Maxwell and the Modes of Consistent Representation

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I

In this paper[1] the problem of the analysis of MAXWELL'S theories of electricity and magnetism will be approached by a discussion of the way in which he developed FARADAY'S ideas. MAXWELL'S statement in the *Treatise on Electricity and Magnetism* (1873) that "I ... translated what I considered to be Faraday's ideas into a mathematical form"[2] was his own indication of the origin of the concepts which he employed, and the starting-point for the elucidation of the historical development of MAXWELL'S thought must be the interpretation of this remark of MAXWELL'S. The concepts which lie at the foundation of his electrical thought were derived from FARADAY, and his use of these concepts determined the structure of his theories of electricity. This paper is an attempt to follow the process by which MAXWELL adopted concepts from FARADAY, and the manner in which these concepts were subsequently transformed by MAXWELL.

A fundamental conceptual dichotomy in MAXWELL'S thought will be delineated to distinguish two very different physical models which he adopted for the representation of nature. Both these modes of representation were formulated on the basis of concepts first introduced by FARADAY. In "On Faraday's Lines of Force" (1856) MAXWELL employed the theory of the primacy of lines of force which was characteristic of FARADAY'S later thought. Like FARADAY in his later period, MAXWELL emphasized that the lines of force represented a real physical state and were not to be understood as fictitious entities, though he did not discuss the nature of the physical state to which the lines of force corresponded. It was in the attempt to specify the nature of this physical state in "On Physical Lines of Force" (1861/62) that MAXWELL came to adopt quite a different physical model, one in which the particles of matter and ether were conceived to be in a state of polarization, opposite parts being in opposite electrical states. FARADAY had used this theory in his earlier period, but had abandoned it in favour of the notion of the primacy of lines of force. The theory of particulate polarization was retained in "A Dynamical Theory of the Electromagnetic Field" (1865) and the *Treatise on Electricity and Magnetism* (1873). This theory was not without its attendant difficulties, and it was to avoid these that MAXWELL reintroduced the

[1] I wish to express my gratitude to Mr J. E. MCGUIRE, of the University of Leeds, for his advice during the preparation of this paper.

[2] JAMES CLERK MAXWELL, *A Treatise on Electricity and Magnetism* (2 vols., Oxford, 1873), x. All references are to the first edition. This work will be referred to as the *Treatise* (followed by an article number). MAXWELL'S posthumously published *Elementary Treatise on Electricity*, (ed.) W. GARNETT (Oxford, 1881), will be referred to as *Elementary Treatise*. MAXWELL'S collected papers, *The Scientific Papers of James Clerk Maxwell*, (ed.) W. D. NIVEN (2 vols., Cambridge, 1890), will be referred to as *Papers*.

notion of the primacy of lines of force in the "Note on the Electromagnetic Theory of Light" (1868) and the *Elementary Treatise on Electricity* (1881). Here however MAXWELL transformed the theory of the primacy of the lines of force by incorporating features first used in the theory of particulate polarization. Thus, the categories of particulate polarization and the primacy of lines of force do not conform to a distinction between earlier and later ideas in MAXWELL's thought. Each of MAXWELL's attempts to formulate a theory of electricity and magnetism represents a different approach to the problem of the representation of nature, and the distinctions between these theories and the nature of the transitions between them will be the concern of this paper.

The two different sets of theories which can be distinguished in MAXWELL's electrical thought were indicated by G. T. WALKER in his Adams Prize essay *Aberrations and Some Other Problems Connected with the Electromagnetic Field* (1900). WALKER classified electromagnetic theories as "tubular" or "molecular", the former class of theories being based on lines of force and the latter on particulate polarization. He traced the tubular view to MAXWELL's 1868 "Note on the Electromagnetic Theory of Light" and the molecular view to the *Treatise on Electricity and Magnetism*[3]. A feature of MAXWELL's ideas which emerges from WALKER's analysis is that there was no ontological separation of matter and the ether in MAXWELL's thought. WALKER emphasized that ether and matter could be considered together as both tubular or molecular, and this is how they were conceived by MAXWELL[4]. The consideration of the electromagnetic field as a system independent of matter is not to be found in MAXWELL's theories but in LORENTZ's theory of the field[5]. WALKER himself preferred the view in which the ether was considered as tubular and matter as molecular, which lead to LORENTZ's theory[6], rather than supposing both ether and matter as tubular or both ether and matter as molecular, which correspond to MAXWELL's view. HERTZ followed MAXWELL in this, opposing the view in which "the electromagnetic conditions of the ether and of the tangible matter at every point in space [were considered] as being in a certain sense independent of each other"[7].

[3] G. T. WALKER, *Aberrations and Some other Problems Connected with the Electromagnetic Field* (Cambridge, 1900), 1f. WALKER represented the two conceptions as considering the medium either "as consisting of particles with polar properties" or as being "regarded as continuous and completely filling space", *ibid.*, vi.

[4] WALKER, *ibid.*, 12ff. See E. T. WHITTAKER, *A History of the Theories of Aether and Electricity* (2 vols., London, 1951—1953), **1**, 259, where it is noted that MAXWELL's custom was "to treat matter as if it were merely a modification of the aether".

[5] H. A. LORENTZ, "La Théorie électromagnetique de Maxwell et son application aux corps mouvants", *Archives néerlandaises des sciences exactes et naturelles* 1892, **25**, 363—552. Reprinted in H. A. LORENTZ, *Collected Papers* (9 vols., The Hague, 1934—1939), **2**, 164—343. LORENTZ separated the ether from matter, and then sought to explain the relations between ether and matter by supposing matter as composed of charged particles, *Collected Papers*, **2**, 228. The concept of a stationary ether and matter as composed of charged particles enabled LORENTZ to formulate a theory in which the independence and mutual interactions of matter and ether were clearly established. LORENTZ's ideas have been discussed in two articles by TETU HIROSIGE: "Lorentz's Theory of Electrons and the Development of the Concept of the Electromagnetic Field", *Jap. Stud. Hist. Sci.*, 1962, **1**, 101—110; "Electrodynamics before the Theory of Relativity, 1890—1905", *ibid.*, 1966, **5**, 1—49.

[6] See WALKER, *op. cit.* (note 3), 20.

The approach which characterizes all FARADAY'S thought was the denial of action at a distance, and both MAXWELL[8] and FARADAY[9] quoted NEWTON'S third letter to BENTLEY — that it was inconceivable that matter could interact without mutual contact, and that it was absurd to suppose gravity an essential property of matter[10] — with great approval. MAXWELL pointed out that it was COTES who had expressed this absurdity[11], describing him as "one of the earliest heretics bred in the bosom of Newtonianism"[12], and argued that it was "more philosophical to admit the existence of a medium which we cannot at present conceive, than to assert that a body can act at a place where it is not"[13]. He rejected the tenet of action at a distance, which he called the "dogma of Cotes", that no explanation could be more intelligible than the fact that "action at a distance is one of the primary properties of matter"[14], for MAXWELL wished to construct a representation of the manner in which the action took place. He argued that any physical theory must embody what he termed a "consistent representation" of the phenomena, and this term expressed his idea of the nature of physical theory. MAXWELL derived this term from a letter of GAUSS to WEBER which he translated in the *Treatise*, stating that GAUSS had "a subjective conviction that it would be necessary in the first place to form a consistent representation [*construirbare Vorstellung*] of the manner in which the propagation [of electric action] takes place"[15].

[7] H. HERTZ, "Ueber die Grundgleichungen der Electrodynamik für bewegte Körper", *Ann. Phys.*, 1890, **41**, 370; *Electric Waves*, (trans.) D. E. JONES (London, 1893), 242. See WHITTAKER, *op. cit.* (note 4), **1**, 329. See HERTZ, *op. cit.*, 242, 268, for a different view.

[8] J. C. MAXWELL, "Action at a Distance" [1873], *Papers*, **2**, 316; "Attraction" [1875], *ibid.*, 487.

[9] MICHAEL FARADAY, *Experimental Researches in Electricity* (3 vols., London, 1839—1855), **3**, 532n, 571.

[10] I. B. COHEN (ed.), *Isaac Newton's Papers and Letters on Natural Philosophy* (Cambridge, 1958), 302f. MAXWELL (*Papers*, **2**, 316) argued that NEWTON himself wished to explain gravity by means of the impulse of an aetherial medium, quoting COLIN MACLAURIN, *An Account of Sir Isaac Newton's Philosophical Discoveries* (2 ed., London, 1750) to the effect that "this was his opinion early; and if he did not publish it sooner, it proceeded from hence only, that he found he was not able, from experiment and observations, to give a satisfactory account of this medium, and the manner of its operation, in producing the chief phaenomena of nature", *op. cit.*, 116f. MAXWELL was clearly searching for a NEWTONIAN pedigree for his denial of action at a distance.

[11] *Sir Isaac Newton's Mathematical Principles of Natural Philosophy*, (trans.) MOTTE-CAJORI (Berkeley, 1934), xxvi.

[12] See L. CAMPBELL & W. GARNETT, *The Life of James Clerk Maxwell* (London, 1882), 437.

[13] MAXWELL, *Papers*, **2**, 312.

[14] MAXWELL, *Treatise*, § 865.

[15] *Treatise*, § 861. GAUSS' letter was published in *Carl Friedrich Gauss Werke* (12 vols., Göttingen, 1863—1933), **5** (1867), 629. The expression was also employed by J. J. THOMSON, *Notes on Recent Researches in Electricity and Magnetism* (Oxford, 1893), 1 as "concrete representation", and by JOSEPH LARMOR, *Aether and Matter* (Cambridge, 1900), 319 as "working representation". See JOSEPH TURNER, "Maxwell on the Logic of Dynamical Explanation", *Philosophy of Science*, 1956, **23**, 36—47 for a discussion on consistent representation, in which it is argued that a "physical hypothesis satisfies the condition of *consistent representation* if the hypothesis is proved consistent with the fundamental principles of dynamics which include, for example, Newton's laws of motion and the principle of the conservation of mechanical energy", *op. cit.*, 37. This sense of "consistency" was certainly important for MAXWELL, but he used the term "consistent representation" in a more restricted sense.

MAXWELL argued that the propagation of electric action in time led "to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action"[16].

Thus, MAXWELL meant that any theory must provide a physical explanation of the phenomena, and such was the importance of this principle in his thought that his achievement in formulating his theories of electricity can be described as an attempt to provide different modes of consistent representation. Successive theories were proposed to avoid difficulties or inadequacies in the theories he had already formulated, for he did not succeed to his own satisfaction. However, he emphasized that a purely physical description led to a "rashness in assumption"[17], but though he valued a mathematical analysis as in his use of the LAGRANGEAN formalism of dynamics in the *Treatise*, he made it clear that any symbolic representation must provide a physical interpretation of nature[18]. Thus, in the *Treatise* he stated that while LAGRANGE'S method was "free from the intrusion of dynamical ideas" his own purpose was to "cultivate our dynamical ideas"[19], and he sought to "retranslate the principal equations of the method into language which may be intelligible without the use of symbols"[20]. Nevertheless, this was not sufficient, for it "kept out of view the mechanism by which the parts of the system are connected"[21] and his aim was a theory in which "the whole intermediate mechanism and details of the motion, are taken as the objects of study"[22]. This concern with providing a physical explanation — a "consistent representa-

[16] MAXWELL, *Treatise*, § 866.

[17] MAXWELL, "On Faraday's Lines of Force", *Papers*, 1, 155f.

[18] The importance of the relation between mathematical and physical representation can be seen in his discussion of the problem of continuity in the *Treatise*. The equations of the *Treatise* involved quantities which were continuous functions of their variables, and MAXWELL discussed the question of the relation between a particulate physical model and equations of continuous action in a section of the *Treatise* on "Physical Continuity and Discontinuity" (§ 7). He made it clear that his notion of continuity in the *Treatise* was one in which "A quantity is said to vary continuously if, when it passes from one value to another, it assumes all the intermediate values", and he illustrated this by referring to "the continuous existence of a particle of matter in time and space". In a manuscript "On Physical Continuity and Discontinuity" (University Library, Cambridge, Add. MSS. 7655), which relates to this section of the *Treatise*, he described this kind of continuity in which the path of a particle described a continuous line in space, its coordinates being continuous functions of the time, as "physical continuity". He contrasted this with "mathematical continuity", which "refers rather to the form of the function than to its particular values". He illustrated the difference by referring to the equation of continuity, the example taken in the *Treatise*, stating that "The 'continuity' which is defined by the 'Equation of continuity' is the continuous existence of the moving particle of a medium, not the continuity of the form of the functions expressing their velocity". Thus the continuity expressed by the equations represented the continuous existence of a particle in space and time, physical continuity. I am grateful to Mr A. E. B. OWEN of the University Library, Cambridge, for his help with MAXWELL MSS.

[19] MAXWELL, *Treatise*, § 554.

[20] *Treatise*, § 567.

[21] *Ibid.*

[22] *Treatise*, § 574.

tion" — of nature can be seen throughout his work, even though he did not succeed in this in "Faraday's Lines of Force". In connection with this paper he said that he had been "planning and partly executing a system of propositions about lines of force &c which may be *afterwards* applied to Electricity, Heat, or Magnetism or Galvanism, but which is in itself a collection of purely geometrical truths embodied in geometrical conceptions of lines, surfaces &c"[23] for he wished to present "the mathematical ideas to the mind in an embodied form, as systems of lines or surfaces, and not as mere symbols"[24], but this was not a physical explanation, even though the lines of force had physical existence. In addition he valued the use of analogies which would direct the mind "to lay hold of that mathematical form which is common to the corresponding ideas in ... two sciences"[25], again indicating the importance of mathematical representation. However, MAXWELL was concerned to stress that no physical explanation could be in perfect correspondence with reality, for there was a distinction between any "consistent representation" and the structure of nature itself[26].

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[23] MAXWELL to WILLIAM THOMSON (Lord KELVIN), 13 September 1855, published in J. LARMOR (ed.), *The Origins of Clerk Maxwell's Electrical Ideas, as Described in Familiar Letters to William Thomson* (Cambridge, 1937), 17. This was first published in *Proc. Camb. Phil. Soc.*, 1936, **32**, 695—750.

[24] MAXWELL, "Faraday's Lines", *Papers*, **1**, 187.

[25] MAXWELL, "Address to the Mathematical and Physical Sections of the British Association" [1876], *Papers*, **2**, 219. See below and note 84 for a discussion of MAXWELL's treatment of the limitations of the use of analogies. In an interesting essay on "Analogies in Nature" written in 1856 for the "Apostles" at Cambridge, MAXWELL developed his ideas on analogy by arguing that "although pairs of things may differ widely from each other, the *relation* in the one pair may be the same as that in the other. Now, as in a scientific point of view the *relation* is the most important thing to know, a knowledge of the one thing leads us a long way towards a knowledge of the other", CAMPBELL & GARNETT, *Life of James Clerk Maxwell* (London, 1882), 243. G. E. DAVIE has suggested, in *The Democratic Intellect: Scotland and her Universities in the Nineteenth Century* (2nd ed., Edinburgh, 1964), 192ff, that these remarks, and his whole approach to the problem of analogy in science, show evident traces of the Scottish philosophical abstractionist approach of MACLAURIN, HUME and REID, which came to him by way of his teacher of philosophy at Edinburgh, WILLIAM HAMILTON. DAVIE also argues that MAXWELL's concern that a theory should embody a union of mathematical and physical ideas can be traced to the influence of J. D. FORBES, his teacher of natural philosophy at Edinburgh, as expressed in a review of the third edition of WILLIAM WHEWELL's *History of the Inductive Sciences* (3 vols., London, 1858) in *Fraser's Magazine*, 1858, **57**, 283—294. See also JOSEPH TURNER, "Maxwell on the Method of Physical Analogy", *Brit. J. Phil. Sci.*, 1955, **6**, 226—238 for a discussion of MAXWELL's views on analogy.

[26] Thus, in his "Address to the Mathematical and Physical Sections of the British Association" in 1870 he stated that "molecules have laws of their own, some of which we select as most intelligible to us and most amenable to our calculation. We form a theory from these partial data, and we ascribe any deviation of the actual phenomena from this theory to disturbing causes. At the same time we confess that what we call disturbing causes are simply those parts of the true circumstances which we do not know or have neglected, and we endeavour in future to take account of them. We thus acknowledge that the so-called disturbance is a mere figment of the mind, not a fact of nature, and that in natural action there is no disturbance", *Papers*, **2**, 228f. Compare the remarks made by HEINRICH HERTZ, *Principles of Mechanics*, (trans.) D. E. JONES & J. T. WALLEY (London, 1899), lf.

