[63] From the standpoint of the first generation of Maxwellian physicists, J. J. Thomson and Heaviside among them, there was no reason to deal with Lorenz's theory. All the same, two of the period's prominent Maxwellians did comment on the theory. FitzGerald became a leading proponent of retarded potentials, which he first applied to the case of electrodynamics in a paper of 1883. He took over the concept from Lord Rayleigh, who in his *Theory of Sound* from

1877 had used it to describe the propagation of waves. Although FitzGerald was probably unaware of Lorenz's work in 1883, he later became very interested in it. In a letter to Joseph Larmor of 1897 he wrote: "Have you seen the other (Copenhagen?) Lorenz's simultaneous-with-Maxwell's-work with these $f(t - r/c)$ functions? It is quite interesting. He entirely escaped the muddle in Maxwell about the forces at one time obeying $\Delta^2 = 0$ and at another $\Delta^2 = a^2d^2/dt^2$."[64]

In a biographical essay of 1901, Oliver Lodge referred to the then modern idea of scalar and vector potentials propagating from the sources. "The very same scheme," Lodge commented, "had been proposed as early as 1867 by L. Lorenz … [in] equations which would equally well represent the facts of electrodynamics for bodies at rest so far as at that time known, while it would also include an electric theory of light consistent with existing optical knowledge."[65] Lodge considered Lorenz's theory a "brilliant and powerful attempt at generalization … precisely parallel analytically with the form of Maxwell's theory" and yet he concluded that it was self-contradictory from a physical point of view. His objection is worth recording *in extenso*:

> All true current was taken to be current of conduction, dielectric polarization not being contemplated; thus even in free space it was necessary to have conductivity, otherwise true current could never become established there. Yet if a distribution of current could be supposed thus established, it would travel according to the same laws as the light-vibration, as a result of the initial postulate that all the effects are propagated in time with a common velocity—provided however that the conductivity is small enough to be neglected in this latter connexion. But on the other hand, if the conductivity is indefinitely small it could never give rise to any electric flux to be so propagated; thus the hypothesis of free propagation in time is inconsistent with conduction.

Lodge suggested that if Lorenz had added a dielectric constant for matter, this would "at once bring his theory into conformity with Maxwell's, at the stage in which the latter was left by its author."

In 1902 the French physicist, chemist, and polymath Pierre Duhem published a critical study of Maxwell's theory in which he included a detailed summary of Lorenz's electrodynamic theory. Duhem disliked Maxwell's electrodynamics, which to his mind was plagued by inconsistencies and relied much too heavily on mechanical models and analogies. It was unsatisfactory from a logical and methodological point of view and yet Duhem admitted that it could not be ruled out on empirical grounds. As to Lorenz's theory, Duhem found it to be "certainly seductive" but he also argued that it faced great difficulties. His most serious objection was not unlike the one raised by Lodge: "According to the previous theory [Lorenz's], in any very poor conductive medium, transverse electric currents always propagate with a speed equal to the speed of light in a vacuum. On the contrary, in a transparent medium, light travels with a speed that characterizes this medium and which is less than the speed of light in a vacuum; and we see no easy way to change the hypothesis of the previous theory so that this contradiction disappears."[66] Based on this objection Duhem summarily concluded that one must "condemn irrevocably the electromagnetic theory of light proposed by L. Lorenz."

Although Lorenz's theory of electrodynamics was not warmly received, neither was it ignored or completely marginalized. Contrary to the silence of Neumann, Clausius, and Helmholtz, in Germany the prominent but also controversial Leipzig physicist and astronomer Carl Friedrich Zöllner referred to the theory in some detail. A staunch advocate of Weber's force law, Zöllner developed it into an atomistic theory according to which a neutral body consisted of an equal amount of positive and negative units with charges $\pm e$. With m denoting the mass of the hypothetical particle (and assuming $m_+ = m_-$) he derived in 1882 the remarkable relation $e^2/Gm^2 \cong 3 \times 10^{40}$, which in a sense explained Newton's gravitational constant G in terms of electrical particles.[67] In a memoir of 1876, Zöllner reviewed Lorenz's *Annalen* paper, quoting extensively and approvingly from it. As he noted, "L. Lorenz in Copenhagen … proceeds in his work essentially mathematically and denies explicitly all physical hypotheses." Zöllner apparently considered Lorenz's theory to provide support for his own view that "the so-called light ether is nothing but an aggregate of electrical molecular currents consisting of a very small number of electrical particles, at least of two combined into a doublet."[68]

