Fortschr. Phys. 34 (1986) 7, 457–503

The Ampere-Neumann Electrodynamics of Metallic Conductors

PETER GRANEAU

Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

Abstract

This is a review of the old electrodynamics which prevailed during the first half of the 165-year history of electromagnetism. Ampere's principal achievement was the deduction of his empirical force law from experiments with several current balances. Faraday then discovered electromagnetic induction. This prompted F. E. Neumann to work out a quantitative explanation of induction based on Ampere's force law. It involved the concept of the electrodynamic potential which, as we know now, is the same entity as magnetic energy. With the newtonian principle of virtual work, Neumann found his potential yielded the correct mechanical forces on metallic current circuits. Neumann's theory contains a physical quantity which today is called the magnetic vector potential and treated as a mathematical contrivance.

Neumann's mutual inductance formula has become a powerful tool of inductance calculations. Maxwell made a major contribution to the Ampere-Neumann electrodynamics by developing the mean-geometric-distance method for calculating the inductance of conductors of finite cross-sections. This became particularly useful after Sommerfeld solved Neumann's double integral for parallel, straight wires. Maxwell built all of Neumann's mathematical theory into his field equations but the lingo changed. Electrodynamic potential became kinetic energy of the field; conductor element interactions became flux linkage; and so on. Maxwell's equations do not contain a magnetic force law. He believed both Ampere's law and the law currently in use, which was first suggested by Grassmann in 1845, were compatible with field theory. Lorentz later found that the motion of charges in vacuum obeyed only Grassmann's law and not Ampere's. From then onward the old electrodynamics fell into disuse and field theory has reigned supremely ever since.

Recent developments have shown the conflict between Ampere's and Grassmann's law to be related to the nature of the electric current. Conduction currents in metals obey Ampere's law and convection currents in vacuum obey Grassmann's law. Both laws agree on the reaction forces between closed metallic circuits, because the relativistic contribution from the Grassmann law then integrates to zero. This fact appears to have mislead Lorentz in believing that the drifting electron in vacuum is magnetically equivalent to the current element of metals.

An examination of the long debate concerning the validity of Newton's third law of motion in electromagnetism proves the Ampere-Neumann electro-dynamics to be valid for metallic circuits while the theory of special relativity and field momentum conservation are required for convecting charges in vacuum. This conclusion is strongly supported by experimental evidence. It demands a change in the concept of the metallic current element.

Contents

1. Introduction

The re-discovery of longitudinal, mechanical forces of electromagnetic origin acting along current streamlines in metallic conductors [1] has created interest in the original electromagnetic theory founded by AMPERE in France and F. E. NEUMANN in Germany during the first half of the nineteenth century [2, 3]. Relativistic electromagnetism has difficulties in explaining this type of experiment without contravening Newton's third law [4] and momentum conservation [5] and thereby implying the possibility of hardly credible means of space propulsion. It is the freedom from these conflicts which provides the incentive for re-examining the old electrodynamics.

The Ampere-Neumann theory is strictly limited to electric current phenomena occurring in metallic conductors. But in this restricted area it rests on a strong empirical basis which makes it virtually infallible. A child of newtonian mechanics, it shares with the latter theory the wide applicability to technology. Curiously, the restriction to metallic conductors now appears to open a door through which the old electrodynamics may be linked to modern solid state physics. It even holds out some hope for a deterministic quantum mechanics. The crucial new component in this undertaking is a better understanding of the nature of the current-element. Longitudinal mechanical forces are inconsistent with the hypothesis that the current-element is merely a moving electron. The basic element must also involve the metal ion and therefore the lattice of solid conductors.

Ampere's and Neumann's original papers have not been translated into English. Modern textbooks ignore the old electrodynamics. At least two or three generations of physicists and engineers have been educated without any knowledge of it. This paper has been written to provide a very abbreviated treatment of the theory while, at the same time, outlining its modern development.

2. Ampere's Force Law

In 1820 Ampere set out to create an electromagnetic theory in the newtonian tradition. For this he required a fundamental law of particle interaction. He suspected this would turn out to be an inverse square law, akin to those of Newton and Coulomb which — to use his own words in English translations [6] — "... opened a new highway into the sciences which have natural phenomena as their object of study".

However difficult it may appear today, the question of what constitutes the elementary particle of electrodynamics apparently posed no problem to Ampere, nor to his contemporaries Biot and Savart. They concurrently plunged for the *current-element.* It is not certain who may have thought of this concept first. Ampere clearly recognized that, unlike the elementary particles of gravitation and electrostatics, which have the scalar magnitudes of mass or charge, the current-element would be a more complex entity, having a direction and a magnitude which is the product of length and current.

On the basis of his first electrodynamic experiments showing the attraction and repulsion of straight and parallel current-carrying wires, Ampere was led to believe that the mechanical force $\Delta F_{m,n}$ between two current-elements $i_m dn$ and $i_n dm$ was to be of the general form

$$\Delta F_{m,n} = -i_m i_n (dm \cdot dn/r_{m,n}^2)\, F(\alpha, \beta, \varepsilon) \tag{1}$$

where $r_{m,n}$ is the distance between the elements and F a function of the three angles α, β and ε, representing the inclinations of the two elements to the distance vector while ε is the inclination between the elements themselves. If $F(\alpha, \beta, \varepsilon)$ is positive, the force is negative which Ampere meant to stand for attraction whreas a positive force indicated repulsion.

With respect to the proportionality of the elemental force to the lengths and currents of the two elements Ampere said:

"First of all, it is evident that the mutual action of two elements of electric current is proportional to their lengths; for assuming them to be devided into infinitesimal equal parts along their lengths, all attractions and repulsions of these parts can be regarded as directed along one and the same straight line, so that they necessarily add up. This action must also be proportional to the intensities of the two currents."

The inverse square factor $(1/r_{m,n}^2)$ Ampere confirmed with his three-circle experiment shown in fig. 1. For the sake of clarity this diagram does not show the current leads to the three parallel and coaxial current circles which are arranged in vertical planes. Go-and-teturn leads were kept close together for minimum disturbance of the measured forces. The three circles were connected in series to ensure equal current intensity in all of them. Ampere's method of compensating for the effect of the earth's magnetic field has also been omitted in fig. 1. The radii of, and the distances between, the three circles

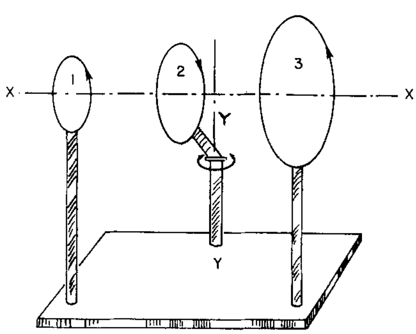


Fig. 1. Ampere's three-circle experiment

were chosen such that the geometric relationship of circle 1 to circle 2 was similar to the relationship of circle 2 to circle 3. In other words, the only difference between the 1—2 and 2—3 combinations was the linear scale factor. Circles 1 and 3 were fixed to the laboratory frame, while circle 2 was held at the level of the other two but with its insulator arm free to rotate about the vertical axis YY. The experiment proved that, if the currents in 1 and 3 encircle the common axis XX in the same direction, circle 2 will remain stationary, it being either attracted or repelled equally strongly by the two adjacent wire circles. When the currents in circle 2 flows in the opposite sense of the two other currents, the mutual forces were attractions.

This experiment confirms that the reciprocal forces between two current loops are independent of the size of the loops. This must also be true for all elemental forces into which the total force may be divided. Hence the geometrical factor $dm \cdot dn/r^2_{m,n}$ of equ. (1) must indeed be the dimensionless number furnished by the inverse square law.

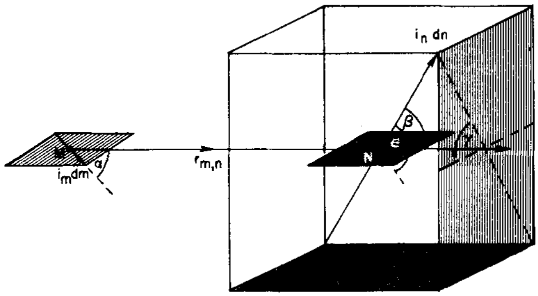


Fig. 2. Angles in Ampere's force formula

Ampere's most challenging task proved to be the determination of the angle function of equ. (1). The three angles are defined by fig. 2. On this diagram M and N are the locations of two unequal current-elements. The distance between them should be treated as a vector. It may be chosen to point from M to N or vica versa. The current-elements are also vectors pointing in the direction of current flow. The angle between $i_m dm$ and $r_{m,n}$ is α. Similarly, the angle between $i_n dn$ and $r_{m,n}$ is β and the inclination of the two elements toward each other is ε.

Each element combined with the distance vector lies in a different plane. These two planes intersect in $r_{m,n}$. Of the two complimentary angles between the planes, γ is that angle through which one plane would have to be turned in order to make the components of the current-elements which are perpendicular to $r_{m,n}$ point in the same direction.

To see how Ampere determined $F(\alpha, \beta, \varepsilon)$, we resolve the two current-elements of fig. 2 into their cartesian components, as shown in fig. 3. These components are given by

$$\begin{aligned} &m(x) = i_m dm \cos\alpha; \quad m(y) = i_m dm \sin\alpha; \quad m(z) = 0; \\ &n(x) = i_n dn \cos\beta; \quad n(y) = i_n dn \sin\beta \cos\gamma; \quad n(z) = i_n dn \sin\beta \sin\gamma. \end{aligned} \tag{2}$$

Now each component of m interacts with each component of n, resulting in a total of six contributions to the elemental force between two current-elements. According to a rule first mentioned by Ampere, four of them are zero. Ampere explained the rule as follows:

"An infinitely small portion of current exerts no action on another infinitely small portion of a current which is situated in a plane which passes through the midpoint and which is perpendicular to its direction. In fact, the two halves of the first element produce equal actions on the second, the one attractive and the other repellent, because the current tends to approach the common perpendicular in one of these halves and to move away from it in the other. These two equal forces form an angle which tends to two right angles according as the element tends to zero. Their resultant is therefore infinitesimal in relation to these forces and in consequence it can be neglegted in the calculations."

Accordingly, the four vanishing force contributions of the element components in fig. 3 are

$$\Delta F_{m(x),n(y)} = \Delta F_{m(x),n(z)} = \Delta F_{m(y),n(x)} = \Delta F_{m(y),n(z)} = 0. \tag{3}$$

A corollary of Ampere's rule is that the mechanical interaction of two current-elements arises from two sets of parallel element components, one of them being the set

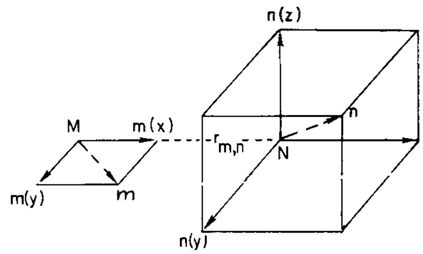


Fig. 3. Resolved-component vector representation of the two general current elements of fig. 2

which lies along the line connecting the elements, and the other is the set perpendicular to that line. Ampere then assumed that the two non-vanishing force contributions may be expressed by

$$\Delta F_{m(y),n(y)} = -m(y)\, n(y)/r^2_{m,n} \tag{4}$$

$$\Delta F_{m(x),n(x)} = -km(x)\, n(x)/r^2_{m,n} \tag{5}$$

where k is a numerical constant and the element components are defined by equ. (2).

With (4) and (5), equ. (1) may now be written

$$\Delta F_{m,n} = -i_m i_n (dm \cdot dn/r^2_{m,n}) (\sin \alpha \sin \beta \cos \gamma + k \cos \alpha \cos \beta). \tag{6}$$

Ampere then introduced the trigonometric equation

$$\cos \varepsilon = \cos \alpha \cos \beta + \sin \alpha \sin \beta \cos \gamma \tag{7}$$

which he proved with reference to a spherical triangle. But this relationship may also be derived from the direction cosines of the two general current-elements. These cosines are

for $i_m dm$: $\cos \theta_{x,m} = \cos \alpha$; $\cos \theta_{y,m} \sin \alpha$; $\cos \theta_{z,m} = 0$;

for $i_n dn$: $\cos \theta_{x,n} = \cos \beta$; $\cos \theta_{y,n} = \sin \beta \cos \gamma$; $\cos \theta_{z,n} = \sin \beta \sin \gamma$.

Equation (7) then follows from the rule

$$\cos \varepsilon = \cos \theta_{x,m} \cos \theta_{x,n} + \cos \theta_{y,m} \cos \theta_{y,n} + \cos \theta_{z,m} \cos \theta_{z,n}.$$

With equ. (7) the force formula may be transformed to

$$\Delta F_{m,n} = -i_m i_n (dm \cdot dn/r^2_{m,n}) \{\cos \varepsilon + (k - 1) \cos \alpha \cos \beta\}. \tag{8}$$

After this step Ampere converted the cosines to partial differentials of $r_{m,n}$ with respect to small displacements of the position M and N along the line of action. The partial differentials are further defined by fig. 4. In the limit as the displacements tend to zero, and writing r for $r_{m,n}$, we find that

$$\cos\alpha = \partial r/\partial m; \qquad \cos\beta = -\partial r/\partial n. \tag{9}$$

Furthermore, if the coordinates of M and N are x_m, y_m, z_m and x_n, y_n and z_n, we have

$$r^2 = (x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2. \tag{10}$$

Differentiating this with respect to m results in

$$r(\partial r/\partial m) = (x_m - x_n)(\partial x_m/\partial m) + (y_m - y_n)(\partial y_m/\partial m) + (z_m - z_n)(\partial z_m/\partial m) \tag{11}$$

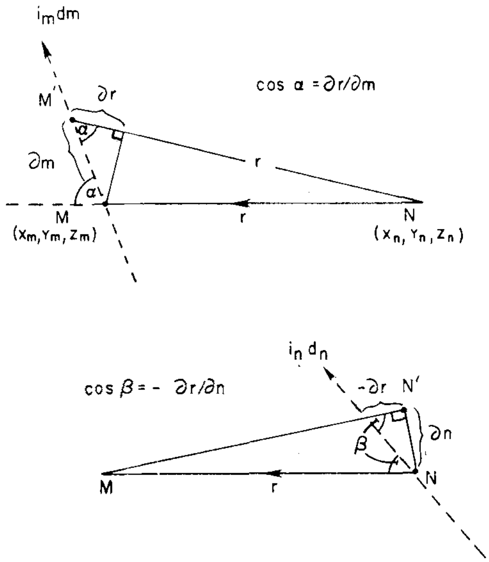


Fig. 4. Partial differentials of the distance vector with respect to the displacement of M and N in the direction of the current elements

and a second differentiation with respect to n results in

$$r(\partial^2 r/\partial m\,\partial n) + (\partial r/\partial m)(\partial r/\partial n) = -(\partial x_m/\partial m)(\partial x_n/\partial n) - (\partial y_m/\partial m)(\partial y_n/\partial n) - (\partial z_m/\partial m)(\partial z_n/\partial n). \tag{12}$$

But the right-hand side of equ. (12) consists of the negative products of the direction cosines of the two current-elements. Therefore

$$\cos\varepsilon = -r(\partial^2 r/\partial m\,\partial n) - (\partial r/\partial m)(\partial r/\partial n). \tag{13}$$

Substituting equs. (9) and (13) into the force equation (8) transforms the latter to

$$\Delta F_{m,n} = i_m i_n (dm \cdot dn/r^2)\,\{r(\partial^2 r/\partial m\,\partial n) + k(\partial r/\partial m)(\partial r/\partial n)\}. \tag{14}$$

This may also be written

$$\begin{aligned}\Delta F_{m,n} &= i_m i_n (dm \cdot dn/r^2)\,(1/r^{k-1})\,(\partial/\partial n)\,\{r^k(\partial r/\partial m)\}\\ &= i_m i_n r^{-(k+1)}(\partial/\partial n)\,\{r^k(\partial r/\partial m)\}\,dm\,dn.\end{aligned} \tag{15}$$

Ampere then invokes the result of another of his null-experiments to determine the value of k. The essential features of this experiment are shown in fig. 5. It will be referred to as Ampere's wire-arc experiment. It proves the mechanical force on a circular arc section of a current-carrying circuit 1, due to a separate closed circuit 2 of any shape and disposition, to be entirely perpendicular to the arc. As shown in fig. 5, Ampere floated the arc section on two mercury throughs and left it free to rotate on the insulator arm OX about the pivot 0. As circuit 2 was brought up to it and moved around, the arc remained stationary. From this Ampere concluded that the tangential force on the wire arc was zero.