II

The nature of FARADAY's ideas relevant to MAXWELL's work must now be discussed[27]. As a result of his discovery of electromagnetic induction in 1831 FARADAY was led to explain the induction of an electric current between two coils of wire wound round an iron ring in terms of the creation of a "peculiar condition"[28] in the ring. He called this "electrical condition of matter" the "*electrotonic* state"[29], regarding it as a state of tension in the particles of the ring, the creation and dissolution of the electrotonic state causing the induction of the current[30]. He referred to electrical action as involving a "peculiar state of tension or polarity"[31], and here defined polarity quite unambiguously as a state in which "a molecule acquires opposite powers on different parts"[32]. Thus, his physical view of the electrotonic state was connected to that of molecules being polarized as the result of an electric force, and by polarity he here meant opposite electrical states on different parts. Shortly after its formulation FARADAY replaced this concept of the electrotonic state by a theory of lines of force[33], as a result of his belief that there was a "*singular independence* of the magnetism and the bar in which it resides"[34]. He now conceived magnetism in terms of magnetic curves or lines[35], and already at this early stage in the development of his thought there was the suggestion that lines of force were entities existing independently of the particles of matter. These two modes of representation, that of lines of force and that of the electrotonic state, were to dominate FARADAY's thought, and were conceived as alternatives. As he later said, the "*electrotonic state* ... would coincide and become identified with that which would then constitute the physical lines of magnetic force"[36]. The idea of the tension of the particles of matter was not renounced for long, and in his early thought the lines of force tended to be conceived as imaginary entities denoting the disposition of the individual particles, as the result of an electric force being transmitted from particle to particle[37]. This can be seen from his important papers on electrostatic induction of 1837 and 1838, where he

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27 For a more complete account of FARADAY's electrical ideas with full documentation see my paper "Faraday's Theories of Matter and Electricity", forthcoming in *Brit. J. Hist. Sci.* I do not accept the view of L. PEARCE WILLIAMS, *Michael Faraday* (London, 1965), that FARADAY's views were determined by an adherence to BOSCOVICH's *Theoria Philosophiae Naturalis* (Vienna, 1758). For a critical discussion of WILLIAMS' interpretation, see J. BROOKES SPENCER, "Boscovich's Theory and its Relation to Faraday's Researches: An Analytical Approach", *Arch. Hist. Ex. Sci.*, 1967, **4**, 184—202.

28 MICHAEL FARADAY, *Experimental Researches in Electricity* (3 vols., London, 1839—1855), **1**, paragraph 61. I will refer to this work in future as *Electricity*, followed by a volume number, and, where appropriate, Faraday's paragraph number, as in the following example: *Electricity*, **1**, par. 61. Otherwise page numbers will be given, for example: *Electricity*, **2**, 284.

29 *Electricity*, **1**, par. 60.

30 *Electricity*, **1**, par. 71, 73.

31 *Electricity*, **1**, par. 949.

32 *Electricity*, **1**, par. 1304.

33 *Electricity*, **1**, par. 218—219.

34 *Electricity*, **1**, par. 220.

35 *Electricity*, **1**, par. 231, 238.

36 *Electricity*, **3**, par. 3269.

37 *Electricity*, **1**, par. 1304.

emphasized the role of the particles of the ambient medium, the dielectric. He spoke of induction "being an action of the contiguous particles of the dielectric which [are] ... thrown into a state of polarity and tension"[38], the inductive action taking place in curved lines. He went on to say that "I use the term *line of inductive force* merely as a temporary conventional mode of expressing the direction of the power"[39], and it is clear that at this stage the idea of the polarity of the particles of matter described an existential condition of matter, whereas lines of force were imaginary and merely expressed the direction in which the condition was manifested, a line of contiguous particles under tension.

In FARADAY'S account of electrostatic induction, action at a distance was denied, the phenomena being produced by "the action of the contiguous particles"[40] of matter. The particles were polarized, and again he was quite clear as to what he meant by this, stating that "induction appears to consist in a certain polarized state of the particles, into which they are thrown by the electrified body sustaining the action, the particles assuming positive and negative points or parts"[41], and he spoke of this state as a "forced" state[42]. Thus, electrostatic action occurred by means of the polarization of the particles of the dielectric medium, and he again returned to the idea of the electrotonic state to describe the action of the particles.

There was one point, though, which caused FARADAY great difficulty; this was his explanation of the mode of transmission of the tension from one polarized particle to another. He argued that by contiguous particles he did not mean particles which touched one another but merely neighbouring particles. As he put it, he meant "those which are next to each other, not that there is *no* space between them"[43]. He considered that if there was a vacuum between contiguous particles there was no reason why the particles should not act across a distance of "half an inch"[44]. However, as ROBERT HARE argued, in an intervention which was crucial to FARADAY'S development, FARADAY'S whole theory involved a denial of action at what Faraday called "sensible distances", and HARE asked FARADAY "what is a sensible distance, if half an inch is not?"[45]. In his reply FARADAY stated that he had considered "*ordinary* induction" to be "an action of contiguous particles ... at insensible distances", and went on to argue by analogy that though induction across a vacuum was not an ordinary instance "yet I do not perceive that it cannot come under the same principles of action"[46].

There was a fundamental difficulty in FARADAY'S conception of contiguous action; for if action at a distance was denied it would not help to suppose that the distance between contiguous particles was insensible, for short-range forces — whose mode of action was undefined — would still have to be supposed as acting

[38] *Electricity*, **1**, par. 1224.
[39] *Electricity*, **1**, par. 1231.
[40] *Electricity*, **1**, par. 1224, and also par. 1295.
[41] *Electricity*, **1**, par. 1298.
[42] *Electricity*, **1**, par. 1298, 1671.
[43] *Electricity*, **1**, par. 1665n; see also par. 1615, 1164n.
[44] *Electricity*, **1**, par. 1616.
[45] R. HARE, "A Letter to Prof. Faraday, on certain Theoretical Opinions", *Phil. Mag.*, 1840, **27**, 45; *Electricity*, **2**, 252.
[46] *Electricity*, **2**, 267.

across the insensible distances. In his reply to HARE, FARADAY implicitly admitted that his theory implied this, for he argued that the same principles of action applied to induction across a vacuum as to induction across insensible distances. Thus, FARADAY had at once denied action at a distance and yet admitted the possibility of forces acting — under the same, but unspecified, principles of action — across sensible and insensible distances.

It is likely that it was the recognition of this basic difficulty in his theory of electric action that led FARADAY to alter completely his physical view of nature. His representation of electrostatic induction by means of action between contiguous particles across insensible distances was conceived in terms of a particulate theory of matter, but in his next discussion of the problem, in the "Speculation touching Electric Conduction and the Nature of Matter" of 1844, he adopted quite a different point of view. He began by pointing out that according to the atomic theory, as usually conceived, material atoms would not be in contact with one another. He now denied the possibility of forces acting across insensible distances, ascribing a role to the intermediate spaces between the atoms to account for the communication of electric action. Thus, the space between atoms was "taken as the only continuous part"[47], but this led to the absurdity that "space may be proved to be a non-conductor in non-conducting bodies, and a conductor in conducting bodies"[48] if the interatomic void played a role in the communication of forces. FARADAY concluded that the solution was to suppose matter as filling all space, and he argued that it was the system of powers and forces round the atomic centres of matter which endowed the atoms with their properties, stating that all knowledge of the atoms was limited to ideas of powers. He asserted that "the substance consists of the powers"[49], and supposed the "mutual penetrability of matter"[50], suggesting that "matter will be *continuous* throughout"[51] all space. This overcame HARE's problem, for by their forces the atoms would penetrate to "the very centres"[52] of force, and electric action could be explained without supposing particles acting across insensible distances.

Thus, FARADAY had abandoned the particulate theory of matter in favour of conceiving matter as forces diffused through space, and already in the "Thoughts on Ray-vibrations" (1846) he had begun to think that the replacement of the theory of action propagated by the polarization of the particles of matter by a theory of the interaction of forces indicated that electric action could be explained by means of lines of force[53]. In the years following the "Speculation" FARADAY's discoveries led him to develop the theory of the primacy of lines of force, which he conceived as entities independent of the particles of matter. This can be seen

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[47] *Electricity*, **2**, 286.

[48] *Electricity*, **2**, 287.

[49] *Electricity*, **2**, 290. Ideas of this kind on "powers" and "forces" can be found, for example, in JOSEPH PRIESTLEY, *Disquisitions Relating to Matter and Spirit* (2nd ed., 2 vols., Birmingham, 1782), **1**, 22, 27, 36; and THOMAS EXLEY, *Principles of Natural Philosophy* (London, 1829), 470, 474.

[50] *Electricity*, **2**, 292. A similar statement can be found in PRIESTLEY's *Disquisitions* (2nd ed., 1782), **1**, 26.

[51] *Electricity*, **2**, 291.

[52] *Electricity*, **2**, 292.

[53] *Electricity*, **3**, 447.

in his explanation of the action of magnets on crystals, the magnecrystallic force, where he argued that the crystal was aligned along the lines of magnetic force[54], and he questioned the existence of a state of polarity in the crystal. This trend of thought was continued in his work on diamagnetism, where he attempted to detect the polarity of diamagnetics, without success[55]. This led to a new emphasis in his ideas, for in 1838 the lines of force had been used to distinguish the direction of a chain of molecules under tension, the particles being supposed to be polarized. However, his failure to detect the polarity of diamagnetics led him to argue that the molecules of diamagnetics were not polarized, but that the diamagnetics merely interacted with the lines of force[56]. FARADAY's new view of the fundamental importance of lines of force as entities distinct from the particles of matter can be seen in his explanation of paramagnetism and diamagnetism in terms of the relative magnetic conductibility of the bodies and the surrounding medium, that is, in terms of the propensity of lines of force to pass through the bodies[57].

Thus, he no longer supposed the polarization of molecules, and in a paper of 1852 "On the Physical Character of the Lines of Force"[58] he argued that the lines of force had a real physical existence, though their nature remained unclear. In replacing the polarization of the particles of matter by the primacy of lines of force FARADAY was supposing that polarity did not exist as a state of matter. However, he continued to use the *term* polarity, speaking of "the polarity of each line of force"[59], but he used the term to represent the direction of the lines of force not the polarization of particles. As he said, "my view of polarity is founded upon the character in direction of the force itself"[60]. Again, he pointed out that electrostatic lines of force would terminate on charges, opposite charges being at opposite ends of each line, but this was not associated with the polarization of molecules of matter. He emphasized that "no condition of *quality* or *polarity* has as yet been discovered"[61] in the lines of electrostatic force. Thus, even though electrostatic lines of force had charged ends, there was no polarization within the lines. This was quite at variance with his abandoned theory of molecular polarization, where ontologically the lines of force represented lines of polarized particles, whereas he now regarded the lines of force as representing a physical state.

The lines of force were in the medium, for a magnet "could not exist without a surrounding medium or space"[62], but the nature of this medium remained unclear. Though he did speculate that lines of force were transmitted by an action

[54] *Electricity*, 3, par. 2479.

[55] *Electricity*, 3, par. 2640—2701.

[56] FARADAY rejected WEBER's theory of diamagnetism, which supposed that molecules of paramagnetic and diamagnetic substances could exhibit opposite polarites under the same conditions of excitation (*Wilhelm Webers Werke* (6 vols., Berlin, 1892—1894), 3, 255—268; *Scientific Memoirs*, 1852, 5, 477—488). See below for a discussion of WEBER's theory.

[57] *Electricity*, 3, par. 2806—2835.

[58] *Electricity*, 3, par. 3243—3299.

[59] *Electricity*, 3, par. 3361.

[60] *Electricity*, 3, par. 3307.

[61] *Electricity*, 3, par. 3249.

[62] *Electricity*, 3, par. 3361. See also par. 3277: "I conceive that when a magnet is in free space, there is such a medium (magnetically speaking) around it".

which was "a function of the aether"[63] he was sceptical as to the existence of the ether. For example, he remarked in the "Thoughts on Ray-vibrations" that "the view which I am so bold as to put forth considers, therefore, radiation as a high species of vibration in the lines of force ... It endeavours to dismiss the aether"[64], but the problem of what transmitted the lines of force remained. He endeavoured to extend the concept to account for gravitation, arguing that the lines of gravitational force gave rise to "a constant necessary condition to action in space, when as respects the sun the earth is *not* in place, and of a certain gravitating action as the result of that previous condition when the earth *is* in place". The lines of gravitational force spread out through space and "the power is always existing around the sun and through infinite space, whether secondary bodies be there to be acted upon by gravitation or not; and not only around the sun, but around every particle of matter which has existence"[65]. FARADAY's final theory was, then, a theory in which lines of force interacted with one another in space; a theory of the primacy of lines of force.

III

In his first attempt to formulate a theory of electricity and magnetism MAXWELL went straight to FARADAY's theory of lines of force. MAXWELL began working on this in 1854 and the tenor of his thought can be seen from a letter to WILLIAM THOMSON in which he spoke of FARADAY's use of magnetic lines of force, suggesting that "something might be done by considering 'magnetic polarization' as a property of a 'magnetic field' or space, and developing the geometrical ideas according to this view"[66]. The approach MAXWELL was following, then, was to consider the polarization in terms of the lines of force, and in the same letter he stated that "I use the word 'polarization' to express the fact that at a point of space the south pole of a small magnet is attracted in a certain direction with a certain force"[67]. In another letter he wrote that he had appropriated "Faraday's theory of polarity which ascribes that property to every portion of the whole sphere of action of the magnetic or electric bodies"[68]. Thus, the theory of "polarity" that MAXWELL was adopting was the theory as used by FARADAY

[63] *Electricity*, 3, 3075.

[64] *Electricity*, 3, 451.

[65] *Electricity*, 3, 574. One of FARADAY's criteria for the physical reality of the lines of force was that they should exhibit a limitation of action in space, and he applied this to the analysis of gravitational action. He argued that if the sun were considered as existing in space exerting no force of gravitation and then another similar sphere in space were to be brought towards it, this was to assume a creation of power, and "their dissociation ... would be equivalent to the annihilation of force" (*Electricity*, 3, 572). It was to avoid this difficulty, which he was to bring up again in his 1857 paper "On the Conservation of Force", *Experimental Researches in Chemistry and Physics* (London, 1859), 443—460, that led him to advance the idea that lines of gravitational force always existed diffused in space.

[66] MAXWELL to WILLIAM THOMSON, 13 November 1854, published in J. LARMOR (ed.), *The Origins of Clerk Maxwell's Electric Ideas* (Cambridge, 1937), 8. See note 23. The term "magnetic field" was first used by FARADAY in 1845, *Electricity* 3, par. 2252 and later by WILLIAM THOMSON, "On the Theory of Magnetic Induction", *Phil. Mag.*, 1851, 1, 179 (*Reprint of Papers on Electrostatics and Magnetism* (London, 1872), 467).

[67] LARMOR, *Origins of Clerk Maxwell's Electric Ideas*, 8.

[68] MAXWELL to WILLIAM THOMSON, 13 September 1855, *ibid.*, 17.

in his later period, to represent the direction of the force, and his use of the terms polarity and polarization did not signify a state of the molecules of matter. In "Faraday's Lines of Force" (1856)[69] he stated that "we might find a line passing through any point of space, such that it represents the direction of the force acting"[70], and he went on to say that a line drawn so as to coincide with the direction of this force was called a line of force, and "we might in the same way draw other lines of force, till we had filled all space with curves indicating by their direction that of the force at any assigned point"[71]. The lines of force filled space, and in an early draft of the paper he noted that "Faraday treats the distribution of forces in space as the primary phenomenon, and does not insist on any theory as to the nature of the centres of force round which these forces are generally but not always grouped"[72]. The lines of force themselves were the fundamental entities, having physical existence — though their physical nature remained undefined — and the particles of matter, the "centres of force", were not considered, and he stated that "we should thus obtain a geometrical model of the physical phenomena"[73]. By this he meant that while the direction of the forces were represented no theory as to the physical nature of the lines of force had been proposed; such a theory, so important for a consistent representation of the phenomena, remained a task for the future. However, though he was unwilling to speculate on the physical nature of the lines of force he did not regard them as fictitious entities, but as having physical existence in space.

Despite MAXWELL's adoption of FARADAY's theory of lines of force as the primary entities he did not accept FARADAY's idea of matter filling all space by its forces. Referring to FARADAY's 1844 "Speculation", he later wrote that to avoid action at a distance FARADAY "even speaks of the lines of force belonging to a body as in some sense part of itself"[74], but he went on to argue that this notion was "not a dominant idea with Faraday", and he represented FARADAY's theory of lines of force as stating that the field was full of lines of force whose arrangement depended on the bodies in the field[75]. For MAXWELL the lines of

[69] MAXWELL, "On Faraday's Lines of Force", *Trans. Camb. Phil. Soc.*, 1856, **10**, 27—83; *Papers*, **1**, 155—229.

[70] *Papers*, **1**, 158.

[71] *Ibid.*

[72] "On Faraday's Lines of Force", University Library Cambridge, Add. MSS. 7655, an early draft of the published paper.

[73] *Papers*, **1**, 158.

[74] *Treatise*, § 529.

[75] It is important to emphasize that FARADAY's concept of matter filling space continuously by its forces did not involve the *identification* of matter — or force — with space. He pointed out (in 1850) that in arguing that "the lines of magnetic force can traverse space" he did not wish "to confound space with matter", for "mere space cannot act as matter acts", *Electricity*, **3**, par. 2787, and he emphasized that "space therefore comports itself independently of matter", *ibid.*, par. 2789. MAXWELL too was concerned to emphasize this. In *Matter and Motion* (London, 1877), 12, he wrote that "Absolute space is conceived as remaining always similar to itself and immovable", and though "there is nothing to distinguish one part of space from another except its relation to the place of material bodies", nevertheless, as he remarked in a draft, he wished "to render distinct the idea of space as independent of matter" for the geometrical properties of space were independent of matter (draft "On Absolute Space", U.L.C. Add. MSS. 7655). Given this commitment, his comments

force were effects of matter, while for FARADAY in the "Speculation" there was no distinction between "matter" and his notion of its "forces" diffused through space. Thus, MAXWELL did not accept the concept of materiality as "force"[76]. In a manuscript he wrote that "some have thought it more philosophical to speak of ... [atoms] as centres of force without attributing to them any finite extension. This would be quite legitimate, provided each centre of force is admitted to have mass"[77].