Lorenz's theory also appeared in a major French textbook of 1882 on electricity and magnetism based on lectures given at Collège de France. The two distinguished authors, Éleuthère Mascart and Jules Joubert, noted that Lorenz's theory was based on the notion of retarded actions and that it led to the consequence that electrical vibrations propagate in space with the velocity of light. "It thus leads to results which are completely similar to those which Maxwell has deduced from an entirely different theory," they wrote.[69] Two years later, in a lecture series given at the University of Kiel, Heinrich Hertz also referred to the results that Lorenz had achieved "without any guidance." According to Hertz, "By proceeding more in the direction of Riemann than that of Maxwell yet unaware of their results, he too realized that one can conceive of electrical oscillations as existing in the ether; and further, that if they exist they must propagate in accordance with the laws of optics.… Those who later entered the field followed in the footsteps of these [three physicists]—the field had been discovered."[70]

In a paper from the same year, Hertz commented on Lorenz's 1867 theory in relation to what he conceived as Maxwell's superior field theory: "Similar laws for the propagation of potentials were proposed by Riemann in 1858 and by Lorenz in 1867, wanting to unify optical and electrical phenomena within the same framework. These investigators recognized that the laws involve the addition of new terms to the forces which actually occur in electrodynamics; and they justify this by pointing out that these new terms are too small to be experimentally observable. But we see that the addition of these terms is far from needing any apology as their absence would necessarily involve contradiction of generally accepted principles."[71] Yet, to Hertz, who disliked the electromagnetic potentials and was convinced of the truth of Maxwell's theory, the Riemann-Lorenz approach was of historical interest only.

Maxwell's field theory did not enjoy wide support until the 1880s, not even in England, while in German-speaking Europe the theory was neither well known nor much appreciated until the end of the decade. Although a German translation of Maxwell's *Treatise* appeared in 1883, it was only with the publication of textbooks by Ludwig Boltzmann, Paul Drude, and August Föppl in the 1890s that Maxwell's theory came to dominate the field.[72] "The study of electricity, magnetism, light and radiant heat is comprised as a whole by the physics of the ether," declared Drude three years after Lorenz's death.[73] What Drude called the physics of the ether was just another name for Maxwell's theory of electromagnetism. While Drude chose to disregard Lorenz's electrodynamic theory, a few years earlier it could still be considered comparable to Maxwell's and worthy of mention.

Thus, the German physicist Paul Volkmann, a professor at the University of Königsberg, was aware of Lorenz's theory, to which he referred in a textbook on optical theory from 1891. Published at a time when electrodynamics was not yet equated with Maxwell's field equations, at least not in Germany, the book gives a fascinating insight in this transitional phase of optics. Referring to the recent development, Volkmann wrote that it demonstrated "the possibility of interpreting light as an electrical or respectively magnetic state of oscillation, that is, to conceive of a theory of light based on electricity and magnetism. This was the way in which the *electrodynamic theory of light* arose due to [the works] of Maxwell and L. Lorenz."[74] To Volkmann, apparently, Lorenz's theory was almost as significant as Maxwell's. Also another textbook of 1891, written by the Viennese physics professor Viktor von Lang, included Lorenz's theory alongside Maxwell's. As Lang pointed out, the 1867 theory was based on Kirchhoff's work published ten years earlier.[75]