Taking equ. (15) and substituting for $\partial r/\partial m$ from equ. (9) gives

$$\Delta F_{m,n} = i_m i_n dm\, r^{-(k+1)}(\partial/\partial n)\,(r^k \cos\alpha)\, dn. \tag{16}$$

The component of this mutual force acts tangentially on $i_m dm$ is obtained by multiplying equ. (16) by cos α. In order to agree with the wire arc experiment, this tangential

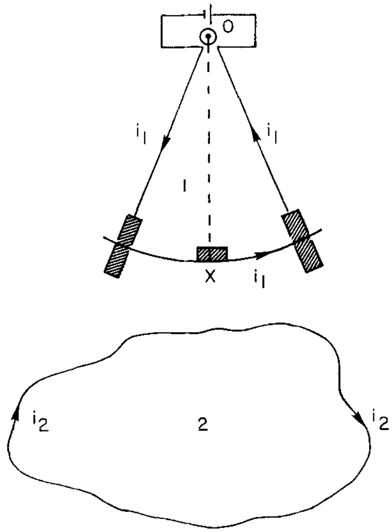


Fig. 5. Ampere's wire-arc experiment

force, when integrated over all elements $i_n dn$ on circuit 2, must come to zero. Hence

$$\int_2 \Delta F_{m,n} \cos\alpha = i_m i_n dm \int_2 r^{-(2k+1)}\, r^k \cos\alpha\,(\partial/\partial n)\,(r^k \cos\alpha)\, dn = 0. \tag{17}$$

For integration by parts we let $\int u\, dv = uv - \int v\, du$

$$u = r^{-(2k+1)}; \qquad \partial u/\partial n = -(2k+1)\, r^{-2(k+1)}(\partial r/\partial n)$$

$$v = 0.5\, r^{2k} \cos^2\alpha; \qquad dv = r^k \cos\alpha\,(\partial/\partial n)\,(r^k \cos\alpha)\, dn.$$

Therefore

$$\int_2 \Delta F_{m,n} \cos\alpha = 0.5 i_m i_n dm \{\cos^2\alpha/r\}_{n'}^{n} + (2k+1) \int_2 (\cos^2\alpha/r^2)\, dr\}. \tag{18}$$

The limits n and n' of the first term are adjacent infinitely short elements on circuit 2 and therefore the first term of equ. (18) vanishes. However, as Ampere did point out, many closed circuits can be imagined for which the integral in the second term of equ. (18) will not be zero. To comply with the wirearc experiment we must then have

$$2k + 1 = 0 \quad \text{or} \quad k = -(1/2). \tag{19}$$

Using this value of k in equ. (8), Ampere expressed his force law by

$$\Delta F_{m,n} = -i_m i_n (dm \cdot dn/r_{m,n}^2) \{\cos \varepsilon - (3/2) \cos \alpha \cos \beta\}. \tag{20}$$

This defined his electrodynamic unit of current in terms of a mechanical force. Later the electromagnetic unit of current (absolute ampere) was introduced. Its measure was given by

$$1 \text{ absolute ampere} = \sqrt{2} \cdot 1 \text{ electrodynamic unit of current.}$$

This converted Ampere's law to the modern form

$$\Delta F_{m,n} = -i_m i_n (dm \cdot dn/r_{m,n}^2) (2 \cos \varepsilon - 3 \cos \alpha \cos \beta). \tag{21}$$

It gives the elemental force in dyn provided the currents are expressed in absolute ampere.

Ampere successfully tested his law with many different experiments. In 160 years it has never been proved wrong when applied to metallic circuits, for which it was devised. This makes it an empirical law, like Newton's law of gravitation. Ampere's method of deriving it is only of academic interest. The proof of the law rests on a firm experimental basis. But it must be remembered that the empirical basis does not extend to currents outside metallic substances, e.g. in plasma and vacuum.

One more of Ampere's experiments should be mentioned because of the attention it received over the years. Before considering this we note that when two current-elements lie on the same straight line and point in the same direction, we have $\cos \varepsilon = \cos \alpha = \cos \beta = 1$. The interaction force then is

$$\Delta F_{m,n} = i_m i_n (dm \cdot dn/r_{m,n}^2). \tag{22}$$

This is always positive and therefore represents repulsion. As a consequence of equ. (22) current-carrying wires should find themselves in tension and jet-reactions should arise at places where the current passes through a liquid-solid metal interface. Both these phenomena have been observed [1, 7].

Ampere's sketch of the apparatus with which he demonstrated the action of longitudinal forces is shown in fig. 6. $ABCD$ was a mercury-filled dish with the liquid metal being divided in two pools by the insulation barrier AC. Current leads m ans s dipped into the pools and a current source — which at Ampere's time would have been a collection of galvanic cells — had to be connected to the terminals E and F. An insulated copper wire $npqr$ with bare ends r and n, in the shape of a hairpin, was floating on the mercury with the legs straddling the insulation barrier and the bend passing over it.

Ampere found, as he expected, that when the current was turned on the hairpin would float away from the terminals toward C. He ascribed this primarily to the longitudinal reaction forces between the current in the mercury portions of the circuit and the legs of the hairpin.

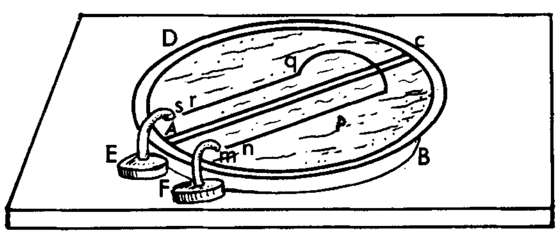


Fig. 6. Ampere's sketch of an experiment demonstrating the action of longitudinal forces

In today's electromagnetic theory the motion of the hairpin has to be attributed to the Lorentz force acting on the bend and reacting against the field [8]. The jets of mercury streaming away from the hairpin ends must then also be caused by Lorentz forces on the diverging current streamline pattern in the liquid metal reacting, not against the hairpin, but the magnetic field. The notion that the magnetic field — that is vacuum — can support large mechanical forces is in conflict with Newton's third law which requires a force balance on elements of matter. With devices such as railguns [9] the vacuum would be required to sustain many tons of thrust. To believe this to be true manifests extraordinary faith in a man-made theory.

Pappas [5] discovered another difficulty with the field theoretic explanation of Ampere's hairpin experiment. He measured the momentum imparted to the hairpin by a pulse of current. Then he calculated the energy that has to be placed in the field for 'non-material' momentum conservation. It turned out to be much more energy than his battery could possibly have supplied. Furthermore, the current required to dispatch this energy from the circuit to the field should have destroyed the conductors, but they easily survived.

3. Neumann's Electrodynamic Potential and Virtual Work

Twenty years elapsed between the conclusion of Ampere's work and F. E. Neumann's first memoir in 1845. In the meantime Faraday [10] had discovered both electromagnetic and electrostatic induction. In searching for a quantitative explanation of electromagnetic induction in terms of Ampere's force law, Neumann discovered the significance of a quantity which may be written

$$P_{m,n} = -(1/2)\, i_m i_n \int_m \int_n (\cos \varepsilon / r_{m,n})\, dm \cdot dn . \tag{23}$$

This he called the electrodynamic potential of two circuits m and n carrying currents i_m and i_n. The double integral involves each $dm \cdot dn$ pair of elements twice while Ampere's theory required them to interact only once. This is taken into account by the factor of (1/2).

Neumann is best remembered by his mutual inductance formula

$$M_{m,n} = -\int_m \int_n (\cos \varepsilon / r_{m,n})\, dm \cdot dn \tag{24}$$

which arises directly from the electrodynamic potential. In equ. (24) the (1/2) factor has been dropped but it must be remembered that element interactions must not be counted twice. Today virtually all precise inductance calculations are based on Neumann's formula (24). It has been dealt into field theory by interpreting $M_{m,n}$ as the magnetic flux linking the two closed curves.

Comparing the electrodynamic potential (23) with Ampere's force law (21) it will be seen that the dimensions of the potential are force $\times$ distance $=$ work or energy. Any change in the currents or the relative position of the current-elements requires energy transfer. If the positions of the circuits change such that the potential increases, then work has to be done by a mechanical source and magnetic energy will be stored. Conversely, if the potential is reduced, stored energy will be transformed to mechanical work or Joule heat or both.

Energy as such was not considered in Neumann's mathematical analysis. $P_{m,n}$ appears to be potential energy. Although both Ampere and Neumann used the term electric *current*, neither of them ascribed to it momentum, as Maxwell would do later.

Between his first and second paper [3] NEUMANN changed his mind about the sign of equ. (23). In potential theory there have always existed difficulties in agreeing on a universal sign convention. KELLOG [11] points out that the most popular rule is to assign negative potential energy to elements of like sign which attract each other and positive potential energy to elements of like sign which repel each other. Gravitating particles are an example of the former class and electrical charges are an example of the latter. But current-elements of certain orientations repel each other and in other orientations the mutual force is attraction. Anyway, what is meant by negative potential energy? We cannot conceive of less than no energy. Hence positive and negative energy must

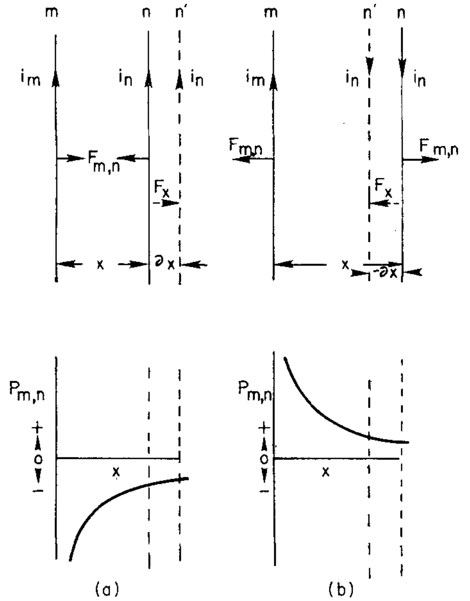


Fig. 7. Electrodynamic potential of straight and parallel currents in wires

be two kinds of energy, like positive and negative charge are two kinds of electricity. One kind of potential energy has to be associated with forces of attraction and the other with repulsion.

To illucidate this point further, let us examine the particular case of two very long, parallel, straight wires m and n, as sketched in fig. 7. In case (a) they carry currents in the same direction and we know from experience that they will then attract each other. In potential theory they will be associated with negative energy. Now assume an externally applied force F_x tends to increase the distance of separation x and brings about the displacement ∂x by moving n to n'. This external force has to do work and expend an amount of energy equal to $F_x\, \partial x$. At first it may be thought that this energy is being added to the stored potential (magnetic) energy. But this cannot be so because the magnitude of $P_{m,n}$ decreases when n is moved to n' as a result of the lengthening of $r_{m,n}$ of every relevant element pair contributing to equ. (23). Not only does the mechanical source sustaining F_x supply energy, but the potential energy store also gives up energy. What absorbes these two streams of energy? As the currents are assumed to remain constant, no additional Joule heat will be dissipated in the wires. Therefore all this energy must flow to the two electrical current sources and relieve them from furnishing part of the Joule heat.

In the case of fig. (7b), where the currents flow in opposite directions and the conductors repel each other, the displacement ∂x from n to n' again requires the supply of energy by the mechanical source sustaining F_x. However now the magnitude of the stored energy increases because $r_{m,n}$ of every relevant element pair becomes shorter. It is now possible that all the energy provided by the mechanical source is being stored as potential energy, and then the electric current sources may not be involved in the energy transactions.

It was Neumann who originated the virtual work concept when he related the reciprocal force of repulsion or attraction between two circuits m and n to the mutual potential $P_{m,n}$ by

$$(F_{m,n})_x = -\partial P_{m,n}/\partial x \tag{25}$$

where x denotes a particular direction in which the virtual displacement ∂x takes place. Combining equs. (23), (24) and (25) we obtain the well-known formula

$$(F_{m,n})_x = i_m i_n \, \partial M_{m,n}/\partial x . \tag{26}$$

Furthermore, when $P_{m,n}$ refers to the mutual energy stored between two parts of the same circuit, the circuit having a selfinductance L and carrying the current i, we find

$$(F_{m,n})_x = i^2 \, \partial L/\partial x . \tag{27}$$

For the proof that Neumann's electrodynamic potential follows directly from Ampere's force law the reader is referred to Ref. [3]. It would take up too much room to reproduce it here.

Neumann's virtual work principle also predicts that the closed circuits m and n, carrying currents i_m and i_n, will exert a mutual torque upon each other. If one of the circuits is given a virtual angular displacement $\partial \Psi_x$ about an arbitrarily chosen x-axis, the mutual torque opposing the displacement is

$$(T_{m,n})_x = -\partial P_{m,n}/\partial \Psi_x = i_m i_n \, \partial M_{m,n}/\partial \Psi_x . \tag{28}$$

4. Neumann's Law of Electromagnetic Induction

Neumann started with the assumption that the induction of electromotive force (e.m.f.) is an interaction between conductor elements. He realized that it could not be a mutual or reciprocal interaction like the one underlying Ampere's force law because the element experiencing the induction might not carry any current with which it could react back on the element which caused the induction. So he reasoned that only the fraction $\Delta F_{m,n}/i_n$ of equ. (21) would be active in setting up an e.m.f. in the element dn through the action of $i_m dm$. He first considered the case of a motionally induced e.m.f. as indicated in fig. 8. There the conductor element dn moves with velocity v_x along the direction x re-

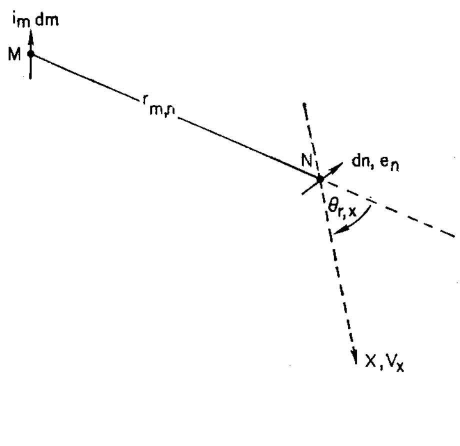


Fig. 8. Diagram for equation (29)

lative to the current-element $i_m dm$. From the experimental facts provided by Faraday and others, Neumann then found that it was $\Delta F_{m,n}/i_n$ multiplied by the relative velocity component parallel to the line connecting the elements which determined the induced e.m.f. Δe_n in the conductor element dn. With Lenz's law of the direction of induced e.m.f.'s he could write

$$\Delta e_n = -(\Delta F_{m,n}/i_n)\, v_x \cos \theta_{r,x}. \tag{29}$$

The angle $\theta_{r,x}$ is shown in fig. 8. Equation (29) is known as Neumann's law of induction.

If Δe_n initiates an induced current i_n in dn, then the power flow to this element is $\Delta e_n i_n$. This quantity Neumann equated to the rate of change of the electrodynamic potential, obtaining the far more general law of induction

$$\Delta e_n = (1/i_n)\,(\partial/\partial t)\,\Delta P_{m,n}. \tag{30}$$

Since

$$\partial r_{m,n}/\partial t = v_x \cos \theta_{r,x} \tag{31}$$

it can be shown easily that equ. (30) agrees with (29).