The key problem in this mode of representation was in employing lines of force quantitatively, to express the forces. MAXWELL achieved this by considering the lines of force as forming tubes carrying an inertialess, incompressible fluid

on Riemann's "Ueber die Hypothesen, welche der Geometrie zu Grunde liegen", first published in 1868 — see *Gesammelte Mathematische Werke* (2 ed., Leipzig, 1892), 272—287, and translated by W. K. CLIFFORD, in *Nature*, 1873, **8**, 14—17, 36f. — in a letter of 11 November 1874 to P. G. TAIT, are extremely interesting. MAXWELL wrote that the aim of the "Riemannsche Idee" of curved space was "to make its curvature uniform everywhere, that is over the whole of space whether that space is more or less than ∞. The *direction* of the curvature is not related to one of the xyz more than another, or to $-x-y-z$ so that as far as I understand we are once more in a pathless sea, starless, mindless and poleless totus teres atque rotundus" (U.L.C. Add. MSS. 7655). MAXWELL here picked out the problem of the definition of co-ordinates in RIEMANN's paper, and his commitment to absolute space was such that he did not find in RIEMANN's remark that "we must seek the ground of ... [the] metric relations [of space] outside it, in binding forces which act upon it", *Nature*, 1873, **8**, 37, the geometric basis for a theory of lines of force. Thus, for MAXWELL, the lines of force were *in* space, and were not space itself. His Latin quotation is from HORACE, *Satires*, Bk. II, 7, line 86.

[76] MAXWELL's notion of "force" was quite different to FARADAY's ideas on the "forces" of matter. In a letter of 9 November 1857 MAXWELL commented on FARADAY's paper "On the Conservation of Force" (see note 65), remarking that "Force is the tendency of a body to pass from one place to another". This letter is at the Institution of Electrical Engineers, London (I am grateful to Mr J. E. WRIGHT, Librarian, for his help), and was printed in the second edition of CAMPBELL & GARNETT's *Life of Maxwell* (London, 1884), 202ff. FARADAY replied, pointing out that by the word "force" he meant "the *source* or *sources* of all possible actions of the particles or materials of the universe; these being often called the *powers* of nature when spoken of in respect of the different manners in which their effects are known", FARADAY to MAXWELL 13 November 1857, U.L.C. Add. MSS. 7655. This letter was printed in both the first and second editions of CAMPBELL & GARNETT's *Life of Maxwell* (1st ed., 288ff.; 2nd ed., 205f.). FARADAY's use of the words "force" and "power" was in line with a long tradition, exemplified by PRIESTLEY and EXLEY (see note 49 and my paper cited in note 27). In a note (1858) appended to his "Conservation of Force" paper FARADAY defined the notion of force as the tendency of a body to pass from one place to another as "mechanical force", his own meaning of force being "the *cause* of a physical action", *Experimental Researches in Chemistry and Physics* (London, 1859), 460. FARADAY's remarks in this note clearly reflect MAXWELL's point.

[77] MAXWELL, a draft "On the Dynamical Explanation of Electric Phenomena", U.L.C. Add. MSS. 7655. This was from an early draft of "A Dynamical Theory of the Electromagnetic Field" (1865). The reference here was to views such as those of BOSCOVICH. Elsewhere MAXWELL noted that BOSCOVICH "did not forget, however, to endow his mathematical points with inertia", "Action at a Distance", *Papers*, **2**, 317. WILLIAM WHEWELL remarked that "a collection of mere centers of force can have no inertia", *Philosophy of the Inductive Sciences* (2nd ed., 2 vols., London, 1847), **1**, 433n, using this as an argument against BOSCOVICH.

which was, as he put it, "merely a collection of imaginary properties ... in a way ... more applicable to physical problems than that in which algebraic symbols alone are used"[78]. The notion of the lines of force forming a tubular surface in fact was derived from FARADAY[79], and this enabled MAXWELL to obtain what he called a "geometrical model" which defined the motion of the fluid by dividing the space it occupied into tubes. The tubes were mere surfaces directing the motion of the fluid which filled all space, and the forces were represented by the motion of the fluid. This was not a physical representation of the lines of force, and the fluid was "not even a hypothetical fluid" but merely a collection of imaginary properties for the expression of mathematical theorems[80]. In this paper MAXWELL was limiting his theory "to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena", but his ultimate aim was a theory in which "physical facts will be physically explained"[81]. Thus, this mode of representation, the primacy of lines of force, did not provide a physical theory of the nature of the lines of force.

The geometrical model was conceived with reference to THOMSON'S representation of the analogy between heat flow and electrostatic action[82], for though, as MAXWELL remarked, "the two subjects will assume very different aspects" if their resemblance was pushed too far, "the mathematical resemblance of their laws will remain, and may still be made useful in exciting appropriate mathematical ideas"[83]. As he remarked in a draft of the paper, he had assumed a purely imaginary fluid because "while the mathematical laws of the conduction of heat derived from the idea of heat as a substance are admitted to be true, the theory of heat has been so modified that we can no longer apply to it the idea of substance"[84], an illustration of the importance MAXWELL attached to the clear distinction between mathematical and physical ideas[85].

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[78] MAXWELL, *Papers*, **1**, 160.

[79] FARADAY wrote that the "power about a magnet, which ... [was] worked in its direction by the lines of magnetic force, may be considered as disposed in sphondyloids, determined by the lines, or rather shells of force", *Electricity*, **3**, par. 3271. Thus, a magnet was surrounded by a "sphondyloid of power", *ibid.*, par. 3276. MAXWELL referred to FARADAY'S use of the word sphondyloid as defining a "tubular surface formed by a system of such lines [of force]", *Papers*, **1**, 192.

[80] *Papers*, **1**, 160.

[81] *Papers*, **1**, 159.

[82] WILLIAM THOMSON, "On the Uniform Motion of Heat in Homogeneous Solid Bodies, and its Connection with the Mathematical Theory of Electricity", *Cambridge Mathematical Journal*, 1842, **3**, 71—84. Reprinted in *Phil. Mag.* 1854, **7**, 502—515, and in W. THOMSON'S *Reprint of Papers on Electrostatics and Magnetism* (London, 1872), 1—14. In a letter of 13 September 1855 MAXWELL informed THOMSON that he had used THOMSON'S "allegorical representation of the case of electrified bodies by means of conductors of heat", *Origins of Clerk Maxwell's Electric Ideas*, 17.

[83] *Papers*, **1**, 157.

[84] "On Faraday's Lines of Force", U.L.C. Add. MSS. 7655, an early draft (see note 72). In the *Treatise* he emphasized that "while we derive great advantage from the recognition of the many analogies between the electric current and a current of material fluid, we must carefully avoid making any assumption not warranted by experimental evidence" (§ 574), for while a fluid was a substance heat was not a substance, and so "we must be careful not to let the one or the other analogy suggest to us that electricity is either a substance like water, or a state of agitation like heat" (§ 72). Thus despite the analogy between the conduction of heat and the con-

In this paper there was no suggestion of molecularity or particulate action. This can be seen from his treatment of electrostatics where positive and negative electricity was said to be on the surface of the dielectric, where the lines of force entered and emerged[86]. In the *Elementary Treatise*, where his representation was also based on the primacy of lines of force, he rejected this theory of electrostatics, but in "Faraday's Lines" he followed the representation of electrostatics of FARADAY's later period, where, though the ends of the lines of force terminated on charges, there was no condition of polarization within the tubes of force. This theory was quite at variance with FARADAY'S earlier theory of molecular polarization, where each line of force represented a line of polarized particles. MAXWELL's viewpoint in "Faraday's Lines" was one of the primacy of lines of force.

An important part of the paper was his discussion of paramagnetism and diamagnetism. This is particularly interesting because of the important role the study of diamagnetism had in leading FARADAY to assert the primacy of lines of force. MAXWELL discussed both FARADAY'S and WEBER'S theories of diamagnetism. WEBER had argued that there was a difference between the molecules of paramagnetic and diamagnetic substances, and his theory was based on AMPÈRE's theory that a magnet consisted of molecules within which electric currents circulated[87]. WEBER suggested that the molecules of diamagnetic substances did not have any electric currents circulating within them but that such currents were *induced* by a magnet. He assumed that "in the single molecules, or around them, closed paths exist in which the ... [electrical] fluids can move without resistance"[88]. According to WEBER, then, currents circulated within the molecules

duction of electricity, in which "flow of electricity corresponds to flow of heat" (§ 243) there were limitations to the analogy, because the term "flow" could not be applied in the same way to electricity as to the case of the transmission of heat. Thus, the "difference between the phenomena consists in the fact that bodies are capable of absorbing and emitting heat, whereas they have no corresponding property with respect to electricity" (§ 244). As he later put it, "a set of electrified bodies placed in a perfectly insulating medium might remain electrified for ever", *Elementary Treatise*, 53. For another example of the analogy see *Treatise*, § 331.

[85] THOMSON had discussed the mathematical treatment of lines of force in a paper "On the Mathematical Theory of Electricity in Equilibrium", *Cambridge and Dublin Mathematical Journal*, 1846, **1**, 75—95, which was reprinted in *Phil. Mag.*, 1854, **8**, 42—62, and in *Reprint of Papers in Electrostatics and Magnetism* (London, 1872), 15—37. The physical conception of lines in force in this paper — which was written in 1845 — was that of FARADAY's earlier period, for only after 1845 did FARADAY assert the primacy of lines of force, and abandon the theory of molecular polarization. Thus, THOMSON spoke of the propagation of electric action "by means of molecular action among the contiguous particles" of the dielectric, *Reprint of Papers on Electrostatics and Magnetism*, 26, and of the polarity of "every portion" of the dielectric *ibid.*, 32, though he argued that a "physical hypothesis" of the action should be avoided, *ibid.*, 29. THOMSON discussed the analogy between heat and electricity, *ibid.*, 28f, and went on to discuss FARADAY's notion of "curved lines of inductive action", *ibid.*, 30.

[86] MAXWELL, *Papers*, **1**, 177.

[87] See A. M. AMPÈRE, *Mémoires sur l'électrodynamique* (2 vols., Paris, 1885—1887), **1**, 140, 214, 404.

[88] WILHELM WEBER, "On the Connexion of Diamagnetism with Magnetism and Electricity", *Scientific Memoirs, Natural Philosophy*, (ed.) J. TYNDALL & W. FRANCIS (London, 1853), 166. First published as "Über den Zusammenhang der Lehre von Diamagnetismus mit der Lehre von dem Magnetismus", *Ann. Phys.*, 1852, **87**, 145—189; in *Wilhelm Webers Werke* (6 vols., Berlin, 1892—1894), **3**, 555—590.

of paramagnetic bodies but not within the molecules of diamagnetic bodies, and he argued that paramagnetic and diamagnetic bodies exhibited opposite polarities under the same conditions of induction, as the currents in the paramagnetic molecules circulated in the opposite direction to the induced currents in diamagnetic molecules[89]. Thus, polarity as a state of the molecules of matter was an essential feature of WEBER's theory, and for this reason the theory had been rejected by FARADAY.

Now, in 1855 — the year in which MAXWELL wrote the paper "On Faraday's Lines of Force" — the question of the polarity of diamagnetics had been the subject of TYNDALL's Bakerian lecture published that year[90], in which TYNDALL, while regarding WEBER's theory as so "artificial ... that the general conviction of its truth cannot be very strong"[91], had nevertheless concluded that "*the diamagnetic force is a polar force, the polarity of diamagnetic bodies being opposed to that of paramagnetic ones under the same conditions of excitement*"[92]. As TYNDALL made quite clear[93] his conclusion was in contradiction to the theory FARADAY was

[89] See WILHELM WEBER, "Über die Erregung und Wirkung des Diamagnetismus nach den Gesetzen inducirter Ströme, *Ann. Phys.*, 1848, **73**, 241—256 (*Werke*, **3**, 255—268); "On the Excitation and Action of Diamagnetism according to the Laws of Induced Currents", *Scientific Memoirs*, (ed.) R. TAYLOR (London, 1852), **5**, 477—488. WEBER argued that whereas the effects of diamagnetism could be explained either by AMPÈRE's molecular currents or by magnetic fluids, the *causes* of diamagnetism could only be explained by supposing the currents were *induced* on diamagnetizing the body. Because the induction effects of paramagnetics and diamagnetics were opposite, the currents in diamagnetics were the reverse of those in paramagnetics. Thus, the molecular currents — or magnetic fluids — could not have existed previously in the diamagnetic bodies, for under the same conditions of induction a magnet could not align existing currents or fluids in paramagnetics one way and those in diamagnetics the other. The currents already existed in the molecules of paramagnetics but were induced in the molecules of diamagnetics, *ibid.*, 486, and the difference between paramagnetics and diamagnetics arose because a magnetic force "tends to give such a direction to an *existing current* that its course is exactly opposed to that of a *current induced* by" the magnetic force, *ibid.*, 488.

[90] JOHN TYNDALL, "On the Nature of the Force by which bodies are repelled from the poles of a magnet", *Phil. Trans.*, 1855, **145**, 1—51. Reprinted in TYNDALL, *Researches on Diamagnetism and Magnecrystallic Action, including the question of Diamagnetic Polarity* (London, 1870), 89—153.

[91] TYNDALL, *Researches on Diamagnetism*, 138.

[92] *Ibid.*, 135. In a later paper "Further Researches on the Polarity of the Diamagnetic Force", *Phil. Trans.*, 1856, **146**, 237—259, he stated that "diamagnetic polarity ... [may be considered] among the most firmly established truths of science", *Researches on Diamagnetism*, 179. He went on to discuss magnecrystallic action in a paper "On the Relation of Diamagnetic Polarity to Magne-crystallic Action", *Phil. Mag.*, 1856, **11**, 125—137, arguing that by assuming diamagnetic polarity magnecrystallic action could be explained, and so "the whole domain of magnecrystallic action is thus transferred from a region of mechanical enigmas to one in which our knowledge is as clear and sure as it is regarding the most elementary phenomena of magnetic action", *Researches on Diamagnetism*, 198.

[93] TYNDALL pointed out that his conclusions indicated that FARADAY's first theory of diamagnetism — based on polarity — was the true one, *Researches on Diamagnetism*, 137. FARADAY had at first concluded that "an explanation of the movements of the diamagnetic bodies, and all the dynamic phenomena consequent upon the actions of magnets upon them, might be offered in the supposition that magnetic induction caused in them a contrary state to that which it produced in magnetic matter", *Electricity*, **3**, par. 2429.

proposing at that time. In fact, FARADAY had subjected the whole question to a renewed discussion in his "Some Points of Magnetic Philosophy" which appeared in the *Philosophical Magazine* in 1855[94], and the problems of polarity and FARADAY'S theory of magnetism as the propensity of a body to conduct lines of force relative to the surrounding medium were discussed in a series of letters between TYNDALL, FARADAY, THOMSON, and WEBER, which were published in the *Philosophical Magazine*[95].

MAXWELL'S interest in this problem can be seen from letters to THOMSON of this period[96], and in his paper he expressed a clear preference for FARADAY'S theory though without committing himself definitely to it, noting that "as the theory of lines of force admits of the most precise, and at the same time least theoretic statement we shall allow it to stand for the present"[97]. Since he based his theory on lines of force, it was "less theoretic" to maintain this approach rather than to introduce new concepts. Thus, his representation of magnetism is quite in accordance with the conceptual picture he adopted in this paper, the primacy of lines of force, and his failure to commit himself firmly on the question of magnetism was only due, as he emphasized, to his belief that insufficient experiments had been performed to render any theory more than hypothetical.

The second part of "Faraday's Lines" was given over to a treatment of FARADAY'S concept of the electrotonic state. This concept was used by FARADAY to represent a condition of electrical tension of matter, and was associated with his earlier physical viewpoint of particulate polarization. However, FARADAY regarded the electrotonic state as an alternative to lines of force, and MAXWELL referred to this both in "Faraday's Lines"[98] and in the *Treatise*, where he remarked that FARADAY found he could dispense with the electrotonic state by a method

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[94] FARADAY, "On some Points of Magnetic Philosophy", *Phil. Mag.*, 1855, **9**, 81—113 (*Electricity*, **3**, par. 3300—3362).

[95] TYNDALL, "On the Existence of a Magnetic Medium in Space", *Phil. Mag.*, 1855, **9**, 205—209; FARADAY, "Magnetic Remarks", *ibid.*, 253—255; THOMSON, "Observations on the 'Magnetic Medium' and on the Effects of Compression", *ibid.*, 290—293; WEBER, "On the Theory of Diamagnetism", *ibid.*, 1855, **10**, 407—410. These letters were reprinted in TYNDALL, *Researches on Diamagnetism*, 213—229. TYNDALL and FARADAY re-stated their representations of diamagnetism, and THOMSON supported FARADAY. WEBER defended his theory of diamagnetic polarity against TYNDALL'S remark as to its artificiality by claiming that his theory was not an "arbitrary assumption", but was "a necessary conclusion from the theory of Ampère", *ibid.*, 227.

[96] In a letter to THOMSON of 17 December 1856 MAXWELL remarked that "Tyndall's paper on Diamagnetism is satisfactory", *Origins of Clerk Maxwell's Electric Ideas*, 30, presumably referring to TYNDALL'S papers of that year (see note 92). In a letter of 22 February 1856 he asked "Do you think my paper on Faraday's lines too long for the *Phil. Mag.*? I would like to put it in because Faraday reads it and so does Tyndall", *ibid.*, 25, and he may well have wanted TYNDALL to see his treatment of diamagnetism. In a letter of the same date to G. G. STOKES he asked the same question as to the suitability of his paper for the *Phil. Mag.*, and again said that "I want to get Tyndall and Faraday to read it", J. LARMOR (ed.), *Memoir and Scientific Correspondence of the Late Sir George Gabriel Stokes, Bart.* (2 vols., Cambridge, 1907), **2**, 4.

[97] MAXWELL, *Papers*, **1**, 180.

[98] MAXWELL, *Papers*, **1**, 188, referring to FARADAY, *Electricity*, **3**, par. 3269.

