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ON THE OBSERVABILITY OF THE FIELD $B^{(3)}$: RELATIVISTIC EFFECTS IN MAGNETO-OPTICS

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It is shown that the novel vacuum field $B^{(3)}$ is an experimental observable, and several methods of observation are suggested: these include the pulsed microwave magnetization of a plasma, the optical Aharonov-Bohm effect, and the microwave frequency optical Faraday effect. The effect of $B^{(3)}$ is presented in the form of relativistically corrected semi-classical theory.

Key words: $B^{(3)}$ field, observability.

1. INTRODUCTION

The semi-classical theory [1–5] of magneto-optics relies on the well-known conjugate product $B^{(1)} \times B^{(2)}$ [3,4] of circularly polarized radiation and has been reviewed, for example, by Zawodny [5]. In this Letter, we work into the theory relativistic details which emerge from a rigorous classical [6] development of the trajectory of one electron (e) in the circularly polarized electromagnetic field, represented by the usual four-potential A_μ. This theory [6] has led to the centrally important inference that there exists in the vacuum a longitudinal magnetic field $B^{(3)}$, the spin field [6–10], which is related to the ordinary plane waves $B^{(1)} = B^{(2)*}$ in a well-defined Lie algebra [6]. The simple method proposed here shows conclusively that $B^{(3)}$ can be isolated from $B^{(1)} = B^{(2)*}$ through the investigation at

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microwave frequencies of the inverse and optical Faraday effects and other magneto-optic effects. Under these conditions, the quantum theory of magnetic effects in paramagnetic media can be adopted straightforwardly for the description of magneto-optic effects, the role of the static magnetic field being played by $\boldsymbol{B}^{(3)}$ suitably corrected by a simple relativistic factor. The basic assumption is that the rigorous one electron theory [6] can be applied in the first approximation to the case where there is a free electron in a paramagnetic material, such as a doped glass [11], or in an atomic gas such as H. This leads to the inference that the inverse Faraday effect, for example, in a paramagnetic should show a magnetization which depends on the *square root* of the beam power density I_0 (W m^{-2}). This method is outlined in Sec. 2 for the inverse Faraday effect, and is extended in Sec. 3 to the optical Faraday effect. Section 4 qualitatively explains the rigorous approach, which must be based on the Dirac equation, i.e., the relativistically correct version of the quantum mechanical wave equation. The rigorous approach, however, will almost certainly be numerical in nature, because as soon as we depart from the single electron, the Hamiltonian must contain potential-energy terms as, for example, in the H atom. These terms almost always make the Dirac equation insoluble analytically, so approximations have to be used.

2. THE RELATIVISTIC TREATMENT OF THE INVERSE FARADAY EFFECT IN PARAMAGNETIC MATTER

The simplest theory of the inverse Faraday effect, i.e., of magnetization by circularly polarized electromagnetic radiation, can be built up from the relativistic Hamilton-Jacobi equation [6] of e in A_μ. This classical, relativistic type of theory can be traced back to Kroll [12], and is described by Landau and Lifshitz [13]. Reference 6 however, uses it to show that the magnetization in the inverse Faraday effect is determined *completely* by the novel field $\boldsymbol{B}^{(3)}$, whose existence was first inferred (using independent methods) only in 1992 [7]. In the limit

$$\omega \ll \frac{e}{m} B^{(0)} \tag{1}$$

the magnetization is given for N non-interacting electrons by

$$\boldsymbol{M}^{(3)} \sim -\frac{Ne^2c^2}{4m\omega^2}\left(1 + x^2 + \frac{x^4}{2} + \ldots\right)\boldsymbol{B}^{(3)}, \tag{2}$$

where $x = (m\omega)/(eB^{(0)})$. Here e/m is the charge to mass ratio of the electron, which is known with precision [14]; ω is the angular frequency of the beam; and $B^{(0)}$ its flux density amplitude (tesla). Therefore $\boldsymbol{M}^{(3)}$ is determined *completely* by $\boldsymbol{B}^{(3)}$ except for fundamental constants and beam parameters. Obviously, if $\boldsymbol{B}^{(3)}$ were zero, nothing would be observable, and, conversely, $\boldsymbol{B}^{(3)}$ is just as obviously an experimental observable. We recall that this is the result of the principle of least action, on which the relativistic Hamilton-Jacobi equation is based. Using the relation between $B^{(0)}$ and I_0,

$$B^{(0)} = \left(\frac{I_0}{\epsilon_0 c^3}\right)^{\frac{1}{2}}, \tag{3}$$

we find that

$$|\boldsymbol{M}^{(3)}| \sim -\frac{Ne^2c^2}{4m\omega^2}\left(1 + x^2 + \frac{x^4}{2} + \ldots\right)\left(\frac{I_0}{\epsilon_0 c^3}\right)^{\frac{1}{2}}, \tag{4}$$

i.e., $|\boldsymbol{M}^{(3)}|$ is proportional to the *square root* of I_0. Here ϵ_0 is the vacuum permittivity as usual [6].