When calculating Δe_n not only for one inducing current-element $i_m dm$ but for all elements on a closed circuit, Neumann was first to prove [3] that the angle function $2 \cos \varepsilon - 3 \cos \alpha \cos \beta$ may be simply written $\cos \varepsilon$ because the closed path integral of $(\cos \varepsilon - 3 \cos \alpha \cos \beta)/r_{m,n}$ is zero. Therefore, if the inducing elements will eventually be integrated around a closed filament of which $i_m dm$ is one element, the induced e.m.f. per unit length may be expresses as

$$\Delta e_n/dn = -(\partial/\partial t)\,\{(i_m dm \cos \varepsilon)/r_{m,n}\}. \tag{32}$$

Now the magnetic vector potential $\boldsymbol{A}_{m,n}$ due to current-element $i_m \boldsymbol{dm}$ at the position of dn is

$$\boldsymbol{A}_{m,n} = i_m \boldsymbol{dm}/r_{m,n}. \tag{33}$$

But only the component $A_{m,n} \cos \varepsilon$ parallel to the element dn is effective in inducing the e.m.f. at the location of dn. Hence equ. (32) may be written

$$\Delta e_n/dn = -(\partial A_{m,n}/\partial t) \cos \varepsilon. \tag{34}$$

This shows that the magnetic vector potential is an essential part of the Ampere-Neumann electrodynamics.

When the two conductor elements dm and dn belong to the same closed circuit we may speak of selfinduction. As the elements do not interact with themselves, the e.m.f. computation does then not involve closed path integrals. This means in selfinductance calculations we are not justified to reduce the angle function to $\cos \varepsilon$. The most general form of equ. (32) for both mutual and selfinduction therefore is

$$\Delta e_n/dn = -(\partial/\partial t)\,\{(i_m dm/r_{m,n})\,(2 \cos \varepsilon - 3 \cos \alpha \cos \beta)\} \tag{35}$$

and the most general expression for the induced e.m.f. per unit length in terms of the vector potential becomes

$$\Delta e_n/dn = -(\partial A_{m,n}/\partial t)\,(2 \cos \varepsilon - 3 \cos \alpha \cos \beta). \tag{36}$$

So long as the inducing circuit is a closed curve (mutual induction), any quantity may be added to the vector potential of equ. (33) which has a zero closed-path integral. This arbitrariness underlying gauge invariance may be eliminated by redefining the elemental magnetic vector potential as follows

$$\begin{aligned} \boldsymbol{A}_{n,m} &= (i_m/r_{m,n}) \{\cos \varepsilon - (3 \cos \alpha \cos \beta/\cos \varepsilon)\} \boldsymbol{dm} \\ \boldsymbol{A}_{m,n} &= (i_n/r_{m,n}) \{\cos \varepsilon - (3 \cos \alpha \cos \beta/\cos \varepsilon)\} \boldsymbol{dn}. \end{aligned} \tag{37}$$

These last equations may be applied to any pair of conductor elements, in conjunction with equ. (34), regardless of whether they belong to separate closed circuits (mutual induction) or the same circuit (selfinduction). The rules of gauge invariance are then superfluous. The three angles α, β and ε are not independent of each other. One could be expressed in terms of the other two. Retaining the three angles has the advantage that everyone appears in a cosine which makes it unecessary to adhere to an angle sign convention.

Finally it should be pointed out that the relationship between Neumann's electrodynamic potential and the magnetic vector potential of a pair of *metallic* conductor elements is most clearly expressed by the dot products

$$\Delta P_{m,n} = -i_m \boldsymbol{dm} \cdot \boldsymbol{A}_{n,m} = -i_n \boldsymbol{dn} \cdot \boldsymbol{A}_{m,n}. \tag{38}$$

5. The Calculation of Inductances

The lasting value of Neumann's theory has been that, with equ. (24), it provides the basic tool for inductance calculations relating to metallic circuits. Even though this formula refers explicitely to mutual inductances, it is equally useful for determining selfinductances. The latter parameters are commonly interpreted as being special cases of mutual inductance with primary and secondary circuit merged into one conductor. Both quantities have the same dimension which in electromagnetic units is length. It is the natural dimension of mutual inductance which depends only on the length and disposition of lines in space. In contrast to this all selfinductance formulas apply to three-dimensional conductors. The reason for this will be revealed shortly.

Maxwell [12] made an important contribution to the Ampere-Neumann electrodynamics by inventing the mean-geometric-distance method of inductance calculation which is embodied in the following analysis of selfinductance. Consider a wire loop as shown in fig. 9, part of which may take the form of a coil. The loop current is assumed to

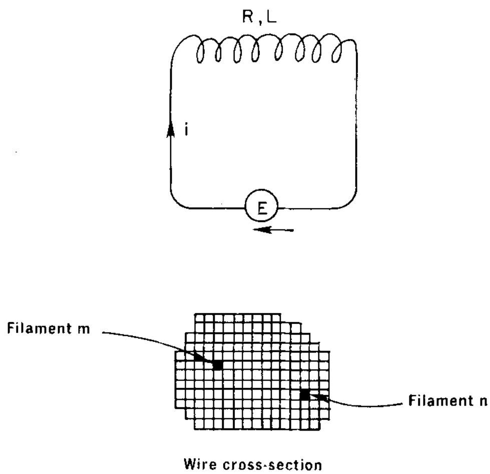


Fig. 9. Wire loop connected in series with an externally generated e.m.f.

be due to an externally generated e.m.f. E applied in series with the circuit. So long as no charge accumulation can occur in the circuit, i.e. when we are dealing with a metallic conductor, no restrictions need be placed upon the shape or size of the loop and its conductor cross-section, nor on the homogeneity of the material or the rate of change of the applied e.m.f.

If R is the series resistance of the loop and L its selfinductance, then the loop current i is determined by the well-known equation

$$iR = E - (d/dt)\,(Li)\,. \tag{39}$$

Let us now look at any two closed filaments m and n passing through any wire cross-section. All filaments must be very thin tubes of flow filled with conducting matter. In fig. 9 the filaments have square cross-sections, but other shapes could have been chosen provided they leave no empty space between filaments. According to the classical definition of a tube of flow, its cross-sectional area may vary along its length, but each conductor section must carry the same current. The current i_m flowing in filament m may be calculated from

$$i_m R_m = E \sum_n (d/dt)\,(M_{m,n} i_n) \tag{40}$$

where R_m is the resistance of the m-th filament and i_n the current in the n-th filament, while $M_{m,n}$ is the mutual inductance between the two general filaments. The summation in equ. (40) covers all possible positions n in the wire cross-section, *including* that position in which n coincides with m. This coincidence defines the selfinductance of an individual filament as

$$L_m = M_{m,m}\,. \tag{41}$$

Bearing in mind that

$$1/R = \sum_m (1/R_m)\,; \qquad i = \sum_m i_m = \sum_n i_n \tag{42}$$

equ. (40) can be solved for i_m. Summing this solution for all filament currents in accordance with equ. (42) gives

$$i = E \sum_m (1/R_m) - (d/dt) \sum_m (1/R_m) \sum_n M_{m,n} i_n\,. \tag{43}$$

Substitution of equ. (42) into (43) and multiplication by R results in

$$iR = E - (d/dt) \left\{ \sum_m (R/R_m) \sum_n M_{m,n} i_n \right\}. \tag{44}$$

A comparison of equ. (44) with (39) defines the selfinductance of the wire loop as

$$L = \sum_m (R/R_m) \sum_n M_{m,n} (i_m/i)\,. \tag{45}$$

This is the most general and exact expression of selfinductance in terms of the filament self and mutual inductances of which the closed metallic circuit is composed.

The expression for L takes on a more simple form when dealing with a wire of constant cross-section and uniform resistivity and the wire being very long compared to its cross-sectional dimension. All filaments are then substantially of the same length and they may all be made to have the same cross-section. The resistance and current ratios in equ. (45) then depend merely on the total number of filaments. If this number is g we have

$$R/R_m = i_m/i = 1/g\,. \tag{46}$$

This argument also implies that the current distribution over the conductor section is constant which will be approximately true for sufficiently slow rates of change of the applied voltage.

With equ. (46) the expression for the loop selfinductance reduces to

$$L = (1/g^2) \sum_m \sum_n M_{m,n}. \tag{47}$$

In this form L is seen to be the mean of all possible mutual inductance permutations of the g filaments, including a total of g combinations in which positions m and n coincide.

MAXWELL [12] recognized that the mutual inductance of a pair of straight parallel lines is largely a function of the logarithm of the distance of separation d. For the purpose of mutual inductance calculations he further assumed that each conductor filament of finite thickness could be represented by a line coinciding with the filament axis. Then the average value of all the mutual inductances of filaments making up a long straight conductor is determined by the average value of $\ln d$. Since there are d filaments involved we have to deal with $g(g-1)/2$ different *mutual* inductances. A geometric-mean-distance (GMD) d' for all filament pair combinations may now be defined by

$$\ln d' = [2/\{g(g-1)\}] \sum_{g(g-1)/2} \ln d. \tag{48}$$

Once the GMD of the conductor cross-section has been found, it becomes possible to equate the selfinductance of this conductor — in accordance with equ. (47) — to the mutual inductance of a single pair of lines separated by d'. Maxwell demonstrated how to compute the GMD of a variety of conductor cross-sections. Furthermore, his GMD technique may equally well be applied to deriving the mutual inductance of a pair of straight parallel conductors, each of them being subdivided into filaments. This latter computation requires the GMD of one conductor cross-section from the other.

It is often forgotten that Maxwell's GMD method, strictly speaking, applies only to very long straight conductors and even in this restricted case it involves SOMMERFELD's approximation [13]. Sommerfeld was first to solve Neumann's mutual inductance formula (24) for a pair of parallel, straight filaments of finite length L and spacing d. His result is

$$M_{m,n} = 2\left\{L \ln\left[\left(L + \sqrt{L^2 + d^2}\right)/d\right] - \sqrt{L^2 + d^2} + d\right\}. \tag{49}$$

When L is very much greater than d, equ. (49) simplifies to the approximation

$$M_{m,n}/(2L) \simeq -1 + \ln(2L) - \ln d. \tag{50}$$

Only in this last approximation is the filament mutual inductance per unit length proportional to $\ln d$, as assumed in Maxwell's GMD method.

We may represent the selfinductance of a conductor of g filaments by a $g \times g$ square mutual inductance matrix which has the filament selfinductances of equ. (41) as the elements of the principal diagonal. Equation (47) is the sum all the matrix elements divided by the number of matrix elements. The same result would be obtained if we took the sum of the mutual inductances of all filament pair combinations, instead of permutations, added to them the elements of the principal diagonal and then divided by $g(g+1)/2$. Now equ. (48) can be thought of being based on a $g \times g$ matrix of the logarithms of filament distances d. If we took filament pair combinations, instead of permutations, we would have to deal again with $g(g+1)/2$ elements of this latter matrix. But in equ. (48) the summation is over $g(g-1)/2$ combinations, indicating that the diagonal elements of the matrix have been ignored. These elements would all be $\ln(0)$ and therefore of indeterminate magnitude. In this way Maxwell's GMD method avoids facing the difficult question of what is the magnitude of the selfinductance of an infinitely thin current filament. The formulas which have been derived with the GMD method all suggest that it should be infinite which must be wrong for it would preclude the starting of any current in a metallic conductor.

To resolve this problem within the framework of the Ampere-Neumann electrodynamics we ask the question: what is the mutual inductance of a pair of conductor elements? According to equ. (24) it should be $-(dm \cdot dn/r_{m,n}) \cos \varepsilon$. However this implies that sooner or later the term will be integrated around a closed loop. In the case of the selfinductance of an isolated closed filament — and adhering to the newtonian principle that elements of matter do not interact with themselves — there is no closed loop integral involved. We therefore have to express the mutual inductance of a pair of conductor elements by the most general expression

$$\Delta M_{m,n} = -(dm \cdot dn/r_{m,n})\,(2 \cos \varepsilon - 3 \cos \alpha \cos \beta). \tag{51}$$

By the same token, each pair of current-elements must be associated with an amount of stored magnetic energy of

$$\Delta P_{m,n} = -i_m i_n \Delta M_{m,n}. \tag{52}$$

We could not imagine element pairs being associated with flux linkage and there is, therefore, no place for equ. (51) and (52) in electromagnetic field theory. But in the Ampere-Neumann electrodynamics which is built on the mechanical force between two current-elements it is natural and logical to associate mutual inductance and stored magnetic energy with element pairs.

With equ. (51) the selfinductance L_f of an isolated filament of a total of x conductor elements takes the form

$$L_f = (1/2) \sum_{m=1}^{x} \sum_{n=1}^{x} \Delta M_{m,n}; \qquad (m \neq n). \tag{53}$$

The (1/2) factor allows for counting each element-pair contribution twice. The two general elements m and n must never be allowed to coincide which is being indicated by $m \neq n$. No solutions of equ. (53) by open-path integration have been found. Fortunately the computer makes it possible to solve the equation numerically. This latter technique was not available to Neumann and Maxwell and most subsequenti nvestigators of inductance calculations.

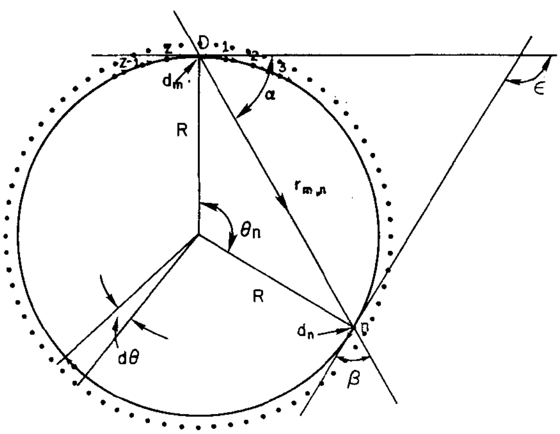


Fig. 10. Construction for calculating the selfinductance of a wire circle.
$\alpha = \beta$; $\varepsilon = \alpha + \beta = 2\alpha = 2\beta$; $\varepsilon = \theta_n$

The computations for a filament circle of radius R may be carried out as follows. Divide the circle into $(z + 1)$ equal elements of arc, as indicated in fig. 10. Take two general elements dm and dn positioned at m and n respectively and let these elements successively occupy each possible arc position on the circle. All the contributions to the circle selfinductance may be tabulated on a $(z + 1) \times (z + 1)$ square matrix with zeros along the principal diagonal. The zeros acknowledge that individual conductor elements do not interact with themselves. To obtain L(circle) we must take the sum of half the off-diagonal elements. It will be recognized, furthermore, that the sum of each row of matrix elements is identical to the sum of any other row because of the symmetry of the circle. Hence L(circle) is given by half the row-sum of the mutual inductance matrix multiplied by the number of rows $(z + 1)$. That is

$$L(\text{circle}) = (1/2)\,(z + 1)\,dm \cdot dn \sum_{n=1}^{z} (3 \cos \alpha \cos \beta - 2 \cos \varepsilon)/r_{m,n}. \tag{54}$$

From the geometrical parameters noted on fig. 10 it follows that

$$3 \cos \alpha \cos \beta = 3 \cos^2 (\theta_n/2) = 1.5(1 + \cos \theta_n) \tag{55}$$

$$r_{m,n}^2 = 2R(1 - \cos \theta_n) \tag{56}$$

$$dm = dn = R\, d\theta = \{2\pi/(z + 1)\}\, R \tag{57}$$

$$\theta = n\, d\theta. \tag{58}$$

Finally, because of the symmetry of the circle, we need only sum half-way around it and then multiply the result by two. The expression to be evaluated by computer therefore is

$$L(\text{circle}) = \left\{\sqrt{2}\, \pi^2 R/(z + 1)\right\} \sum_{n=1}^{z/2} \{3 - \cos (n \cdot d\theta)\}/\sqrt{1 - \cos (n \cdot d\theta)}. \tag{59}$$

We are now faced with the decision of what the number z should be. To probe this question, equ. (59) was evaluated fo ra range $300 \leqq z \leqq 20{,}000$. The results in terms of the dimensionless number L/R are listed on table 1 and plotted on fig. 11. A curve was fitted to the results by regression analysis. It was found to correspond closely to the logarithmic relationship

$$L/R = 13.56 + 12.63 \ln (z + 1). \tag{60}$$

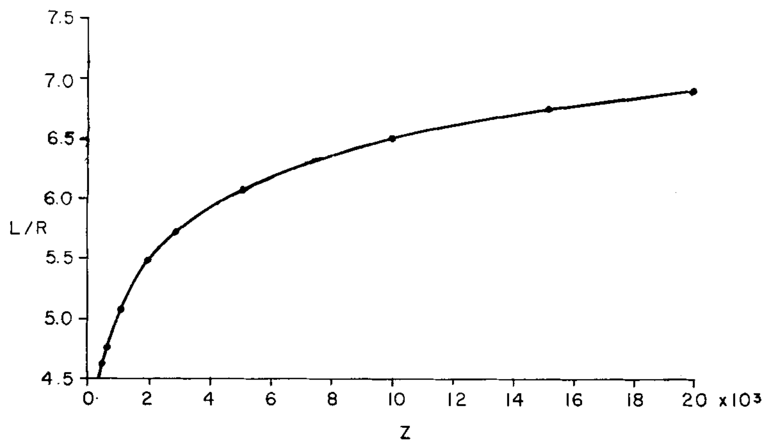


Fig. 11. Selfinductance per unit radius of a circular filament. Curve from equ. (60); points from equ. (59)

Logarithmic relationships are commonplace in inductance computations. Equation (60) tells us that z must not approach infinity as the Ampere-Neumann electrodynamics would then become absurd. In any case, since we are dealing with matter interactions and matter is not infinitely divisible, it is only reasonable that the conductor element size should have a finite lower limit. Whilst equ. (60) demands only the length of the element to be finite, it would indeed be surprising if it could be infinitely thin. From what we know about the atomic structure of metals one would expect the length-to-width ratio of the ultimate conductor element to be of the order of one.