"which, in FARADAY'S hands, was far more powerful"[99]: "by means of considerations founded on the lines of magnetic force"[100]. Thus, MAXWELL considered the electrotonic state as an alternative to lines of force; as he put it in "Faraday's Lines", the electrotonic state was "measured by the number of these lines"[101]. However, he regarded the electrotonic state as expressing a "physical truth" about a "state of bodies"[102], but pointed out that he used the electrotonic state because "it reduces to one principle not only the attraction of currents ... but also the attraction of electrified bodies without any new assumption"[103]. The electrotonic state enabled the consideration of the number of lines of force passing through a point to be avoided in favour of determining the electrical condition at the point by means of functions at the point. The electrotonic state provided "the means of avoiding the consideration of the quantity of the magnetic induction [that is, the number of lines of force] which *passes through* the circuit. Instead of this artificial method we have the natural one of considering the current with reference to quantities existing in the same space with the current itself"[104]. Nevertheless, though he made it clear that the electrotonic state expressed a physical truth, he emphasized that his representation of the electrotonic state "involves no physical theory, it is only a kind of artificial notation"[105]. From these remarks it seems that in "Faraday's Lines" MAXWELL employed the electrotonic state merely as a means of providing a mathematical representation of the lines of force, and that he did not replace the lines of force by the electrotonic state. Thus, the lines of force were the fundamental physical entities, and by-passing them in this way by the electrotonic state did not imply the abandonment of his viewpoint of the primacy of lines of force.

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[99] MAXWELL, *Treatise*, § 541.

[100] MAXWELL, *Treatise*, § 540.

[101] *Papers*, **1**, 187. The way in which the electrotonic state was equivalent to representation by the lines of force can be seen from a statement later in the paper where he argued that the "electro-tonic intensity ... measures ... the number of lines of magnetic force", *ibid.*, 206.

[102] *Ibid.*, 187.

[103] MAXWELL to THOMSON, 13 September 1855, *Origins of Clerk Maxwell's Electric Ideas*, 18. In "Faraday's Lines" MAXWELL stated that "the recognition of certain mathematical functions as expressing the 'electro-tonic state' of Faraday ... is, as far as I am aware, original; but the distinct conception of the possibility of the mathematical expressions arose in my mind from the perusal of Prof. W. Thomson's papers", *Papers*, **1**, 209, and one of the papers mentioned was THOMSON'S "Mathematical Theory of Magnetism", *Phil. Trans.*, 1851, **141**, 243—285 (*Reprint of Papers on Electrostatics and Magnetism*, 341—404), and in particular MAXWELL mentioned Articles 78ff., where THOMSON introduced quantities F, G, H which "are three functions to a certain extent arbitrary", *Reprint of Papers*, 402. He defined F, G, H in terms of "$\alpha, \beta, \gamma, \ldots$ the components of the intensity of magnetization", *ibid.*, 384, writing $\alpha = \frac{dH}{dy} - \frac{dG}{dz}$, $\beta = \frac{dF}{dz} - \frac{dH}{dx}$, $\gamma = \frac{dG}{dx} - \frac{dF}{dy}$, *ibid.*, 402. MAXWELL was to use this method in his later work: see *Papers*, **1**, 476, 556; and *Treatise*, § 591 (see note 122).

[104] MAXWELL, *Papers*, **1**, 203.

[105] *Ibid.*, 205.

IV

The programme of MAXWELL'S second paper on electromagnetism, "On Physical Lines of Force" (1861/62)[106], was foreshadowed in "Faraday's Lines" when he declared his intention of achieving a "mechanical conception"[107] of the electrotonic state. The physical nature of the lines of force remained undefined in "Faraday's Lines", but his attempt to develop a physical explanation of the nature of lines of force in "Physical Lines" was to lead him to a radical alteration in his conceptual viewpoint. In 1857 he told FARADAY that the extension of FARADAY'S theory involved "questions relating to the connexion between magneto-electricity and certain mechanical effects which seem to me to be opening up quite a new road to the establishment of principles in electricity and a possible confirmation of the physical nature of magnetic lines of force. Professor W. Thomson seems to have some new lights on this subject"[108]. He was referring to a recently published paper of THOMSON'S in which FARADAY'S discovery of the rotation of the plane of polarization of polarized light by magnets[109] was explained by a theory of magnetism as the rotation of molecular vortices in a fluid ether[110].

[106] MAXWELL, "On Physical Lines of Force", *Phil. Mag.*, 1861, **21**, 161—175, 281—291, 338—348; *ibid.*, 1862, **23**, 12—24, 85—95 (*Papers*, **1**, 451—513).

[107] MAXWELL wrote that "by a careful study of the laws of elastic solids and of the motions of viscous fluids, I hope to discover a method of forming a mechanical conception of this electro-tonic state adapted to general reasoning", *Papers*, **1**, 188, referring to THOMSON'S paper "On a Mechanical Representation of Electric, Magnetic, and Galvanic Forces", *Cambridge and Dublin Math. Journal*, 1847, **2**, 61—64; *Mathematical and Physical Papers* (6 vols., Cambridge, 1882—1912), **1**, 76—80. The fact that THOMSON had attempted to represent electric and magnetic forces by means of a mechanical medium clearly had an effect on MAXWELL, in his attempt to represent the nature of the lines of force, as he indicated in "Physical Lines"(*Papers*, **1**, 453). When THOMSON was working on this paper he wrote a remarkable letter to FARADAY, which could well have served to set out MAXWELL'S own programme in "Physical Lines". THOMSON wrote that "I enclose the paper which I mentioned to you as giving an analogy for electric and magnetic forces, by means of the *strain* propagated through an elastic solid. What I have written is merely a sketch of the mathematical analogy. I did not venture event to hint at the possibility of making it the foundation of a physical theory of the propagation of electric and magnetic forces, which, if established at all, would express as a necessary result, the connection between electrical and magnetic forces, and would show how the purely *statical* phenomenon even of magnetism may originate either from electricity in motion, or from an inert mass such as a magnet. If such a theory could be discovered, it would also, when taken in connection with the undulatory theory of light, in all probability explain the effect of magnetism on polarized light", THOMSON to FARADAY 11 June 1847, letter at the Institution of Electrical Engineers, London. In S. THOMPSON, *The Life of William Thomson* (2 vols., London, 1910), **1**, 203f.

[108] MAXWELL to Faraday, 9 November 1857, letter at the Institution of Electrical Engineers (see note 76).

[109] FARADAY, *Electricity*, **3**, par. 2146—2187, 2221—2229.

[110] W. THOMSON. "Dynamical Illustrations of the Magnetic and Helicoidal Rotary Effects of Transparent Bodies on Polarized Light", *Phil. Mag.*, 1857, **13**, 198—204. MAXWELL wrote to inform THOMSON about "Physical Lines" on 10 December 1861, stating that "I have been trying to develop the dynamical theory of magnetism as an affection of the whole magnetic field according to the views stated by you", referring to this paper in the *Phil. Mag.*, *Origins of Clerk Maxwell's Electric Ideas*, 34. It was this paper of THOMSON'S rather than the 1847 paper on the strain of an elastic

The requirement of formulating a physical theory of the nature of the lines of force led MAXWELL to a re-assessment of the place of lines of force in his theory, for in "Physical Lines" he supposed that in a magnetic field the medium was in rotation about the lines of magnetic force, the rotation being performed by molecular vortices having their axes parallel to the lines of force. The magnitude of the magnetic force at each point was equal to the velocity of the outermost portion of the vortices, and its direction to that of the axis of the vortex[111]. Thus, in "Physical Lines" MAXWELL did not refrain from speculating as to the nature of the lines of force; but by the introduction of the vortical mechanism to explain their nature, the lines of force had lost their primacy. Though MAXWELL spoke of the "polarity" of the lines of force[112] there was no explicit suggestion that he meant more by this than he had meant in "Faraday's Lines" in using the same term: that is, to express the direction of the lines of force. However, by introducing the vortices to provide a physical representation of the lines of force he had not only departed from the geometrical model and the primacy of lines of force of "Faraday's Lines", but he had also begun to move away from the concept of polarity of that paper. He stated that the vortices possessed polarity, and the "polarity" of the lines of force was represented by means of the polarity of the vortices which constituted them. Since each of these possessed polarity, each element of the line had acquired a separate polarity. There was no suggestion of this in "Faraday's Lines", where there was no condition of polarity *within* the lines of force. Thus, he had taken a step towards the abandonment of the concept of polarity of "Faraday's Lines", for polarity now implicitly expressed a state of the lines of force, not merely their direction. However, in this first part of the paper, published in the March 1861 issue of the *Philosophical Magazine*[113], MAXWELL adhered to FARADAY's theory of paramagnetism and diamagnetism[114]. Lines of force were therefore no longer the fundamental entities, but in seeking to explain the physical nature of the lines of force by the hypothesis of molecular vortices MAXWELL had done more than

solid (see note 107) which was to provide MAXWELL with a crucial physical model, though he did refer to the 1847 paper in "Physical Lines" (*Papers*, **1**, 453). THOMSON argued that "the magnetic influence on light discovered by Faraday depends on the direction of motion of moving particles", *Phil. Mag.*, 1857, **13**, 199, and he concluded that magnetism possessed a rotatory character, referring to "Mr. Rankine's hypothesis of 'molecular vortices'", *ibid.* RANKINE had suggested a vortical atomic model to explain heat effects, the planar vortices possessing circular motion: W. J. M. RANKINE, "On the Centrifugal Theory of Elasticity and Applied to Gases and Vapours", *Phil. Mag.*, 1851, **2**, 509—542, and "On the Mechanical Action of Heat, especially in Gases and Vapours", *Transactions of the Royal Society of Edinburgh*, 1853, **20**, 147—190.

[111] MAXWELL, *Papers*, **1**, 467f.

[112] *Ibid.*, 454f. MAXWELL spoke of the lines of force as being "dipolar". In the *Treatise* he distinguished "dipolarity" from "unipolarity", the latter being the polarity associated with the polarization of particles (§ 381 and footnote). Thus, he did not associate "the dipolar character of the line of force", *Papers*, **1**, 455, with molecular polarization. I will argue that MAXWELL's thought was to undergo a transformation *during* the composition of "Physical Lines", and this can be seen from the fact that in the first part of this paper he stated that "every vortex is essentially dipolar", *ibid.*, not, at this stage, *explicitly* moving towards a concept of polarization in accord with FARADAY's early view (see below).

[113] MAXWELL, *Phil. Mag.*, 1861, **21**, 161—175 (*Papers*, **1**, 451—466).

[114] *Papers*, **1**, 461.

abandon the primacy of lines of force; he had begun to move away from the concept of polarity associated with the lines of force model, and this was to lead him to a new conceptual viewpoint.

In the second part of the paper, which was published in the April and May numbers of the *Philosophical Magazine*[115], he attempted to explain the transmission of rotation in the same direction from vortex to vortex, and the occurrence of electric currents. He suggested that contiguous vortices were separated by a layer of spherical particles each revolving on its own axis in the opposite direction to the neighbouring vortices[116]. This was the hypothesis of "idle wheels", and if adjacent vortices were not revolving at the same rate the idle wheel particles would acquire a translatory motion; the flow of these particles constituted the electric current, and the tangential pressures resulting corresponded to the electromotive force. As MAXWELL made clear this model was "provisional"[117] and "may appear somewhat awkward. I do not bring it forward as a mode of connexion existing in nature, or even as that which I would assent to as an electrical hypothesis"[118], but he suggested it merely for heuristic purposes. On the other hand, he considered that the theory of molecular vortices in the first part of the paper was probably true[119].

The way in which the abandonment of the primacy of lines of force and the replacement of this model by a mechanical representation of the lines by the molecular vortices affected MAXWELL'S conceptual framework can be seen in his treatment of the electrotonic state in this part of the paper, for here he defined the electrotonic state in terms of the motion of the vortices. Having explained the electromotive force in terms of the forces exerted by the vortices on the particles between them[120] he defined the electromotive force as the time rate of change of the electrotonic state[121]; the magnetic force was also expressed in terms of the

[115] *Phil. Mag.*, 1861, **21**, 281—291, 338—348 (*Papers*, **1**, 467—488).

[116] *Papers*, **1**, 468.

[117] *Ibid.*

[118] *Ibid.*, 486.

[119] MAXWELL stated that he wished to separate "by way of provisional answer" his explanation of the method of rotation of the vortices, from "the mechanical deductions" which resolved the question of condition of the medium by supposing a state of stress in the medium "and the hypothesis of vortices which gave a probable answer" to the question of the cause of this stress, *ibid.*, 468.

[120] *Ibid.*, 475. The velocity of the vortices was given by α, β, γ (the magnetic force), and the electromotive force P, Q, R was defined by equations $\frac{dQ}{dz}-\frac{dR}{dy}=\frac{\mu\, d\alpha}{dt}$, $\frac{dR}{dx}-\frac{dP}{dz}=\frac{\mu\, d\beta}{dt}$, and $\frac{dP}{dy}-\frac{dQ}{dx}=\frac{\mu\, d\gamma}{dt}$, where μ was the magnetic inductive capacity, to be defined in the *Treatise* by $\boldsymbol{B}=\mu \boldsymbol{H}$ (§ 614), where $\boldsymbol{B}$ was the magnetic induction and $\boldsymbol{H}$ the magnetic force (in component notation: α, β, γ). In vector notation, MAXWELL was writing the relation $\dot{\boldsymbol{B}}=\text{curl}\,\boldsymbol{E}$, that is, "Faraday's Law", which relates the time rate of change of magnetic induction ($\boldsymbol{B}$) to the electromotive force ($\boldsymbol{E}$).

[121] The electromotive force P, Q, R was defined in terms of quantities F, G, H: $P=\frac{dF}{dt}$, $Q=\frac{dG}{dt}$, $R=\frac{dH}{dt}$. MAXWELL defined "the quantities, F, G, H as the resolved parts of that which Faraday has conjectured to exist, and has called the *electrotonic state*", *ibid.*, 476. MAXWELL clearly derived these symbols from THOMSON'S 1851 paper, and he was to use them (see note 122) in a similar way to THOMSON (see note 103). In vector notation, MAXWELL was writing $\boldsymbol{E}=\dot{\boldsymbol{A}}$, where $\boldsymbol{A}$ is the vector potential (electrotonic state).

electrotonic state[122]. The electrotonic state had quite a different role in "Physical Lines" than in "Faraday's Lines" where the lines of force were the fundamental entities. In "Physical Lines" MAXWELL regarded the molecular vortices as the real entities, and these determined the nature of the lines of force. In expressing the relations for the magnetic and electromotive forces in terms of the electrotonic state MAXWELL related the electrotonic state to the molecular vortices, for the magnetic and electromotive forces were themselves defined with respect to the molecular vortices.

The radical nature of this change in conceptual viewpoint can be seen from the third part of the paper, concerned with electrostatics, which was published in January 1862[123]. From his correspondence it is clear that this paper was not complete until the autumn of 1861[124], about six months after Part II appeared in print. In his theory of electrostatics he proposed a different model of the ether. Instead of a hydrodynamic model he used a model of an elastic solid in which the ethereal substance formed spherical cells endowed with the property of elasticity, the cells being separated by electric particles which by their action on the elastic substance of the cells would distort the cells. Thus, the effect of an electromotive force was to distort the cells by a change in position of the electric particles, and call into play an elastic force. As he put it:

> "According to our hypothesis, the magnetic medium is divided into cells, separated by partitions formed of a stratum of particles which play the part of electricity. When the electric particles are urged in any direction, they will, by their tangential action on the elastic substance of the cells, distort each cell, and call into play an equal and opposite force arising from the elasticity of the cells. When this force is removed the cells will recover their form, and the electricity will return to its former position"[125].

MAXWELL pictured the distortion of the cell as a displacement of electricity within each molecule in a given direction, the effect over the whole dielectric being to produce "a general displacement of the electricity in a given direction"[126]. This then was the origin of the central concept of displacement, and it had a crucial significance on his mode of representation, for as he did not fail to emphasize, "electromotive force acting on a dielectric produces a state of polarization of its parts similar in distribution to the polarity of the particles of iron under the influence of a magnet, and, like the magnetic polarization, capable of being

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[122] MAXWELL defined the magnetic force α, β, γ by equations $\frac{dG}{dz} - \frac{dH}{dy} = \mu\alpha$, $\frac{dH}{dx} - \frac{dF}{dz} = \mu\beta$, and $\frac{dF}{dy} - \frac{dG}{dx} = \mu\gamma$, *ibid.* In vector notation, $\boldsymbol{B} = \operatorname{curl} \boldsymbol{A}$, where $\boldsymbol{B} = \mu\boldsymbol{H}$ (see note 120).

[123] MAXWELL, *Phil. Mag.*, 1862, **23**, 12—24 (*Papers*, **1**, 489—502).

[124] MAXWELL wrote to Faraday on 19 October 1861 (letter at the Institution of Electrical Engineers), CAMPBELL & GARNETT, (*Life of Maxwell* (2nd ed., 1884), 243—245, and to THOMSON on 10 December 1861, *Origins of Clerk Maxwell's Electric Ideas*, 34f., informing them that he had worked out this part of the paper in the country, that is, during the summer, which he spent at Glenlair.

[125] MAXWELL, *Papers*, **1**, 492. Displacement was conceived in terms of the change of position of the electric particles.