In the opposite limit to (1), i.e.,

$$\omega \gg \frac{e}{m}B^{(0)}, \tag{5}$$

it is found that [6]

$$|\boldsymbol{M}^{(3)}| \sim -\frac{Ne^3c^2}{4m^2\omega^3}\left(1 + \frac{1}{x^2} + \frac{1}{2x^4} + \ldots\right)\frac{I_0}{\epsilon_0 c^3}, \tag{6}$$

and so $|\boldsymbol{M}^{(3)}|$ is dominated by an I_0 dependence. The condition (5) occurs at *visible* frequencies; the condition (1) occurs at *microwave* frequencies first used experimentally by

Deschamps *et al.* [15], with megawatt pulses setting up an electron plasma in helium gas.

In order to extend these results to paramagnetic materials, it is assumed, for the sake of simplicity, that the inverse Faraday effect can be described by the *optical* field

$$\boldsymbol{B} := \boldsymbol{B}^{(3)}\left(1 + x^2 + \frac{x^4}{2} + \ldots\right), \tag{7}$$

which is an ordinary magnetic field corrected by the relativistic factor $\left(1 + x^2 + \frac{x^4}{2} + \ldots\right)$, valid in the limit (1). In this approximation, therefore, the well-developed theory of the Faraday effect [16] can be adopted [8] for its first-order optical equivalent by replacing the ordinary static magnetic field by the relativistic *light magnet* [17] described in Eq. (7). By magnetizing paramagnetic matter with pulsed microwaves [15], the $I_0^{1/2}$ dependence expected from Eq. (4) can be isolated experimentally, demonstrating the existence of $\boldsymbol{B}^{(3)}$ in the vacuum. Systematic experimental development should be based on the methods used by Deschamps *et al.* [15] for plasma magnetization. Under the conditions reported by these authors, however:

$$\omega \sim 5\frac{e}{m}B^{(0)} \tag{8}$$

and in consequence an I_0 dependence of the magnetization is expected theoretically [6,18] from condition (5), and was observed experimentally [15]. To obtain condition (1) experimentally, and to isolate $\boldsymbol{B}^{(3)}$ through its characteristic $I_0^{1/2}$ dependence, the peak power of the microwave pulses used by Deschamps *et al.* [15] must be increased by a factor of at least fifty for the 3 GHz frequency used. With contemporary technology (e.g., synchronized oscilloscopes which can detect nanosecond pulses or shorter) this appears to be straightforward.

3. THE OPTICAL FARADAY EFFECT

Similarly, the use of pulsed microwaves appears to be essential, in a correctly relativistic theory, to isolate the optical Faraday effect [6,19] due to $\boldsymbol{B}^{(3)}$ at first order. This effect is the rotation of the plane of polarization of a probe with $\boldsymbol{B}^{(3)}$, relativistically corrected as in Eq. (7). At visible frequencies, condition (5) holds in the relativistic theory, and the optical Faraday effect will be dominated by the term in I_0, i.e., a term in $B^{(0)}\boldsymbol{B}^{(3)}$. This is because the theory of the ordinary Faraday effect [16] depends on a perturbation of the antisymmetric polarizability which is to first order in the applied magnetic field. This field is $\boldsymbol{B}^{(3)}$ in relativistic magneto-optics and is defined by the condition (1). In the opposite limit (5), the perturbation of the antisymmetric polarizability is proportional to the term $B^{(0)}\boldsymbol{B}^{(3)}$, and is to order I_0, not $I_0^{1/2}$. Experimental work on the optical Faraday effect, and on magneto-optics in general, has been carried out in the overwhelming majority of cases with pulsed *visible* frequency lasers. In consequence, an I_0 dependence is always observed, because condition (5) is always in effect. In other words, the relativistic factor reduces at visible frequencies to

$$(m^2\omega^2 + e^2B^{(0)2})^{\frac{1}{2}} \rightarrow m\omega\,. \tag{9}$$

The correctly relativistic theory of the optical Faraday effect, as for the inverse Faraday effect, must be based on the Dirac equation, as discussed in the next section.