In contrast to this, Ampere and Neumann assumed the current-element to be infinitely divisible. This is understandable because without the infinitesimal calculus they would have been unable to obtain useful quantitative results for mutual inductances where the minimum distance between element pairs is usually large compared to the conductor cross-section. That infinitely small conductor and current-elements are not admissible emerges only in the calculation of inductive and mechanical interactions between elements of the same circuit where neighboring elements come as close together as the element size permits. This discovery had to wait until computer-aided finite element analysis became a reality.

If we make dm and dn equal to the atomic spacing, say 10 angstrom, then circles ranging in diameter from 1 mm to 1000 m would involve 10^6 to 10^{12} elements. Equation (60) permits the calculation of the selfinductance per unit radius for these two large numbers of elements. The results are included in table 1 and it will be seen that the selfinductances derived in this way are not unreasonably large. This admits the possibility that the atom itself is the basic conductor and current-element.

Inductance calculations should be as much affected by the lateral distance between neighboring conductor elements as by their longitudinal separation. The lateral spacing determines the number of filaments into which the conductor must be resolved. If the atom is the basic element, then longitudinal and lateral spacing will be about equal. The element shape would then be given by the lattice cell and, to a first approximation, could be taken to be a cube.

In electromagnetic units, the formula usually quoted for the selfinductance of a circle of radius R made of round wire of radius 'a' is

$$L/R = 4\pi\{\ln (8R/a) - k\} \tag{61}$$

where the suggested values for the constant k range from 1.75 to 2.5. The uncertainty with regard to k indicates some difficulty in making equ. (61) to agree with experiment. If we take a square wire of $a \times a$ cross-section and resolve it into approximately cubic conductor elements we would have

$$R/a = (z + 1)/(2\pi). \tag{62}$$

Substituting this into equ. (61) and solving for a range of z-values gave the results recorded in the last column of table 1. We find order of magnitude agreement with equs. (59) and (60). Since equ. (61) is not well confirmed by experiment, it seems cubic conductor elements in conjunction with equs. (59) and (60) represent an adequate first approximation — the single filament approximation — to the selfinductance of a wire circle. For greater accuracy the conductor has to be resolved into a number of parallel filaments.

The foregoing procedure provides a method of calculating the selfinductance of rectangular and other non-circular wire-loop or magnet-coil shapes for which there exist no reference formulas. In every case it helps to set up the complete square matrix of motual inductances between all element pairs with zeros along the principal diagonal and use cubic elements throughout.

Table 1

Selfinductance per unit radius of a circular filament

z	from equ. (59)	from equ. (60)	from equ. (61) $R/a = (z + 1)/2\pi$ $k = 2$
300	42.8	42.8	49.6
400	44.6	44.6	53.2
500	46.1	46.0	56.0
600	47.2	47.2	58.3
1000	50.4	50.4	64.7
2000	54.8	54.8	73.4
3000	57.4	57.3	78.5
5000	60.6	60.6	84.9
10,000	64.9	64.9	93.6
15,000	67.5	67.5	98.7
20,000	69.3	69.3	102.4
10^6		94.0	151.5
10^{12}		181.3	325.1

6. Computing Ampere Tension

An infinitely long straight conductor could be, and has been, treated as a closed circuit. Yet it would be futile to analyze it because, even with finite size elements, the Ampere formula would give infinite tension at every point. To prove anything about Ampere tension, the investigation has to concern itself with closed solid metallic circuits of finite size.

Consider the example illustrated by fig. 12 in which a square circuit carries a steady current i and is adequately cooled to ensure uniform constant temperature everywhere in the metal. Sides BC, CD and AD of the circuit are firmly embedded in a dielectric structure which is rigidly anchored to the laboratory frame. AB is a free length of wire resting against a wall meant to absorb the lateral force on AB.

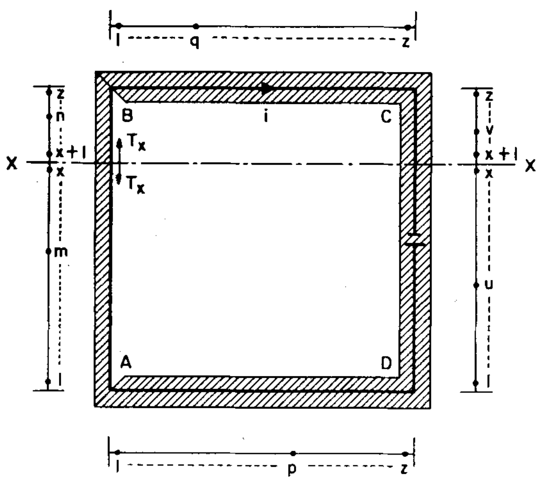


Fig. 12. Square circuit with one free side

Let T_x/i^2 be the specific tension in atomic bonds across plane X intersecting wire AB. Each side of the square is assumed to be divided into z equal length elements thin enough so that the conductor may be treated as a single filament. The major contribution to T_x will come from the respulsions exerted by the general elements m in AX on the general elements n in XB. This will be given by equ. (22). As the equation is independent of the unit of length we may choose this to be

$$dm = dn = 1 \text{ unit of length.} \tag{63}$$

With the labelling of current-elements indicated on fig. 12, the distance between the two general elements may be written

$$r_{m,n} = n - m. \tag{64}$$

The specific tension contribution made by the $m - n$ element combinations is

$$T_1/i^2 = \sum_{m=1}^{x} \sum_{n=x+1}^{z} \{1/(n-m)^2\}. \tag{65}$$

This will be a maximum when $x = z/2$.

Next we consider the interactions of current-elements in AB with elements in sides BC and AD. These interactions are all repulsions because the angle function of Ampere's force law is negative for all element pair combinations. Interactions between BC and BX, on the one hand, and between AD and AX, on the other, do not exert tension on the atomic bonds across plane X. But the repulsions between BC and AX, as well as between AD and XB, add to T_x. By resolving the latter repulsions along AB we obtain the second contribution to the specific tension across plane X, that is

$$T_2/i^2 = \sum_{n=x+1}^{z} \sum_{p=1}^{z} (3/r_{p,n}^2) \cos^2 \alpha_n \sin \alpha_n + \sum_{m=1}^{x+1} \sum_{q=1}^{z} (3/r_{q,m}^2) \cos^2 \alpha_m \sin \alpha_m \tag{66}$$

where

$$r_{p,n}^2 = (n - 0.5)^2 + (p - 0.5)^2 \tag{67}$$

$$r_{q,m}^2 = (z - m + 0.5)^2 + (q - 0.5)^2 \tag{68}$$

$$\cos \alpha_n = (n - 0.5)/r_{p,n}; \qquad \sin \alpha_n = (p - 0.5)/r_{p,n} \tag{69}$$

$$\cos \alpha_m = (z - m + 0.5)/r_{q,m}; \qquad \sin \alpha_m = (q - 0.5)/r_{q,m}. \tag{70}$$

The 0.5 terms arise from the fact that the position of the current-element is a point half-way along its length.

The third contribution ot T_x derives from the interactions between AB and CD. The angle function for this side pair has everywhere $\cos \varepsilon = -1$ and $\cos \beta = -\cos \alpha$. Furthermore, since α varies from 45 to 135°, $2 \cos \varepsilon - 3 \cos \alpha \cos \beta = -2 + 3 \cos^2 \alpha$. This is never positive and then, because of the negative sign in Ampere's force law, all interactions are again repulsions.

It is convenient to split CD by the plane X (see fig. 12) with general element u on one side and v on the other. Symmetry ensures that every elemental repulsion with an upward longitudinal component is offset by a symmetrical interaction with a corresponding downward component. Therefore actions of XC on XB do not contribute to T_x. The same is true for actions of DX on AX. But tensile forces will be produced in AB by the actions of XC on AX and by DX on XB. They contribute

$$T_3/i^2 = \sum_{m=1}^{x} \sum_{v=x+1}^{z} (-1/r_{m,v}^2)(-2 + 3 \cos^2 \alpha_v) \cos \alpha_v$$

$$+ \sum_{n=x+1}^{z} \sum_{n=1}^{x} (-1/r_{n,u}^2)(-2 + 3 \cos^2 \alpha_u) \cos \alpha_u \tag{71}$$

where $r_{m,v}^2 = (v - m)^2 + z^2$ (72)

$$r_{n,u}^2 = (n - u)^2 + z^2 \tag{73}$$

$$\cos \alpha_v = (v - m)/r_{m,v} \tag{74}$$

$$\cos \alpha_u = (n - u)/r_{m,u}. \tag{75}$$

The total specific tension in the wire AB may then be obtained by adding equs. (65), (66) and (71).

$$T_x/i^2 = T_1/i^2 + T_2/i^2 + T_3/i^2. \tag{76}$$

Figure 13 is a plot of the three tension components and their sum for $z = 1000$.

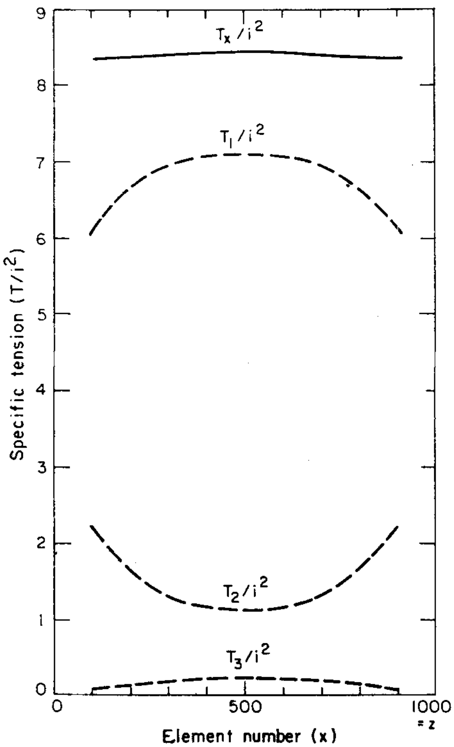


Fig. 13. Specific tension in free side of square circuit

It can easily be shown that the computed tension increases with the number z of elements into which one side is divided. This appears to be an unsatisfactory outcome of the Ampere theory. As in the case of selfinductance, the difficulty can be overcome by making a reasonable assumption about the length-to-width ratio of the current-element. As a first step we calculate the most important tension contribution, given by equ. (65), for different values of z. Table 2 lists the results for z varying from 20 to 200 and $x = z/2$. A regression analysis performed on this data revealed a very close fit to

$$T_1/i^2 = 0.19 + \ln z. \tag{77}$$

Table 2

Computer evaluation of equation (65) for z varying from 20 to 200 and $x = z/2$

z	T_1/i^2
20	3.188
30	3.593
40	3.880
50	4.103
60	4.285
70	4.396
80	4.573
90	4.691
100	4.796
110	4.891
120	4.978
130	5.058
140	5.133
150	5.202
160	5.266
170	5.327
180	5.384
190	5.438
200	5.489

It can be shown that the specific tension contributions T_2/i^2 and T_3/i^2 obey similar logarithmic laws. Hence T_x/i^2 will also be a logarithmic function of z. For $z = 100$ equ. (77) yields 7.098 as compared to 7.099 obtained by finite element analysis. Hence equ. (19) can be extrapolated confidently to much larger values of z.

Because of equ. (77) and the logarithmic approach to infinity of the Ampere tension, we find once more that the Ampere electrodynamics becomes absurd if the current-element is taken to be infinitely divisible. The same is actually true for an electrodynamics based on the Lorentz force law. We really have no choice but to accept elements of finite size. Could the lower element size limit be set by the distance between neighboring atoms? In metal lattices this would be of the order of 10^{-7} cm. No less than 10^9 current-elements would be required if AB of fig. 12 were 100 cm long. Equation (77) then gives a specific tension of 20.91 which is only three times the tension obtained for $z = 1000$. It is not an unreasonable number and therefore lends some support to the idea that the atomic lattice cell is the basic current-element.

Any reduction in current-element length from macroscopic to microscopic dimensions should be accompanied by a similar reduction in the size of the element cross-section. In other words, the specific tension of 20.91 applies to a conductor of 10^{-7} cm diameter and one meter long. For conductors of larger diameters a bunch of parallel filaments have to be substituted, each being essentially a string of atoms.

What will be the tension in two adjacent strings of atoms which share the current i? To obtain an answer consider the two square-section filaments of fig. 14. They have been subdivided into four portions a, b, c, and d. Each portion consists of $z/2$ cubic current-elements with their vectors all pointing in the same direction. The shape of cubic elements has been chosen for the convenience with which a solid conductor may be divided into cubic cells, and not because it is the actual shape of the atomic cell. Let us now determine T_1/i^2 across the midplane of the filament combination when each filament carries the current $i/2$. The tensile force due to the interaction of portions a and b can be calculated with equ. (77). An equal component will arise from the interaction of portions c and d. Let these two components be $T_{a,b}$ and $T_{c,d}$, then

$$T_{a,b} = T_{c,d} = (1/4)\,(0.19 + \ln z)\,i^2. \tag{78}$$

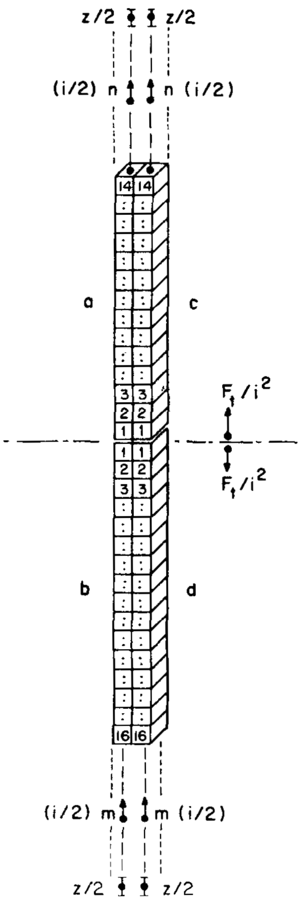


Fig. 14. Tension across the perpendicular midplane of two parallel and straight filaments

For the calculation of $T_{a,b}$ and $T_{c,d}$ which, because of symmetry, are equal to each other, we find from fig. 14

$$r_{m,n}^2 = (m + n - 1)^2 + 1 \tag{79}$$

$$\cos \varepsilon = 1 \tag{80}$$

$$\cos \alpha = \cos \beta = (m + n - 1)/r_{m,n}. \tag{81}$$

Applying Ampere's law to filament portions a and d and resolving the elemental interaction forces in the direction of the current, we obtain

$$T_{a,d} = T_{c,b} = (1/4)\,i^2 \sum_{m=1}^{z/2} \sum_{n=1}^{z/2} (1/r_{m,n}^2)\,(2\cos\varepsilon - 3\cos\alpha\cos\beta)\cos\alpha. \tag{82}$$

Solving the simultaneous equations (79) to (82) by computer and applying regression analysis to the results revealed the logarithmic relationship

$$T_{a,d} = T_{c,b} = (1/4)\,(-1.64 + \ln z)\, i^2. \tag{83}$$

Hence the tension T_1 across the midplane of the filament combination is

$$T_1 = 2T_{a,b} + 2T_{a,d} = (-0.73 + \ln z)\, i^2 \tag{84}$$

which is smaller than the force given by equ. (77). This result demonstrates that the amperian tension is reduced when the current divides between adjacent filaments. It will be referred to as longitudinal force dilution, and it largely compensates for the apparent Ampere tension increase when the macroscopic element size is reduced.