Lorenz's Silence

To use Lodge's phrase, Lorenz's electrodynamic theory of light was "brilliant," a work of genius. However, it was isolated in the sense that he did not follow up upon it or seek to develop it in his later works. In 1869 Lorenz published his important work on refractivity leading to the Lorenz-Lorentz law, but without using his electrical light theory or referring to it. Only in a paper of 1879 on electrical oscillations did he briefly refer to the connection between light and electrical vibrations that he had investigated so thoroughly twelve years earlier.[76] Nor did he ever respond to the remarks made by Maxwell, Zöllner, and Hertz. Lorenz's apparent lack of interest in his own theory is puzzling. In a perceptive study of 1956, the physicist Léon Rosenfeld argued that Lorenz's identification of light with oscillating electrical currents was "by no means an isolated incident, a stroke of luck in Lorenz's career; it is the first, and most successful, stage in the quest of very wide scope pursued with remarkable singleness of purpose."[77] But although Lorenz's theory of 1867 was not a stroke of luck, neither was it a first stage of a planned research program. It was the product of his earlier phenomenological wave theory of light and insofar as it was an electrodynamic theory it was both the first stage and the last stage.

As discussed above, it took several years until Maxwell's electromagnetic theory of light attracted serious attention among physicists on the other side of the Channel. Certainly, in 1867 Lorenz was unaware of Maxwell's theory. Although this is not remarkable, it is remarkable that he never referred to the electromagnetic theory of the Scottish genius. His silence was not rooted in ignorance, though, for some of Lorenz's unpublished notes document that he had studied the theory and also that he was aware of Maxwell's critical remarks to his own theory. He presumably realized that although his own equations for electricity corresponded to those in Maxwell's theory, the two theories differed completely in form and physical interpretation.

Key elements in Maxwellian electrodynamics, such as dielectric polarization and the displacement current, were absent in Lorenz's theory, which, although a theory of propagation of electrical signals, did not employ the notion of fields in the sense of Faraday and Maxwell. To Lorenz, the propagation of light was a result of conduction currents and therefore limited to a medium with some degree of uniform conductivity. The conductivity could be very small, but not zero. Contrariwise, Maxwell's theory applied to a perfectly non-conducting medium, a perfect dielectric, through which the oscillating fields propagated.

What the theories of Maxwell and Lorenz had in common was that they both denied the existence of action-at-distance forces of the kind favored by many German physicists in the tradition of Weber. Lorenz argued that light was a manifestation of electrical currents, but he did not justify his theory in terms of its experimental consequences; nor did he suggest or predict that there might exist electromagnetic waves at other than optical wavelengths. In this respect Maxwell was a bit more specific, as he concluded in 1865 that "light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagating through the electromagnetic field according to electromagnetic laws."[78]

It was this prediction that Hertz verified in his brilliant experiments nearly a quarter of a century later and that soon turned Maxwell's theory into a standard theory of electromagnetism. Maxwell did not himself speculate any more than Lorenz did about the possibility of actually observing the propagation of electromagnetic waves. Another of Maxwell's theoretical predictions, dating from his *Treatise* of 1873, was that light should exert a tiny pressure on a macroscopic body which for strong sunlight he calculated to be 8.8×10^{-8} lb per square foot. Maxwell did not consider the predicted effect to be measurable and it was only confirmed experimentally in the early years of the new century. Lorenz's paper contained no similar prediction.

Only on one occasion did Lorenz apply his own theory along the line of Maxwell's, and then without publishing the result.[79] In notes of 1887, Lorenz derived the relationship between the dielectric constant and the refractive index that Maxwell had found on the basis of his field theory (figure 6). By using his own theory of electrodynamics, he derived—after lengthy calculations—an expression for the refractive index n and its dependence on the size of the molecular current elements. After having derived a similar expression for the dielectric constant in terms of the vacuum permittivity he concluded that the relative permittivity was given by

$$\varepsilon_r = \varepsilon/\varepsilon_0 = n^2.$$

This is the very same relationship as obtained from Maxwell's theory.