4. THE DIRAC EQUATION IN MAGNETO-OPTICS

The theory of magneto-optics has been based [9] on the semi-classical, perturbative, approach to the Schrödinger equation, and so is not correctly relativistic. This type of theory is not capable of producing the $I_0^{1/2}$ dependence due to $\boldsymbol{B}^{(3)}$, but it produces the I_0 dependence due to the conjugate product $iB^{(0)}\boldsymbol{B}^{(3)*}(= \boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)})$ without the relativistic correction given in the simplest terms (one-electron theory) in Eq. (6). Thus, effects such as light shifts due to circularly polarized electromagnetic radiation [20], the inverse Faraday effect [11,15], the inverse Zeeman effect [21], and the inverse Cotton-Mouton effect [22] have always appeared to have an I_0 profile. It is important to note that such a profile is in itself *experimental evidence for* $\boldsymbol{B}^{(3)}$, because these magneto-optic effects can be understood even in semi-classical theory, through the conjugate product $iB^{(0)}\boldsymbol{B}^{(3)*}$. The correctly relativistic approach, however, shows that there is also an $I_0^{1/2}$ profile, which dominates at microwave frequencies. It is exceedingly important to try to isolate this experimentally because it isolates the vacuum $\boldsymbol{B}^{(3)}$ itself. As discussed elsewhere [6] this is the first classical vacuum field to be inferred since Maxwell and as such is clearly of central importance in optics, spectroscopy and field theory [23].

Semi-classical perturbative methods are highly developed [9] but are based in the overwhelming majority of cases on the Schrödinger equation. To make the theory relativistic, and suitable for pulsed microwave experiments, and $\boldsymbol{B}^{(3)}$ theory, the Dirac equation must replace the Schrödinger equation. The trajectory of one electron in the electromagnetic field must then be calculated in relativistic quantum field theory, using the classical theory as a starting point. The latter is based on the relativistic Hamilton-Jacobi equation [6] for e in A_μ, and the Dirac equation applied to this problem [6] has shown that the intrinsic electron spin forms an interaction Hamiltonian *directly* with $\boldsymbol{B}^{(3)}$, and with $\boldsymbol{B}^{(3)}$ only. This result shows that it is absurd to assert that $\boldsymbol{B}^{(3)}$ is not an observable or is zero in the vacuum. The next theoretical step might well be, therefore, the calculation of the microwave induced Zeeman effect in atomic H, an effect which must also be determined completely by $\boldsymbol{B}^{(3)}$, and which must give an $I_0^{1/2}$ profile. The relativistic but semi-classical approach to this problem sets up the Dirac equation for H in the presence of the classical A_μ; and the rigorous approach must be based on quantum electrodynamics, with a quantized field. The next step after that might be the calculation of the ESR shift in H due to a circularly polarized microwave pulse, such a calculation and experiment appears to be convenient to carry out, because ESR itself depends on the mature technology of microwave pulses. The latter are also used, of course, as the basis of all NMR technology. Ultimately, a new understanding of ESR and NMR might emerge, one based on $\boldsymbol{B}^{(3)}$ rather than the usual transverse plane waves.

It might also be appropriate to extend the $\boldsymbol{B}^{(3)}$ calculations, using the Dirac equation, to the paramagnetic doped glasses used by Pershan *et al.* [11] in their well-known experimental demonstration of the inverse Faraday effect, and to repeat their experiment with microwave pulses in order to isolate the $I_0^{1/2}$ profile of $\boldsymbol{B}^{(3)}$. Overall, therefore, a systematic effort with contemporary library software, such as MOTECC [24], is needed to solve the Dirac equation of atoms with spare electrons in the presence of the circularly polarized electromagnetic field. This theory for one electron in the field has been given in Ref. 6.

The existence of $\boldsymbol{B}^{(3)}$ implies that of the optical equivalent [6] of the Aharonov-Bohm effect, in which the vector potential associated with $\boldsymbol{B}^{(3)}$ produces a physical effect in regions where $\boldsymbol{B}^{(3)}$ is itself excluded. As shown in the original paper by Aharonov and Bohm [25], their effect is quantum relativistic in nature, and full account of this must be taken when looking for the equivalent effect due to $\boldsymbol{B}^{(3)}$ and in attempts to isolate it from any concomitant effects at second order in $B^{(0)}$. This may well require the use of circularly polarized microwave pulses rather than a circularly polarized laser, but this remains an open question.

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