7. Macroscopic Current-Element Analysis

During the eighty years from 1820 to 1900, when the Ampere law was in wide use, current-elements were treated as being infinitely divisible. The result was a continuum theory which led to singularities in the integration of tensile forces because $r_{m,n}$ across the interface of conductor portions approached zero. This probably explains why so little was written in the nineteenth century about amperian tension in electric conductors.

Large current-elements of a cross-section equal to the whole conductor section have often been successfully employed to calculate the reaction forces between two complete circuits which were separated by at least ten current-element lengths. It therefore seemed worthwhile to investigate if single filament representations of practical conductors can be helpful in estimating the magnitude of Ampere tension.

To do this a 100×40 cm rectangular circuit, made up of 0.25 inch diameter copper rod and two liquid mercury links, was set up in a vertical plane. As shown in fig. 15,

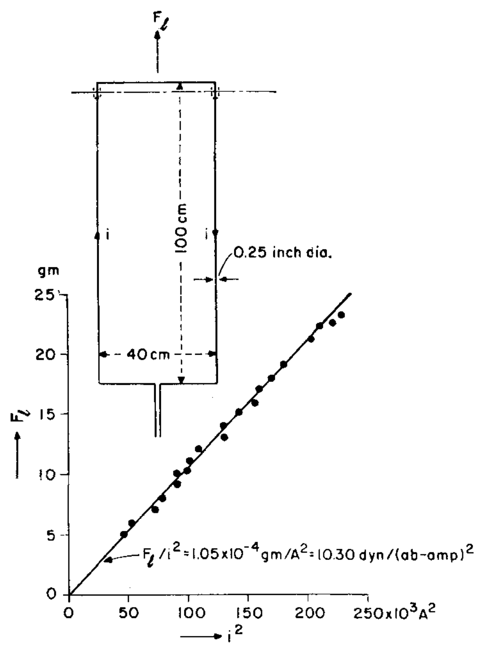


Fig. 15. Lift-force measurements on a rectangular circuit

the uppermost side and three centimeter of each vertical leg were cut off and reconnected with liquid mercury contained in dielectric cups attached to the bottom portion of the circuit. The liquid metal gaps were less than one millimeter long. The electromagnetic lift force F_l on the upper portion of the circuit was measured with a beam balance. As the results plotted on fig. 15 indicate, the reaction force was found to be proportional to the square of the current, giving a specific force of $F_l/i^2 = 10.30$. According to accepted pinch force theory [14], the liquid mercury is responsible for a specific upward thrust of 1.00. Hence Ampere's force law should account for 9.30 of the measured specific force.

In the macroscopic current-element analysis of the lift force, the circuit was modelled as a single filament of one centimeter long elements, making perfect right-angled joints at each corner. Apart from computing the Ampere lift force, consisting partly of longitudinal components in the vertical legs and partly of transverse forces on the horizontal branch, the Lorentz force on the horizontal conductor was also computed with the same one centimeter long elements and the formula

$$\Delta \boldsymbol{F}_m = (i^2/r^2_{m,n})\, \boldsymbol{dm} \times (\boldsymbol{dn} \times \boldsymbol{1}_r) \tag{85}$$

where $\boldsymbol{1}_r$ is the unit distance vector drawn from $\boldsymbol{dn}$ to $\boldsymbol{dm}$. The results were as follows

Ampere:	$F_l/i^2 = 11.20$	(47 percent longitudinal)
Lorentz:	$F_l/i^2 = 11.24$	(all transverse)
Experiment:	$F_l/i^2 = 9.30$.	

Hence the single filament representation with current-elements of a length even greater than the conductor diameter gave a rough guide to the magnitude of the reaction forces between parts of the same circuit. In the chosen example the Ampere law appeared to overestimate the actual lift force by 20 percent. Part of this discrepancy may have been due to experimental errors.

The other interesting relevation was that, for the same element size, both Ampere's and the Lorentz force theory predicted almost identical tension forces. But in Ampere's electrodynamics 47 percent of the tension was due to longitudinal forces, while the Lorentz force was of course entirely a transverse force which produced tension indirectly. This coincidence and the prominence given to rectangular circuits in the past has no doubt contributed to the belief that Ampere's force law is not needed.

How should one proceed from here? The founders of the Ampere-Neumann electrodynamics were masters of analysis, but obviously did not find an analytical solution for the directly induced conductor tension. Computer-aided finite element analysis was not available to scientists of the nineteenth and the first half of the twentieth century. This new tool has therefore been explored a little further.

The macroscopic current-element has to be of a definite shape and the cube lies close at hand. Consider the $a \times a$ square-section conductor of length l. If $l \gg a$, the important midplane tension is largely independent of any further increase in length. In the case of fig. 13, where $l/a = z = 1000$, over eighty percent of the midplane tension is being contributed by the repulsion of in-line elements. Therefore, when dealing with very long straight conductors, we may also ignore the return circuit and remember that this will underestimate the Ampere tension. For $z = 10{,}000$ the speicfic midplane tension from equ. (77) comes to 9.40. This would apply, for example, to a 100 m long conductor of one square-centimeter cross-section. Assuming a current of 1000 A = 100 ab-amp, the single filament model predicts a tension of only 96 gram, or 0.096 kg/cm^2 tensile stress. It would produce a negligible amount of strain in spite of the high continuous current density of 1000 A/cm^2. Not surprisingly, Ampere tension almost had to go unnoticed in ordinary wires and cables used for the transmission and distribution of electrical energy.

The tensile stress would become more severe in fully loaded cryogenically cooled conductors. For instance, an aluminium conductor of one square-centimeter cross-section held at the temperature of liquid nitrogen could possibly carry 10,000 A continuously, when it would be subject to 9.6 kg/cm^2 tensile stress. Even then the strain is quite modest.

Ampere tension is likely to be of critical importance in superconductors. Type II superconducting filaments embedded in a copper matrix and cooled with superfluid liquid helium can support current densities of the order of 100,000 A /cm^2 over the combined copper and superconductor area. In the 100 m long, one square-centimeter conductor this would give rise to 960 kg/cm^2 tensile stress which not only would produce noticeable strain but very likely change the superconducting properties of the rod, which are known to be strain sensitive.

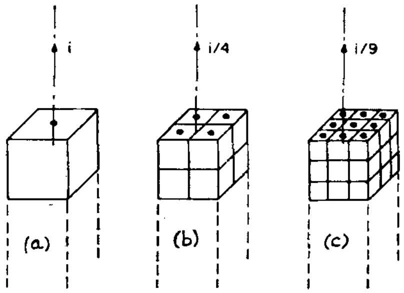


Fig. 16. Cubic element subdivision of a linear conductor

The single filament representation of the linear conductor is the crudest model one can use. Finer subdivision of the conducting matter into smaller cubes should result in better approximations to the specific tension. Therefore, let every element of fig. 16(a) be subdivided into eight smaller cubes, as shown by (b). This multiplies the computational work by at least a factor of 64. Angles α and β are then no longer zero for all relevant current-element combinations and equ. (82) has to be used in addition to (65). To obtain a quantitative indication of the force dilution resulting from filament subdivision we analyze a relatively short conductor of 2 m length and 1 cm^2 cross-section. The return circuit would make a significant contribution to the maximum tension in this short conductor but we will not compute this. The midplane tension due only to the straight portion was found to be

Single filament, fig. 16(a):	$T_1/i^2 = 5.49$
Four filaments, fig. 16(b):	$T_1/i^2 = 4.71$
Nine filaments, fig. 16(c):	$T_1/i^2 = 4.55$.

This example illustrates that the computed specific Ampere tension converges quite rapidly as the number of parallel filaments is increased. Hence only a modest degree of subdivision will give good approximations. Although the ultimate current-element of the Ampere-Neumann electrodynamics is likely to be of atomic size, the usefulness of manageable calculations involving quite large macroscopic current-elements has hereby been demonstrated. Considerable liberties can be taken with the shape and size of macroscopic current-elements when computing forces between separate circuits. It will be realized, however, that the mutual force between two circuits does not involve Ampere tension.

8. The Experimental Evidence for Longitudinal Ampere Forces

8.1 Ampere's hairpin experiment

Were it not for the longitudinal forces, the Ampere-Neumann electrodynamics would largely be history. It would add nothing to the development of modern physics and new technology that could not be provided by relativistic electromagnetism. Experiments purporting to show the existence of longitudinal Ampere forces have been known for 160 years and fuelled a certain amount of controversy. Many a heated argument could have been avoided had it always been recognized that the validity of the early electrodynamics is confined to metallic conductors. The conduction electron travelling through the metallic lattice appears to be subject to a more complex force system than the free electron travelling through vacuum. For this reason Lorentz was compelled to drop Ampere's force law when he tried to explain the dynamics of isolated electric charges in vacuum tubes. He found it necessary to substitute GRASSMANN's law [16] which later became known as the Lorentz force law.

Long before Lorentz made the change, Ampere himself appears to have been under some pressure to provide an explicit demonstration of longitudinal forces. He had taken the view that his empirical law was the generalization of many experimental results, collected mainly by himself, which all implied the existence of longitudinal forces and no explicit demonstration was necessary. However when Ampere visited the Swiss scientist de LaRive, both men performed a famous test, which here is called Ampere's hairpin experiment, to distinguish it from his many other demonstrations. Ampere's sketch of the apparatus which was used in Geneva is reproduced in fig. 6. Figure 17 shows the circuit arrangement with which the author in 1981 performed the hairpin experiment at MIT.

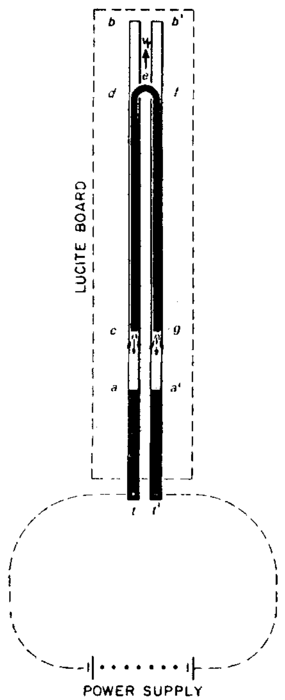


Fig. 17. Modern version of Ampere's hairpin experiment

With reference to fig. 17, cdefg is an insulated copper hairpin with bare ends floating on two liquid mercury channels *ab* and *a'b'*. When more than 200 A of current is passed around this circuit the hairpin moves to the end of the channels bb'. According to Ampere's law the motive force in this experiment is being provided by the repulsion between the hairpin leg *cd* and the mercury portion *ac* and, equally, between the leg *gf* and the mercury section *a'g*.

A new observation was made at MIT which was not reported by Ampere and de LaRive, nor by anyone else who repeated their experiment. When the forward motion of the hairpin was blocked by placing an obstacle in the way of the hairpin bend, strong jets of liquid mercury could be seen to emanate from the hairpin ends *c* and *g*. The turbulence in the liquid gave the distinct impression that the hairpin was subject to jet-propulsion [1]. The area of strongest turbulence was quite narrowly confined to the hairpin ends and it did not extend outward to the region where the current streamline pattern in the mercury must have shown the greatest divergence.

It was further noticed that some turbulence occurred in the liquid mercury near its interface with the one-half inch square fixed copper bars at *a* and *a'*. The overall turbulence in the liquid mercury sections could be increased, at constant current, by moving the hairpin closer to the copper bars.

To investigate the interaction of oppositely directed jets at the ends of a liquid metal section a little further, a separate experiment was performed. Two lengths of 0.5×0.5 inch square copper bars were cemented into the same trough with a butt gap of 1/8th inch between them. The gap was filled with liquid mercury. When current was passed across the gap a limited amount of turbulence could be seen in the liquid. Above 900 A the liquid metal would actually bulge upward with depressions in the liquid level near the copper faces. With a further increase in current up to approximately 1000 A, the liquid mercury would be thrown upward and ejected out of the open channel. As the temperature rise of the copper conductors was less than 100 °C, the ejection of the mercury was likely to be the result of strong longitudinal repulsion from the two copper bars.

Ampere's critics of recent times have held that the motive force on the hairpin was the Lorentz force on the bend *e* (fig. 17) passing over the dielectric barrier between the two mercury troughs. HILLAS' argument is typical in this respect [8]. This transverse force on the hairpin bend is also predicted by Ampere's formula. But in the Ampere electrodynamics it is cancelled by longitudinal reaction forces in the hairpin legs and therefore unable to accelerate the hairpin with respect to the mercury on which it floats. The magnetic field at the bend is primarily due to the current in the hairpin legs. One might expect that the reaction to the Lorentz driving force should also reside in the hairpin legs. This is, however, impossible because the Lorentz force on the legs is everywhere perpendicular to the direction of relative motion.

Could the Lorentz reaction force reside in the power supply branch of the circuit? For the sake of simplicity, let this branch be a short connection between *t* and *t'* of fig. 17. It is easily shown that the Lorentz repulsion between the short sides of a long rectangular circuit is very small compared with the total Lorentz force on each of the short sides. It therefore follows that most of the reaction force to the Lorentz force on the hairpin bend — which according to Ampere's critics is responsible for the hairpin motion — must reside outside the conducting matter which forms the closed hairpin circuit.

This reaction force difficulty was actually overcome by the special theory of relativity which associates momentum with the magnetic energy stored and travelling in the electromagnetic field. It is claimed that changes in this momentum can support mechanical reaction forces in vacuum. A clear exposition of this mechanism was provided by CULLWICK [15]. Paradoxically, it tries to preserve the validity of Newton's third law in relativity theory which was meant to supersede the newtonian dynamics. In connection with this it is too often forgotten that all three of Newton's laws refer to forces on matter. Vacuum forces are therefore meaningless in a world based on Newton's laws. Hence forces said to be associated with the change of field-energy momentum do not contribute to the force balance required by Newton's third law.

Quite recently PAPPAS [5] performed a new version of Ampere's hairpin experiment. This is illustrated by fig. 18. A rather large metal hairpin was suspended horizontally from the laboratory ceiling to constitute an impulse pendulum. The pendulum communicated with the current leads of a battery through two mercury cups. When the circuit

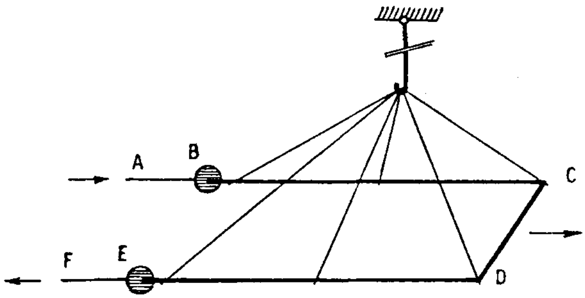


Fig. 18. Pappas' impulse pendulum experiment. *BCDE* is the copper hairpin and *B* and *E* are the mercury cups

was closed the electromagnetic forces imparted an impulse to the pendulum and made it swing away from the static mercury cups, thereby interrupting the current. The amplitude of the swing and the time it took to reverse its motion was measured by Pappas and used to calculate the mechanical momentum that was imparted to the hairpin.

In order to conserve momentum, on the relativistic basis, a certain amount of magnetic energy had to have been deposited in the field and accelerated to the velocity of light. Pappas calculated the energy required for this purpose and compared it with the energy stored in his battery. He found that it was much more energy than his battery could have supplied. Besides, the current required to radiate so much energy into the field in a brief moment would have melted the conductors of his circuit. This appears to have been the first time that relativistic momentum conservation was tested *with a metallic circuit*. The result casts serious doubts on the hypothesis that the field, i.e. vacuum, can support mechanical forces that are very much greater than radiation pressure.