The Fate of Lorenz's Theory

During the first half of the twentieth century, there were a few attempts to revive interest in Lorenz's half-forgotten theory of electrodynamics. Although the theory was recognized to belong to the past, at least it deserved to be known as an original contribution to physics. According to the mathematician Edmund Whittaker's monumental *History of the Theories of Aether and Electricity* (first published in 1910), "the theory of L. Lorenz is practically equivalent to that of Maxwell, so far as concerns the propagation of electromagnetic disturbances through free aether."

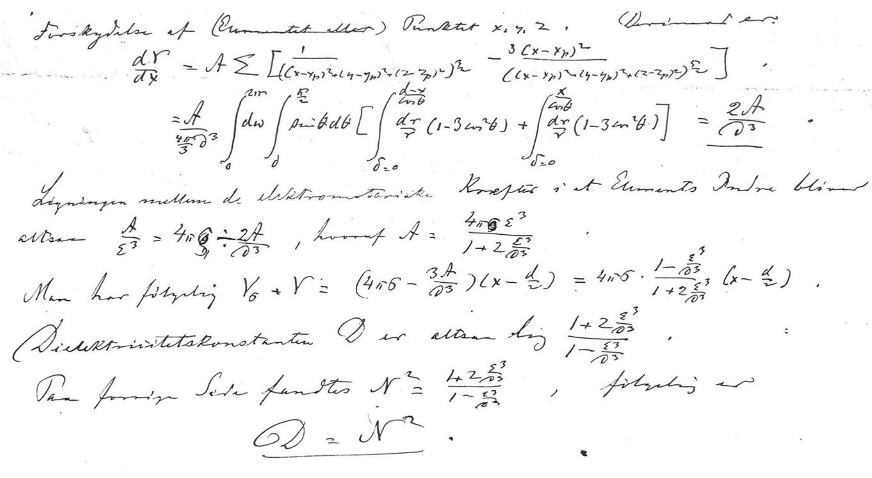

Fig. 6. Lorenz's unpublished derivation of the relationship between the dielectric constant (D) and the specific refractivity (N). Source: Lorenz Papers, Royal Danish Academy of Sciences and Letters

Yet he also pointed out that Lorenz's "theory lacks the rich physical suggestiveness of Maxwell's."[80] At about the same time, Wilhelm Wien, a recipient of the 1911 Nobel Prize in physics, wrote a survey article on electromagnetic theories of light in the multi-volume *Encyclopädie der Mathematischen Wissenschaften* in which he included a section on Lorenz's 1867 paper. Although Wien found Lorenz's theory to be interesting and close to Maxwell's theory, he objected that the theory of 1867 was not really an electro-*magnetic* theory: "It is inferior [to Maxwell's theory] in so far that it only takes into consideration the electrical and not the magnetic forces. In Lorenz's theory there is no appreciation of the important fact that, apart from the electrical force a magnetic force also exists which in transverse waves is orthogonal to the former. In this way one of the most important advantages of Maxwell's theory is lost, namely that the latter unifies the two complementary systems of the older theory, whose vectors too had to be assumed to be orthogonal."[81] In fact, Lorenz consistently wrote of light as *electrical* vibrations and referred nowhere in his paper to magnetism or magnetic vibrations. The words simply did not appear in his 1867 paper and nor did terms such as "electromagnetic" and "electromagnetism." Nor, as discussed above, did the term "potential" appear.

Whereas Wien and Whittaker, and with them practically all physicists, recognized the superiority of Maxwell's theory over Lorenz's, the Irish physicist Alfred O'Rahilly was not so sure. In an ambitious but ill-fated attempt to reform electrodynamics, he cited large passages from Lorenz's 1867 memoir, arguing that the theory of the Danish physicist was preferable to that of Maxwell on grounds of logic and simplicity. "Starting with the Lorenz-Riemann generalisation from electrodynamics," he wrote in 1938, "we immediately deduce Maxwell's equations for vacuum—without so much as mentioning the so-called displacement currents." O'Rahilly obviously rated Lorenz's theory highly: "As a matter of fact, the view of Lorenz is accepted universally today, while there appears to be little or no realisation of the elementary inference that Maxwell's displacement-current is thereby rendered unnecessary. Everyone now uses the retarded potentials introduced by L. Lorenz."[82] However, O'Rahilly's unorthodox attempt to establish electromagnetism on a non-Maxwellian basis in partial accordance with the views of Lorenz fell on deaf ears.