[126] *Ibid.*, 491.

described as a state in which every particle has its poles in opposite conditions"[127]. Thus, displacement was introduced as a model for dielectric polarization[128], and MAXWELL had now moved quite explicitly to a view in which polarization was described as a state in which opposite parts of a particle were in opposite conditions. Therefore in Part III of the paper he moved even further away from FARADAY's theory of the primacy of lines of force; not only did he represent lines of force by means of molecular vortices, but now he fully accepted a theory of molecular polarization, a theory quite contradictory to the viewpoint of "Faraday's Lines". Having begun "Physical Lines" with an attempt to enrich the model of "Faraday's Lines" by explaining the physical nature of the lines of force, with this complete change in his theory of the nature of dielectric polarization he had completely abandoned the viewpoint of the earlier paper. In attempting to achieve a consistent representation by a mechanical model he had returned to FARADAY's abandoned position. His basic framework was now particulate, and the fundamental concepts in "Physical Lines" were the molecular vortices and displacement, conceived as a polarization of the molecules of the dielectric. Polarization was defined quite unambiguously, here again relating to FARADAY's earlier thought.

It was in this part of the paper that MAXWELL achieved his first derivation of the theory of light, a derivation in which his mechanical model of the ether and his relation between electromotive force and displacement played a vital role[129]. He assumed the ether was an elastic solid, and calculated the velocity of propagation of transverse disturbances through it, finding that the value agreed with the value for the velocity of light. He wrote that he could "scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena*"[130].

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[127] *Ibid.* In vector notation, MAXWELL gave the relation between the electromotive force $\boldsymbol{E}$ and the displacement $\boldsymbol{D}$, as $\boldsymbol{E} = -4\pi E^2 \boldsymbol{D}$ where E is a constant varying with the nature of the dielectric.

[128] This has been noted by PIERRE DUHEM, *Les théories électriques de J. Clerk Maxwell* (Paris, 1902), 110—113, and by JOAN BROMBERG, "Maxwell's Displacement Current and His Theory of Light", *Arch. Hist. Ex. Sci.*, 1968, **4**, 220.

[129] In his derivation of a wave-equation the key role was the double interpretation of the equation $\boldsymbol{E} = -4\pi E^2 \boldsymbol{D}$ as an equation of elasticity and as an electrical equation. Thus, $\boldsymbol{E}$ is an electromotive force in the direction of the displacement, and an elastic restoring force opposite to it. This dual meaning was related to his physical model, and when he abandoned the mechanical ether model in "Dynamical Theory" the elastic restoring force did not appear and the negative sign in the equation — which he had been able to retain by an error in one of his proofs — disappeared. See JOAN BROMBERG, *op. cit.* (note 128), 218—234.

[130] MAXWELL, *Papers*, **1**, 500. MAXWELL formulated his theory of light without being aware of a paper by W. WEBER & R. KOHLRAUSCH, "Electrodynamische Maasbestimmungen, insbesondere Zurückführung der Stromintensitätsmessungen auf mechanisches Maass" (1857), *Wilhelm Webers Werke*, **3**, 609—676, in which the ratio between the electrostatic and electrodynamic units of charge was determined. This ratio has the dimensions of a velocity, the velocity of propagation of electric action. WEBER's constant referred to electrodynamic rather than electromagnetic units, and so his ratio came out as $\sqrt{2}\times$the velocity of light. That MAXWELL was unaware of this paper is clear from a letter to FARADAY of 19 October 1861 in which he stated that "I have determined the velocity of propagation of transverse vibrations ... The coincidence [between velocities] is not merely numerical. I worked out the

MAXWELL gave FARADAY an account of his concept of molecular polarization and the distortion of the cells of the medium in a letter in October 1861, written before the paper was published, and informed him that "I think I have been able to get hold of some of your ideas, such as the electrotonic state, action of contiguous parts, etc., and my chief object in writing to you is to ascertain if I have got the same ideas which led you to see your way into things, or whether I have no right to call my notions by your names"[131]. This letter seems to indicate that MAXWELL wished to test FARADAY'S reaction to his return to FARADAY'S abandoned conceptual viewpoint. In the letter there was no mention of lines of force; the electrotonic state and the action of contiguous parts were listed as being concepts which MAXWELL had employed and derived from FARADAY. FARADAY'S reaction remains unknown.

In the same letter he told FARADAY that he had not found "any determination of the rotation of the plane of polarization by magnetism in which the absolute intensity of magnetism at the place of the transparent body was given. I hope to find such a statement by searching in libraries, but perhaps you may be able to put me on the right track". FARADAY pencilled the name "Verdet" to this, for VERDET had shown that the rotation was proportional to the magnetic force[132], and this problem was considered in detail by MAXWELL in the fourth part of his paper, published in the February 1862 issue of the *Philosophical Magazine*[133]. A full account of all this — much as it appeared in the published paper — was given by MAXWELL in a letter to THOMSON in December 1861[134], and both in this letter and in the published paper an even more important result of VERDET'S was discussed, his discovery that paramagnetic and diamagnetic substances rotated the plane of polarization of polarized light in opposite directions[135]. MAXWELL

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formulae in the country before seeing Weber's number ... and I think we have now strong reason to believe, whether my theory is a fact or not, that the luminiferous and electromagnetic medium are one" (see note 124). This statement, and the way in which his wave-equation was derived from the model, clearly show the unexpectedness of the result. See also a letter to Thomson of 10 December 1861 where he repeated this statement: "I made out the equations in the country before I had any suspicion of the nearness between the two values of the velocity of propagation of magnetic effects and that of light" (see note 124). For a discussion of WEBER'S ideas on this question see K. H. WIEDERKEHR, *Wilhelm Eduard Weber: Erforscher der Wellenbewegung und der Elektrizität* (Stuttgart, 1967), 140f. The first indication of the numerical equivalence of the velocities of propagation of light and electricity was by G. KIRCHHOFF in 1857, in a paper translated as "On the Motion of Electricity in Wires", *Phil. Mag.*, 1857, **13**, 393—412. See KIRCHHOFF, *Gesammelte Abhandlungen* (Leipzig, 1882), 131—154. MAXWELL seems to have been unaware of this paper, in which little significance was attached to the numerical equivalence. WEBER too did not consider KIRCHHOFF'S result as significant (*Werke*, **4**, 157).

131 MAXWELL to FARADAY, 19 October 1861. (See note 124).

132 ÉMILE VERDET, "Recherches sur les propriétés optiques développées dans les corps transparents par l'action du magnétisme", *Annales de Chimie et de Physique*, 1854, **41**, 370—412; *ibid.*, 1855, **43**, 37—44.

133 MAXWELL, *Phil. Mag.*, 1862, **23**, 85—95. (*Papers*, **1**, 502—513).

134 MAXWELL to THOMSON, 10 December 1861 (see note 124).

135 VERDET, "Note sur les propriétés optiques des corps transparents soumis à l'action du magnétisme", *Comptes Rendus*, 1856, **43**, 529—532; "Note sur les propriétés optiques des corps magnétiques", *ibid.*, 1857, **44**, 1209—1213.

argued that "we must admit the diamagnetic state to be the *opposite* of the paramagnetic", for the vortices "revolve in the opposite direction". He concluded that "this result agrees so far with that part of the theory of M. Weber which refers to the paramagnetic and diamagnetic conditions", but he noted that this result did not "require us to admit either M. Weber's theory of the mutual action of electric particles in motion, or our theory of cells and cell-walls"[136]. Nevertheless, despite his caution over WEBER's theory of molecular electric currents he had clearly abandoned FARADAY's theory of magnetism, and he remarked that the "behaviour [of iron] may be explained on our hypothesis of molecular vortices, by supposing that the particles of the *iron itself* are set in rotation by the tangential action of the vortices"[137].

The results of this part of the paper were to confirm him in his belief that the hypothesis of molecular vortices was true, for magnetic phenomena were explained in terms of the rotation of the vortices, and he concluded that "other phenomena in nature seem to lead to the conclusion that all substances are made up of a number of parts, finite in size, the particles composing these parts being themselves capable of internal motion"[138]. In the second part of the paper he had speculated that "the size of the vortices is indeterminate, but is probably very small as compared with that of a complete molecule of ordinary matter"[139], but he did not follow this up by proposing a theory of the constitution of molecules. The rotation of the plane of polarization of light by magnets might require an hypothesis of molecular vortices, but the nature of the vortices remained unknown. As he said in his lecture on "Molecules" in 1873, science was not "debarred from studying the internal mechanism of a molecule"[140], but it was clear — as he noted in the *Treatise* — that a satisfactory theory of the magnetic action on light required knowledge of "something more definite about the properties which must be attributed to a molecule"[141].

In Part I of the paper he had adopted FARADAY's theory of magnetism while in Part IV he had abandoned FARADAY's theory and agreed with WEBER that paramagnetic and diamagnetic phenomena were due to opposite states, though he did not adopt WEBER's theory of molecular currents. This inconsistency was due to the time-span in which the paper was written, in which he moved further away from the position of "Faraday's Lines" towards FARADAY's abandoned

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136 MAXWELL, *Papers*, **1**, 507.

137 *Ibid.*

138 *Ibid.*, 508.

139 *Ibid.*, 485.

140 *Ibid.*, **2**, 376.

141 MAXWELL, *Treatise*, § 830. MAXWELL did speculate further at this time on the problem of the size of atoms and vortices. Thus in the letter to FARADAY of 19 October 1861 (see note 124) he argued that the rotation of the plane of polarization was "proportional to the *diameter* of the vortices", and he made a similar suggestion in the published paper, *Papers*, **1**, 506f., though as he admitted to THOMSON in the letter of 10 December 1861 (see note 124), this was "not yet capable of proof". In another letter to Thomson, of 17 December 1861, he stated that "I shall be glad to know the max^{m} breadth of atoms", and he referred to experiments to determine "the maximum breadth of a vortex of magnetism", *Origins of Clerk Maxwell's Electric Ideas*, 39. He had made a similar remark about such experiments in the letter to FARADAY mentioned above.

viewpoint of molecular polarization. WEBER'S theory of magnetism, which involved molecular polarization, was in accordance with the theory of polarization in Part III of MAXWELL'S paper, in which the molecules of the dielectric were said to be polarized. MAXWELL emphasized the implications of VERDET'S results for FARADAY'S theory of magnetism in a letter to P. G. TAIT in 1867, where he wrote that:

> "I do not understand how Verdet's discovery that the paramagnetic bodies produce rotation of the plane of polarization in the opposite direction to diamagnetic bodies *confirms* Faraday's doctrine that a diamagnetic body is only less paramagnetic than the field. It is a pretty doctrine but I do not think that Faraday thought it certain and Verdet's phenomena appear to me to be the strongest thing against it. I am myself sorry to part with it"[142].

V

As his ideas reported in the successive parts of "Physical Lines" developed, MAXWELL'S view on the ontological status of the ether also changed[143]. In Part I of "Physical Lines" there was no indication that MAXWELL necessarily believed in the reality of the ether[144], but in Part III — as a result of the derivation of the theory of light — he argued that the inference that the optical and electromagnetic ethers were identical could scarcely be avoided, and he began to regard the ether as a real entity. Its new position in his thought was retained in "A Dynamical Theory of the Electromagnetic Field" (1865)[145], where he argued that the rotation of the plane of polarization of polarized light, the phenomena of optics, and the polarization of dielectrics all led him to the conclusion that there was "an aethereal medium pervading all bodies, and modified only in degree by their presence", and that this "complicated mechanism" was "subject to the general laws of Dynamics"[146]. The ether pervaded all bodies and was not conceived as an entity ontologically separate from matter.

This paper, "A Dynamical Theory of the Electromagnetic Field", offered a new approach, for MAXWELL derived his basic equations of electromagnetism from general equations of mechanical systems without employing any physical model; but there is an essential continuity between it and "Physical Lines". The theory was a theory of the electromagnetic field because "it has to do with the space in the neighbourhood of the electric or magnetic bodies", and it was a dynamical theory[147] "because it assumes that in that space there is matter in

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142 MAXWELL to P. G. TAIT, 23 December 1867, U.L.C. Add. MSS. 7655.

143 See JOAN BROMBERG, *op. cit.* (note 128), 227f.

144 Thus, he stated his object was "to clear the way for speculation" about a physical state in the medium by "pointing out the mechanical consequence of such hypotheses", in the hope that he would be "of some use to those who consider the phenomena as due to the action of a medium", *Papers*, **1**, 452, rather than introducing a theory which he wished to be understood as a representation of reality.

145 "A Dynamical Theory of the Electromagnetic Field", *Phil. Trans.*, 1865, **155**, 459—512; *Papers*, **1**, 526—597.

146 *Ibid.*, 532f.

147 Throughout this paper the term "dynamical" will be used as MAXWELL used it in his work on electricity. For MAXWELL dynamics was the science in which "special attention is paid to force as the cause of motion", MAXWELL, *Matter and Motion*

motion, by which the observed electromagnetic phenomena are produced"[148]. Despite his clearly expressed belief that there was a complicated motion in the medium, in "Dynamical Theory" he avoided any consideration of the nature of this motion. He made it quite clear that in his use of mechanical terms such as electric elasticity his purpose was merely heuristic[149]; nevertheless the principle of mechanism extended throughout the paper, for, as he said, "in speaking of the Energy of the field, however, I wish to be understood literally. All energy is the same as mechanical energy"[150]. The energy manifested itself as magnetic and electric polarization, and, as in "Physical Lines", the concept of molecular polarization was fundamental, the action of an electromotive force on a dielectric producing "a state of polarization of its parts similar in distribution to the polarity of a mass of iron under the influence of a magnet ... a state in which every particle has its opposite poles in opposite conditions"[151]. Thus, once again he proposed a particulate theory of electric action, and though he abandoned the mechanical model of "Physical Lines" he did not fail to note that "magnetic polarization and electric polarization ... [were], according to a very probable hypothesis ... the motion and the strain of one and the same medium"[152]. That the elimination of the mode of the mechanism did not lead to the denial of the principle of mechanism can also be seen from his treatment of the electrotonic state, which played a vital role in the paper, and was defined in terms of mechanical principles. MAXWELL pointed out that just as every change of momentum involved the action of a mechanical force, every change of the electrotonic state involved the action of an electromotive force, and he now called the electrotonic state the "electromagnetic momentum"[153]. He conceived the relation between the electrotonic state and the electromotive force as if it were a mechanical relation. Once again, as in "Physical Lines", he attempted to achieve a representation by means of a mechanical interpretation — though in this case the interpretation was not linked to a mechanical model — and, as in "Physical Lines", his conceptual picture corresponded to FARADAY's abandoned approach of the polarization of the particles of matter rather than the primacy of lines of force.

The difference between these two modes of representation can be seen from his brief discussion of the problem of gravitation in "Dynamical Theory". In 1857 he had reassured FARADAY that:

> "I for my part cannot realise your dissatisfaction with the law of gravitation provided you conceive it according to your principles ... [for] lines of force

(London, 1877), 26, the nature of the forces being defined by NEWTON's laws. Thus, "when a physical phenomenon can be completely described as a change in the configuration and motion of a material system, the dynamical explanation of that phenomenon is said to be complete", *Papers*, **2**, 418, and "the equations of dynamics completely express the laws" of such an explanation, *ibid.*, 374. Thus, "dynamical" meant considering matter in terms of the laws of motion and impact.

148 *Papers*, **1**, 527.

149 *Ibid.*, 563f.

150 *Ibid.*, 564.

151 *Ibid.*, 531.

152 *Ibid.*, 564.

153 *Ibid.*, 542.

can 'weave a web across the sky' and lead the stars in their courses without any necessarily immediate connexion with the objects of their attraction. The lines of Force from the Sun spread out from him and when they come near a planet *curve out from it* so that every planet diverts a number depending on its mass from their course and substitutes a system of its own so as to become something like a comet, *if lines of force were visible*"[154].

Thus, MAXWELL was arguing that the theory of the primacy of lines of force — which was the mode of representation adopted by MAXWELL, in "Faraday's Lines", when he wrote this letter — could be extended to gravitation. However, MAXWELL's changing view of the place of lines of force for the representation of electromagnetic phenomena necessitated a reassessment of this explanation of gravity. In "Dynamical Theory" he pointed out that the distinguishing feature of gravitation, that it was always an attractive force, had the consequence that the energy of any gravitational field of a material constitution was less wherever there was a resultant gravitational force; hence, those parts of space in which there was no resultant force would possess an enormous energy. He admitted that "I am unable to understand in what way a medium can possess such properties"[155]. Once again MAXWELL was attempting to extend his theory to explain gravity, but he was obliged to announce that he would refrain from further speculation for he was unable to represent the physical nature of lines of gravitational force[156].

The nature of his interpretation in "Dynamical Theory", and his reasons for abandoning the mechanism of "Physical Lines", can be more readily seen from the *Treatise on Electricity and Magnetism* (1873), where the approach of "Dynamical Theory" was preserved and applied to a wider range of problems. In the *Treatise* he observed that "the problem of determining the mechanism required to establish a given species of connexion between the motions of the parts of a system always admits of an infinite number of solutions"[157], and this had far-reaching implications for the theory of "Physical Lines". In "Physical Lines" the derivation of the wave equation was achieved as a consequence of the model of the ether he employed. If there were an infinite number of such models, a derivation of the wave equation based so firmly on one such model was somewhat questionable. In fact, as early as December 1861, shortly before his derivation of

[154] MAXWELL to FARADAY, 9 November 1857 (see note 76). MAXWELL was here taking up FARADAY's point, *Electricity*, **3**, 574, that lines of gravitational force always existed diffused in space, and MAXWELL was reassuring FARADAY over the difficulties he had raised in "On the Conservation of Force" (see note 65).

[155] MAXWELL, *Papers*, **1**, 571.