Pappas demonstrated another interesting fact. When the current leads from the battery to the mercury cups are tilted, each in its own vertical plane, the momentum imparted to the hairpin was changed. He argued correctly that the Lorentz force on the hairpin bend should be essentially independent of the angle of tilt of the current leads. Therefore it could not have been this Lorentz force which was responsible for the hairpin acceleration, as claimed by Ampere's critics.

Pappas was satisfied that his findings were compatible with Ampere's force law. A significant aspect of the tilt experiment is that it disproves the Lorentz force explanation even when it is accepted that relativistic electromagnetism does not comply with Newton's third law.

A few more words need be said about amperian jet-propulsion or the repulsion of liquid from solid current-elements in the direction of current flow. Accepting the view expressed by HILLAS [8] that the motion of the liquid behind the hairpin ends is not the cause of the hairpin motion, there remain discrepancies between the calculable distribution of Lorentz forces on the diverging current streamline pattern and the observed behavior of the liquid metal. The jets are far too concentrated spatially to provide hope that they may comply with the Lorentz force pattern. More serious is the fact that the jet action does not disappear when the current streamlines in the liquid metal are forced to remain straight and parallel. This is the case in a conductor of uniform cross-section arranged horizontally in a dielectric trough with part of the solid metal being replaced by liquid mercury. The Lorentz forces should then pinch the conductor. But since it is incompressible, they cannot give rise to flow turbulence. Yet experiment with a conductor arrangement of this kind clearly indicates liquid turbulence, longitudinal flow, and finally the lateral expulsion of liquid as if two jets were colliding.

MAXWELL [12] discussed Ampere's hairpin experiment in his Treatise. He did not perform the experiment himself and was not aware of the jet phenomena. It was his opinion that the experiment was compatible with both Ampere's and the Lorentz force laws. The latter he knew as GRASSMANN's formula [16] which had been first published in 1845. Maxwell argued that both laws predict the same forces for closed circuits. He did not realize that this is true only for the mutual force between two separate circuits and not for the reaction force between two parts of the same circuit.

At times the question has been asked: could the hairpin propulsion be due to local heating at the solid-liquid interfaces. When two dry solid conductor surfaces are brought together, the electric current is known to flow through a few contact points. The high resistance of the narrow contact necks causes sufficient Joule heating for the contact points to melt and spot weld together. No evidence has been provided which shows that the contact point mechanism is also active at a liquid-solid conductor interface. Without it the heat generated at the interface is quite small and certainly incapable of producing relative motion. Any chance of contact point formation was eliminated in an experiment carried out by TAIT [18]. He used a 'liquid mercury hairpin' contained in a glass tube and found that it was propelled just the same as a copper hairpin with bimetallic interfaces. As CLEVELAND [17] and others have shown, force measurements on rectangular circuits involving copper-mercury interfaces are in reasonable agreement with both the old and the new electrodynamics, leaving no room for a third theory based on heat propulsion.

The evidence gathered over the years with the hairpin experiment appears to be overwhelmingly in favor of Ampere's force law. In 160 years no experiment has come to light which disproves any of the predictions of Ampere's formula for metallic circuits. Of quite a number of positive longitudinal force experiments recorded in the scientific literature, we will mention only three more. One was devised by F. E. Neumann, another by Hering became the operating principle of the first magnet-less electromagnetic liquid metal pump, and the last one demonstrates the action of Ampere tension in solid wires.

8.2. Neumann's demonstration of longitudinal forces

The existence of longitudinal forces was fully accepted during most of the nineteenth century. Neumann had a classroom experiment with which he demonstrated them routinely to his students. Figure 19 is a diagram of his demonstration as recorded by his pupil VONDERMUEHLL [19].

A, *B* and *C* are mercury throughs and *D* and *E* are copper wire bridges from *A* to *B* and from *B* to *C*. When current is passed along the troughs, the two pieces of wire move away from each other. They must therefore be subject to repulsion. This experiment has at times be critized because of the small hooks at the ends of the wires which dip into the liquid metal. Transverse forces on these short vertical sections could contribute to, or cause, the wire repulsion. To clarify this issue the author analyzed the conductor run *ABCDEFGHKL* of fig. 20 with both the Ampere and the Lorentz force laws. The broken lines *AB*, *EF*, and *KL* of this diagram represent liquid mercury connections while the solid lines are the two wire bridges of Neumann's experiment. For the purpose of the finite element analysis, the horizontal portions of the wire bridges were divided into 200 current-elements and the vertical dips into four elements. The analysis did not cover the return circuit which is known to have little influence on the longitudinal repulsion between the two wires.

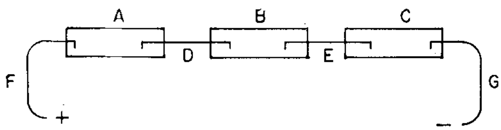


Fig. 19. Neumann's diagram of a demonstration of longitudinal forces

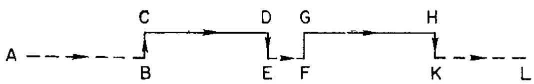


Fig. 20. Liquid conductor portions with solid wire bridges. Liquid portions: $AB = KL = 200$ elements. Solid portions: $BC = DE = FG = HK = 4$ elements. $CD = GH = 200$ elements

Table 3

Finite element computations for Neumann's experiment.

Grassmann Formula

Horizontal force (in dyn for 1 ab.-amp.) on:

		FG	*GH*	*HK*	*FGHK*
Due to	*AB*	$+53.30 \times 10^{-6}$	0	-10.28×10^{-6}	
	BC	-255.98×10^{-6}	0	$+79.011 \times 10^{-6}$	
	CD	-1531.1×10^{-6}	0	$+44.241 \times 10^{-6}$	
	DE	$+6390.4 \times 10^{-6}$	0	-255.98×10^{-6}	
	EF	$+2.9477$	0	-35.991×10^{-6}	
	FG	0	0	$+b$	
	GH	$-a$	0	$+a$	
	HK	$-b$	0	0	
	KL	$+74.986 \times 10^{-6}$	0	-2.9493	
Total					2.95×10^{-3}
		Ampere Formula			
Due to	*AB*	$+132.71 \times 10^{-6}$	$+219.68 \times 10^{-3}$	-30.855×10^{-6}	
	BC	-511.87×10^{-6}	$+132.71 \times 10^{-6}$	$+158.02 \times 10^{-6}$	
	CD	-4.589×10^{-3}	$+1.0055$	$+132.71 \times 10^{-6}$	
	DE	$+12.762 \times 10^{-3}$	-4.589×10^{-3}	-511.87×10^{-6}	
	EF	$+3.9795$	$+1.8594$	-107.96×10^{-6}	
	FG	0	0	0	
	GH	0	0	0	
	HK	0	0	0	
	KL	$+224.93 \times 10^{-6}$	-2.7713	-3.9840	
Total					$+0.3112$

Table 3 lists the results of these calculations. Both formulas are seen to predict a horizontal repulsion force on the bridge *FGHK*. The Ampere repulsion is more than one hundred times as large as the Lorentz repulsion. Furthermore, the absolute value of the Lorentz force for currents up to 500 A is so small that it would hardly be capable of overcoming the strong adhesion of copper to liquid mercury. In the description of Neumann's experiment no information is given of the wire diameter or the current magnitude. It is unlikely that Neumann could have passed more than 500 A around the circuit. For this upper current bound the horizontal Lorentz force on *FGHK* would only be 7.38 dyn. The Ampere formula, on the other hand, predicts as much as 0.8 gram. From this evidence alone one would have to conclude that Neumann's experiment favors Ampere's law over the Lorentz force law.

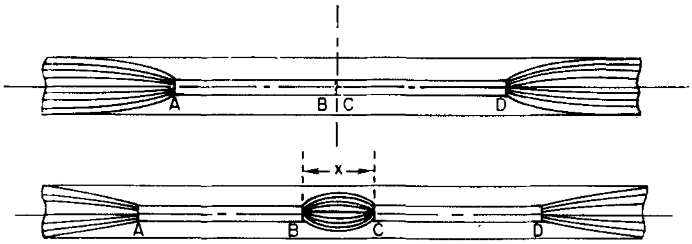


Fig. 21. Rod positions on liquid mercury surface before and after the passage of 450 A of current

To eliminate the vertical hooks on the wire bridges, the author devised a modified version of Neumann's test for longitudinal forces. The apparatus consisted of a straight-through liquid mercury trough of 30.5 cm length and 1.27 × 1.27 cm square cross-section. The liquid conductor was continued in both directions with 30.5 cm long copper bars. The circuit was closed by a remote return conductor trough a 500 A dc current supply. Two *insulated* copper rods AB and CD, as shown in fig. 21, of 5 cm length and 0.3 cm diameter with bare end faces were laid end-to-end on the mercury surface in the middle of the trough.

When a current of 450 A was made to flow along the trough, the rods would submerge and separate axially. As soon as the current was switched off, 10—20 seconds later, the rods would surface, being separated by the distance x shown in fig. 21. Because of the 50:1 resistivity ratio of mercury to copper, the rods carry a substantial fraction of the current in their section of the trough. As discussed elsewhere [1], the attraction between the parallel rod and mercury currents urges the rods toward the center of the trough section and is therefore responsible for their submersion. The distance of separation was of the order of 2.5 cm. We now consider the longitudinal forces that could have caused the rod separation.

Since the copper rods were coated with magnet wire insulation, but for the end faces, the current in them must have been directed axially and all Lorentz forces acting directly on the copper rods would have been normal to the direction of relative motion. With no liquid metal at the $B-C$ interface of the two touching rods, the pinch thrust at faces A and D should have pushed the rods together. In the more likely event of liquid metal filling a short gap between B and C, the pinch thrust on these faces should have counteracted that at A and D. Either way, the pinch forces in the mercury could not explain the separation of the rods. The Lorentz forces on the diverging current streamlines in the mercury (see fig. 21) could conceiveably cause mercury circulation near A and D. But no such circulation could take place in the very short $B-C$ gap because there the streamlines are diverging by a negligible amount. Hence the initial separation of the rods, if at all attributable to magnetohydrodynamic actions, would have had to be caused by the circulations at A and D *pulling* the rods apart. This appears to be a far-fetched explanation.

Over the years there have been suggestions that heat generated at the solid-liquid contact faces could set up propulsion forces. This was investigated by TAIT [18] in connection with the hairpin experiment. He obtained a negative result. But even if a thermal propulsion mechanism did exist, it should result in symmetrically disposed opposing forces on all rod ends and therefore be unable to produce relative motion between the rods. Furthermore, if for some unknown reason the thermal propulsion forces

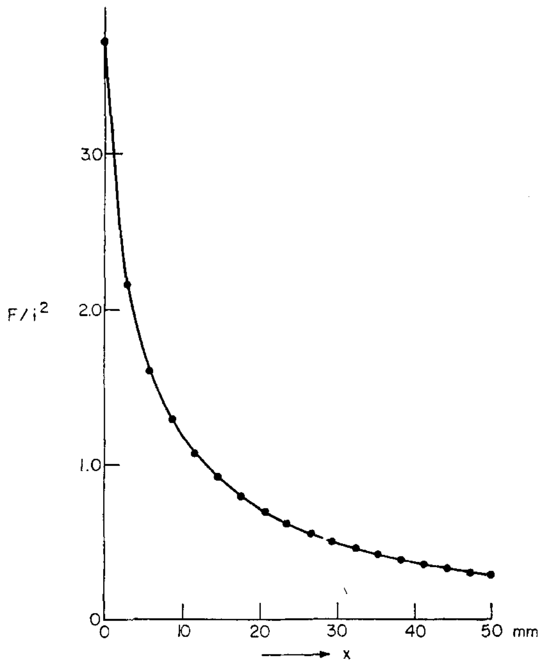


Fig. 22. Specific rod separation forces as a function of x

were not equal and opposite, the motion of the rods should not stop until they strike the end of the trough. There appears to exist no plausible explanation why the intensity of the thermal action should relent as the rods separate.

Figure 22 is a plot of the specific Ampere repulsion force (F/i^2) acting between the rods as a function of the distance of separation x. The points on this graph were calculated with finite current-element analysis in which the rods were replaced by single filaments of elements, the element length being equal to the rod diameter. According to this graph the rods should strongly repel each other while they are in contact and the repulsion force should fall off quite sharply with the distance of separation. For 450 A = 45 absolute ampere, the maximum repulsion comes to 7.5 gram. This decreases to one gram at $x = 2.5$ cm which could well be the adhesion drag resisting motion of the rods through the liquid mercury. Hence the Ampere force law provides a natural explanation of the observed phenomenon.

8.3. One of Hering's longitudinal force experiments

At the turn of the century Carl Hering designed, built and operated furnaces in which liquid metal was heated by the passage of large currents through molten pools. In the course of this work he was the first to discover the electromagnetic pinch effect. He also observed what he called stretch effects which were in fact the result of longitudinal Ampere forces. In his widely quoted paper [20] of 1923 he writes, as follows, about the technological applications of these effects.

"By passing currents, especially at high current densities, through such very mobile conductors as mercury or molten metals in some types of electric furnaces, the writer many years ago noticed the existence of some heretofore unrecognized electromagnetic forces which tended to move the conductors, and being mobile liquids they responded much more readily to such forces than solid conductors do. Some of these new forces were very formidable, for like most of such forces, they presumably increase with the square of the current. The writer then made use of them in electric furnaces (to pump liquid metal), many of which are in daily use, these new forces being the absolutely essential factor, showing their industrial importance."

In this quotation Hering referred to the invention of the first electromagnetic liquid metal pump. As the metal was driven by longitudinal forces, the Hering pump did not require external magnet coils. The present generation of electromagnetic pumps has been designed on the basis of transverse Lorentz forces and must, therefore, rely on external magnets.

In discussing his experiments, Hering made no distinction between the Ampere-Neumann electrodynamics and modern field theory. This led to much confusion and detracted from what might otherwise have become a strong case for the Ampere theory.

The most decisive of Hering's many longitudinal force experiments is illustrated in fig. 23. There *ABCDEFGH* is a rectangular circuit standing in a vertical plane and having a power supply connected in the *AH* branch. The current i leaving the supply is split at *C*, with i_1 passing along the vertical branch *CG* and i_2 completing the journey around the large rectangle. Three mercury cups *B*, *C* and *D* make it possible for the horizontal conductor section *BD* to move along its length. The vertical branch *CG* dips into the cup *C* and it restrained from moving with the cup. The weight of the horizontal section *BD*, including that of the mercury cup *C*, is taken up by two long insulating filaments attached to the laboratory ceiling. With this suspension, a longitudinal force of a few gram can easily move *BCD* along its length.

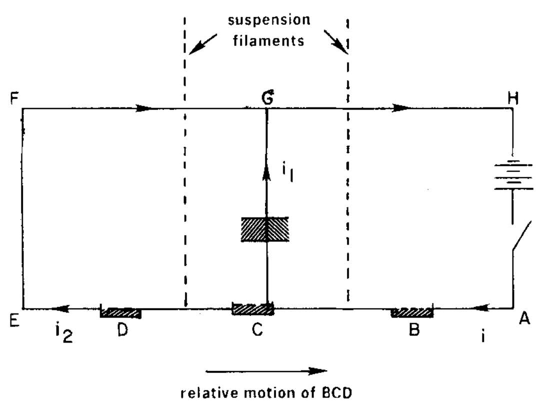


Fig. 23. One of Hering's many longitudinal force experiments

When a current of several hundred ampere is switched on, the mobile horizontal conductor section *BCD* moves vigorously in the direction from *D* to *B*. Figure 24 serves to explain how longitudinal Ampere forces account for the observed relative motion. We consider the three current-elements $i_1 \cdot dm$, $i_2 \cdot dn$ and $i \cdot do$. The interaction of elements *dm* and *do* results in the mutual force of repulsion $\Delta F_{m,0}$ which has a component in the direction of the observed relative motion. The interaction of elements *dm* and *dn* results in the attraction $\Delta F_{m,n}$ which also has a component in the direction of relative motion. In fact every combination of one element on the vertical branch *CG* and one on the horizontal portion *BCD* contributes a longitudinal force component acting from *D* toward *B*. These are the dominant Ampere forces which fully explain the result of Hering's experiment.