Apparently without knowing of O'Rahilly's book, the Danish physicist Mogens Pihl completed a doctoral dissertation in 1939 on Lorenz's contributions to physics, including a detailed reconstruction of the electrodynamic theory. "Lorenz's system of equations is equivalent to Maxwell's [and] … the Lorenz equations are nothing but the well-known relativistically invariant formulation of the Maxwell equations," he wrote.[83] Although Pihl found Lorenz's derivation of the electromagnetic theory of light to be "more natural" than Maxwell's, he admitted that the physical structure of the latter theory was deeper and richer. Moreover, since Lorenz's theory assumed media for which Ohm's law is valid, it was more restrictive than Maxwell's. In a review of Pihl's work, the American physicist Ronold King, making the connection to O'Rahilly's point of view, wrote: "If

Maxwell had never lived, modern electromagnetism could have been developed from the work of Lorenz."[84] However, such a counterfactual scenario appears highly unlikely.

Ether/Or? Discussion and Conclusion

In this paper, the electrical theory of light proposed by Lorenz in 1867 has been examined with respect to content, context, and reception. Despite its scientific qualities, from a sociological point of view the theory was unsuccessful because it failed to attract serious interests among physicists. Though never a rival to Maxwell's fully developed theory of light, its poor reception during the 1870s and 1880s cannot be explained in terms of overwhelming competition from Maxwell's standard theory. At the time, there was no standard theory.

It seems to me that Lorenz's indifference to Maxwell's field theory and his strange decision not to develop or even defend his own alternative was a major reason for its fate. Lorenz was a loner who insisted on developing his own ideas independently of mainstream physics. While such an attitude may on rare occasions lead to success, in general it only leads to trouble and oblivion. In an autobiographical note of 1877, Lorenz stated that he defended a "realist" view of science and tried to avoid "any physical theory that does not follow by necessity from the phenomena." Moreover, "I have no confidence in the customary views and physical theories such as, for example, the now generally accepted theory of heat as molecular motion."[85] Apparently his lack of confidence also included the major electromagnetic theories of the period such as Helmholtz's and Maxwell's.

Lorenz's electrodynamic theory of light was in several ways remarkable, not least because it explicitly denounced the concept of the ether as a carrier of light as "unreasonable." It may in this respect appear very modern, even Einsteinian. However, Lorenz's dismissal of the luminiferous ether was more rhetorical than real because his theory would not work in an absolute vacuum. In a sense, he reintroduced the ether in his own way, namely by filling the vacuum with weak conduction currents instead of conceiving it as completely void.[86] This feature in his theory was recognized by Zöllner in his review of 1876, although Zöllner's version of "molecular currents" is not to be found in Lorenz.

As mentioned above, at the end of his 1867 paper Lorenz assumed that "in the so-called vacuum there is sufficient matter to form an adequate substratum for the motion [of electricity]." He had in mind some kind of highly rarefied gas "[not] different from the known gases," as he expressed it in his Danish paper. Conceptions of the ether as a rarefied gas were known at the time and later in the century. For example, Edlund assumed in the 1870s an ethereal electric fluid which he identified with a gaseous substance.[87] Lorenz's idea of the vacuum filled with a tiny amount of matter was clearly a physical hypothesis, contrary to his bold methodological claim that the theory had been established "without the assumption of a physical hypothesis." Revealingly, two years later, in the paper in which he first stated the Lorenz-Lorentz formula, he defined empty space as "a space in which there is no *recognizable* amount of matter" (emphasis added).[88] It seems that Lorenz's theory was after all an ether theory of a sort, though of a sort very different from other ether theories in the period.

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