[156] MAXWELL returned to this question in his article on "Attraction", arguing that for a system of two bodies we can "express the fact that there is attraction ... by saying that the energy of the system ... increases when their distance increases. The question, therefore, Why do the two bodies attract each other? may be expressed in a different form. Why does the energy of the system increase when the distance increases?", *ibid.*, **2**, 486. This may be compared with FARADAY's argument — about the creation and annihilation of "force" — in "On the Conservation of Force" (see note 65). He continued by discussing his theory of stress in a medium as applied to gravity, concluding that "we have not ... been able to imagine any physical cause for such a state of stress", *ibid.*, 489. He concluded by emphasizing that any explanation of gravity would have to be subject to energy conservation.

[157] MAXWELL, *Treatise*, § 831.

the wave equation was published, he wrote to a friend that he was "trying to form an exact mathematical expression for all that is known about electromagnetism without the aid of hypothesis"[158]. By October 1864 he had succeeded in this, for he informed STOKES of his success, for his calculation of the velocity of transmission of magnetic disturbances was no longer dependent on "any hypothesis about the structure of the medium or any mechanical explanation of electricity and magnetism"[159]. Thus, his derivation was now from a purely electromagnetic theory, the fundamental assumption being that the energy was supposed to reside in the electromagnetic field, "in the space surrounding the electrified and magnetic bodies, as well as in those bodies themselves"[160]. Nevertheless, all energy was mechanical energy.

In "Dynamical Theory" and in the *Treatise* he developed the theory on the basis of mechanical principles, and in the *Treatise* he did this quite explicitly from the LAGRANGEAN formalism of dynamics, in which the connexions of the motions of the medium with the variables were eliminated from the equations, so that the equations were "independent of the particular form of these connexions"[161]. Even if an infinite number of mechanical models could be constructed the LAGRANGEAN formalism of dynamics was independent of any particular model. Nevertheless he emphasized that the formalism was to be interpreted in dynamical terms, for "we must keep constantly in mind the ideas appropriate to the fundamental science of dynamics"[162]. In a draft of "Dynamical Theory" he wrote that "when any physical phenomenon can be completely described as a change in configuration or the motion of a material system, the dynamical explanation is said to be complete. We cannot conceive any further explanation to be either necessary, desirable, or possible"[163]. The passage concluded with the words: "hence the process by which physical science is 'unified' must ..." but here he broke off. The implication was clear, that a dynamical explanation was a final, complete explanation, and this idea dominated "Dynamical Theory" and the *Treatise*. Though MAXWELL did not develop his theory in terms of a mechanical model this dynamical explanation was conceived as a physical explanation of the

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[158] CAMPBELL & GARNETT, *Life of Maxwell* (1st ed., London, 1882), 330.

[159] J. LARMOR (ed.), *Memoir and Scientific Correspondence of the late Sir George Gabriel Stokes, Bart.* (2 vols., Cambridge, 1907), **2**, 26.

[160] MAXWELL, *Papers*, **1**, 564.

[161] MAXWELL, *Treatise*, § 554. HENRI POINCARÉ argued that "Maxwell does not give a mechanical explanation of electricity and magnetism ... [but] limits himself to demonstrating that such an explanation is possible", *Électricité et Optique* (2 vols., Paris, 1890/91), **1**, vii, and considered this "Maxwell's fundamental idea", *ibid.*, xiv. POINCARÉ found MAXWELL's use of the LAGRANGEAN formalism of dynamics the key to the *Treatise*, regarding MAXWELL's programme as being to demonstrate the possibility of mechanical explanation by constructing an interpretation of the phenomena in terms of LAGRANGE's equations, and then to compare the equations with the experimental relations, the aim being to demonstrate the principle of mechanical explanation, *ibid.*, xff. This was not MAXWELL's programme however, for he found the dynamical explanation of the *Treatise* unsatisfactory, for it did not satisfy his own criterion of "consistent representation" (see below).

[162] MAXWELL, *Treatise*, § 567.

[163] Draft, "On the Dynamical Explanation of Electric Phenomena" (see note 77). This may be compared with his remarks in his lecture "On the Dynamical Evidence of the Molecular Constitution of Bodies" [1875], *Papers*, **2**, 418 (see note 147).

phenomena, as he made perfectly clear in advancing his notion of "consistent representation" in the *Treatise.*

As MAXWELL'S successors recognized, the *Treatise* lacks unity[164], but the essential conceptual ideas of the *Treatise* were founded on the particulate viewpoint. Among key features of the *Treatise* are MAXWELL'S representation of the energy of the field and the stress in the medium; as he made clear, he only demonstrated the possibility of representing a state of stress in a medium and asserted nothing about "the mode in which this state of stress is originated and maintained in the medium"[165]. The representation of the energy of the field was of fundamental importance in the *Treatise,* and MAXWELL expressed the kinetic or electromagnetic energy of the field in terms of the electrotonic state and the electric current, or alternatively, in terms of the magnetic force and the magnetic induction[166]; the electrostatic or potential energy was expressed in terms of the electric force and the displacement current[167]. Now, by their very nature, these quantities were closely bound up with the theory of polarization of the medium which he maintained in the *Treatise,* and some attention must be paid to his treatment of polarization, which he regarded as representing the stress in the medium[168], and of the electrotonic state.

As in "Physical Lines" he defined polarization in terms of equal and opposite charges at opposite ends of a particle[169], as a "forced state" of the medium[170],

[164] Thus, HERTZ wrote that "Many a man has thrown himself with zeal into the study of Maxwell's work, and, even when he has not stumbled upon unwonted mathematical difficulties, has nevertheless been compelled to abandon the hope of forming for himself an altogether consistent conception of Maxwell's ideas", *Electric Waves* (London, 1893), 20. HERTZ was referring to the *Treatise* as was LORENTZ when he wrote that "it is not always easy to comprehend Maxwell's ideas. One feels a lack of unity in his book due to the fact that it records faithfully his gradual transition from old to new ideas", "Clerk Maxwell's Electromagnetic Theory" (Rede Lecture 1923), *Collected Papers,* **8**, 356.

[165] MAXWELL, *Treatise,* § 645.

[166] MAXWELL used the concept of magnetic induction in "Faraday's Lines", *Papers,* **1**, 192, which he used as a means of representing the lines of force, by dividing electric and magnetic magnitudes into "quantities" and "intensities", *ibid.,* 192, magnetic induction being of the former and magnetic force of the latter kind. In his lecture "On the Mathematical Classification of Physical Quantities" [1871] he changed the names of these two classes to "fluxes" and "forces", *ibid.,* **2**, 261, electric displacement and magnetic induction being fluxes, electric and magnetic force being forces, *ibid.,* 262. See also *Treatise,* § 12 (in the second edition MAXWELL used the terms electric and magnetic "intensities", and his two classes became "fluxes" and "intensities").

[167] Thus, MAXWELL defined the electrostatic or potential energy of the field as $W=\frac{1}{2}\iiint(Pf+Qg+Rh)\,dx\,dy\,dz$, where P, Q, R and f, g, h are components of the electric force and electric displacement, respectively, and the electromagnetic or kinetic energy as $T=\frac{1}{8\pi}\iiint(a\alpha+b\beta+c\gamma)\,dx\,dy\,dx$, where a, b, c and α, β, γ are the components of the magnetic induction and magnetic force, respectively. See *Treatise,* § 638. See also "Dynamical Theory", *Papers,* **1**, 563.

[168] MAXWELL said of his explanation of electrostatic stress that it was "precisely that to which Faraday was led", quoting *Electricity,* **1**, par. 1297 and 1298, that "Induction appears to consist in a certain polarized state of the particles", *Treatise,* § 109.

[169] *Treatise,* § 111. See also "Dynamical Theory", *Papers,* **1**, 531, 554, and "Physical Lines", *ibid.,* 491.

[170] *Treatise,* § 60.

and he made it clear that he attributed this manner of representing the stress in the medium to FARADAY, quoting FARADAY's description of polarization as a forced state of the particles of the medium and relating his concepts to FARADAY's earlier approach. As before, dielectric polarization was represented by the electric displacement, the variations of which constituted electric currents[171]. In the *Treatise* MAXWELL emphasized once again that in his theory the particles of a magnet were polarized, and he now developed a theory of magnetic polarization in a manner analogous to his theory of dielectric polarization; once again polarization was defined quite unambiguously by a particulate theory[172], though he did not suggest a mechanical model for polarization[173].

In his treatment of the electrotonic state he did not return to the viewpoint of "Faraday's Lines", but referred to the electrotonic state as "the fundamental quantity in the theory of electromagnetism"[174] — which, he noted, FARADAY found he could dispense with by means of lines of force — and he went on to make the significant observation that, despite the value of the electrotonic state, FARADAY had used another method, which "in Faraday's hands was far more powerful ... [the method of] lines of force"[175]. However, though he made it clear that the electrotonic state at a point was equivalent to the number of lines of force passing through the point, he did not replace the electrotonic state by the lines of force. In contrast to his treatment in "Faraday's Lines", where the lines of force remained the fundamental entities, in the *Treatise* he argued that instead of referring to the number of lines of force as representing the electrotonic state "we may speak of the magnetic induction"[176]. In the *Treatise* the magnetic induction and the electrotonic state were fundamental quantities, and the significance of this remark is that in the *Treatise* the magnetic induction defined the lines of force, whereas in "Faraday's Lines" he used this concept merely to represent the lines of force[177].

Now, the displacement, the magnetic induction, and the electric and magnetic forces were all defined as vector quantities[178]. The electrotonic state was also defined as a vector quantity, and the electrotonic state was renamed the vector

[171] *Treatise*, § 60. See also "Dynamical Theory", *Papers*, **1**, 531, 554.

[172] *Treatise*, § 381.

[173] Thus, displacement is no longer linked to the change of position of rolling particles as in "Physical Lines". In "Dynamical Theory" and the *Treatise* he defined displacement by the motion of electricity (*Papers*, **1**, 554; *Treatise*, § 60), as in "Physical Lines" (*Papers*, **1**, 491). The rolling particle model is not used in "Dynamical Theory" or in the *Treatise*, and MAXWELL defined displacement in terms of the quantity of charge crossing a specified area. I am grateful to Dr. JOAN BROMBERG for a discussion of this point. The relation of displacement to change in MAXWELL's thought is an extremely complex question (see JOAN BROMBERG, "Maxwell's Electrostatics", *American Journal of Physics*, 1968, **36**, 142—151 for a discussion of some of the problems), but MAXWELL clearly associated displacement with particulate polarization in "Physical Lines", "Dynamical Theory" and the *Treatise* (see note 169).

[174] *Treatise*, § 540.

[175] *Treatise*, § 541.

[176] *Ibid.* See also § 406 for a definition of the electrotonic state in terms of magnetic induction. In vector notation, $\boldsymbol{B}=\text{curl}\,\boldsymbol{A}$, where $\boldsymbol{B}$ is the magnetic induction, and $\boldsymbol{A}$ the vector potential (electrotonic state).

[177] See above, and also note 166.

[178] *Treatise*, § 11, 12.

potential in the *Treatise*, the term deriving from the formal properties of the quantity[179]. The replacement of lines of force by the vector quantities of magnetic induction and vector potential had an important consequence, for MAXWELL employed vectorial representation (he used quaternions[180]) because he found vectorial representation conducive to his geometrical approach. He regarded it as "a method of thinking" because "it calls upon us at every step to form a mental image of the geometrical features represented by the symbols"[181]. It was valuable as a method of representing directed quantities in space rather than as a method of calculation[182], but it was the kind of geometrical representation this entailed that was so important. In the *Treatise* he stated that if a quantity was a vector quantity "then any body or particle to which this directed quantity or vector belongs may be said to be Polarized, because it has opposite properties in the two opposite directions or poles of the directed quantity"[183]. Now, in "Faraday's Lines" the geometrical property of the lines of force only represented the direction of the forces in space, and his concept of polarity in that paper was limited to the expression of a directional property and excluded any notion of "opposite properties in the two opposite directions". Thus, by using vectors in the *Treatise* and by replacing the lines of force by the vectorial quantities of vector potential (electrotonic state) and magnetic induction, MAXWELL expressed geometrical properties of a different kind for these quantities. They did not merely represent direction, for their different "poles" represented states which had opposite polarity. MAXWELL was so committed to particulate polarization in the *Treatise* that he took the directed properties of vectors as a means of representing particulate polarization, in which opposite parts of the particles were in opposite states.

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179 *Treatise*, § 406. See ALFRED M. BORK, "Maxwell and the Vector Potential", *Isis*, 1967, **58**, 218f.

180 MAXWELL used HAMILTON's operator ∇ (W. H. HAMILTON, "On Quaternions", *Phil. Mag.*, 1847, **31**, 279—293) in "Dynamical Theory", *Papers*, **1**, 578, for brevity in deriving the wave-equation. He first referred to quaternions in a letter to TAIT of 7 March 1865, asking "Does any one write quaternions but Sir W. Hamilton & you?" referring to TAIT's *Elementary Treatise on Quaternions* (Oxford, 1867). TAIT encouraged MAXWELL to study quaternions, stating that "If you read the last 20 or 30 pages of my book I think you will see that 4 nions are worth getting up, for there it is shown that they go into that $\triangleleft$ business like greased lightning" (13 December 1867), for "it was for ∇ alone that I took to Q[uaternions] originally" (5 April 1871). These letters and others, show that MAXWELL's real interest in quaternions began in 1870 when he was writing the *Treatise* (letters at U.L.C., Add. MSS. 7655). The name, and mode of writing, of HAMILTON's operator was unsettled at this time. See MICHAEL J. CROWE, *A History of Vector Analysis* (London, 1967), 32, 146.

181 [JAMES CLERK MAXWELL], "Quaternions" [review of P. KELLAND & P. G. TAIT, *Introduction to Quaternions* (London, 1873)], *Nature*, 1873, **9**, 137.

182 MAXWELL stated that "The invention of the calculus of Quaternions is a step towards the knowledge of quantities related to space which can only be compared for its importance, with the invention of triple coordinates by Descartes", *Papers*, **2**, 259, though "The ideas of this calculus, as distinguished from its operations and symbols, are fitted to be of the greatest use in all parts of science", "On the Mathematical Classification of Physical Quantities", *Papers*, **2**, 259 (see also *Treatise*, § 10). For a discussion of MAXWELL's ambiguous attitude to quaternions see CROWE, *op. cit.* (note 180), 127—139.

183 *Treatise*, § 381. In discussing polarization here he distinguished between unipolar and dipolar quantities (see note 112).

The quantities of displacement, electrotonic state, magnetic induction, and the electric and magnetic forces, were fundamental quantities in the *Treatise*, not only for the expression of the energy of the field but also in the general equations of the electromagnetic field[184]. All these quantities were vector quantities and therefore they all expressed states of "polarization", having opposite properties in opposite directions, and so this concept of polarization was fundamental to the mode of representation of the *Treatise*. Thus, his mode of mathematical representation in the *Treatise* — by vectors — was in consonance with his adherence to the theory of molecular polarization.

In his explanation of the action of magnetism on light in the *Treatise* MAXWELL retained the interpretation of "Physical Lines", and he again argued that some rotatory motion occurred in the medium and that the rotation could not be of a portion of the medium of sensible dimensions but was of "very small portions of the medium"[185]. The motion of the medium produced a disturbance of the vortices, which affected the mode of propagation of the ray. While he conceded that the hypothesis of molecular vortices was unproved[186] he made it clear that he regarded it as true, whereas the theory of electricity as the motion of idle wheel particles was merely an illustration of a possible mechanism[187]. As in "Physical Lines" he noted the significance of VERDET'S experiments on the opposite rotation of polarized light by paramagnetic and diamagnetic substances, which indicated that the two classes of bodies were really opposite[188], and he went on to develop a theory of magnetism[189]. He now accepted WEBER'S theory of magnetism (derived from AMPÈRE) that electric currents in paramagnetics circulated in the opposite direction to the induced currents in diamagnetic bodies[190]; in "Physical Lines" he had accepted WEBER's theory that paramagnetics and diamagnetics were in opposite conditions, but did not adopt his hypothesis of molecular currents. Weber's theory involved states of opposite

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184 *Treatise*, § 591—619.

185 *Treatise*, § 822. Thus, MAXWELL continued to use the magnetic vortices to explain the magnetic action on light. This phenomenon was not discussed in "Dynamical Theory", so the vortices were not employed in that paper. Already in "Physical Lines" he had made it clear that he believed the idea of vortices to be probably true (see note 119).

186 He stated that the "theory proposed [of the magnetic rotation of light] ... is evidently of a provisional kind, resting as it does on unproved hypotheses relating to the nature of molecular vortices", *Treatise*, § 830.

187 He emphasized that "I think we have good evidence for the opinion that some phenomenon of rotation is going on in the magnetic field, that this rotation is performed by a great number of very small portions of matter ... by means of some kind of mechanism connecting them", but that the "attempt which I then made [in 'Physical Lines'] to imagine a working model of this mechanism must be taken for no more than it really is, a demonstration that mechanism may be imagined capable of producing a connexion mechanically equivalent to the actual connexion of the parts of the electromagnetic field", *Treatise*, § 831.

188 *Treatise*, § 809.

189 He emphasized that he adopted AMPÈRE's theory (see note 87) that considered "a magnet, not as a continuous substance ... but as a multitude of molecules, within each of which circulates a system of electric currents", *Treatise*, § 834.