Additional longitudinal repulsion forces arise across the two mercury cups *B* and *D*. They do not cancel each other because i is greater than i_2. The net force from these two sources actually opposes the relative motion once the ends of *BD* have passed the mid-

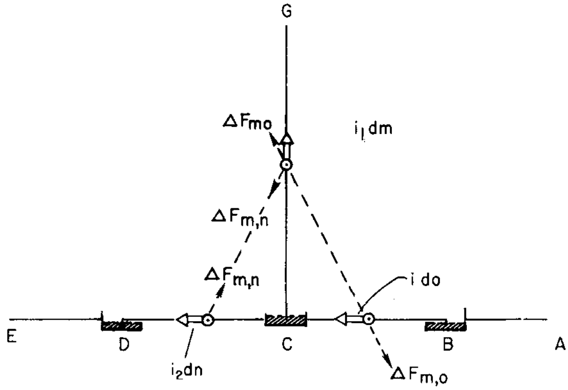


Fig. 24. Dominant Ampere forces in Hering's experiment

points in the mercury cups *B* and *D*. A further set of disturbing forces in cups *B* and *D* are the pinch forces which exert opposing thrusts on the ends of the movable conductor. Since $i > i_2$, the net longitudinal force due to pinch is opposing the relative motion and could not be the cause of it.

Finally we must examine the effect of the transverse forces on the short vertical hooks dipping into the liquid mercury at *B* and *D*. Because of the inequality of the currents at the two ends of the movable conductor, the combined transverse forces on the hooks should have a resultant in the direction of the observed motion. This type of positive disturbing force has already been investigated in connection with Neumann's longitudinal force experiment and in particular with the help of fig. 20. It was then found that the longitudinal Ampere force was more than one hundred times as large as the disturbing Lorentz force. Furthermore, for currents up to 500 A the absolute value of the net force on the hooks was so small that it could not have overcome the strong adhesion of copper to liquid mercury.

The Hering experiment, therefore, furnishes further evidence for the existence of longitudinal Ampere forces. As a matter of interest it may be pointed out that the sum of the transverse components $\Delta F_{m,0}$ and $\Delta F_{m,n}$ on the element *dm* is equal to the longitudinal force exerted on elements do and *dn*. This illustrates how the Ampere forces comply with Newton's third law. On the other hand, the Lotentz force on element *dm* would have to have its reaction force in the field.

8.4. Wire fragmentation experiments

In the past 160 years almost all experimental evidence for non-lorentzian forces has been collected with circuits containing some liquid metal. The liquid portions permitted the measurement of forces with balances, or the observation of relative motion, while the current was generally less than 1000 A. Very much higher currents are required to establish the existence of Ampere tension in solid conductors. Currents of the order of 100 kA have been employed in exploding wire experiments. It therefore seemed worthwhile to search the literature of wire explosions for any indication of Ampere tension.

From the point of view of the Ampere electrodynamics one would expect straight and curved wire sections to rupture somewhere along their lengths before any melting can take place. An air arc should immediately bridge the fracture gap so that tension would continue to exist and cause further breaks in the hot metal of rapidly rising temperature. Finally the sum of the voltage drops along the series-connected bridging arcs should severely limit, if not extinguish, the current. Premature current extinction has indeed been observed by many investigators of exploding wire phenomena.

However it is usually found that, after a brief current pause, an overall arc of modest voltage drop will strike, discharging the remainder of the energy stored in capacitors and evaporating the test wire. If during the current pause the ignition of an overall arc could be prevented, by using a very long wire, the surviving wire fragments should show signs of tensile fracture which would demonstrate the effect of tensile forces of electromagnetic origin. The major difficulty of designing an experiment of this kind was to ensure that the wire fragments had time to separate before the longitudinal force disappeared or the wire melted.

While pondering this problem the author found Nasilowski's paper [21] describing an experiment in which, quite unintentionally, the wire explosion was permanently arrested at precisely the correct moment. The experiment produced the wire fragmentation expected from the Ampere electrodynamics. Two new features of Nasilowski's wire explosions were longer wires and longer current pulses than normally employed in this research.

NASILOWSKI [21] shows six photographs. Two of them depict a collection of fragments of his 1 mm diameter, 1.5 m long copper wire. The diameter of the fragments looks the same as that of the original wire and the fracture faces are nearly perpendicular to the wire axis. The length of the fragments varies between 3 and 10 mm. The other four photographs are of metallurgical sections through some fragments, in the longitudinal direction, to reveal the internal grain structure and visible signs of crack initiation.

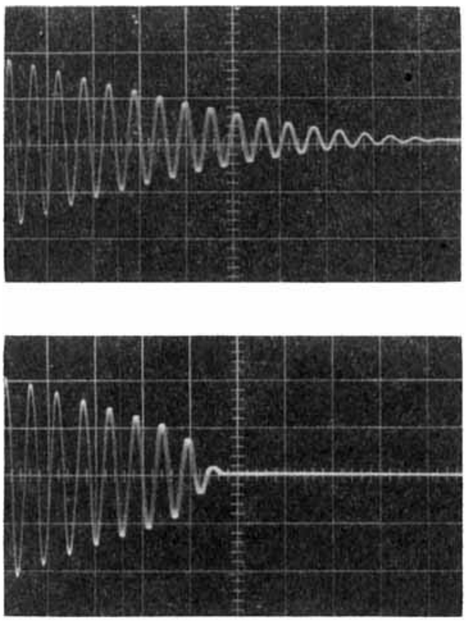
BAXTER [22] has measured the temperature distribution along fuse wires subjected to current pulses. Except for end-effects, he found that the wire temperature rose uniformly over the wire length right up to the melting point. Nasilowski's work confirmed that melting also occurred uniformly along the wire, with the boundary between the molten and the solid phase being a cylindrical surface moving radially inward. Hence wire fragmentation could not be the result of preferred melting at some locations along the wire.

The metallurgical evidence indicated that in some of Nasilowski's experiments fracture had taken place without any change in the grain structure throughout the body of the wire and therefore without any prior melting. But the electric arc bridging the gap between adjacent fragments, subsequent to fracture, produced a small amount of freshly molten material which adhered to the fracture faces and was recognizable by its dendritic structure. Nasilowski's metallurgical tests clearly indicated that the wire had parted in the solid state under the action of tension.

The author repeated Nasilowski's experiment in modified form at the Massachusetts

Institute of Technology [23]. Aluminium wires of 99 percent purity and 1.2 mm diameter were subjected to current pulses of 5000 to 7000 A amplitude. The current was derived from a high-voltage capacitor bank and passed through an inductor to allow it to ring down at 2000 Hz over a period of five to ten milliseconds. When the capacitor bank was charged to 60 kV, the discharge current would decay approximately exponentially, as shown in the oscillogram of fig. 25(a), without breaking the wire. It has been estimated that the 60 kV discharge was accompanied by a wire temperature rise of several hundred degrees centigrade which resulted in a thermal extension of the order of one percent.

By subsequent increases of the discharge voltage in steps of 2 kV, a pulse current level was reached at which the wire broke in one or more places. The hot fragments



a

b

Fig. 25. Discharge current oscillograms; $y = 3$ kA/cm, $x = 1$ ms/cm.
(a) At 60 kV. (b) At 68 kV

would fall to the floor and be distorted on impact. When repeating the test with a new wire and 2 kV additional voltage, the wire would break into a greater number of pieces. At the 66 and 68 kV levels a one meter long wire would fragment into 20—30 pieces. Finally, at 70 kV, the test wire would show clear signs of melting which obliterated much of the tensile break evidence. The oscillogram of fig. 25(b) indicates discharge current limiting and quenching due to the many arcs across fracture gaps.

The strongest indication of Ampere tension was obtained with straight one meter long wires mounted vertically, as shown in fig. 26. The wires were held in position with cotton threads, leaving 1 cm long arc-gaps in air between the wire ends and two terminations of the capacitor-inductor series circuit. When the discharge circuit was closed with a mechanical switch, the two one-centimeter arc-gaps would break down, allowing the current pulse to flow through the test wire. The purpose of the arc-gaps was to allow distortion-free thermal expansion and disconnect the wire mechanically from the fixed discharge circuit.

Figure 27(a) shows a collection of aluminium wire fragments produced by these experiments. Their distortion was caused by impact on the laboratory floor while they were hot. Photograph (b) clearly depicts transverse fractures which were spot-welded together again by arcing across the fracture gaps. Figure 27(c) is an optical micrograph of one fragment end, illustrating the brittle nature of the fracture. The last photograph (d) of a fracture face was taken by scanning electron microscopy. Similar micrographs of greater magnification revealed micron-deep melting of the fracture surface, consistent with arcing across the fracture gap.

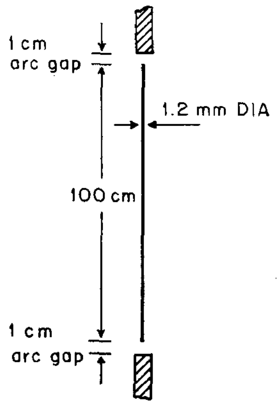


Fig. 26. Positioning of the wire before fragmentation

Provided the wire is treated as a bundle of filaments, the transverse pinch force may be calculated with Ampere's or the Lorentz force law. Pinch forces are potentially able to extrude soft wire. But NORTHRUP's analysis [14] shows that the extrusion force is less than one-tenth of the magnitude of the Ampere tension. For this reason, and even more because no significant diameter reduction has been observed near the fracture faces, it seems certain that pinch-off was not the cause of wire fragmentation.

Considerable thought has been given to the possibility of the wire fractures being the result of travelling stress waves or thermal shock. The velocity of sound in the conductor metals is of the order of 5000 m/s. Hence a stress wave could travel the length of the wires used by Nasilowski and the author in 0.1 to 0.3 ms. Tensile stress magnification by multiple reflections from the wire ends or anchors is therefore not out of the question. Nasilowski employed a unidirectional current pulse. The radial pinch force was not removed from his wire until the current ceased to flow. Only after the end of the pulse could travelling stress waves have created excessive tension. But with the aid of voltage drop measurements Nasilowski found that the wire broke well before the end of his 20 ms pulse. This was further confirmed by the formation of arcs across the fracture gaps. Hence Nasilowski's results cannot be explained with travelling stress waves. This cannot be said with the same degree of certainty of the MIT experiments.

A dynamic thermal shock model has been examined. Consider a straight aluminium wire of one meter length. When heated close to the melting point its length will increase approximately 1.4 cm. During the pulse period each end of the wire would be displaced by 0.7 cm. If the pulse lasts for for 5 ms, as in the shortest of the MIT experiments, the last centimeter on either end of the wire would attain an average velocity of 140 m/s. For a wire of 1.2 mm diameter, the mass per centimeter comes to 0.03 g, giving a kinetic

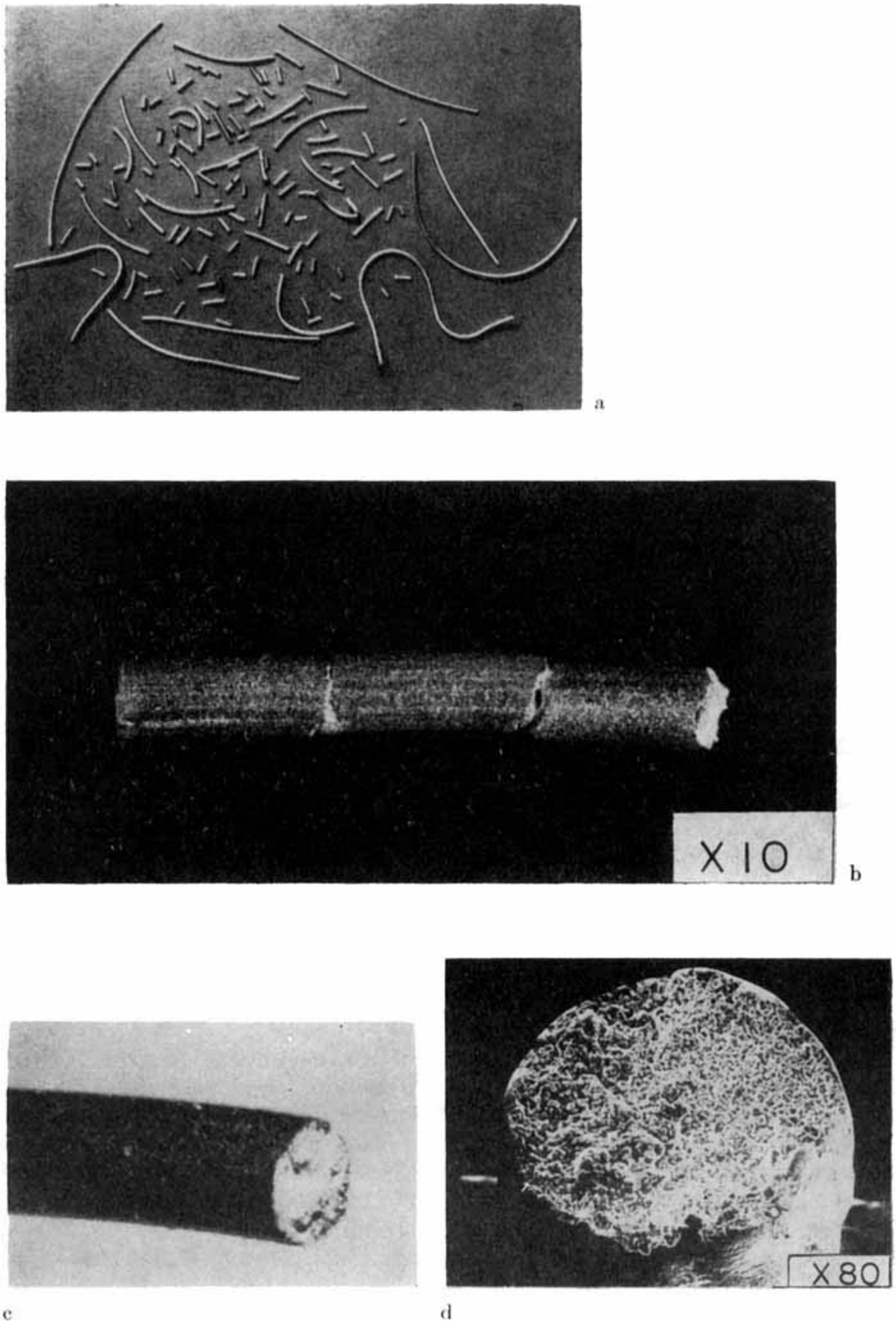


Fig. 27. Fragments of a 1.2 mm diameter aluminium wire.
(a) Collection of fragments from several experiments.
(b) Fragments reconnected by arc spot welding.
(c) Optical micrograph of fracture face.
(d) Scanning electron micrograph of fracture face

energy of the last centimeter of 300 erg. Allowing for only ten percente longation of hot aluminium before fracture, the average tensile force required to absorb the kinetic energy comes to only 3 g, which is negligible.

9. Recent Developments

9.1. Experimental

One of the most important new experimental developments in support of the Ampere-Neumann theory has been PAPPA's electrodynamic impulse pendulum [5] depicted by fig. 18. Using a vertical, parallel filament suspension, the technique has been refined and made fully quantitative in the author's laboratory at the Massachusetts Institute of Technology [24]. Perhaps the most significant modification from Pappas' procedure was the substitution of a high-voltage capacitor bank for the battery current source. The discharge of high-voltage capacitors made it possible to arc across short gaps in air. This avoided the practical difficulties with mercury cups.

The energy stored in the capacitors before the discharge was known accurately. Depending on experimental circumstances. it turned out that this energy supplied by the capacitors to a pendulum experiment fell short, by factors ranging from 1000 to 2000, of providing the amount of field energy required for relativistic momentum conservation. There can, therefore, be no doubt left that reaction forces in the field (vacuum) counteracting Lorentz forces on metallic conductors simply do not exist. If Newton's third law is to be obeyed, the reaction forces have to reside in the metallic lattice of the conductors which carry the current responsible for the observable electrodynamic phenomena. Only the Ampere electrodynamics furnishes a credible model of the newtonian reaction mechanism.