190 *Treatise*, § 838, 843.

polarity, and there was a complete acceptance of the concept of polarity as involving opposite states of the molecules of a substance in the *Treatise.*

Both displacement and the vector potential (electrotonic state) were fundamental to the general equations of the field and in his formulation of the electromagnetic theory of light. Both in "Dynamical Theory" and the *Treatise* MAXWELL was faced with the problem of obtaining a condition of transversality for the wave propagation. His arguments here involved the vector potential, for he was concerned to show that the components of the vector potential were propagated as transverse waves. In vector notation, he was attempting to argue that $\operatorname{div} \boldsymbol{A}=0$ (where $\boldsymbol{A}$ is the vector potential), for this was the condition for transversality. Now the arguments he used to justify this in the published text of "Dynamical Theory", the manuscript of the paper, and the *Treatise* were all different, which indicates that MAXWELL found some difficulty with this problem[191]. In "Dynamical Theory", after giving his justification, he went on to state that the equations of the electromagnetic field showed that "transversal vibrations only can be propagated"[192], but he went on to refer to the problem of longitudinal vibrations in the optical ether. This problem had been discussed by STOKES in a British Association "Report" in 1862[193] and STOKES had made it clear that any theory of the optical ether would have to be able to explain why only transversal vibrations were propagated[194]. MAXWELL remarked that both optical and electrical sciences

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[191] MAXWELL wrote $J=\frac{dF}{dx}+\frac{dG}{dy}+\frac{dH}{dz}$ where F, G, H are the components of the vector potential ($\boldsymbol{A}$). Thus, in vector notation, $J=\operatorname{div} \boldsymbol{A}$ (See *Papers*, **1**, 578; *Treatise*, § 783). The condition for transversality was $J=0$. In the manuscript of "Dynamical Theory" he justified this by arguing that "Here J is either zero or it continually increases or diminishes with the time, if e [the quantity of free electricity] remains constant, which no physical quantity can do. Hence J is zero, and the only disturbance is ... wholly transversal", MS, "A Dynamical Theory of the Electromagnetic Field", Royal Society, *Phil. Trans.* **72**, 7 (I am grateful to the Library staff of the Royal Society for their help). In the printed text of the paper he stated that "Since the medium is a perfect insulator, e, the free electricity, is immovable, and therefore $\frac{dJ}{dt}$ is a function of x, y, z, and the value of J is either constant or zero, or uniformly increasing or diminishing with the time [t]; so that no disturbance depending on J can be propagated as a wave", *Papers*, **1**, 582. In the *Treatise* he stated that if the medium was a non-conductor "$\nabla^2\psi$ [ψ is the electric potential], which is proportional to the volume density of free electricity, is independent of t. Hence J must be a linear function of t, or a constant, or zero, and we may therefore leave J and ψ out of account in considering periodic disturbances" (§ 783). In modern terminology, he was attempting to impose a "gauge" condition on the vector potential, so $\operatorname{div} \boldsymbol{A}=0$. See also a note by P. F. CRANEFIELD, *Annals of Science*, 1954, **10**, 361.

[192] MAXWELL, *Papers*, **1**, 582.

[193] G. G. STOKES, "Report on Double Refraction", *Report of the Thirty-Second Meeting of the British Association for the Advancement of Science; held at Cambridge in October 1862* (London, 1863), 253—282.

[194] Thus, in discussing A. L. CAUCHY's paper "Mémoire sur la théorie de la lumière", *Mém. de l'Acad.*, 1830, **10**, 293—316, STOKES stated "That theory should point to the necessary existence of such a wave consisting of strictly normal [*i.e.* longitudinal] vibrations, and yet to which no known phenomenon can be referred, is bad enough; but in the present theory the vibrations are not even strictly normal, except for waves in a direction perpendicular to any one of the principal axes", Stokes, *op. cit.* (note 193), 256. Against GEORGE GREEN's argument, in "On the Laws

were at a loss "when called on to affirm or deny the existence of normal [i.e. longitudinal] vibrations"[195], hinting that he did not find his justification of transversality entirely convincing.

VI

It may have been this problem, in part, that led MAXWELL to abandon the approach of "Dynamical Theory" — but which he was to return to in the *Treatise* — and attempt quite a different mode of representation in his next paper before the *Treatise*, the "Note on the Electromagnetic Theory of Light" (1868)[196]. In this paper he abandoned the electrotonic state altogether, and developed the electromagnetic theory of light on the basis of the primacy of the lines of force. Thus, he avoided the difficulty of justifying the transversality of the propagated wave by the div $A=0$ condition by the elimination of the electrotonic state (vector potential)[197], and he returned to the approach of "Faraday's Lines".

of Reflexion and Refraction of Light", *Trans. Camb. Phil. Soc.*, 1838, **7**, 1—24, 113—120 (*Mathematical Papers of the Late George Green*, (ed.) N. M. FERRERS (London, 1871), 245—269, 283—290) that longitudinal waves would be suppressed, STOKES argued that in fact "The only way ... of getting over this difficulty, is by making the perfectly gratuitous assumption that the medium, though perfectly transparent for the more nearly transversal vibrations, is intensely opaque for those more nearly normal", *op. cit.*, 258. STOKES concluded that though GREEN's paper "On the Propagation of Light in Crystallized Media", *Trans. Camb. Phil. Soc.*, 1839, **7**, 121—140 (*Mathematical Papers*, 293—311) obtained a condition of transversality on the single supposition of the incompressibility of the optical ether, GREEN's theory was not completely successful for GREEN was obliged either to assume conditions about the mode of vibration of the medium which were contrary to experience, or to assume that the medium was subject to conditions of initial stress, which was a "forced relation", *op. cit.*, 265. STOKES advanced similar objections against MACCULLAGH's theory, "An Essay Towards a Dynamical Theory of Crystalline Reflection and Refraction" [1839], *Trans. Roy. Irish Acad.*, 1848, **21**, 17—50 (*The Collected Works of James MacCullagh*, (ed.) J. H. JELLETT & S. HAUGHTON (Dublin, 1880), 145—184), which he regarded as not being dynamically sound, *op. cit.*, 266f. Thus, STOKES found that no theory of the optical ether had successfully explained the absence of longitudinal vibrations.

[195] MAXWELL, *Papers*, **1**, 582. MAXWELL's concern with the problems of the optical ether at this time can be seen from his letter to STOKES of 15 October 1864 (see note 159) where he remarked that "I am trying to understand the conditions at a surface for reflexion and refraction". See L. ROSENFELD, "The Velocity of Light and the Evolution of Electrodynamics", *Nuovo Cimento*, 1956, **4** (supp. 5), 1661.

[196] MAXWELL, "On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Forces; with a Note on the Electromagnetic Theory of Light", *Phil. Trans.*, 1868, **158**, 643—657 (*Papers*, **2**, 125—143).

[197] It is interesting that OLIVER HEAVISIDE, who eliminated the vector potential in his formulation of electrodynamics, associated this elimination with the transversality problem. Thus, he remarked that "not even Maxwell himself quite understood how [the vector potential] ... operated in his 'general equations of propagation'", *Electromagnetic Theory* (3 vols., London, 1893—1912), **1**, 69, and he referred to $J[=\mathrm{div}\,A]$ as a "parasite" of the vector potential (*ibid.*). HEAVISIDE argued that the vector potential should be eliminated, for it was the electric and magnetic forces, $\boldsymbol{E}$ and $\boldsymbol{H}$ which "actually represent the state of the medium anywhere ... it is $\boldsymbol{E}$ and $\boldsymbol{H}$ that are propagated", *Electrical Papers* (2 vols., London, 1892), **2**, 483, for "when the electric force itself is made the subject of investigation, the question of divergence of the vector-potential does not present itself at all", *ibid.*, 363. Thus, HEAVISIDE

He developed the theory from four theorems: the first two related electromotive force and magnetic force in terms of the lines of force, and the second two expressed relations between electromotive force and displacement, and displacement and electric current[198]. Thus, he retained the concept of displacement, which he defined once again as dielectric polarization, but this concept had been introduced in "Physical Lines" (and retained in "Dynamical Theory") as a means of representing the polarization of the particles of the dielectric. However, there was no mention of molecular or particulate polarization in the 1868 "Note", and the way in which he harmonized the concept of displacement with his lines of force model can be seen from a letter which he wrote to THOMSON in the following year (1869)[199]. In the published paper he represented the action of an electromotive force on a dielectric as leading to a displacement of electricity between the surfaces of the dielectric, and in the letter he represented equipotential surfaces of the dielectric as enclosing "cells" of the dielectric, the cells being bounded by lines of force so as to form tubes of force[200]. A displacement of electricity took place within each cell enclosed by a tube of force, so the concept of displacement was connected to the lines of force.

A similar approach can be seen in his unfinished *Elementary Treatise on Electricity* written shortly after the publication of the *Treatise on Electricity and Magnetism* in 1873, and published posthumously in 1881. In the "Preface" to the *Elementary Treatise* he wrote that in the *Treatise* he had employed methods which were necessary to the study of the mathematical theory of electricity but that "I have since become more convinced of methods akin to those of Faraday, and have therefore adopted them from the first"[201]. In fact, it is representation by lines and tubes of force which characterizes the *Elementary Treatise;* it has already been noted that in the *Treatise* he stated that FARADAY'S particular methods in-

was here — and in his remark on the vector potential in the equations of propagation — referring to the problem of the $J=0$ condition. HEAVISIDE considered that the fact there were no longitudinal waves in MAXWELL'S theory was a triumph of the theory. He stated that there were "no 'longitudinal' waves in Maxwell's theory analogous to sound waves. Maxwell took good care that there should not be any ... the phenomena of light indicated the absence of longitudinal waves; to get rid of them was a difficulty in elastic solid theories ... Now Maxwell's theory went of itself in the direction required", *Electromagnetic Theory*, **2**, 493. Thus, by eliminating A and its "parasite" J, HEAVISIDE also eliminated the unwanted longitudinal waves.

198 MAXWELL, *Papers*, **2**, 138f. MAXWELL'S first two theorems were as follows: "Theorem A. If a closed curve be drawn embracing an electric current, then the integral of the magnetic intensity taken round the closed curve is equal to the current multiplied by 4π ... Theorem B. If a conducting surface embraces a number of lines of magnetic force, and if, from any cause whatever, the number of these lines is diminished, an electromotive force will act round the circuit, the total amount of which will be equal to the decrement in the number of lines of magnetic force in unit of time", *ibid.*, 138. Thus, Theorem B was stated explicitly in terms of lines of force, and that this also applied to Theorem A can be seen from the fact that he went on to state that the "number of lines of magnetic force may be otherwise defined" in terms "of the integral of the magnetic intensity", *ibid.*, thus defining Theorem A in terms of lines of force.

199 MAXWELL to THOMSON, 5 June 1869, *Origins of Clerk Maxwell's Electric Ideas*, 45.

200 The notions of "tubes of force" and "cells" had been used by MAXWELL in "Faraday's Lines", *Papers*, **1**, 160, 165, and derived from FARADAY (see note 79).

201 MAXWELL, *Elementary Treatise*, viii.

volved representation by lines of force. As before, a tube of force was defined as a system of lines of force forming a tubular surface[202], and he wrote that he had constructed a "geometrical model of the field of electric force" by means of the tubes of force[203]. Thus, as in "Faraday's Lines", the aim was to represent the field by a geometrical model of lines and tubes of force. As in the 1869 letter to THOMSON, he stated that the tubes of force were cut into "unit cells"[204] by equipotential surfaces, and again he represented the effect of electromotive force as causing a displacement of electricity within the cells[205], and he made it clear that "a tube of induction is defined with respect to ... electric displacement"[206]. Thus, he clearly connected displacement to the idea of tubes of force. The notion of unit cells, like tubes of force, was first used by MAXWELL in "Faraday's Lines" where he related these ideas to his treatment of the imaginary fluid. In the *Elementary Treatise* MAXWELL represented the energy of the field as being stored in the unit cells[207], so again his representation was in accordance with the lines of force model.

The interpretation of displacement as taking place within the tubes of force in the *Elementary Treatise* and the 1869 letter to THOMSON was quite different from the way in which displacement was represented in "Physical Lines", "Dynamical Theory", and the *Treatise*, where displacement was connected to the concept of particulate polarization. In "Faraday's Lines" the concept of displacement was not introduced, but in that paper MAXWELL made it clear that there was no condition of polarity within the tubes of force, and there was no suggestion of the polarization of molecules. Thus, the approach in the *Elementary Treatise* represents something of a synthesis between the lines of force model as employed in "Faraday's Lines" and the viewpoint founded on the polarization of particles as employed in "Physical Lines", "Dynamical Theory", and the *Treatise*. The necessity of incorporating displacement into the model of lines of force, because this concept was required for his derivation of the wave equation, led MAXWELL to modify his lines of force approach; there was now a condition of polarity within each tube of force due to the displacement. However, here MAXWELL did not admit molecular polarization, and the lines of force approach was preserved by representing polarization as occurring within each tube of force. The sections on electrostatics in the *Elementary Treatise* may be compared with the corresponding sections in the *Treatise;* while he discussed lines and tubes of force in the latter work there was no connection of these concepts with displacement, and the model he adopted was one of molecular polarization[208]. There was no mention of the electrotonic state in the *Elementary Treatise*, but the work was incomplete, and the electrotonic state does not appear in the corresponding sections of the *Treatise*. Nevertheless, it is possible that he would have retained the approach of the 1868 "Note" and avoided the concept altogether. The approach in the

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[202] *Ibid.*, 46.

[203] *Ibid.*, 50.

[204] *Ibid.*, 47.

[205] *Ibid.*, 49.

[206] *Ibid.*, 57.

[207] *Ibid.*, 47f.

[208] On lines of force see *Treatise*, § 47, 82; on displacement and molecular polarization see § 60, 111.

Elementary Treatise was, therefore, a geometrical, non-mechanical mode of representation, and was presented as an alternative to the dynamical explanation of the *Treatise.*

VII

This dichotomy between lines of force and molecular polarization can also be seen in the work of MAXWELL's successors who attempted to achieve formulations of "Maxwell's theory". It is interesting that two of MAXWELL's followers, J. H. POYNTING and J. J. THOMSON, adopted the lines of force approach. Both related their work to FARADAY[209] and POYNTING, in adopting the idea of tubes of force and unit cells, also related his ideas to the *Elementary Treatise* and represented the energy of the field as being stored within the unit cells[210]. They both expressed the fundamental relations of electromagnetism in a manner analogous to the first two theorems of MAXWELL's 1868 "Note", and this was particularly explicit in POYNTING's theory[211]. They both avoided displacement, which J. J. THOMSON replaced by a quantity which he defined in terms of the number of tubes of force passing through a plane surface[212]. POYNTING's treatment of the electrotonic state resembled MAXWELL's in "Faraday's Lines", for he defined it in terms of the tubes of force[213]. The concept was not used at all by THOMSON. Despite minor differences of exposition POYNTING and THOMSON both adopted the viewpoint of the primacy of lines of force. POYNTING made it clear that he took the tubes as fundamental entities, their ultimate nature being unknown[214], and THOMSON also remarked that "we have not attempted any theory of the constitution of these tubes"[215], and emphasized that in FARADAY's

[209] J. H. POYNTING, "On the Connection between Electric Current and the Electric and Magnetic Inductions in the Surrounding Field", *Phil. Trans.*, 1885, **176**, 277n; *Collected Scientific Papers* (Cambridge, 1920), 194n. J. J. THOMSON, *Notes on Recent Researches in Electricity and Magnetism* (Oxford, 1893), 2.

[210] POYNTING, *Collected Scientific Papers*, 196. This was POYNTING's first general principle.

[211] POYNTING (*loc. cit.*) stated two further general principles, which in fact were Theorems A and B of MAXWELL's 1868 "Note" (*Papers*, **2**, 138, see note 198 above), and he then proceeded to state his modifications of them. This modification was carried out in terms of the concept of tubes of force, and POYNTING related the magnetic force to the "tubes of electric induction" and the electric force to the "tubes of magnetic induction", POYNTING, *op. cit.* (note 210), 197f. In doing this he was to express the analogy between Theorems A and B of MAXWELL's 1868 "Note" which was only implicit in MAXWELL's formulation. J. J. THOMSON obtained equations for the magnetic force in terms of the number and velocities of the tubes of force, *Recent Researches*, 8, and equations for the electric force in terms of the motions of the tubes of force, *ibid.*, 10.

[212] J. J. THOMSON, *Recent Researches*, 6.

[213] POYNTING defined magnetic induction ($\boldsymbol{B}$) in terms of the tubes of force passing through a surface, and he was able to show that the equation $\boldsymbol{B} = \operatorname{curl} \boldsymbol{A}$ (POYNTING did not use vector notation) would follow if the components of the vector potential ($\boldsymbol{A}$) F, G, H were defined in terms of the tubes of force. Thus, he stated that "We should obtain Maxwell's equation of we defined F, G, H to be the number of tubes which would cut the axes per unit length", *Collected Scientific Papers*, 213.

[214] POYNTING, "An Examination of Prof. Lodge's Electromagnetic Hypothesis" [*Modern Views of Electricity* (London, 1889)], *Electrician*, 1893, **31**, 636; *Collected Scientific Papers*, 268.

[215] J. J. THOMSON, *Recent Researches*, 52.

theory the tubes were the fundamental entities and had "an existence apart from the molecules of the dielectric, though these were polarized by the tubes when they passed through the dielectric"[216]. They both made it clear that their theories were non-mechanical representations of electromagnetism, and THOMSON wrote that the theory of tubes of force was "geometrical rather than dynamical"[217]. Thus, they both adopted a mode of representation based on the primacy of lines of force.