Another discovery made with the electrodynamic impulse pendulum was that the momentum imparted to the hairpin conductor of fig. 18 was much smaller than expected from the Lorentz and Ampere formulas. Eventually it was realized that the discrepancy was caused by the elastic deformation of the rails behind the pendulum. The rails buckled under the influence of the longitudinal recoil forces. In deflecting sideways they were able to store elastic energy which would otherwise have been converted to kinetic pendulum energy. Stiffening the rail structure did increase the pendulum momentum for the same current pulse. This provided further confirmation of the validity of the Ampere electrodynamics. Recoil buckling of the rails has practical consequences for the design of railgun accelerators [9].

It is of course of great interest to learn whether the Ampere-Neumann electrodynamics applies to superconductors and also semiconductors and plasmas. All we know with ertainty today is that it does not hold for convecting charges in vacuum. It would seem reasonable to expect the amperian current element to contain the conduction electron and the associated positive atomic ion. Electronic conduction through a positive ion space charge in superconductors, semicondutors and plasma seems to have the ingredients required for the Ampere force mechanism to operate.

At the time of writing two types of superconductivity experiments are being planned. One will search for Ampere tension. The other arises from the following consideration [25]. Neumann's electrodynamic potential energy is equivalent to the magnetic energy stored in the field. From the Meissner effect we know that superconductors will expel magnetic flux, and with it the magnetic energy, from their interior regions. One would also expect Neumann's potential energy to be expelled from the inside of superconductors. Now we can think of regions where the magnetic vector potential is curl-free, as for example outside a very long solenoid. If a superconductor it placed in such a region it could not experience the normal Meissner effect because the magnetic field inside it is in any case zero. However, if the magnetic energy is the kind of potential energy prescribed by Neumann's theory, we may still find that persistent supercurrents are set up in the surface of the superconductor in order to eliminate stored magnetic energy inside the body. This question will be put to a test in the second superconductivity experiment.

The first evidence of the existence of longitudinal Ampere forces in plasmas has been obtained with a series of experiments [26] involving underwater arcs. It is generally believed that the shockwave produced by an arc in water is the result of the sudden creation of high-pressure steam in the arc column. Lorentz forces acting on the arc are either too small to cause the shock or they are actually containment forces opposing the explosion. Without any knowledge of longitudinal Ampere forces, which are capable of promoting an explosion, investigators had little choice but to conclude underwater arc explosions were thermodynamic in nature.

In the reporeted experiments [26] nine joules of energy stored in capacitors was repeatedly discharged through a small volume ($\simeq 3\ \mathrm{cm}^3$) of water. Any individual discharge would either ignite an arc and cause an explosion or, with saltwater, it would merely give rise to electrolytic conduction (convection current) without in any way producing a visible disturbance of the water. It seems unlikely that in one case the nine joules of energy were capable of fuelling a thermodynamic explosion, while in the other they were impotent to do so. A more likely explanation of the experimental findings is that Ampere forces in the arc plasma were the cause of the explosion and that these forces are absent in convection currents.

9.2. Theoretical

Perhaps the most illuminating theoretical development of recent years has been the demonstration [29] that both Ampere's and Grassmann's (or Lorentz's) force law are compatible with Neumann's virtual work formula (25) when, and only when, the reaction force between two closed circuits is being calculated. In this particular case the relativistic force contributions from Grassmann's law integrate to zero and the remainder is equal to the prediction made by the Ampere law. This can be shown as follows.

Referring to fig. 2, the force ΔF_m on element $i_m dm$ due to element $i_n dn$ in currently taught relativistic electromagnetism is given by equ. (85). This is not a symmetrical force formula and therefore ΔF_n acting on $i_n dn$ will differ from ΔF_m. The two unsymmetrical forces may be written

$$\begin{aligned} \Delta \boldsymbol{F}_m &= (i_m i_n / r_{m,n}^2)\, \boldsymbol{dm} \times (\boldsymbol{dn} \times \boldsymbol{1}_{r,m}) \\ \Delta \boldsymbol{F}_n &= (i_m i_n / r_{m,n}^2)\, \boldsymbol{dn} \times (\boldsymbol{dm} \times \boldsymbol{1}_{r,n}) \end{aligned} \tag{86}$$

where $\boldsymbol{1}_{r,m}$ is unit vector along $r_{m,n}$ drawn toward dm. It will be realized that equ. (86) is expressed in fundamental electromagnetic units where the currents must be inserted in absolute ampere to give the force in dyne.

The triple vector product of (86) may be expanded to

$$\begin{aligned} \Delta \boldsymbol{F}_m &= \boldsymbol{dn}(1/r_{m,n}^2)\, i_m i_n dm \cos \alpha_m - \boldsymbol{1}_{r,m}(1/r_{m,n}^2)\, i_m i_n dm dn \cos \varepsilon \\ \Delta \boldsymbol{F}_n &= \boldsymbol{dm}(1/r_{m,n}^2)\, i_m i_n dn \cos \alpha_n - \boldsymbol{1}_{r,n}(1/r_{m,n}^2)\, i_m i_n dm dn \cos \varepsilon . \end{aligned} \tag{87}$$

In fig. 2 the angles α_m and α_n are

$$\alpha_m = \pi - \alpha ; \qquad \alpha_n = \beta . \tag{88}$$

Equations (86) and (87) were first suggested by GRASSMANN [16] in 1845. Maxwell referred to them as Grassmann's force law. Fifty years later Lorentz incorporated them in what today is being called the Lorentz force.

When $i_m dm$ belongs to a closed circuit m and i_n to another closed circuit n, the net reaction force between the two circuits is either an attraction (negative) or repulsion (positive) force given by

$$F_{m,n} = -i_m i_n \int_m \int_n (\cos \varepsilon / r_{m,n}^2)\, dm dn. \tag{89}$$

That the last equation follows from (87) was proved by WHITTAKER [30] and others. It indicated that the first term of equ. (87) vanishes under the integrations. Precisely this term, which does not stand for attraction or repulsion, has been responsible for the disagreement of Grassmann's law with Newton's third law of motion. The second term of equ. (87) is a newtonian term which is also contained in the Ampere force law. Therefore, when computing merely the reaction force between two closed metallic circuits, the Lorentz force sheds its relativistic trimmings and predicts the the same force as the Ampere formula.

In the calculation of reaction forces between parts of an isolated circuit not all integrals are closed-path integrals and the contributions from the relativistic component of equ. (87) do not add up to zero. It is precisely in those situations that we find disagreement between the Ampere electrodynamics and relativistic electromagnetism. This explains why Lorentz force calculations agree with Neumann's virtual work formula (26), involving the mutual inductance between two circuits, but they generally disagree with equ. (27) containing the selfinductance.

While trying to understand how the old electrodynamics could succeed where the Lorentz force seems to have failed, one inevitably has to take a closer look at the microscopic nature of the electric current and particularly at the metallic current element. Ampere treated this element like a piece of metal or wire. His electrodynamics forces were supposed to act directly on the material and not the "electric fluid" passing through the wire. The current strength was related to the fluid motion, but the quantitative laws of force and induction of the Ampere-Neumann electrodynamics did not involve the velocity of charges.

WEBER [27] proposed the first change in the current element model of metals. He considered each element to consist of a mobile positive and negative charge drifting past each other in opposite directions. By postulating mutual forces between charges which were functions of charge velocities he was able to build up an electrodynamics in complete agreement with the laws of Ampere and Neumann. Weber's force formula incorporates the Coulomb force, as Lorentz's formula would do later. It also contained a constant which was equal to the velocity of light. Weber argued the forces on the charges were passed on to the metal but failed to explain how, at the same time, the charges could move freely through the conductor. This inconsistency and the subsequent discovery of the immobility of the lattice ion made Weber's current element untenable.

However, the association of the current element strengths with charge velocities has survived. Lorentz found that the convection current element in vacuum was simply the electric charge multiplied by its velocity. He further argued, by analogy, that the metallic current element is the conduction electron multiplied by its average drift velocity. A force on this lorentzian current element can become a body force when the conduction electron runs up against the potential barrier of the metal surface. Yet Lorentz's current element cannot explain the existence of Ampere tension along the streamlines of current flow where the conduction electron would not encounter potential barriers. Ampere tension inevitably requires further changes in the metallic current element model. Some mechanical link between the conduction electron and the parent lattice ion has to come into play.

Nearly twenty years ago [28] it was realized that Neumann's electrodynamic potential requires the existence of mutual torques between current element pairs. This seemed incompatible with Lorentz's drifting electron element. At the time, the conclusion was drawn that the metallic current element was likely to involve an oriented electron-phonon interaction, reminiscent of the interaction responsible for creating Cooper pairs of electrons in superconductors. In all probability, the electron of the current element would be the conduction electron which should be almost — but not entirely — free to roam through the metal lattice. Some attractive interaction of this electron with the parent atom would give rise to an electrostatic dipole pivoted on the lattice site. It would then be the direction of the dipole, rather than any charge velocity, which determines the direction of the current element. If this rationale should turn out to have any validity, the metal atom itself would also be the current element. Quite different arguments concerned with the size of current elements [7] led to the same conclusion.

Pivoted dipole elements [25] would exist in a metal even when there is no current flowing through it. Analyzing [28] the collective behavior of such elements, in response to Neumann mutual torques, revealed that, when left to themselves, they would all try to counteralign each other and thereby create perfect disorder. A certain degree of order would be established by forced and induced current flow. Both these phenomena would have to be accompanied by the turning of atomic current elements about their nuclear pivots. Diamagnetism could be explained on the same basis. An intriguing side issue is that the dipoles representing two electrons pivoted on the same nucleus would counteralign themselves very strongly which is equivalent to the spin-up and spin-down magnetic pairing of electrons.

This section attempted to sketch some of the current ideas which may, or may not, help to establish a new model of the metallic current element consistent with all experimental facts. The melting pot of new ideas should sooner or later bring forth experiments testing the validity of the pivoted dipole model.

10. Discussion

Few, if any, textbooks on electromagnetism now in print contain an historically correct description of the work of the famous French scientist A. M. Ampere and the first German theoretical physicist F. E. Neumann. Ampere's name is wrongly associated with a circuital law connecting the magnetic field around a conductor with the current passing through it. The magnetic field is the antithesis of Ampere's action-at-a-distance electrodynamics. The magnetic force law expounded in our textbooks was first proposed by Grassmann [16] who intended it to supersede Ampere's law, yet we find that Grassmann's law is often attributed to Ampere. Neumann is correctly remembered by his mutual inductance formula, but his mathematical theory of electromagnetic induction, so well described by Maxwell, is now embodied in what is generally called Faraday's law of induction. Much of the confusion may have arisen because the important papers by Ampere and Neumann were never translated into English. It is hoped that this review will remedy some of the historical distortions.

Ampere's force law, equ. (21), is an empirical law. No experiment with metallic conductors is known which does not comply with this law. Ampere's method of deducing his law has occasionally been criticized. Like the scaffolding of a building, the method of deduction of an empirical law may safely be discarded without in any way impairing the value of the edifice is has helped to erect.

The current element invented by Ampere was an element of matter. In 1820 he still treated matter as an infinitely divisible continuum. With our knowledge of the atomic structure of conductor metals we should not be surprised to find that infinite divisibility leads to absurd results and integration singularities. Ampere had little choice, for without the integration over infinitely small elements his force law would not have yielded quantitative results. Today the computer enables us to use finite-size elements. What first appeared to be a numerical approximation to reality is now becoming more real than the idealistic matter and current continuum.

Few scientists of this century know that Neumann derived the laws of electromagnetic induction directly from Ampere's force formula, and that Maxwell incorporated Neumann's work in its entirity into the field theory. Concepts changed names, but the mathematics has remained the same. Neumann's electrodynamic potential became Maxwell's kinetic energy of the field and is now simply called magnetic energy. The inductive interaction of conductor elements in Neumann's mutual inductance became flux linkage. This latter linkage cannot be applied to a pair of conductor elements. Therein lies the reason why closed circuits became so important. They concealed the relativistic nature of Grassmann's law and its conflict with newtonian physics.

The huge success of Maxwell's field theory rests on the concept of electromagnetic radiation, of which the Ampere-Neumann electrodynamics gave no hint. Maxwell's notions brought optics into the sphere of electromagnetism and sprouted the hardly believable technologies of wireless communication, radio, radar, television, and so on. When Lorentz added Grassmann's force law to field theory, to make sense of the motion of charges in vacuum, electromagnetism seemed complete but for a few flaws which were immediately eradicated by Einstein's special theory of relativity. For the last eighty years this monumental abstract creation of man has remained unchanged. During the second half of the twentieth century most physicists have considered it unnecessary to keep a critical eye on it.

In the last resort science depends on total honesty. The sobering experimental facts concerning energy in the near-field [5, 31] and forces along current streamlines in metals [1, 7] will, at the very least, require some adjustment in field theory, or they could become the pebble releasing an avalanche of change in centuries to come.

11. References

[1] P. GRANEAU, Nature **295**, 311 (1982).
[2] A. M. AMPERE, *Theorie mathematique des phenomenes electro-dynamiques*, Albert Blanchard, Paris, 1958.
[3] F. E. NEUMANN, Ostwald's Klassiker No. 10 (1845); Ostwald's Klassiker No. 36 (1847).
[4] I. A. ROBERTSON, Philosophical Magazine **36**, 32 (1945).
[5] P. T. PAPPAS, Nuovo Cimento **76 B**, 189 (1983).
[6] R. A. R. TRICKER, *Early electrodynamics*, Pergamon Press, Oxford, 1965.
[7] P. GRANEAU, IEEE Transaction *MAG-20*, 444 (1984).
[8] A. M. HILLAS, Nature **302**, 271 (1983).
[9] P. GRANEAU, J. appl. Phys. **53**, 6648 (1982).
[10] M. FARADAY, *Experimental researches in electricity*, Vol. 1, London, 1884.
[11] O. D. KELLOG, *Foundations of potential theory*, Dover, New York, 1953, p. 53.
[12] J. C. MAXWELL, *A treatise on electricity and magnetism*, Oxford University Press, London, 1892, Vol. 2, p. 324.
[13] A. SOMMERFELD, *Electrodynamics*, Academic Press, New York, 1952, p. 107.
[14] E. F. NORTHRUP, Phys. Rev. **24**, 474 (1907).
[15] E. G. CULLWICK, *Electromagnetism and relativity*, Longmanns, London, 1957, p. 227.
[16] H. G. GRASSMANN, Poggendorf's Annalen der Physik und Chemie **64**, 1 (1845).
[17] F. F. CLEVELAND, Phil. Mag. **25**, 416 (1936).
[18] P. G. TAIT, Phil. Mag. **21** 319 (1861).

[19] F. E. NEUMANN, *Vorlesungen über elektrische Strome*, Teubner, Leipzig, 1884.
[20] C. HERING, Journal of the Franklin Institute **194**, 611 (1921).
[21] J. NASILOWSKI, *Exploding wires*, Plenum Press, New York, 1964, Vol. 3, p. 295.
[22] H. W. BAXTER, *Electric fuses*, Arnold, London, 1950.
[23] P. GRANEAU, Phys. Letters **97 A**, 253 (1983).
[24] P. GRANEAU, P. N. GRANEAU, Nuovo Cimento **7 D**, 31 (1986).
[25] P. GRANEAU, Ampere-Neumann electrodynamics of metals, Hadronic Press, Nonantum MA 02195 USA, 1985.
[26] P. GRANEAU, P. N. GRANEAU, Appl. Phys. Letters **46**, 468 (1985).
[27] W. WEBER, *Wilhelm Webers Werke*, Springer, Berlin, 1893.
[28] P. GRANEAU, Internat. J. Electronics **20**, 351 (1966).
[29] P. GRANEAU, Nuovo Cimento **78 B**, 213 (1983).
[30] E. WHITTAKER, *A history of the theories of the aether and electricity*, Nelson, London, 1951, Vol. 1, p. 87.
[31] M. M. NOVAK, Fortschr. Phys. **28**, 285 (1980).