This point of view was emphatically opposed by HERTZ, who stated that the theory of the primacy of lines of force — of FARADAY, POYNTING, and J. J. THOMSON — implied that ontologically the lines of force were fundamental entities, rather than conventional symbols for a state of matter. Thus, he argued that "the conception employed by FARADAY, of a motion of the lines of force relatively to the surrounding medium, is indeed a highly remarkable one, and may be capable of being worked out; but it is entirely different from the view here followed, according to which the lines of force simply represent a symbol for special conditions of matter"[218]. This, then, was his approach, and he noted that a "similar theory [to Poynting's] has also been developed recently by J. J. Thomson ... In so far as this theory and Poynting's lead to Maxwell's equations, I would regard them as special forms of 'Maxwell's theory', although their conceptions are undoubtedly not Maxwell's"[219]. In dissociating the lines of force approach from MAXWELL's own view HERTZ clearly regarded MAXWELL's ideas as those embodied in the *Treatise*, for he emphasized[220] that he had adopted "Maxwell's standpoint", which was that in which "we must conceive each particle of the dielectric as being charged with negative electricity on ... [one] side, and with positive electricity on the ... [other] side"[221]. HERTZ appears not to have known of the 1868 "Note"[222], and he developed his own formulation of the fundamental relations of electromagnetism in a manner based on the electric and magnetic forces alone[223].

HEAVISIDE was also opposed to the use of lines of force, and his formulation of MAXWELL's theory was based on the symmetry between the proportionalities

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[216] *Ibid.*, 2.

[217] *Ibid.*, 52. See also POYNTING, *op. cit.*, 267.

[218] HEINRICH HERTZ, *Electric Waves*, trans. D. E. JONES (London, 1893), 255 (this is in the paper cited in note 7).

[219] *Ibid.*, 277, n. 35. HERTZ was here referring to J. J. THOMSON's paper "On the Illustration of the Properties of the Electric Field by Means of Tubes of Electrostatic Induction", *Phil. Mag.*, 1891, **31**, 149—171, in which the concept of tubes of force was employed.

[220] HERTZ, *Electric Waves*, 27.

[221] *Ibid.*, 26.

[222] However, HERTZ argued that the vector potential did not define a physical quantity and it was therefore superfluous and should be eliminated, "Über die Grundgleichungen der Electrodynamik für ruhende Körper", *Ann. Phys.*, 1890, **40**, 578; *Electric Waves*, 196.

[223] Though HERTZ eliminated displacement he argued that "The expression 'electric force' in these papers is only another name for a state of polarization of space", *Electric Waves*, 27n. HERTZ's reasons for eliminating displacement can be seen from the Introduction to *Electric Waves*. HERTZ argued that MAXWELL's ideas were those of "the pure conception of action through a medium", for "we now rather regard the polarizations [of the particles] as the only things which are really present", *op. cit.*, 25. Thus, HERTZ meant that charge was a manifestation of the polarization

between displacement and electric force, and magnetic induction and magnetic force[224]. This analogy between electricity and magnetism was implied by MAXWELL in his paper "On the Mathematical Classification of Physical Quantities"[225] and relates to his distinction between fluxes and forces in the *Treatise*[226], where he did indicate the analogy between electricity and magnetism. For HEAVISIDE, the fluxes — displacement and magnetic induction — were fundamental quantities, and he stated that "I must, however, wonder at the persistence with which the practitians have stuck to 'the lines' as they call the flux in question"[227]. Thus, HEAVISIDE did not interpret magnetic induction in terms of lines of force, and he made it clear that in his formulation the forces and fluxes were "the objects of immediate attention"[228]. Thus, HEAVISIDE's theory was based on the concepts of the *Treatise*, and though there were differences between his ideas and HERTZ's their formulations of electromagnetism have much in common[229], for in both their theories the equations of the field were expressed as relations between the electric and magnetic forces[230]. Thus, HEAVISIDE and HERTZ developed their theories from the categories of the *Treatise*, despite their differences from the mode of representation in that work[231].

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of the ether; this was stated by MAXWELL in the *Treatise* (§ 111). Now, HERTZ also argued that MAXWELL's equations could be derived from the limiting case of HELMHOLTZ's theory ("Über die Bewegungsgleichungen der Elektricität für ruhende leitende Körper" [1870], *Wissenschaftliche Abhandlungen*, **1**, 543—628) in which the polarization was explained by distance forces, for in the limiting case of HELMHOLTZ's theory all the energy was in the medium and "the distance-forces must become infinitely small", *Electric Waves*, 24. This corresponded to the case where "electricity must therefore behave ... like an incompressible fluid", *ibid.*, as stated by MAXWELL in the *Treatise* (§ 61). Thus, HERTZ's elimination of $\boldsymbol{D}$ can be seen as the result of the problem of the definition of displacement in terms of charge, for he argued that the notion that charge was a manifestation of polarization could not be equated with the idea that electricity behaved like an incompressible fluid, *op. cit.*, 25. By eliminating displacement as a concept independent of the electric force, the problem was avoided. In vector notation (which was not used by HERTZ, HERTZ's equations were operator equations between curl $\boldsymbol{H}$ and $\boldsymbol{E}$ and curl $\boldsymbol{E}$ and $\boldsymbol{H}$, *ibid.*, 201 (paper cited in note 222).

224 HEAVISIDE, *Electromagnetic Theory*, **1**, 20f.

225 MAXWELL, *Papers*, **2**, 262.

226 MAXWELL, *Treatise*, § 12 (see note 166).

227 HEAVISIDE, *op. cit.* (note 224), 30.

228 *Ibid.*, iv.

229 For example, both HEAVISIDE and HERTZ eliminated the vector potential (see notes 197 and 222). HERTZ's theory differed from HEAVISIDE in that HERTZ eliminated displacement as an independent quantity (see note 223).

230 Thus, in HEAVISIDE's paper "The General Solution of Maxwell's Electromagnetic Equations in a Homogeneous Isotropic Medium, especially in regard to the Derivation of special solutions, and the Formulae for Plane Waves", *Phil. Mag.*, 1889, **27**, 29—50; *Electrical Papers*, **2**, 468—485, the equations of the field were expressed as operator equations between curl $\boldsymbol{H}$ and $\boldsymbol{E}$ and curl $\boldsymbol{E}$ and $\boldsymbol{H}$, *Electrical Papers*, **2**, 468. *Cf.* HERTZ's formulation (see note 223). HEAVISIDE employed vector notation. HEAVISIDE emphasized that in his formulation "the electric and magnetic sides of electromagnetism are symmetrically exhibited and connected", *Electromagnetic Theory*, **1**, iiif.

231 For example, HEAVISIDE's two circuital laws (*ibid.*, 34f.), which gave the field equations, correspond to the first two theorems of MAXWELL's 1868 "Note", though for HEAVISIDE these laws were not founded on a lines of force view.

VIII

Further consideration must now be given to an aspect of MAXWELL's thought in the *Treatise* which has already been noted but which raises a number of problems. This is the question of MAXWELL's conception of dynamical explanation, for MAXWELL argued that the dynamical theory of the *Treatise* was not a complete explanation of the phenomena of electromagnetism[232]. MAXWELL's discussion of this problem is particularly important in that the difficulties he discussed may well have contributed to his abandonment of the dynamical explanation of the *Treatise* — and of "Dynamical Theory" — and his return to the lines of force view in the *Elementary Treatise.* In the *Treatise* MAXWELL emphasized that according to the theory employed in the *Treatise* electrical action was "a phenomenon due to an unknown cause", but that in a complete theory of electromagnetism a current would be represented as "the result of known motions of known portions of matter, in which ... the whole intermediate mechanism and details of the motion, are taken as the objects of study"[233]. In saying this MAXWELL pointed out that "a knowledge of these things would amount to at least the beginnings of a complete dynamical theory of electricity"[234]. Though he realized that any number of mechanical models could be constructed so as to represent the phenomena, his desire to achieve a complete explanation of the phenomena led him to consider the possibility of mechanical construction.

However, there is evidence that MAXWELL envisaged certain problems in this mode of representation, in addition to the difficulty that an infinite number of possible mechanical models could be constructed. This evidence consists in a number of passages in the first edition of the *Treatise* which MAXWELL deleted in preparing the second edition for the press, taken in conjunction with certain manuscripts and passages from other published work. In the first edition of the *Treatise* he argued that rather than assume action at a distance, the action could be accounted for by means of the intermediate connexions in the medium, which led to a theory of what he called "internal forces". The internal forces were assumed to act between the particles across distances which though insensible were finite. He went on to make the crucial point that "the observed action at a considerable distance is therefore explained by means of a great number of forces acting between bodies at very small distances, for which we are as little able to account as for the action at any distance however great"[235]. Thus, MAXWELL recognized that to replace forces acting between macro-bodies across sensible distances by internal forces acting between micro-particles was to replace one unknown by another. He admitted this and went on to say that "by establishing the necessity of assuming these internal forces ... we have advanced a step ... which will not be lost, though we should fail in accounting for these

[232] This has been noted by JOSEPH TURNER, "Maxwell on the Logic of Dynamical Explanation", *Philosophy of Science*, 1956, **23**, 36—47. However, MAXWELL's discussion of this question went beyond the idea that a dynamical explanation must be provided. See below.

[233] MAXWELL, *Treatise*, § 574.

[234] *Ibid.*

[235] *Treatise*, § 105.

internal forces"[236]. MAXWELL did not adopt FARADAY's solution of, as he put it, "representing lines of force belonging to a body as in some sense part of itself"[237], in other words speaking of matter extending continuously throughout space by means of its forces, but he retained the particulate approach of the *Treatise*, and postulated the existence of internal forces — which were themselves unexplained — between micro-particles.

In passages which remained in the second edition, MAXWELL went on to say that he had supposed the medium in a state of stress but that he had not explained the nature of this stress[238]. However, he stated that he believed the "next step" to be important, "to account by mechanical considerations for these stresses in the dielectric" and he added that "I therefore leave the theory at this point"[239]. Taken together with the earlier passages as they appeared in the first edition, this next step was the explanation of the internal forces. The implication, in the *Treatise*, was that the internal forces could be defined by the equations of dynamics, for MAXWELL did discuss the possibility of achieving a complete dynamical theory in the *Treatise*, and he said that such a theory was an ultimate explanation of the phenomena[240]. Thus, the implication was that the next step consisted in obtaining some kind of mechanical model, despite his explicit realization that an infinite number of such models could be constructed and that there was still a problem in explaining the nature of the internal forces acting between micro-particles across insensible distances. However, MAXWELL argued that whatever the nature of these forces the LAGRANGEAN formalism of dynamics enabled the nature of the internal forces to be ignored, for he stated that he had assumed the medium to be a moving system "the motion being communicated from one part of the system to another by forces, the nature and laws of which we do not yet even attempt to define, because we can eliminate these forces from the equations of motion by the method given by Lagrange for any connected system"[241]. In other words the problem of the nature of the internal forces could be avoided, for the LAGRANGEAN formalism of dynamics enabled their "nature and laws" to be ignored[242].

Nevertheless, even though the problem could be avoided MAXWELL did not abandon all discussion of the difficulty. In a manuscript on the "Dimensions of Physical Quantities" — probably dating from the early 1870's — MAXWELL distinguished between macro- and micro-phenomena, for he wrote that "when

[236] *Treatise*, § 107.
[237] *Treatise*, § 529.
[238] *Treatise*, § 110.
[239] *Treatise*, § 111.
[240] See above. See his remark that "we cannot conceive any further explanation [than a dynamical explanation] to be either necessary, desirable, or possible" (note 163).
[241] *Treatise*, § 552.
[242] Thus, the problem could be avoided by employing the LAGRANGEAN formalism of dynamics. HERTZ was to make a similar point in arguing that the problems of the nature of charge, displacement, and electricity could be avoided. Thus, he stated that "we have accumulated round the term ... 'electricity' more relations than can be completely reconciled amongst themselves", and though "these painful contradictions are removed" when displacement is eliminated "the question as to the nature of [electricity] ... will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions", *Principles of Mechanics* (London, 1899), 7f.

we come to deal with very small quantities of matter its properties begin to be different from those observed in large masses ... the forces which we call molecular begin to show themselves acting in a different manner from those forces which are alone sensible in their action on great masses. There is therefore a real distinction between very small and very large bodies in nature"[243]. There was a difference between the individual molecule and the forces with which it was associated, and a sensible mass of molecules and the forces — the observed forces — which were associated with them. Though he believed that energy conservation applied both to micro- and to macro-bodies[244], the nature of the molecular forces remained an unexplained problem for him; hence his remark that a satisfactory theory of the magnetic action on light required knowledge of "something more definite about the properties which must be attributed to a molecule"[245]. In this manuscript MAXWELL was questioning the assumption that the equations of dynamics could be employed to describe the motions of insensible particles, for the explicit inference was that molecular forces were of a totally different kind from the forces between sensible bodies. He made a similar remark in his essay on "Science and Free Will" in 1873, where he stated that "a constituent molecule of a body has properties very different from those of the body to which it belongs"[246]. This distinction[247] between sensible and insensible bodies can also be seen in his article on "Atom", dating from about 1875, where he referred to the notion that two atoms could not coincide as "an unwarrantable concession to the vulgar opinion that two bodies cannot co-exist in the same place. This opinion is deduced from our experience of the behaviour of bodies of sensible size, but we have no experimental evidence that two atoms may not sometimes coincide"[248]. These

[243] MAXWELL, MS on "Dimensions of Physical Quantities" (U.C.L. Add. MSS. 7655).

[244] This can be seen from his argument against LE SAGE's hypothesis of "ultramundane corpuscles" to explain gravitation, that it involved a "constant expenditure of work", *Papers*, **2**, 490. Thus, even "ultramundane corpuscles" were subject to energy conservation, for according to LE SAGE's theory "the habitable universe, which we are accustomed to regard as the scene of a magnificent illustration of the conservation of energy as the fundamental principle of all nature, is in reality maintained in working order only by an enormous expenditure of external power", *ibid.*, 477.

[245] *Treatise*, § 830.

[246] CAMPBELL & GARNETT, *Life of Maxwell* (1st ed., 1882), 439.

[247] His remarks on this problem clearly derived from his concern with this problem in his work on the kinetic theory of gases. See my paper "Molecular Forces, Statistical Representation, and Maxwell's Demon", forthcoming in *Studies in History and Philosophy of Science*. However, these remarks also relate to his remarks on "internal forces" in the *Treatise*, so in his work on both electricity and gas theory he was concerned with the problem of molecular forces.

[248] MAXWELL, *Papers*, **2**, 448. MAXWELL was here denying NEWTON's third Rule of Philosophizing, the "foundation of all philosophy", according to which the conclusion that "impenetrability ... was a universal property of all bodies whatsoever" was held to depend on the "analogy of Nature", *Sir Isaac Newton's Mathematical Principles of Natural Philosophy*, (trans.) MOTTE-CAJORI (Berkeley, 1934), 398f. The third Rule had been discussed by WILLIAM WHEWELL, who found it to be "a mode of reasoning far from conclusive", *Philosophy of the Inductive Sciences* (2nd ed., 2 vols., London, 1847), **2**, 289. WHEWELL remarked that according to NEWTON "the properties of bodies depend on the attractions and repulsions of the particles. Therefore, among other properties of bodies, their hardness depends on such forces. But if the hardness

remarks, which date from the same period as the first edition of the *Treatise*, clearly have relation to his statements on internal forces in that edition. Thus, the internal forces between micro-particles were of a different nature to the forces between sensible bodies. The consideration of these problems in the *Treatise* may well have contributed to MAXWELL'S decision to depart from the dynamical theory of the *Treatise* and to return to the lines of force approach in the *Elementary Treatise*, even though consideration of the "nature and laws" of the internal forces could be avoided in using the LAGRANGEAN formulation of dynamics.

IX

MAXWELL'S statement in the *Treatise* that he had provided a mathematical expression for FARADAY'S physical ideas was an over-modest account of his achievement. He had certainly done this, but he had succeeded in accomplishing a great deal more. Though his theories were structured by an adherence to physical concepts first introduced by FARADAY, these concepts were transformed by MAXWELL; for in striving to represent FARADAY'S notions mathematically MAXWELL was led to develop theories of far greater physical subtlety and power than any that FARADAY had proposed. The very complexity of his physical ideas led his successors to delineate crucial ambiguities in his work, and — though there were other central problems in his thought — it seems clear that the efforts of his successors to achieve formulations of "Maxwell's theory" involved attempts to develop a clear understanding of the dichotomy in MAXWELL'S thought between lines of force and particulate polarization. In this paper it has been argued that this conceptual dichotomy — which had its roots in concepts employed by FARADAY — underlies the development of MAXWELL'S thought.

The research reported here was done in the Department of Philosophy, University of Leeds.

of the bodies depends upon the forces, the repulsion, for instance of the particles, upon what does the hardness of the particles depend? What progress do we make in explaining the properties of bodies, when we assume the same properties in our explanation? and to what purpose do we assume that the particles *are* hard?", *ibid.*, **1**, 432. A very different view was taken by JAMES CHALLIS in a long series of papers during the 1850s and 1860s in which CHALLIS was concerned with the problem of molecular forces. For example, see CHALLIS, "A Theory of Molecular Forces", *Phil. Mag.*, 1860, **19**, 88—102. He gave a clear statement of his adherence to the third Rule in his paper "On Newton's 'Foundation of all Philosophy'", *ibid.*, 1863, **26**, 280—292, where he stated that "the experience of the *senses* relative to masses is necessary and sufficient for revealing to us the universal properties of the ultimate constituents of the masses", *op. cit.*, 282. CHALLIS told MAXWELL that his theories were "strictly within the rules of the Newtonian principles of Philosophy", CHALLIS to MAXWELL, 10 June 1861 (U.L.C. Add. MSS. 7655). NEWTON'S arguments on the third Rule are discussed in a number of papers by J. E. MCGUIRE. See "The Origin of Newton's Doctrine of Essential Qualities", *Centaurus*, 1968, **12**, 233—260; "Atoms and the 'analogy of Nature': Newton's third Rule of Philosophizing", *Studies in History and Philosophy of Science*, 1970, **1**, 1ff.

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